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Open and closed support of some star related graphs under addition and multiplication

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ABSTRACT

Let G be a simple graph. *supp(v)* is defined by $\sum_{u\in N(v)} deg\, u$. *supp(G)* is defined by $\sum_{v\in V(G)} supp(v)$. $supp[v]$ is defined by $\sum_{u\in N[v]} deg\, u$. *Supp[G]* is defined by $\sum_{v\in V[G]} supp[v]$. $mult(v)$ is defined by $\prod_{u\in N(v)} deg\, u$. and $mult(G)$ is defined by $\prod_{v\in V(G)} mult(v)\,.\, mult\ [v]$ is defined by $\prod_{u\in N[v]} deg\, u$. $mult\ [G]$ is defined by. $\prod_{v\in V[G]} mult[v]$ In this paper, open and closed support of some of star related graphs under addition and multiplication are studied.

Keywords: Open support of a graph under addition, Open support of a graph under multiplication, closed support of a graph under addition, closed support of a graph under multiplication, Duplication of a vertex, Duplication of an edge, Duplicating of a vertex by an edge, Duplication of an edge by a vertex

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1. INTRODUCTION

Let $G = (V,E)$ be a simple, finite and undirected graph with p vertices and q edges. The degree of a vertex $v \epsilon V(G)$ is the number of edges of G incident with v and is denoted by $deg_G(v)$ or $deg\ v$. The maximum and the minimum degrees of the vertices of G are respectively denoted by $\Delta(G)$ and $\delta(G)$. A vertex of degree 0 in G is called an isolated vertex and a vertex of degree 1 is called a pendent vertex or an end vertex of G. A vertex of a graph G is said to be a vertex of full degree if it is adjacent to all other vertices in G. The neighbourhood of a vertex $v\epsilon V(G)$ is the set $N_G(v)$ of all the vertices adjacent to v in G. For a set $X \subseteq V(G)$the open neighbourhood $N_G(X)$ is defined to be $\cup_{v\epsilon X} N_G(v)$. The neighbourhood of a vertex $v \in V(G)$ is the set $N_G(v)$ of all vertices adjacent to v in G. $N_G[v] = N_G[v] \cup \{v\}$ for a set $X \subseteq V(G)$ the closed neighbourhood $N_G[X]$ is defined to be $\bigcup_{v\in X} N_G[v]$. The concept of open support of a vertex under addition and open support of a graph under addition were introduced in [2, 3]. The concept of open support of a vertex under multiplication and open support of a graph under multiplication were introduced in [4] and further studied in [4]. The concept of closed support of a vertex under addition and closed support of a graph under addition were introduced in [5]. The concept of closed support of a vertex under multiplication and closed support of a graph under multiplication were introduced in [6] and further studied in [6].

In this paper, open and closed support of the graph G is obtained by duplicating all vertices, duplicating all edges, duplicating all the vertices by edges, duplicating all the edges by vertices of star under addition and multiplication are studied. Terms defined in this paper are used in the sense of Harary [14, 23]. A systematic presentation of diverse applications of graph theory is given in [7-13, 16-18, 20, 22-28]. Following definitions are necessary for the present study.

DEFINITION 1.1[1]: Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In the other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

DEFINITION 1.2[1]: Duplication of an edge $e = uv$ of a graph G produces a new graphG' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$; $N(v') = N(v) \cup \{u'\} - \{u\}$.

DEFINITION 1.3[19]: Duplication of a vertex $v_k$by a new edge $e = v_k'v_k''$in a graph G produces a new graph G' such that $N(v_k') = \{v_k, v_k''\}$and $N(v_k'') = \{ v_k, v_k'\}$.

DEFINITION 1.4[19]: Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{ u , v \}$.

DEFINITION 1.5[2]: Let $G = (V, E)$ be a graph. An open support of a vertex v under addition is defined by $\sum_{u\in N(v)} deg\ u$ and it is denoted by $supp(v)$.

DEFINITION 1.6[5]: Let $G = (V, E)$ be a graph. A closed support of a vertex, v under addition is defined by $\sum_{u\in N[v]} deg\ u$ and it is denoted by $supp[v]$.

DEFINITION 1.7[2]: Let $G = (V, E)$ be a graph. An open support of a graph G under addition is defined by $\sum_{v\in V(G)} supp(v)$ and it is denoted by $supp(G)$.

DEFINITION 1.8[5]: Let $G = (V, E)$ be a graph. A closed support of a graph G under addition is defined by $\sum_{v\in V[G]} supp[v]$ and it is denoted by $supp[G]$.

DEFINITION 1.9[4]: Let $G = (V, E)$ be a graph. An open support of a vertex v under multiplication is defined by $\prod_{u\in N(v)} deg\, u$ and it is denoted by $mult(v)$.

DEFINITION 1.10[6]: Let $G = (V, E)$ be a graph. A closed support of a vertex v, under multiplication is defined by $\prod_{u\in N[v]} deg\, u$ and it is denoted by $mult\ [v]$.

DEFINITION 1.11[4]: Let $G = (V, E)$ be a graph. An open support of a graph G under multiplication is defined by $\prod_{v\in V(G)} mult(v)$ and it is denoted by $mult(G)$.

DEFINITION 1.12[6]: Let $G = (V, E)$ be a graph. A closed support of a graph G under multiplication is defined *by* $\prod_{v\in V[G]} mult[v]$ and it is denoted by $mult\ [G]$.

DEFINITION 1.13[21]: Let G be the graph with fixed vertex v and let $(P_m : G)$ be the graph obtained from m copies of G and the path $P_m : u_1, u_2, \dots, u_m$ by joining u_i with the vertex v of the i^{th} copy of G by means of an edge for $1 \le i \le n$.

