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SIR Model for the Spread of Tuberculosis in Kudus Regency

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ABSTRACT

The purpose of this study was to analyze the spread of tuberculosis in Kudus Regency. One of the analyzes that can be done is making a mathematical model of SIR. The SIR mathematical model describes uninfected and susceptible individuals who are infected and can transmit the disease to a number of other individuals (infectious) and individuals who have recovered or are free from disease (Recovered). From 2018 to 2019 according to the Health Profile of Kudus Regency, the spread of tuberculosis that occurred has increased in all cases, namely the number of tuberculosis sufferers in Kudus Regency reached 3,133 patients, and the number of individuals who recovered reached 589 people. Based on the analysis of the SIR model, it is found that the equilibrium point (S, I) = (52632, 8614230) will be stable when R0 > 1, with the final conclusion, the basic reproduction rate is obtained, namely $R_0 = 1{,}6367$ which indicates that one infected individual can infect 2 people on average or individuals susceptible to tuberculosis.

***Keywords*:** SIR Epidemic Models, Tuberculosis, Equilibrium Point, Stability Analysis

1. INTRODUCTION

Tuberculosis (TB) which is also known by the abbreviation TB is an infectious disease that causes the second largest health problem in the world after HIV. This disease is caused by the bacillus of the bacterium Mycobacterium tuberculosis. Tuberculosis itself can attack any

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part of the body, but the most common and most common is tuberculosis infection of the lungs. The spread of this disease can occur through people who already have TB. Then, coughing or sneezing spit out saliva that has been contaminated and inhaled by healthy people whose immune systems are weak against tuberculosis. Although it usually attacks the lungs, this disease can also affect other parts of the body, such as the central nervous system, heart, lymph nodes, and others. Indonesia is a tropical country, where the tropics are more susceptible to contracting infectious diseases than temperate climates.

The main cause is environmental factors. This, can lead to less than optimal public awareness and control of infectious diseases. One of the infectious diseases that occur in Indonesia is tuberculosis or Mycobacterium tuberculosis which is transmitted through sputum. Kudus Regency is one of the cities located in Indonesia, so Kudus is also included in several cities whose residents to this day are affected by the spread of tuberculosis.

Tuberculosis has even caused more deaths than any other infectious disease, including HIV [5]. This has become very scary, especially in tropical countries including Indonesia. If you look at regional internal factors, the lack of houses with good ventilation, poverty and lack of education about this disease are the main factors. In addition, there are other factors such as the number of immigrants who come. In fact, about 95% of the 98% of deaths caused by tuberculosis occur in immigrants. [24]

Several studies were conducted to find out how this virus developed [3], [4]. There are several studies on the prevention and treatment of patients suffering from tuberculosis [7], [13], [22]. Several mathematicians conducted research on the spread of infectious diseases such as Malaria, HIV, and dengue fever, such as those conducted by many scientist [2], [8], [11], [14], [16], [18].

While research with the SIR model for the spread of tuberculosis has been carried out by many scientists [1], [6], [9], [10], [12], [15], [17], [19], [20], [21], [23]. In this study, we are interested in studying the model of the spread of tuberculosis in Kudus Regency using the SIR model and making predictions about its distribution in the next few years.

2. MATERIAL AND METHODS

2. 1. Model Formulation

In this study we used the SIR model of the spread of epidemic diseases. This results in the modeling the total population will be divided into three different groups. The first group is those who have not been infected by the tuberculosis virus, hereinafter referred to as the suspect group. The second group is those who are being infected by the tuberculosis virus. The last group is those who recovered or died after being infected by the tuberculosis virus.

We assume that there is neither immigration nor pure births and deaths, so the number of people who have not been infected will not increase. The number in the second group or the infected group will be influenced by the number of contacts between the first group (people who have not been infected) and the second group (people who are currently infected) and the number of people who recover or die after being infected. The last group will only be affected by the number of people who recover or die after being infected by the tuberculosis virus.

