Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 164 (2022) 108-121 EISSN 2392-2192

Comparison of Newton's Interpolation and Aitken's Methods with Some Numerical Methods for Solving System of First and Second Order Differential Equation

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ABSTRACT

There has been a greater attempt to solve differential equations by numerical methods. In this paper, we will treat the problem of system of two equations of first and second orders by using Newton's Interpolation and Aitken's Methods and we will compare the results with Some Numerical Methods as the method of Euler method, Euler Cauchy method (Modified Euler Method) and Range Kutta method.

***Keywords*:** Differential equation of first and second orders, Numerical method, Newton's Interpolation and Aitken's Methods, Euler Method, Euler Cauchy Method, Modified - Euler Method, Range Kutta Method

2010 MSC: 65B05, 65J15, 65L06

1. INTRODUCTION

Throughout the last years, significant research attention in theoretical physics and other applied mathematics related domains has been concentrated on finding and debating the exact

(Received 14 November 2021; Accepted 03 December 2021; Date of Publication 04 December 2021)

and approximated solutions to nonlinear ordinary differential equations of first and second orders for various potentials. Many numerical issues in real-life events can be represented by a class of ordinary differential equations (ODEs), notably first and second orders equations, necessitating the solution of differential equations. So many branches of physical-mathematical sciences, such as applied mathematics, mechanical engineering, electrical engineering, fluid mechanics, condensed matter physics, theoretical physics, and so on, rely heavily on the study of precise and estimated numerical solutions of nonlinear ordinary differential equations of first and second orders [1]. Several approaches, such as Mathematica, Mathematical Laboratory (MATLAB), and Maple, have recently been developed to evaluate and obtain exact solutions to nonlinear ordinary differential equations. As a result, there is a continuing need to create consistent and effective methods for obtaining an approximate solution to ordinary differential equations of second order. Previous research has struggled to focus on first-order differential equations.

Faith [2] obtained the subsequent dual terms using a mixture of Newton's interpolation and Lagrange procedures, and previously used the three values for y to build a quadratic equation using the Lagrange approach. To acquire the solution of an ordinary differential equation of first order, IDE [3] combined Aitken's and Newton's interpolation methods to extract the second two terms, then used the three values for y to build a linear or quadratic equation using the Aitken interpolation method. As a result, there exist multiple methods for displaying an approximated solution for various types of differential equations. To address nonlinear problems, Temimi and Ansari [4] suggested a semi-analytical iterative approach. It has been used to solve a variety of physics-related differential equations, including second-order nonlinear ordinary differential equations [5]. Using the Laplace transform method, Fatoorehchia and Abolghasemia [6] investigated the exact analytical solution of nonlinear differential equations. The exact solution of nonlinear ordinary differential equations was studied by Mahmoud and Shehu [7], who used the natural decomposition method (NDM) to get exact solutions for three different types of nonlinear ordinary differential equations (NLODEs).

In mathematical physics, Mbagwu, Madububa, and Nwamba [8] examined the series solution of nonlinear ordinary differential equations using the single Laplace transform approach. Mbagwu et al [9] evaluated the analytical and numerical solutions to second order ordinary differential equations using Newton's interpolation and Aitken's methods. To obtain exact solutions, Karwan and Aram [10] employed a new technique to solve a system of first order linear differential equations in matrix form. To acquire the coefficients of the Taylor series of the solution of differential and integral equations, Ahmed et al [11] used the Differential Transform Method to solve a System of Linear First-Order Differential Equations. Mondal, Roy, and Das [12] investigated the Runge-Kutta-Fehlberg Method and its Application for Numerical Solution of First-Order Linear Differential Equations in a Fuzzy Environment.

The exact solution are evaluated to the differential (exact solutions ideas) systems (i)-gH and (ii)-gH. Comprehensive error analysis is also performed after the process. Abraha [13] investigated a Comparison of Numerical Methods for System of First Order Ordinary Differential Equations in order to obtain approximate solutions to first-order ordinary differential equations. Classical Runge-Kutta method, Modified Euler technique, and Euler method are the three methods.