2. MAIN RESULTS

THEOREM 2.1: Let G be the graph obtained by duplicating all the vertices of the star $K_{1,n}$.

Then (i) $supp(G) = 5n\,(n+1)$

(ii) $supp[G] = 5n^2 + 11n$

(ii) $mult(G) = 16^n\ n^{3n}$

(iv) $mult[G] = 2^{5n+1} n^{3n+2}$

PROOF: Let G be the graph obtained by duplicating all the vertices of the star $K_{1,n}$.

Let $v, v_1, v_2, \dots, v_n$ be the vertices of $K_{1,n}$.

Let $u, u_1, u_2, \dots, u_n$ be the corresponding vertices in G.

Then $deg\ v = 2n$, $deg\ u = n$, $deg\ v_i = 2$where $1 \le i \le n$, $deg\ u_j = 1$where $1 \le j \le n$

Now $supp(v) = \sum_{u\in N(v)} deg\ u = \sum_{i=1}^{n} deg\, v_i + \sum_{j=1}^{n} deg\, u_j = 2n + n = 3n$

$supp(u) = \sum_{v\in N(u)} deg\ v = \sum_{i=1}^{n} deg\, v_i = 2n$

$supp(v_1) = \sum_{v\in N(v_1)} deg\ v = deg\ u + deg\ v = n + 2n = 3n$

Similarly $supp(v_i) = 3n$ where $2 \le i \le n$

$supp(u_1) = \sum_{v\in N(u_1)} deg\ v = deg\, v = 2n$

Similarly $supp(u_j) = 2n$where $2 \le j \le n$

Therefore $supp(G) = \sum_{v\in V(G)} supp(v) = supp(v) + supp(u) + \sum_{i=1}^{n} supp(v_i) + \sum_{j=1}^{n} supp(u_j)$

$= 3n + 2n + n\,(3n) + n\,(2n) = 5n + 5n^2 = 5n\,(n + 1)$

Now $supp[v] = \sum_{u\in N[v]} deg\, u = deg\, v + \sum_{i=1}^{n} deg\, v_i + \sum_{j=1}^{n} deg\, u_j = 2n + 2n + n = 5n$

$supp[u] = \sum_{v\in N[u]} deg\, v = deg\, u + \sum_{i=1}^{n} deg\, v_i = n + 2n = 3n$

$supp[v_1] = \sum_{v\in N[v_1]} deg\, v = deg\, v_1 + deg\, u + deg\, v = 2 + n + 2n = 3n + 2$

Similarly $supp[v_i] = 3n + 2$ where $2 \le i \le n$

$supp[u_1] = \sum_{v\in N[u_1]} deg\, v = deg\, u_1 + deg\, v = 1 + 2n$

Similarly $supp[u_j] = 1 + 2n$ where $2 \le j \le n$

Therefore $supp[G] = \sum_{v\in V(G)} supp\,[v] = supp[v] + supp[u] + \sum_{i=1}^{n} supp[v_i] + \sum_{j=1}^{n} supp\left[u_j\right] = 5n + 3n + n(3n + 2) + n(1 + 2n) = 5n + 3n + 3n^2 + 2n + n + 2n^2 = 5n^2 + 11n$

$mult(v) = \prod_{u\in N(v)} deg\ u = \prod_{i=1}^{n} deg\, v_i \times \prod_{j=1}^{n} deg\ u_j = 2^n \times 1 = 2^n$

$mult(u) = \prod_{v\in N(u)} deg\ v = \prod_{i=1}^{n} deg\, v_i = 2^n$

$mult(v_1) = \prod_{v\in N(v_1)} deg\ v = deg\ u \times deg\ v = n \times 2n = 2n^2$

Similarly $mult(v_i) = 2n^2$ where $2 \le i \le n$

$mult(u_1) = \prod_{v\in N(u_1)} deg\ v = deg\, v = 2n$

Similarly $mult(u_j) = 2n$ where $2 \le j \le n$

Therefore $mult(G) = \prod_{v\in V(G)} mult(v) = mult(v) \times mult(u) \times \prod_{i=1}^{n} mult(v_i) \times \prod_{j=1}^{n} mult(u_j)$

$= 2^n \times 2^n \times (2n^2)^n \times (2n)^n = 2^{4n} \times n^{3n} = 16^n\ n^{3n}$

Now $mult[v] = \prod_{u\in N[v]} deg\, u = deg\, v \times \prod_{i=1}^{n} deg\, v_i \times \prod_{j=1}^{n} deg\, u_j = 2n \times 2^n \times 1^n$

$= 2^{n+1} n$

$mult[u] = \prod_{v\in N[u]} deg\, v = deg\, u \times \prod_{i=1}^{n} deg\, v_i = n \times 2^n = 2^n n$

$mult[v_1] = \prod_{v\in N[v_1]} deg\, v = deg\, v_1 \times deg\, u \times deg\, v = 2 \times n \times 2n = 4n^2$

Similarly $mult[v_i] = 4n^2$ where $2 \le i \le n$

$mult[u_1] = \prod_{v\in N[u_1]} deg\, v = deg\, u_1 \times deg\, v = 1 \times 2n = 2n$

Similarly $mult[u_j] = 2n$ where $2 \le j \le n$

Therefore $mult[G] = \sum_{v\in V(G)} mult\,[v] = mult[v] \times mult[u] \times \sum_{i=1}^{n} mult[v_i] \times \sum_{j=1}^{n} mult\left[u_j\right] = 2^{n+1} n \times 2^n n \times [4n^2]^n \times [2n]^n = 2^{5n+1} n^{3n+2}$

