Thermal Analysis of a Convective-Radiative Moving Porous Trapezoidal Fin with Variable Thermal Properties and Internal Heat Generation using Finite Element Method

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ABSTRACT

Trapezoidal fins are widely used especially for aircraft applications where the weight of the fin and safety are important factors in the design of the structures. Therefore, the present work focuses on the analysis of heat transfer in a convective-radiative moving porous trapezoidal fin using finite element method. The simulated results reveal that the dimensionless temperature distribution increases as the surface emissivity constant, power index of heat coefficient, Peclet number, porosity and ambient temperature increase. The present results will help in the design of trapezoidal fin operating in a high temperature and temperature-dependent heat transfer coefficient.

Keywords: Thermal analysis, Convective-Radiative fin, Moving porous fin, Trapezoidal fins, Aircraft applications

1. INTRODUCTION

Fins have been widely applied for heat transfer enhancements in thermal systems. The wide areas of application have led to various research works. Taler and Taler presented steady and unsteady heat transfer through Fins of complex geometry [1]. Ghasemi et al. [2] analytically
investigated the heat transfer in both solid and porous fin with temperature-dependent heat generation using differential transform method (DTM) while in an earlier study, with the aids of grapho-analytical method, Kosarev [3] presented steady state thermal analysis of rectangular fins with variable thermal conductivity. Meanwhile, Laor and Kalman [4] numerically explored the thermal performance and optimum dimension of different cooling fins with variable heat transfer coefficient. Other researchers have presented several studies on the extended surfaces [5-18].

From the reviewed studies, it was established that due to its relative low cost and ease of manufacturing of longitudinal rectangular profile, various works have focused on this shape of fin are widely used in many heat transfer enhancement systems. However, it has been well established that though fins of irregular geometries/shapes are usually more expensive to manufacture, they provide a more efficient utilization of the material for their productions. Therefore, they are lighter in weight than the rectangular fins. Therefore, they are preferable in some applications despite the fact that they are expensive and difficult to manufacture.

![Diagram of Rocket](image)

**Figure 1.** Application of Trapezoidal Moving fins in Space Rockets

Consider model rockets, fin structure is one of the critical components that should be carefully designed. In fact, the apogee and stability of the model rocket is affected by the shape and size of the fins. Longitudinal fins of triangle, trapezoidal, concave parabolic and convex parabolic profiles have been considered. However, aside the relative high cost of production,
the concave and convex parabolic fins of zero tip thickness are relatively and very difficult to be fabricated in practice. Also, the costs of productions of these fins with parabolic profiles discourage their applications in heat enhancement systems. Manufacturing a fin with triangle profile in practice eventually leads to development of fins with trapezoidal profile. Considerations of lightness in weight, cost and safety show that trapezoidal fin is the best option especially for airborne and space rocket applications as shown in Fig. 1. Therefore, recourse can be made to trapezoidal fin for the applications where the weight of the fin is an important factor in the design of the structures, equipment, device and systems.

The importance of this shape has led to several studies. In these studies, Hagen [19] presented perturbation analysis of tapered fins with temperature-dependent thermal properties. With the aids of response surface method, Tola and Nikbay [20] conducted a study examined the impact of basic sizing parameters of a trapezoidal fin on flutter speed. With the aids of hybrid differential evolution algorithm, Das et al. [21] estimated the critical dimensions for a trapezoidal-shaped steel fin. Kharni and Aziz [22] applied homotopy analysis method for heat transfer study in trapezoidal fins with variable thermal conductivity and surface heat transfer coefficient.


In most of these studies on fins with trapezoidal profiles, the thermal conductivity of the fin materials and the heat transfer coefficient were assumed to be uniform. This shows that the need for expanded studies for the relaxation of the assumptions is needed. Also, the accuracy of prediction of the developed thermal models in the previous studies depend on the method of solutions used in analysis the thermal models. Therefore, considering the inherent advantages, wide range of applications and high level of accuracy of the method justify the consideration of the method for the problem under consideration. Also, finite element analysis of heat transfer in a convective-radiative moving porous trapezoidal has not been studied in open literature. Therefore, in this work, Galerkin finite element method is used study the thermal behaviour of convective-radiative moving porous fins of fin with trapezoidal profile. The numerical solutions are used to investigate the effects of fin thermal model parameters on the thermal performance of the fin.