The calculated analytical results show that the classical fourth order Runge-Kutta method provides the closest values. Using the Runge-Kutta Method, Koroche [14] investigated a numerical solution of a first-order ordinary differential equation.

The current method is particularly efficient and practical for solving first-order ordinary differential equations since it approximates the precise solution very well. A very well differential equations problem consists of at least one differential equation and at most one additional equation, such that the system arranged has only one result (existence and uniqueness), referred to as the exact or analytic solution to distinguish it from the approximated numerical solutions.

Furthermore, this analytic solution must be inextricably linked to statistics, in the notion that if the equations are slightly changed, the result does not vary much. We explored this problem(s) using a combination of Aitken's and Newton's interpolation methods [15-19] to obtain the analytic or precise solution of ordinary differential equation(s) of second order and a system of ordinary differential equations of first order. See also [20-29] for some new qualitative conclusions of solutions of nonlinear differential equations of second order. Finally, we completed a number of problems and mathematical answers to demonstrate the competency of the technique described in this work, and we compared it to the Runge Kutta and Euler Methods. We study the following initial value problems (see [9, 13]):

$$y'' = f(x, y'), y(x_o) = y_o,\ y'(x_o) = y_o, \tag{1.1}$$

where $f(x, y') \in C\ (R, R)$,

$$xy'' + 2y' + x = 1, (1) = 2, y'(1) = 1 \tag{1.2}$$

$$y'' + \pi^2 e^y = 0, y(1) = 0, y'(0) = a \tag{1.3}$$

and

$$y_{i+1} = y_i + hf(x_i, y_i), i = 0,1, \dots, n \tag{1.4}$$

$$y_{i+1} = y_i + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h \tag{1.5}$$

The goal of this paper is to compare Newton's interpolation and Aitken's methods with the Runge Kutta and Euler methods in order to provide estimated results for the above initial value problems with respect to nonlinear ordinary differential equations of first and second orders and a system of first order linear differential equations.

2. COMBINED NEWTON'S INTERPOLATION AND LAGRANGE METHODS [2, 3]

This study combines Newton's interpolation and Aitken's Methods. We used newton's interpolation method to find the second two terms then use the three values for y to form a quadratic equation using Lagrange interpolation method as follows;

2. 1. Newton's Interpolation Method [2, 3]

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)(x - x_1)..a_2(x - x_{n-1}) \tag{2.1}$$

where

$$a_0 = y_0 \ , \ \ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} \ , \ \ a_2 = \frac{\dfrac{f(x_2 - x_1)}{(x_2 - x_1)} - \dfrac{f(x_1 - x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \tag{2.2}$$

etc.

2. 2. Lagrange Interpolation Method [2, 3]

$$y_n = \frac{(x - x_1) - (x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0) - (x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0) - (x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \tag{2.3}$$

3. DESCRIPTION OF THE PROPOSED METHOD [2, 3]

This method will combine a Newton's interpolation method and Aitken's method. It used newton's interpolation method to find the second two terms then use the three values for y to form a linear or quadratic equations using Aitken's interpolation method as follows;

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \ldots + a_n(x - x_0)(x - x_1)\ldots a_2(x - x_{n-1}) \tag{3.1}$$

where

$$a_0 = y_0 \ , \ \ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} \ , \ a_2 = \frac{\dfrac{f(x_2 - x_1)}{(x_2 - x_1)} - \dfrac{f(x_1 - x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \tag{3.2}$$

etc.