EXAMPLE 2.2: Consider the star $K_{1,3}$

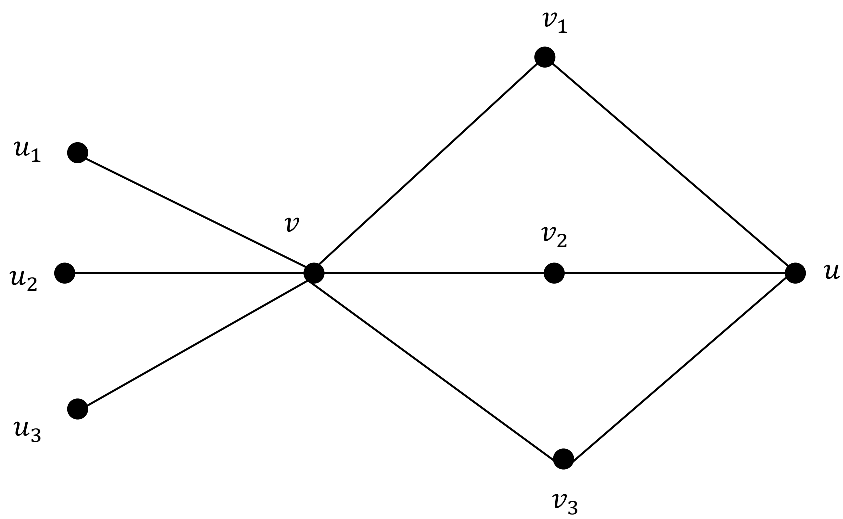

Figure 1

$supp(K_{1,3}) = 5n\,(n+1) = 5 \times 3\,(3+1) = 15\,(4) = 60$

$supp[K_{1,3}] = 5n^2 + 11n = 5(3)^2 + 11(3) = 78$

$mult(K_{1,3}) = 16^{\,n}\ n^{\,3n} = (16)^{\,3}\,(3)^{\,3\times 3} = 80{,}621{,}568.$

$mult\big[K_{1,3}\big] = 2^{5n+1}n^{3n+2} = 2^{5(3)+1}3^{3(3)+2} = 11{,}609{,}505{,}792$

THEOREM 2.3: Let G be the graph obtained by duplicating all the edges of the star $K_{1,n}$ then

(i) $supp(G) = 5n\,(n+1)$

(ii) $supp[G] = 5n^2 + 11n$

(iii) $mult(G) = 16^{\,n}\ n^{\,3n}$

(iv) $mult[G] = 2^{5n+1}n^{3n+2}$

PROOF: Let G be the graph obtained by duplicating all the edges of the star $K_{1,n}$.

Let $v, v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$.

Let $u, u_1, u_2, \ldots, u_n$ be the corresponding vertices in G.

Then $deg\ v = n$, $deg\ u = 2n$, $deg\ v_i = 2$ where $1 \le i \le n$, $deg\ u_j = 1$ where $1 \le j \le n$.

The graph obtained by duplicating all the edges of the star $K_{1,n}$ is same as the graph obtained by duplicating all the vertices of $K_{1,n}$. As in the proof of Theorem 2.1, $supp(G) = 5n\,(n+1)$, $supp[G] = 5n^2 + 11n$, $mult(G) = 16^n\ n^{3n}$ and $mult[G] = 2^{5n+1}n^{3n+2}$.

THEOREM 2.4: Let G be the graph obtained by duplicating all the vertices by edges of the star $K_{1,n}$. Then

(i) $supp(G) = n^2 + 21n + 12$

(ii) $supp[G] = n^2 + 29n + 18$

(iii) $mult(G) = 16\,(432)^n\,(n+2)^{n+2}$

(iv) $mult[G] = 3^{4n}4^{3n+3}(n+2)^{n+3}$

PROOF: Let G be the graph obtained by duplicating all the vertices by edges of the star $K_{1,n}$.

Let $v, v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$.

Let $v', v_1', v_2', \ldots, v_n'$ and $v'', v_1'', v_2'', \ldots, v_n''$ be the corresponding vertices in G.

Then $deg\ v = n+2$, $deg\ v' = 2$, $deg\ v'' = 2$, $deg\ v_i = 3$ where $1 \le i \le n$

$deg\ v_i' = 2$ where $1 \le i \le n$, $deg\ v_i'' = 2$ where $1 \le i \le n$

Now $supp(v) = \sum_{u \in N(v)} deg\ u = \sum_{i=1}^{n} deg\ v_i + deg\ v' + deg\ v'' = 3n + 2 + 2 = 3n + 4$

$supp(v') = \sum_{v \in N(v')} deg\ v = deg\ v + deg\ v'' = (n + 2) + 2 = n + 4$

$supp(v'') = \sum_{v \in N(v'')} deg\ v = deg\ v + deg\ v' = (n + 2) + 2 = n + 4$

$supp(v_1) = \sum_{v \in N(v_1)} deg\ v = deg\ v + deg\ v_1' + deg\ v_1'' = (n + 2) + 2 + 2 = n + 6$