2. THERMAL MODEL PHYSICAL MODEL AND THE GOVERNING EQUATIONS

Consider a porous fin of trapezoidal profile, having length L, width W, a uniform cross-sectional area A, and perimeter P (approximated by 2W) as shown in Fig. 2. Assuming a time-
invariant one-dimensional temperature distribution along the length of the fin and uniform temperature at the base of the fin. Also, it is assumed that porous fin is isotropic, homogeneous, and saturated with the single-phase fluid. The fluid and the solid matrix of the porous fin are in local thermodynamic equilibrium. There is no thermal contact resistance between the prime surface and the base of the fin. The clear fluid and the porous medium interaction are described by the Darcy’s formulation.

Following these assumptions and using the energy balance in Fig. 2 based on energy conservation law, we arrived at

$$\frac{d}{dx} \left( k_{eff} \delta(x) \frac{dT}{dx} \right) - \rho_f c_{p,f} \frac{Kg \beta}{v_f} (T - T_o)^2 - 2(1 - \varphi)h(T)(T - T_o)$$

$$-2(1 - \varphi)\sigma(T)T_b(T^4 - T_o^4) + \delta q(T) = \delta \rho_f c_{p,s} u_x \frac{dT}{dx}$$

(1)

Figure 2. Schematic representation of a porous fin of an unspecified profile and the energy balance [11].

Heat transfer coefficient, surface emissivity, heat generation vary with temperature by the following equations

$$h(T) = h_b \left( \frac{T - T_o}{T_b - T_o} \right)^m$$

(2)
On substituting Eq. (2), (3) and (4) into Eq. (1), we have

\[
\varepsilon(T) = \varepsilon_a \left(1 + \alpha \left(\frac{T - T_a}{T_b - T_a}\right)\right) \tag{3}
\]

\[
q(T) = q_a \left(1 + c_1 \left(\frac{T - T_a}{T_b - T_a}\right) + c_2 \left(\frac{T - T_a}{T_b - T_a}\right)^2 + c_3 \left(\frac{T - T_a}{T_b - T_a}\right)^3\right) \tag{4}
\]

On substituting Eq. (2), (3) and (4) into Eq. (1), we have

\[
\frac{d}{dx}\left\{k_{eff} \delta \frac{dT}{dx}\right\} - \rho_f c_{p,f} \frac{K g \beta}{v_f} (T - T_a)^2 - 2(1 - \varphi) h_b \left(\frac{T - T_a}{T_b - T_a}\right)^m (T - T_a) - 2(1 - \varphi) \sigma \varepsilon_b \left[1 + \alpha \left(\frac{T - T_a}{T_b - T_a}\right)\right] (T^4 - T_a^4)
\]

\[+ \delta q_a \left(1 + c_1 \left(\frac{T - T_a}{T_b - T_a}\right) + c_2 \left(\frac{T - T_a}{T_b - T_a}\right)^2 + c_3 \left(\frac{T - T_a}{T_b - T_a}\right)^3\right) = \delta \rho_f c_{p,f} u \frac{dT}{dx} \tag{5}\]

where the effective thermal conductivity relates the fluid thermal conductivity and solid conductivity as

\[
k_{eff} = \varphi k_f + (1 - \varphi) k_s \tag{6}\]

Using the following non-dimensional parameters,

\[
X = \frac{x}{L}, \psi = \frac{\delta_0}{L}, \theta = \frac{T}{T_b}, k_r = \frac{k_{eff}}{k_f}, \text{Pr} = \frac{\rho_f c_{p,f} v_f}{k_f}, N_{cc} = \frac{2h_b L^2}{k_{eff} \delta_0}, Da = \frac{K}{\delta_0^2}, Ra = Gr \text{Pr}
\]

\[
N_{rc} = \frac{2E_a \sigma T_3 L^3}{k_{eff} \delta_0^3}, Pe = \frac{\rho_s c_{p,s} u_s L}{k_{eff}}, Gr = \frac{\beta g T_b \delta_0^3}{v_f^2}, s_h = \frac{Da Ra}{\psi^2 k_f}, Q_a = \frac{L^2 q_a}{T_b k_{eff}}, \lambda = \frac{\delta}{\delta_0} \tag{7}
\]