Based on the explanation above, we present the above formulation into a diagram as follows.

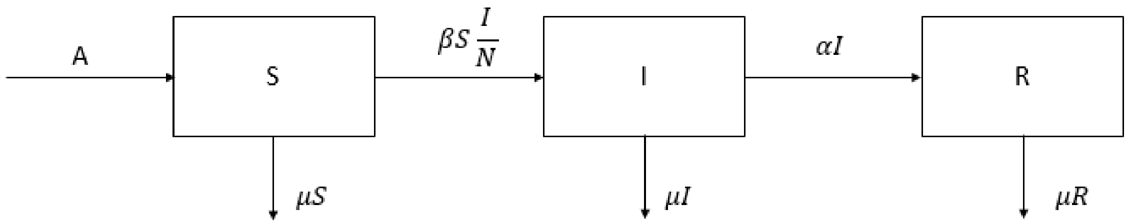

Figure 1. Illustration of SIR Model

From the illustration above, the following SIR model is obtained:

$$\frac{dS}{dt} = A - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - \alpha I - \mu I$$

$$\frac{dR}{dt} = \alpha I - \mu R$$

2. 2. Notations

S	: individuals who are susceptible to tuberculosis.
I	: the number of people infected with Tuberculosis.
R	: the number of individual who recovered from Tuberculosis.
A	: birth or migration rate
β	: transmission rate of tuberculosis
α	: cure rate for tuberculosis
δ	: tuberculosis death rate
λ	: eigenvalues.
R_0	: basic reproduction number.

2. 3. Algorithm

i. Linear differential equations.

The general form of nonlinear differential equations can be systematically written

$$\dot{x} = f(x) \tag{1}$$

ii. The stability of the equilibrium point.

The equilibrium point $\bar{x}$ is said:

a. Stable if for every $\varepsilon > 0$ there is $\delta > 0$ such that $|| x0 - \bar{x} || < \delta$ then $|| x\,(t, x0) - \bar{x} || < \varepsilon$

b. Asymptotically stable if the equilibrium point is stable and there is $\delta_1 > 0$ such that if $|| x0 - \bar{x} || < \delta_1$ then $\lim_{t \to \infty} ||\, \mathrm{x}0 - \bar{x} \,|| = 0$

c. It is unstable if the equilibrium point does not meet (a).

iii. Linearization.
Differential equation

$$\dot{x} = \boldsymbol{Ax}$$

and the jacobian matrix,

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_m} \end{bmatrix} \tag{2}$$

iv. Eigenvalues
The eigenvalues of matrix A are the characteristic roots of the polynomial $det(A - \lambda I) = 0$ or written in the form

$$\alpha_n \lambda^n + \alpha_{(n-1)} \lambda^{n-1} + \cdots + \alpha_1 \lambda + \alpha_0 = 0$$

Eigenvalues different from the characteristic equation are $\lambda_1, \dots, \lambda_j;\ j \leq n$. The equilibrium point $\bar{x}$ is said:

a. Stable, if $\lambda_i < 0$ and there is $\lambda_k = 0$ with $i = 1, \dots, j$ and $k \leq j$.

b. Asymptotically stable, if $\lambda_i < 0, \forall i = 1, \dots, j$.

c. Unstable, if, if $\exists \lambda_i > 0, \forall i = 1, \dots, j$

3. RESULT AND DISCUSSION

3. 1. Descriptive Data Analysis of Tuberculosis Epidemic Model

Tuberculosis data used are data on the population of Kudus Regency, data on the number of Tuberculosis sufferers and data on the number of patients who recovered from Tuberculosis that occurred in Kudus Regency in 2018 and 2019 as an initial analysis to determine the parameters of the Tuberculosis epidemic model. Below is given the data on the population of Kudus Regency.