Similarly $supp(v_i) = n + 6$ where $2 \le i \le n$

$supp(v_1') = \sum_{v \in N(v_1')} deg\ v = deg\ v_1 + deg\ v_1'' = 3 + 2 = 5$

Similarly $supp(v_i') = 5$ where $2 \le i \le n$

$supp(v_1'') = \sum_{v \in N(v_1'')} deg\ v = deg\ v_1 + deg\ v_1' = 3 + 2 = 5$

Similarly $supp(v_i'') = 5$ where $2 \le i \le n$

Therefore $supp(G) = \sum_{v \in V(G)} supp(v) = supp(v) + supp(v') + supp(v'') + \sum_{i=1}^{n} supp(v_i) + \sum_{i=1}^{n} supp(v_i') + \sum_{i=1}^{n} supp(v_i'') = (3n + 4) + (n + 4) + (n + 4) + n\,(n + 6) + 5n + 5n$

$= n^2 + 21n + 12$

Now $supp[v] = \sum_{u \in N[v]} deg\, u = deg\, v + \sum_{i=1}^{n} deg\, v_i + deg\, v' + deg\, v''$

$= n + 2 + 3n + 2 + 2 = 4n + 6$

$supp[v'] = \sum_{v \in N[v']} deg\, v = deg\, v' + deg\, v'' + deg\, v = 2 + 2 + n + 2 = n + 6$

$supp[v''] = \sum_{v \in N[v']} deg\, v = deg\, v'' + deg\, v' + deg\, v = 2 + 2 + n + 2 = n + 6$

$supp[v_1] = \sum_{v \in N[v_1]} deg\, v = deg\, v_1 + deg\, v + deg\, v_1' + deg\, v_1'' = 3 + 2 + 2 + n + 2$

$= n + 9$

Similarly $supp[v_i] = n + 9$ where $2 \le i \le n$

$supp[v_1'] = \sum_{v \in N[v_1']} deg\ v = deg\ v_1' + deg\ v_1'' + deg\ v_1 = 2 + 2 + 3 = 7$

Similarly $supp[v_i'] = 7$ where $2 \le i \le n$

$supp[v_1''] = \sum_{v \in N[v_1'']} deg\ v = deg\ v_1'' + deg\ v_1' + deg\ v_1 = 2 + 2 + 3 = 7$

Similarly $supp[v_i''] = 7$ where $2 \le i \le n$

Therefore $supp[G] = \sum_{v \in V(G)} supp\ [v] = supp[v] + supp[v'] + supp[v''] + \sum_{i=1}^{n} supp[v_i] + \sum_{i=1}^{n} supp[v_i'] + \sum_{i=1}^{n} supp[v_i'']$

$= 4n + 6 + n + 6 + n + 6 + n(n + 9) + 7n + 7n = n^2 + 29n + 18$

Now $mult(v) = \prod_{u \in N(v)} deg\ \ u = \prod_{i=1}^{n} deg\ \ v_i \times deg\ v' \times deg\ v'' = 3^n \times 2 \times 2 = 4 \times 3^n$

$mult(v') = \prod_{v \in N(v')} deg\ \ v = deg\ v \times deg\ v'' = 2\ (n + 2)$

$mult(v'') = \prod_{v \in N(v'')} deg\ \ v = deg\ v \times deg\ v' = 2\ (n + 2)$

$mult(v_1) = \prod_{v \in N(v_1)} deg\ \ v = deg\ v \times deg\ v_1' \times deg\ v_1'' = (n + 2) \times 2 \times 2 = 4\ (n + 2)$

Similarly $mult(v_i) = 4\ (n + 2)$ where $2 \le i \le n$

$mult(v_1') = \prod_{v \in N(v_1')} deg\ \ v = deg\ v_1 \times deg\ v_1'' = 3 \times 2 = 6$

Similarly $mult(v_i') = 6$ where $2 \le i \le n$

$mult(v_1'') = \prod_{v \in N(v_1'')} deg\ \ v = deg\ v_1 \times deg\ v_1' = 3 \times 2 = 6$

Similarly $mult(v_i'') = 6$ where $2 \le i \le n$

Therefore $mult(G) = \prod_{v \in V(G)} mult(v) = mult(v) \times mult(v') \times mult(v'') \times \prod_{i=1}^{n} mult(v_i) \times \prod_{i=1}^{n} mult(v_i') \times \prod_{i=1}^{n} mult(v_i'') = 4 \times 3^n \times 2\ (n + 2) \times 2\ (n + 2) \times [4\ (n + 2)]^n \times 6^{\,n} \times 6^{\,n}$

$= 16\ (432)^{\,n}\ (n+2)^{\,n+2}$.

$mult[v] = \prod_{u \in N[v]} deg\, u = deg\, v \times \prod_{i=1}^{n} deg\, v_i \times deg\, v' \times deg\, v''$

$= (n + 2) \times 3^n \times 2 \times 2 = (4n + 8)3^n$

$mult[v'] = \prod_{v \in N[v']} deg\, v = deg\, v' \times deg\, v'' \times deg\, v = 2 \times 2 \times (n + 2) = 4(n + 2)$

$mult[v''] = \prod_{v \in N[v'']} deg\, v = deg\, v'' \times deg\, v' \times deg\, v = 2 \times (n + 2) \times 2 = 4(n + 2)$

$mult[v_1] = \prod_{v \in N[v_1]} deg\, v = deg\, v_1 \times deg\, v \times deg\, v_1' \times deg\, v_1'' = 3 \times (n + 2) \times 2 \times 2$

$= 12(n + 2)$

Similarly $mult[v_i] = 12(n + 2) where\ 2 \le i \le n$

$mult[v_1'] = \sum_{v \in N[v_1']} deg\, v = deg\, v_1' \times deg\, v_1'' \times deg v_1 = 2 \times 2 \times 3 = 12$