Substituting the non-dimensional quantities into Eq. (6), the resulting non-dimensional energy equation is obtained as

\[
\frac{d}{dX} \left(\frac{\delta}{\delta_0} \frac{d\theta}{dX}\right) - N_{cc} (1 - \varphi) \left(\frac{\theta - \theta_a}{1 - \theta_a}\right)^m (\theta - \theta_a) - N_{rc} (1 - \varphi) \left[1 + \alpha \left(\frac{\theta - \theta_a}{1 - \theta_a}\right)\right] (\theta^4 - \theta_a^4)
\]

\[-S_h (\theta - \theta_a)^2 + Q_a \frac{\delta}{\delta_0} \left[1 + c_1 \left(\frac{\theta - \theta_a}{1 - \theta_a}\right) + c_2 \left(\frac{\theta - \theta_a}{1 - \theta_a}\right)^2 + c_3 \left(\frac{\theta - \theta_a}{1 - \theta_a}\right)^3\right] = \frac{\delta}{\delta_0} Pe \frac{d\theta}{dx} \tag{8}\]

The thickness of the element local fin is expressed as
\[ \delta(x) = (\delta_o - \delta_L) \left(1 - \frac{x}{L}\right) + \delta_L \]  

(9)

Therefore, the Eq. (8) can be expressed as

\[
\frac{d}{dX} \left[ (1-\lambda)(1-X) + \lambda \frac{d\theta}{dX} \right] - N_v(1-\varphi) \left( \frac{\theta - \theta_o}{1 - \theta_o} \right)^n \left( \theta - \theta_o \right) - N_v(1-\varphi) \left[ 1 + a \left( \frac{\theta - \theta_o}{1 - \theta_o} \right) \right] \left( \theta^4 - \theta_o^4 \right) 
- S_v(\theta - \theta_o)^2 + Q_v \left[ (1-\lambda)(1-X) + \lambda \right] \left[ 1 + c_1 \left( \frac{\theta - \theta_o}{1 - \theta_o} \right) + c_2 \left( \frac{\theta - \theta_o}{1 - \theta_o} \right)^2 + c_3 \left( \frac{\theta - \theta_o}{1 - \theta_o} \right)^3 \right] \left[ (1-\lambda)(1-X) + \lambda \right] Pe \frac{d\theta}{dX} 
\]

(10)

where \( \lambda = \frac{\delta}{\delta_0} \)

The boundary conditions for the fins are

\[ \theta |_{X=0} = 1 \]  

(11)

\[ \frac{d\theta}{dX} |_{X=1} = 0 \]  

(12)

3. FINITE ELEMENT METHOD TO THE THERMAL MODEL

The dimensionless thermal model in Eq. (9) is strongly nonlinear which cannot be solved exactly and analytically. Therefore, Galerkin finite element method is used in this work.

Following the procedures of finite element method, the entire computational domain of each profile is partitioned into “n” number of linear elements of equivalent size. A typical element is isolated as shown in Fig. 3.

Then variational formulation of the given problem over the typical element is constructed as given by Eq. (13).

\[
\int_{X_i}^{X_j} w_i \left[ \frac{d}{dX} \left[ (1-\lambda)(1-X) + \lambda \frac{d\theta}{dX} \right] - N_v(1-\varphi) \left( \frac{\theta - \theta_o}{1 - \theta_o} \right)^n \left( \theta - \theta_o \right) - N_v(1-\varphi) \left[ 1 + a \left( \frac{\theta - \theta_o}{1 - \theta_o} \right) \right] \left( \theta^4 - \theta_o^4 \right) 
- S_v(\theta - \theta_o)^2 + Q_v \left[ (1-\lambda)(1-X) + \lambda \right] \left[ 1 + c_1 \left( \frac{\theta - \theta_o}{1 - \theta_o} \right) + c_2 \left( \frac{\theta - \theta_o}{1 - \theta_o} \right)^2 + c_3 \left( \frac{\theta - \theta_o}{1 - \theta_o} \right)^3 \right] \left[ (1-\lambda)(1-X) + \lambda \right] Pe \frac{d\theta}{dX} \right] dX = 0 
\]

(13)

where \( w_i \) is the weight function corresponding to \( \theta(X) \).