$$y_1 = a_0 + a_1(x - x_0) \tag{3.3}$$

$$y_2 = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \tag{3.4}$$

3. 1. Aitken's Interpolation Method [3]

$$P_{o,k}(x) = \frac{1}{x_k - x_o} \begin{vmatrix} y_o & x_o - x \\ y_k & x_k - x \end{vmatrix} \tag{3.5}$$

$$P_{o,1,2}(x) = \frac{1}{x_2 - x_1} \begin{vmatrix} P_{o,1}(x) & x_1 - x \\ P_{o,2}(x) & x_2 - x \end{vmatrix} \tag{3.6}$$

$$y_n = P_{o,1,2,\dots,n}(x) = \frac{1}{x_n - x_{n-1}} \begin{vmatrix} P_{o,1,\dots,(n-1)}(x) & x_{n-1} - x \\ P_{o,1,\dots,(n-2),n}(x) & x_n - x \end{vmatrix} \quad (3.7)$$

4. RUNGE-KUTTA METHOD [8]

For the equation $y' = f(x,y)$ and the initial condition $y(x_o) = y_o$

$$y(x+h) \sim y(x) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \quad (4.1)$$

$$\begin{aligned} K_1 &= h.f(x,y) \\ K_2 &= h.f\left(x + \frac{h}{2}, y + \frac{K_1}{2}\right) \\ K_3 &= h.f\left(x + \frac{h}{2}, y + \frac{K_2}{2}\right) \\ K_4 &= h.f(x+h, y+K_3) \end{aligned} \quad (4.2)$$

4. 1. Solving System of Differential Equation of First Order

Consider the system of simultaneous differential equations of first order (SDE) of the form:

$$y' = f(x,y,u) \quad (4.3)$$

u' = g (x, y, u), with $y(x_0) = y_0$ and $u(x_0) = u_0$

Numerical Examples

We will check the effectiveness of the present technique (Euler Method). First numerical comparison for the following test examples.

Problem 1: Solve, by using Euler method

$$y' = xy + u$$
$$u' = uy + 1$$
$$y(0) = 0, u(0) = 1$$

Here, we see that f (x, y, u) = $xy + u$

g (x,y,u) = uy + 1

Hence, y (0.1) = y (0) + (0.1) f (0,0,1) = 0.1

u (0.1) = u (0) + (0.1) g (0,0,1) = 1.1

Problem 2: Solve same problem1 by using Euler Cauchy method (Modified Euler method).

By Euler method, we have, y (0.1) =0.1 and u (0.1) = 1.1

hence, $y = (0.1) = y(0) + \frac{0.1}{2}[f(0,0,1) + f(0.1,0.1,1.1)] = 0.1055$

$$u = (0.1) = u(0) + \frac{0.1}{2}[g(0,0,1) + g(0.1,0.0155,1.1)] = 0.1058025$$

Problem 3: Solve problem1 by using Range-Kutta method

We have, $K_1 = h.f(x_1, y_1, u_1)$

$$L_1 = h.g(x_1, y_1, u_1)$$

$$K_2 = h.f(x_1 + \frac{h}{2}, y_1 + K_1/2+, u_1 + L_1/2)$$

$$L_2 = h.g(x_1 + \frac{h}{2}, y_1+K_1/2+, u_1 + L_1/2)$$

$$K_3 = h.f(x_1 + \frac{h}{2}, y_1 + K_2/2+, u_1 + L_2/2)$$

$$L_3 = h.g(x_1 + \frac{h}{2}, y_1+K_2/2+, u_1 + L_2/2)$$

$$K_4 = h.f(x_1 + h, y_1 + K_3+, u_1 + L_3)$$

$$L_4 = h.g(x_1 + h, y_1+K_3+, u_1 + L_3)$$

So, we have, $x = (0.1) = x(0) + h = \cdots$

$$y = (0.1) = y(0) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 0.1037773568$$

$$u = (0.1) = u(0) + \frac{1}{6}(L_1 + 2L_2 + 2L_3 + L_4) = 1.10712792$$

4. 2. Numerical Problems of Newton's Interpolation and Aitken's Methods

Various numerical examples were used to demonstrate the applicability of Newton's and Aitken's interpolation methods. These scenarios are used when exact logical findings are required for easier validation. Nonlinearities are elaborated in mathematical models from several disciplines such as physics, mathematics, chemistry, and engineering in the following problems.