Similarly $mult[v_i'] = 12$ where $2 \le i \le n$

$mult[v_1''] = \sum_{v \in N[v_1'']} deg\, v = deg\, v_1'' \times deg\, v_1' \times deg\, v_1 = 2 \times 2 \times 3 = 12$

Similarly $mult[v_i''] = 12$ where $2 \le i \le n$

Therefore $mult[G] = \prod_{v \in V(G)} mult\ [v] = mult[v] \times mult[v'] \times mult[v''] \prod_{i=1}^{n} mult[v_i] \times \prod_{i=1}^{n} mult[v_i'] \times \prod_{i=1}^{n} mult[v_i'']$

$= (4n + 8)3^n \times 4(n + 2) \times 4(n + 2) \times [12(n + 2)]^n \times 12^n \times 12^n = 3^{4n}4^{3n+3}(n + 2)^{n+3}$

EXAMPLE 2.5: Consider the star graph $K_{1,3}$

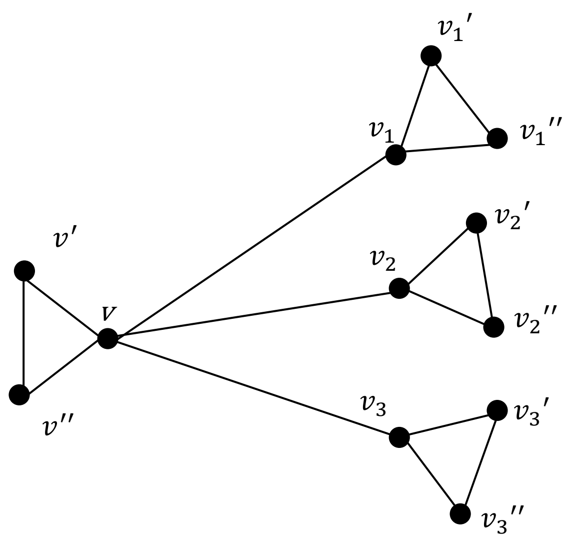

Figure 2

$supp(K_{1,n}) = n^2 + 21n + 12 = 3^2 + 21\ (3) + 12 = 84$

$supp[K_{1,3}] = n^2 + 29n + 18 = (3)^2 + 29(3) + 18 = 114$

$mult(K_{1,3}) = 16\ (432)^{n}\ (n + 2)^{n+2} = 16\ (432)^3\ (3 + 2)^{3+2} = 16 \times (432)^3 \times 5^5$

$= 4{,}031{,}078{,}\ 400{,}000.$

$mult[K_{1,3}]\ = 3^{4n}4^{3n+3}(n + 2)^{n+3} = 3^{4(3)}4^{3(3)+3}(3 + 2)^{3+3} = 4^{12} \times 8{,}303{,}765{,}625.$

THEOREM 2.6: Let G be the graph obtained by duplicating all the edges by vertices of the star $K_{1,n}$. Then

$(i)\ supp(G)\ = 4n\ (n + 2)$

$(ii)\ supp[G] = 4n^2 + 14n$

$(iii)\ mult(G)\ = 64^{\ n}\ n^{\ 2n}$

$(iv)\ mult[G]\ = 2^{8n+1}n^{2n+1}$

PROOF: Let G be the graph obtained by duplicating all the edges by vertices of the star $K_{1,n}$.

Let $v, v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$.

Let $u_1, u_2, \ldots, u_n$ be the corresponding vertices in G.

Then $deg\ v = 2n$,$deg\ v_i = 2$ where $1 \leq i \leq n$,$deg\ u_j = 2$ where $1 \leq j \leq n$

Now $supp(v) = \sum_{u \in N(v)} deg\ u = \sum_{i=1}^{n} deg\ v_i + \sum_{j=1}^{n} deg\ u_j = 2n + 2n = 4n$

$supp(v_1) = \sum_{v \in N(v_1)} deg\ v = deg\ v + deg\ u_1 = 2n + 2$

Similarly $supp(v_i) = 2n + 2$ where $2 \leq i \leq n$

$supp(u_1) = \sum_{v \in N(u_1)} deg\ v = deg\ v + deg\ v_1 = 2n + 2$

Similarly $supp(u_j) = 2n + 2$ where $2 \leq j \leq n$

Therefore $supp(G) = \sum_{v \in V(G)} supp(v) = supp(v) + \sum_{i=1}^{n} supp(v_i) + \sum_{j=1}^{n} supp(u_j)$

$= 4n + n(2n + 2) + n(2n + 2) = 8n + 4n^2 = 4n(n + 2)$

Now $supp[v] = \sum_{u \in N[v]} deg\, u = deg\, v + \sum_{i=1}^{n} deg\, v_i + \sum_{j=1}^{n} deg\, u_j = 2n + 2n + 2n = 6n$

$supp[v_1] = \sum_{v \in N[v_1]} deg\, v = deg\, v_1 + deg\, v + deg\, u_1 = 2 + 2n + 2 = 2n + 4$

Similarly $supp[v_i] = 2n + 4\ where\ 2 \leq i \leq n$

$supp[u_1] = \sum_{v \in N[u_1]} deg\, v = deg\, u_1 + deg\, v + deg\, v_1 = 2 + 2n + 2 = 2n + 4$

Similarly $supp[u_j] = 2n + 4\ where\ 2 \leq j \leq n$

Therefore $supp[G] = \sum_{v \in V(G)} supp\ [v] = supp[v] + \sum_{i=1}^{n} supp[v_i] + \sum_{j=1}^{n} supp[u_j]$