Using the Galerkin Finite Element Method, the shape function is equal to the weight function. Therefore, Eq. (13) is written as
An approximate solution of the variational problem is assumed and the element equations are generated by substituting the assumed solution in the formulation as shown from Eqs. (15) - (23). The element matrix, which is called stiffness matrix, is constructed by using the element interpolation functions as stated as follows:

The finite element approximate solution for a two-node linear element in Fig. 3 is given as

$$\theta(X) = N_i(X) \theta_i + N_j(X) \theta_j$$

(15)

After derivation from the two-node linear element, we have

$$\theta(X) = \left( \frac{X_i - X}{X_j - X_i} \right) \theta_i + \left( \frac{X - X_i}{X_j - X_i} \right) \theta_j,$$

(16)

where

$$N_i = \frac{X_j - X}{X_j - X_i}, \quad N_j = \frac{X - X_i}{X_j - X_i},$$

(17)

$N_i$ and $N_j$ are called shape/interpolation/test/basis functions.

**Figure 3.** A 2-node element

For a two-node element as shown in Fig. 3, with
\[ X_i = 0, \quad X_j = L_e, \quad \Rightarrow X_j - X_i = L_e, \]  

(18)

Therefore, the shape functions can be written as

\[ N_i = 1 - \frac{X}{L_e}, \quad N_j = \frac{X}{L_e}, \]  

(19)

We can therefore write Eq. (16) as

\[ \theta(X) = \left(1 - \frac{X}{L_e}\right) \theta_i + \left(\frac{X}{L_e}\right) \theta_j, \]  

(20)

Then, the variational formulation can be written as

\[ \int_0^{L_e} \left[ \frac{d}{dX} \left[ \theta \frac{d\theta}{dX} \right] - \alpha N_e (1 - \varphi) \left( \frac{\theta - \theta_i}{1 - \theta_i} \right)^n \left( \theta - \theta_j \right) - N_e (1 - \varphi) \left( 1 + \alpha \left( \frac{\theta - \theta_i}{1 - \theta_i} \right) \left( \theta - \theta_j \right) \right) \right] dX = 0 \]  

(21)

On substituting Eq. (15) into Eq. (21), we have

\[ \int_0^{L_e} \left[ \left( 1 - \lambda \right) (1 - \varphi) \frac{d\theta}{dX} + \lambda \right] \left( \frac{d\theta}{dX} \right)^n \left( \theta - \theta_i \right) + N_e (1 - \varphi) \left( \frac{\theta - \theta_i}{1 - \theta_i} \right)^n \left( \theta - \theta_j \right) - N_e (1 - \varphi) \left( 1 + \alpha \left( \frac{\theta - \theta_i}{1 - \theta_i} \right) \left( \theta - \theta_j \right) \right) \right] dX = 0 \]  

(22)

The above Eq. (22) can be written in the finite element model form as

\[ [K^e] \{\theta^e\} = \{f^e\} \]  

(23)
\[
K_{ij}^e = \int_0^{L_e} \left[ \left( \sum_{p=0}^{m+1} \frac{1}{(1-x)^p} P_{m+1} \phi^p \right) N_j \right] \frac{dN_i}{dx} \, dx
\]

where

\[
f_i^e = -\int_0^{L_e} \left[ N_i \frac{d\theta}{dx} \right] \, dx
\]

\[
\bar{\theta}(X) = N_i(X) \bar{\theta} + N_j(X) \bar{\theta}
\]

The matrices are found \( K_{ij}^e \) and \( f_i^e \) after substitution of \( N_i = \frac{1 - X}{L_e} \) and \( N_j = \frac{X}{L_e} \) into the above equations and then integrate. Thus the element matrix is obtained. Algebraic equations are developed from the element matrix which are assembled by imposing the inter-element continuity conditions. Thus, a matrix of the order \( r+I \) is developed. Due to the present of nonlinearity term in the finite element model obtained is nonlinear. Therefore, an iterative method is adopted in the solution of the systems of nonlinear equations. The essential and natural boundary conditions given in Eq. (11) an (12) are imposed on the assembled equations. When the two boundary conditions are imposed, the remaining system contains \( r-I \) equations, and system that is solved by the Gauss-Seidel method using an accuracy of 10^{-6}.

5. RESULT AND DISCUSSION

With the aids of MATLAB for the solution of the systems of algebraic equations and simulation of the results, the following results in Fig. 4-15 are presented.