Problem 4: Let solve the following initial value problem:

$$y'' + {y'}^2 + y^2 = 1 - cos(x), y(0) = 0, y'(0) = 1 \quad (4.3)$$

We suppose the step $h = 0.50$ and take the first initial condition, $y(0) = 0$. Using Newton's interpolation method, we have:

$a_0 = 0 = y_0,$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,0} = 1,$$

$y_1 = 0 + 1(0.50 - 0) = 0.50,$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Which implies that:

$$a_2 = \frac{\left[\frac{dy}{dx}\right]_{0.50,0.50} - \left[\frac{dy}{dx}\right]_{0,0}}{0.60 - 0} = 0.41,$$

$y_2 = 0 + 1(0.60 - 0) + 0.41(0.60 - 0)(0.60 - 0.50) = 0.6246.$

Now, we form the linear and quadratic equations by means of Aitken's method:

From equation (4.3), we obtain:

$P_{0,1}(x) = 1 - x,$

$P_{0,2}(x) = 1 - 1.3854x,$

$P_{0,1,2}(x) = 0.41x^2 - 1.00505x + 1.$

From equation (4.3), we take the second initial condition $y'(0) = 1$

We suppose the step $h = 0.50$. Using Newton's interpolation method, we have:

$a_0 = 1 = y_0,$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,1} = -1,$$

$y_1 = 1 - 1(0.50 - 0) = 0.50,$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Which implies that:

$$a_2 = \frac{\left[\frac{dy}{dx}\right]_{0.50,0.50} - \left[\frac{dy}{dx}\right]_{0,1}}{0.50 - 0} = -1.500,$$

$y_2 = 1 - 1(0.50 - 0) - 1.500(0.50 - 0)(0.50 - 0.50) = -0.2500.$

Now, we form the linear and quadratic equations by means of Aitken's method:

Then, we obtain:

$P_{0,1}(x) = 1 - x,$

$P_{0,2}(x) = 1 - 0.88385x,$

$P_{0,1,2}(x) = -1.500x^2 - 1.00606x + 1.$

Problem 5: Let solve the following initial value problem:

$$y' = (1 - 2t)y, y(0) = 1 \tag{4.4}$$

We suppose the step $h = 0.40$ and take the first initial condition, $y(0) = 1$. Using Newton's interpolation method, we have:

$a_0 = 1 = y_0,$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,1} = -1,$$

$y_1 = 1 - 1(0.40 - 0) = 0.60,$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Which implies that:

$$a_2 = \frac{\left[\frac{dy}{dx}\right]_{0.40,0.60} - \left[\frac{dy}{dx}\right]_{0,0}}{0.60 - 0} = -0.126,$$

$y_2 = 1 - 1(0.60 - 0) - 0.126(0.60 - 0)(0.60 - 0.40) = 0.3849.$

Now, we form the linear and quadratic equations by means of Aitken's method:

From equation (4.4), we obtain:

$P_{0,1}(x) = 1 - x,$

$P_{0,2}(x) = 1 - 0.97385,$

$P_{0,1,2}(x) = -0.126x^2 - 1.00808x + 1.$

Problem 6. Let us consider the following initial value problem:

$$y'' = -2t(y')^2, y(0) = 2, y'(0) = -1 \tag{4.5}$$

We suppose the step $h = 0.30$ and take the first initial condition, $y(0) = 2$. Using Newton's interpolation method, we have:

$a_0 = 2 = y_0,$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,2} = -2,$$

$y_1 = 2 - 2(0.30 - 0) = 1.40,$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Which implies that:

$$a_2 = \frac{\left[\frac{dy}{dx}\right]_{0.30,1.40} - \left[\frac{dy}{dx}\right]_{0,2}}{0.40 - 0} = -0.395,$$

$y_2 = 2 - 2(1.40 - 0) - 0.395(1.40 - 0)(1.40 - 0) = -0.1574.$

Now, we form the linear and quadratic equations by means of Aitken's method:

From equation (4.5), we obtain:

$\mathrm{P}_{0,1}(x) = 1 - x,$

$\mathrm{P}_{0,2}(x) = 1 - 0.77375x,$

$\mathrm{P}_{0,1,2}(x) = -0.395x^2 + 1.03202x + 1.$

From equation (4.5), we take the second initial condition $y'(0) = -1$

Let choose the step $h = 0.40$. Let use Newton's interpolation method. Then, we have:

$a_0 = -1 = y_0,$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,-1} = 1,$$

$y_1 = -1 - 1(0.40 - 0) = -1.40,$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Which implies that:

$$a_2 = \frac{\left[\frac{dy}{dx}\right]_{0.40,-1.40} - \left[\frac{dy}{dx}\right]_{0,-1}}{0.50 - 0} = 0.880,$$

$$y_2 = 1 - 1(-1.40 - 0) + 0.880(-1.40 - 0)(-1.40 - 0) = 0.2125.$$

Now, we form the linear and quadratic equations by means of Aitken's method:

Then, we obtain:

$P_{0,1}(x) = 1 - x,$

$P_{0,2}(x) = 1 - 0.78587x,$

$P_{0,1,2}(x) = 0.880x^2 - 1.22808x + 1.$

4. 3. Solving System of Differential Equation of second Order

Consider the differential equation of second order,

$$y'' = g(x, y, y') \tag{4.6}$$

By taking

$$u = y'$$

We have,

$$u' = y'' = g(x, y, y')$$

So, we have two equations

$$u = y'$$
$$u' = g(x, y, y')$$

Mathematical Problems

We will check the effectiveness of the present technique (Euler Method). First numerical comparison for the following test examples.

Problem 7: Find $y(0.1)$ and $y(0.2)$ by using Euler method

$$\text{Let } y' = u$$
$$u' = -y$$

Or, $y' = f(x, y, u) = u$ & $u' = g(x, y, u) = -y$

So, $y(0.1) = 0.1$

$u(0.1) = 1$

$y(0.2) = 0.2$

$u(0.2) = 0.99$

But, as the analytic solution is, $y = sinx$

We see, $y(0.1) = sin(0.1) = 0.998$

$y(0.2) = sin(0.2) = 0.1986$

Problem 8: Find $y(0.1)$ and $y(0.2)$ by using Euler-Cauchy method

By Euler method we have, $u(0.1) = 1$ & $y(0.1) = 0.1$, hence, by Euler Cauchy we have, $y(0.1) = 0.1$ and $u(0.1) = 0.995$

and by Euler we have $u(0.2) = 0.985$

By Euler Cauchy we have $y(0.2) = 0.199$.

Problem 9: Same problem by Range Kutta Method

We have, $x_0 = 0$, $y_0 = 0$, $u_0 = 0$

So, $K_1 = hu_0 = 0.1$

$L_1 = -hy_0 = 0$

$K_2 = 0.1, L_2 = -0.005, K_3 = 0.09975, L_3 = -0.005$

$K_4 = 0.0995, L_4 = -0.0995$

Hence, $y(0.1) = 0.099833$.

5. CONCLUSIONS

We attempt to solve differential equations using numerical methods in this article, getting more than three nonlinear ordinary differential equations. We used Newton's Interpolation and Aitken's Methods to find exact and analytic solutions to a system of three second-order equations, and we compared the findings to certain numerical methods such as the Euler method, Euler Cauchy method (Modified Euler Method), and Range Kutta method.

The Newton and Aitken interpolation methods, as well as the Euler, Euler Cauchy (Modified Euler Method), and Range Kutta methods, represent a significant advancement in the fields over current methods.

Our long-term goal is to compare Aitken's and Newton's interpolation methods to the Euler method, the Euler Cauchy method (Modified Euler Method), and the Range Kutta method in order to obtain other nonlinear differential equations (PDEs, ODEs) that arise in other fields of science, engineering, and other related fields.

The findings indicate that this process is more effective and reliable than other approaches to tackling the problem.

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