$= 6n + n(2n + 4) + n(2n + 4) = 4n^2 + 14n$

Now $mult(v) = \prod_{u \in N(v)} deg\ u = \prod_{i=1}^{n} deg\, v_i \times \prod_{j=1}^{n} deg\ u_j = 2^n \times 2^n = 4^n$

$mult(v_1) = \prod_{v \in N(v_1)} deg\ v = deg\ v \times deg\ u_1 = 2n \times 2 = 4n$

Similarly $mult(v_i) = 4n$where $2 \leq i \leq n$

$mult(u_1) = \prod_{v \in N(u_1)} deg\ v = deg\, v \times deg\ v_1 = 2n \times 2 = 4n$

Similarly $mult(u_j) = 4n$ where $2 \leq j \leq n$

Therefore $mult(G) = \prod_{v \in V(G)} mult(v) = mult(v) \times \prod_{i=1}^{n} mult(v_i) \times \prod_{j=1}^{n} mult(u_j)$

$= 4^n \times (4n)^n \times (4n)^n = 64^n\ n^{2n}$

Now $mult[v] = \prod_{u \in N[v]} deg\, u = deg\, v \times \prod_{i=1}^{n} deg\, v_i \times \prod_{j=1}^{n} deg\, u_j = 2n \times 2^n \times 2^n$

$= 2^{2n+1} n$

$mult[v_1] = \prod_{v \in N[v_1]} deg\, v = deg\, v_1 \times deg\, u \times deg\, u_1 = 2 \times 2n \times 2 = 8n$

Similarly $mult[v_i] = 8n\ where\ 2 \leq i \leq n$

$mult[u_1] = \prod_{v \in N[u_1]} deg\, v = deg\, u_1 \times deg\, v \times deg\, u_1 = 2 \times 2n \times 2 = 8n$

Similarly $mult[u_j] = 8n \; where \; 2 \le j \le n$

Therefore $mult[G] = \prod_{v \in V(G)} mult\,[v] = mult[v] \times \prod_{i=1}^{n} mult[v_i] \times \prod_{j=1}^{n} mult[u_j]$

$= 2^{2n+1} n \times [8n]^n \times [8n]^n = 2^{8n+1} n^{2n+1}$

EXAMPLE 2.7: Consider the star $K_{1,3}$

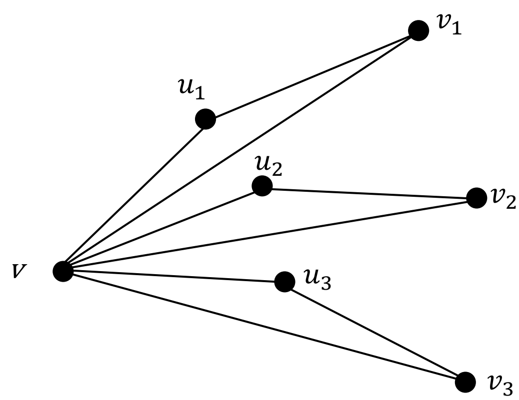

Figure 3

$supp(K_{1,4}) = 4n\,(n+2) = 4 \times 3\,(3+2) = 60$

$supp[K_{1,3}] = 4n^2 + 14n = 4(3)^2 + 14(3) = 78$

$mult(K_{1,n}) = 64^{\,n}\, n^{\,2n} = (64)^3 \times 3^{(2 \times 3)} = 191{,}102{,}976.$

$mult[K_{1,3}] = 2^{8n+1} n^{2n+1} = 2^{8(3)+1} (2)^{2(3)+1} = 4{,}294{,}967{,}296$

THEOREM 2.8: Let $G = (P_m : K_{1,n})$. Then

(i) $supp(G) = 10m + mn\,(n+3) - 10$

(ii) $supp[G] = mn^2 + 5mn + 13m - 8$

(iii) $mult(G) = 1296\,(3)^{3m-10}\,(n+1)^{m\,(n+1)}$

(iv) $mult[G] = 2^6\, 3^{4m-8} (n+1)^{mn+2m}$

PROOF: Let $G = (P_m : K_{1,n})$.

Let $V(G) = \{ v_i, u_i, v_{ij} \,/\, 1 \le i \le m,\ 1 \le j \le n \}$

Then $deg\ u_i = 2$ where $i = 1, m$, $deg\ u_i = 3$ where $2 \le i \le m-1$

$deg\ v_i = n + 1$ where $1 \le i \le m$, $deg\ v_{ij} = 1$ where $1 \le i \le m,\ 1 \le j \le n$

Now $supp(u_1) = \sum_{v \in N(u_1)} deg\ v = deg\ v_1 + deg\ u_2 = (n+1) + 3 = n + 4$

Similarly $supp(u_m) = n + 4$

$supp(u_2) = \sum_{v\in N(u_2)} deg\ v = deg\ u_1 + deg\ u_3 + deg\ v_2 = 2 + 3 + (n + 1) = n + 6$

Similarly $supp(u_{m-1}) = n + 6$

$supp(u_3) = \sum_{v\in N(u_3)} deg\ v = deg\ u_2 + deg\ u_4 + deg\ v_3 = 3 + 3 + (n + 1) = n + 7$

Similarly $supp(u_i) = n + 7$ where $4 \le i \le m - 2$

$supp(v_1) = \sum_{v\in N(v_1)} deg\ v = deg\ u_1 + \sum_{j=1}^{n} deg\ v_{1j} = n + 2$

Similarly $supp(v_m) = n + 2$

$supp(v_2) = \sum_{v\in N(v_2)} deg\ v = deg\ u_2 + \sum_{j=1}^{n} deg\ v_{2j} = n + 3$