In Figs. 4, the effects of fin material on the temperature distribution in trapezoidal fins is presented. Although, all the three geometries considered follow the same trend. Also, the figure illustrates the effects of fins geometry on the thermal response of the fins when subjected to the same conditions. The figures show that copper porous fin has the highest values of dimensionless temperature distribution while stainless steel has the least dimensionless temperature distribution. The is due to higher thermal conductivity and lowest thermal resistance of copper material as compared to that of aluminium, silicon nitrides and stainless
steel material. The lowest temperature recorded for the stainless steel is on the basis of its highest thermal resistance and lowest thermal conductivity.

Figure 4. Effect of different materials on dimensionless temperature distribution of trapezoidal fin.

Figure 5. Effect of surface emissivity on temperature distribution of silicon nitride porous fin
Fig. 5 shows the impacts of surface emissivity constant and power index of heat coefficient on the dimensionless temperature distribution of trapezoidal silicon nitride porous fin while the influence of the power index of heat coefficient on the dimensionless temperature distribution of a concave aluminium porous fin is presented in Fig. 6. It can be inferred that the dimensionless temperature distribution increases as the surface emissivity constant and power index of heat coefficient increase. The reason for the behaviour in Fig. 5 is because the radiative heat dissipation increases from the porous surface of the fin increases as the surface emissivity constants which in consequent increases the dimensionless temperature distribution.

It can be stated in Fig. 6, the dimensionless temperature distribution marked \( m=0 \) represents the dimensionless temperature distribution in the concave aluminium porous fin when forced convection occurs on the fin surface. Also, the dimensionless temperature distribution profile marked \( m=0.25 \) represents the thermal behaviour in the concave aluminium porous fin when laminar natural convection occurs on the fin surface. It should also be that the dimensionless temperature distribution profile marked with \( m=2 \) presents the thermal response of the concave aluminium porous fin when nucleated boiling process (which lead to heat loss reduction) occurs on the porous fin surface. The same trends and behaviours in Fig. 8 and 9 are also recorded in the other materials and geometry of the fin.

The effects of moving parameter, the Peclet number on the temperature distribution in the convex stainless steel porous fin is shows in Fig. 7. The figure reveals the effect that the Peclet has on the fin is highly significant as the figure shows a rapid increase in temperature distribution with increase in Peclet number for the fin. The increase in the Peclet number shows increased speed in the moving porous fin. As the speed of the moving fin is increases, the heat
transfer time between the porous fin surface and the ambient fluid get shorter, and more heat are retained in the fin and the temperature distribution in the fin is increased.

**Figure 7.** The effect of Peclet number on the temperature distribution of a copper porous fin

**Figure 8.** The effect of internal heat generation on the temperature distribution of a copper porous fins
Fig. 8 and 9 reveal the effects of internal heat generation, $Q_a$ and heat generation parameters ($c_1$, $c_2$ and $c_3$) on the convex copper porous fin, respectively. It is shown from the
figures that the temperature distribution in the moving porous fin increases as the internal heat generation, and the effect of heat generation parameters ($c_1$, $c_2$ and $c_3$) increase. Such response is because as the internal heat generation and internal heat generation parameters are increased, more heat is generated in the fin, thereby the temperature distribution in the fin is increased.

**Figure 11.** The effect of convective conductive parameter ($N_{cc}$) on the temperature distribution of a silicon nitride porous fin.

The effects of porosity on the temperature distribution in the fin is presented in Fig. 10. It is shown that the dimensionless temperature distribution increases with increase in porosity. This behaviour is due to the void increment in the fin which consequently increases the porosity of the fin, and in return, the rate of heat transfer through the fin as well as the effective thermal conductivity of the porous fin decrease. However, the more the porosity of the fin, the more the convective and radiative heat enhancement capacity of the fin due to large surface area.

The ratio of convective heat dissipation to that of conductive heat transfer in the fin is connotated by convective-conductive parameter ($N_{cc}$). Therefore, Figs 11 displays the effect of convective-conductive parameter on the temperature distribution in the fin. Also, the ratio of radiative heat dissipation to that of conductive heat transfer in the fin is connotated by radiative-conductive parameter ($N_{rc}$). So, Figs 12 displays the effect of radiative-conductive parameter on the temperature distribution in the fin. It could be stated as shown that as the convective-conductive parameter of the fin decreases, convective heat loss from the surface of the porous fin decreases. However, as radiative-conductive parameter decreases, the fin temperature increases as shown in Figs 12 & 13. Fig. 12. This contrast behaviour on fin temperature of the radiative-conductive parameter as compared to convective-conductive parameter is due to the fact that the heat loss through the fin by radiation is directly proportional to the difference in the fourth power of fin temperature and ambient temperature. The impact of ambient
temperature on the concave aluminium porous fin temperature is illustrated in Fig. 13. The figure shows that as ambient temperature increases, the dimensionless temperature distribution in the concave aluminium porous fin increases. This is due to reduction in the temperature differentials between the environment and the porous fin.