Similarly $supp(v_i) = n + 3$ where $3 \le i \le m - 1$

$supp(v_{11}) = \sum_{v\in N(v_{11})} deg\ v = deg\ v_1 = n + 1$

Similarly $supp(v_{1j}) = n + 1$ where $2 \le j \le n$ and $supp(v_{ij}) = n + 1$ where $2 \le i \le m$, $1 \le j \le n$

Therefore $supp(G) = \sum_{v\in V(G)} supp(v) = supp(u_1) + supp(u_2) + \sum_{i=3}^{m-2} supp(u_i) + supp(u_{m-1}) + supp(u_m) + supp(v_1) + \sum_{i=2}^{m-1} supp(v_i) + supp(v_m) + \sum_{i=1}^{m} \sum_{j=1}^{n} supp(v_{ij}) = (n + 4) + (n + 6) + (m - 4)(n + 7) + (n + 6) + (n + 4) + (n + 2) + (m - 2)(n + 3) + (n + 2) + mn(n + 1) = 10m + mn(n + 3) - 10$

Now $supp[u_1] = \sum_{v\in N[u_1]} \deg v = \deg u_1 + \deg v_1 + \deg u_2 = 2 + n + 1 + 3 = n + 6$

Similarly $supp[u_m] = n + 6$

$supp[u_2] = \sum_{v\in N[u_2]} \deg v = \deg u_2 + \deg v_2 + \deg u_3 + \deg u_1 = 3 + n + 1 + 3 + 2$
$= n + 9$

Similarly $supp[u_{m-1}] = n + 9$

$Supp[u_3] = \sum_{v\in N[u_3]} \deg v = \deg u_3 + \deg v_3 + \deg u_4 + \deg u_2 = 3 + n + 1 + 3 + 3$
$= n + 10$

Similarly $supp[u_i] = n + 10$ where $4 \le i \le m - 2$

$supp[v_1] = \sum_{v\in N[v_1]} \deg v = \deg v_1 + \deg u_1 + \sum_{j=1}^{n} \deg v_{1j} = n + 1 + 2 + n = 2n + 3$

Similarly $supp[v_m] = 2n + 3$

$supp[v_2] = \sum_{v\in N[v_2]} \deg v = \deg v_2 + \deg u_2 + \sum_{j=1}^{n} \deg v_{2j} = n + 1 + 3 + n = 2n + 4$

Similarly $supp[v_i] = 2n + 4$ where $3 \le i \le m - 1$

$supp[v_{11}] = \sum_{v\in N[v_{11}]} \deg v = \deg v_{11} + \deg v_1 = 1 + n + 1 = n + 2$

Similarly $supp\left[v_{1j}\right] = n + 2$ where $2 \le j \le n$ $supp\left[v_{ij}\right] = n + 2$ where $2 \le i \le m, 1 \le j \le n$

Therefore $supp[G] = \sum_{v\in V(G)} supp\ [v] = supp[u_1] + supp[u_2] + \sum_{i=3}^{m-2} supp[u_i]\ supp[u_{m-1}] + supp[u_m] + supp[v_1] + \sum_{i=2}^{m-1} supp[v_i] + supp[v_m] +$

$\sum_{i=1}^{m}\sum_{j=1}^{n} supp[v_{ij}] = (n+6)+(n+9)+(m-4)(n+10)+(n+6)+(n+9)(2n+3)+(m-2)(2n+4)+(2n+3)+m[n(n+2)] = mn^2+5mn+13m-8$

$mult(u_1) = \prod_{v\in N(u_1)} deg\ v = deg\ u_2 \times deg\ v_1 = 3\,(n+1)$

Similarly $mult(u_m) = 3\,(n+1)$

$mult(u_2) = \prod_{v\in N(u_2)} deg\ v = deg\ u_1 \times deg\ u_3 \times deg\ v_2 = 2 \times 3 \times (n+1) = 6\,(n+1)$

Similarly $mult(u_{m-1}) = 6\,(n+1)$

$mult(u_3) = \prod_{v\in N(u_3)} deg\ v = deg\ u_2 \times deg\ u_4 \times deg\ v_3 = 3 \times 3 \times (n+1) = 9\,(n+1)$

Similarly $mult(u_i) = 9\,(n+1)$ where $4 \le i \le m-2$

$mult(v_1) = \prod_{v\in N(v_1)} deg\ v = deg\ u_1 \times \prod_{j=1}^{n} deg\ v_{1j} = 2 \times 1^n = 2$

Similarly $mult(v_m) = 2$

$mult(v_2) = \prod_{v\in N(v_2)} deg\ v = deg\ u_2 \times \prod_{j=1}^{n} deg\ v_{2j} = 3 \times 1^n = 3$

Similarly $mult(v_i) = 3$ where $3 \le i \le m-1$

$mult(v_{11}) = \prod_{v\in N(v_{11})} deg\ v = deg\ v_1 = n+1$

Similarly $mult(v_{1j}) = n+1$ where $2 \le j \le n$ and $mult(v_{ij}) = n+1$ where $2 \le i \le m$, $1 \le j \le n$

Therefore $mult(G) = \prod_{v\in V(G)} mult(v) = mult(u_1) \times mult(u_2) \times \prod_{i=3}^{m-2} mult(u_i) \times mult(u_{m-1}) \times mult(u_m) \times mult(v_1) \times \prod_{i=2}^{m-1} mult(v_i) \times mult(v_m) \times \prod_{i=1}^{m}\prod_{j=1}^{n} mult(v_{ij}) = [3(n+1)] \times [6\,(n+1)] \times [9\,(n+1)]^{m-4} \times [6\,(n+1)] \times [3\,(n+1)] \times 2 \times 3^{m-2} \times 2 \times (n+1)^{mn}$