**Figure 12.** The effect of convective radiative parameter (\(N_{rc}\)) on the temperature distribution of a silicon nitride porous fin.

**Figure 13.** Non-dimensional Temperature in concave Aluminium porous fin different ambient temperature.
6. CONCLUSION

Thermal behaviour of a convective-radiative moving porous trapezoidal fin has been studied using finite element method. The simulated results reveal that the dimensionless temperature distribution increases as the surface emissivity constant, power index of heat coefficient, Peclet number, porosity and ambient temperature increase. The study will greatly assist the design of trapezoidal fin operating in a high temperature and temperature-dependent heat transfer coefficient and especially for aircraft applications where the weight of the fin and safety are important factors in the design of the structures.

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Nomenclature

- $A$: cross-sectional Area of the fin ($m^2$)
- $c_1, c_2, c_3$: heat generation parameters
- $c_p$: specific heat capacity ($J \cdot Kg^{-1} \cdot K^{-1}$)
- $Gr$: Grashof number
- $g$: gravitational acceleration ($ms^{-2}$)
- $h$: heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
- $k_{eff}$: efficient thermal conductivity of the materials ($W \cdot m^{-1} \cdot K^{-1}$)
- $k_r$: thermal conductivity at the ambient fluid temperature ($W \cdot m^{-1} \cdot K^{-1}$)
- $k_r$: thermal conductivity ratio
- $k_s$: thermal conductivity of the solid, $W \cdot m^{-1} \cdot K^{-1}$
- $k_b$: thermal conductivity at the base temperature ($W \cdot m^{-1} \cdot K^{-1}$)
- $K$: permeability of the porous fins, $m^2$
- $L$: length of the fin
- $m$: power index of heat transfer coefficient
- $m$: mass flow rate passing through the fin pores, $Kg \cdot S^{-1}$
- $N_{cc}$: convective-conductive parameter
- $N_{rc}$: radiative-conductive parameter
- $N$: non-linear differential operator
- $Pe$: Peclet number
- $Pr$: Prandtl number
- $Q_a$: non-dimension heat generation at ambient temperature
- $q$: heat generation ($W \cdot m^{-3}$)
- $Ra$: Rayleigh number
- $S_h$: porous parameter
T \text{ temperature (K)} \\
u_x \text{ speed of moving porous fin, m·s}^{-1} \\
v_w \text{ velocity of fluid passing through the porous fin, m·s}^{-1} \\
W \text{ width of the fin, m} \\
x \text{ coordinate in x-direction, m} \\

\textbf{Greek symbols} \\
\alpha \text{ coefficient of surface emissivity} \\
\beta \text{ coefficient of thermal expansion} \\
\varepsilon \text{ surface emissivity} \\
\eta \text{ fin efficiency from newtons law of cooling} \\
\eta' \text{ volume adjusted fin efficiency} \\
\theta \text{ dimensionless temperature} \\
v_f \text{ kinematic viscosity of the fluid. m}^2\cdot\text{s} \\
\rho \text{ density, Kg·m}^{-3} \\
\sigma \text{ Stefan-Boltzman constant W·m}^{-2}\cdot\text{K}^{-4} \\
\varphi \text{ porosity of the porous medium} \\

\textbf{Subscripts} \\
0 \text{ values at } x = 0 \\
a \text{ values at ambient fluid} \\
b \text{ values at fin base} \\
e \text{ values at fin tip} \\
f \text{ value of fluid} \\
L \text{ values at } x = L \\
s \text{ values of solid materials} \\
\xi \text{ dimensionless coordinate} \\
\lambda \text{ Lagrange multiplier} \\
\lambda' \text{ the slope of thermal conductivity-temperature curve (K}^{-1}) \\
\psi \text{ thermo geometric fin parameter} \\
\tau \text{ integration variable} \\

\textbf{References} \\