$= 1296 \times (3)^{2(m-4)+m-2}\ (n+1)^{m(n+1)} = 1296\,(3)^{3m-10}\,(n+1)^{m(n+1)}$

Now $mult[u_1] = \prod_{v\in N[u_1]} \deg v = \deg u_1 \times \deg v_1 \times \deg u_2 = 2 \times (n+1) \times 3 = 6(n+1)$

Similarly $mult[u_m] = 6(n+1)$

$mult[u_2] = \prod_{v\in N[u_2]} \deg v = \deg u_2 \times \deg v_2 \times \deg u_3 \times \deg u_1 = 3 \times (n+1) \times 3 \times 2$

$= 18(n+1)$

Similarly $mult[u_{m-1}] = 18(n+1)$

$mult[u_3] = \prod_{v\in N[u_3]} \deg v = \deg u_3 \times \deg v_3 \times \deg u_4 \times \deg u_2 = 3 \times (n+1) \times 3 \times 3 = 27(n+1)$

Similarly $mult[u_i] = 27(n+1)$ where $4 \le i \le m-2$

$mult[v_1] = \prod_{v\in N[v_1]} deg\ v = deg\ v_1 \times deg\ u_1 \times \sum_{j=1}^{n} deg\ v_{1j} = (n+1) \times 2 = 2(n+1)$

Similarly $supp[v_m] = 2(n+1)$

$mult[v_2] = \prod_{v\in N[v_2]} \deg v = \deg v_2 \times \deg u_2 \times \sum_{j=1}^{n} \deg v_{2j} = (n+1) \times 3 = 3(n+1)$

Similarly $mult[v_i] = 3(n+1)$ where $3 \le i \le m-1$

$mult[v_{11}] = \prod_{v\in N[v_{11}]} \deg v = \deg v_{11} \times \deg v_1 = 1 \times (n+1) = n+1$

Similarly $mult[v_{1j}] = n + 1$ where $2 \le j \le n$, $mult[v_{ij}] = n + 1$ where $2 \le i \le m, 1 \le j \le n$

Therefore $mult[G] = \prod_{v \in V(G)} mult\ [v] = mult[u_1] \times mult[u_2] \times \sum_{i=3}^{m-2} mult[u_i] \times mult[u_{m-1}] \times mult[u_m] \times mult[v_1] \times \sum_{i=2}^{m-1} mult[v_i] \times mult[v_m] \times \prod_{i=1}^{m} \sum_{j=1}^{n} mult[v_{ij}] = 6(n+1) \times 18(n+1) \times (27(n+1))^{(m-4)} \times 18(n+1) \times 6(n+1) \times 2(n+1) \times (3(n+1))^{(m-2)} \times 2(n+1) \times [(n+1)^n]^m = 2^6 3^{4m-8}(n+1)^{mn+2m}$

EXAMPLE 2.9: Consider the graph $(P_4: K_{1,3})$

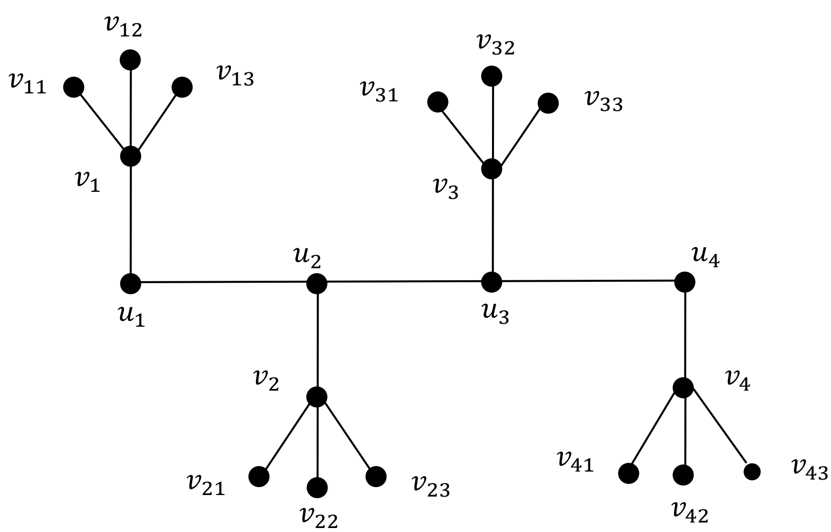

Figure 4

$supp(P_4: K_{1,3}) = 10m + mn\,(n + 3) - 10 = 10(4) + (4 \times 3)\,(3 + 3) - 10 = 102$

$supp[P_4: K_{1,3}] = mn^2 + 5mn + 14m - 12 = (4)(3)^2 + 5(4)(3) + 14(4) - 12 = 140$

$mult(P_4: K_{1,3}) = 1296\ (3)^{3m - 10}\ (n + 1)^{m\,(n + 1)} = 1296\ (3)^{3(4) - 10}\ (3 + 1)^{4(3 + 1)} = 11{,}664 \times 4^{16}$

$mult[P_4: K_{1,3}] = 2^6 3^{4m-8}(n+1)^{mn+2m} = 2^6 3^{4(4)-8}(3+1)^{(4)(3)+2(4)} = 3^8 4^{23}$

3. CONCLUSION

In this paper open and closed support of graph G obtained by duplicating all the vertices, all the edges, vertices by edges and edges by vertices of a star under addition and multiplication are determined.

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