Table 1. Total population of Kudus Regency in 2018 and 2019

Year	Number of inhabitants (people)		Total
	Male	Female	
2018	423985	437445	861430
2019	428815	442496	871311

Table 1 above shows the total population of Kudus Regency in 2018 as many as 861430 people, written N = 861430 with the assumption that the initial population susceptible to tuberculosis is 861430, written $S(0) = S(2018) = 861430$. Then the total population in 2018 was 861340 people with the assumption that the population after 1 year who was susceptible to tuberculosis was 871311, written $S(2019) = 871311$.
Data on the number of Tuberculosis patients in Kudus Regency.

Table 2. Number of Tuberculosis sufferers in Kudus Regency in 2018 and 2019

Year	Number of Infected (Soul)		Total
	Male	Female	
2018	1612	1318	2930
2019	1629	1504	3133

Table 2. above shows the total population of Kudus Regency infected with tuberculosis in 2018 as many as 2930 people, written I(t) = 2930 with the assumption that the number of patients infected with Tuberculosis is 2930, written I(0) = I(2018) = 2930 . Then the number infected with Tuberculosis in 2019 was 3133 people, it can be written I(2019) = 2930.

Furthermore, the data on the number of patients recovered from Tuberculosis in Kudus Regency.

Table 3. Number of Tuberculosis patients recovered from Kudus Regency in 2018 and 2019

Year	Total healed (Soul)		Total
	Male	Female	
2018	240	294	534
2019	292	297	589

Table 3, above shows the total population of Kudus Regency who recovered from tuberculosis in 2018 as many as 534 people, written R = 534 with the assumption that the number of patients recovered from Tuberculosis is 534, written R(0) = R (2018) = 534. While 589 people recovered from Tuberculosis in 2019, it can be written R (2019) = 589.

3. 2. Tuberculosis Epidemic Model Assumptions and Parameters

Based on data from the Central Java health office in 2018 and 2019, it can be assumed that the SIR epidemic model is as follows:

1. In the population there is a birth rate in an initial population of Kudus Regency.
2. An increase in the number of infected individuals and individuals susceptible to contracting tuberculosis.
3. The increase in the rate of individuals recovering from tuberculosis.
4. Individuals who recover cannot be susceptible to contracting Tuberculosis again.

Furthermore, the assumptions above are formed by the SIR epidemic model against tuberculosis, namely:

$$\begin{cases} \frac{dS}{dt} & = & A - \alpha SI \\ \frac{dI}{dt} & = & \alpha SI - (\beta + \delta)I \end{cases}$$

With a description of individuals who are susceptible to Tuberculosis (S) and individuals infected with Tuberculosis (I), birth rate or migration (A). While the parameters are for Tuberculosis transmission rate (α), cure rate (β), and death rate due to tuberculosis (δ).

Data on the initial population of Kudus Regency who are susceptible to tuberculosis S(0) = S(2018) = 861430 and S(2019) = 871311 then the number of residents who recovered from tuberculosis R(2019) = 589, so that it can be substituted into the number of individuals who are prone to tuberculosis (S),

$$S(t) \ = \ S(0)e^{\left(\frac{\alpha}{\beta+\delta}\right)R(t)} \tag{3}$$

with $S(t)$, namely the number of susceptible individuals, $S(0)$ the initial number of population and $R(t)$ individuals who recovered, obtained

$$\frac{\alpha}{\beta+\delta} = 19 \times 10^{-6}$$

with the transmission rate $\beta = 1{,}9 \times 10^{-6}$, assuming the recovery rate in 10 years is $\alpha = \delta = \frac{1}{10}$, then $\alpha = 10^{-1} = 0{,}1$. As a result, an epidemic model of SIR infected with tuberculosis was obtained,

$$\begin{cases} \frac{dS}{dt} & = & 861430 - 1{,}9 \ \times 10^{-6} S(t)I(t) \\ \frac{dI}{dt} & = & 1{,}9 \ \times 10^{-6} S(t)I(t) - 0{,}1I(t) \end{cases}$$

3. 3. The Equilibrium Point of the Tuberculosis Epidemic Model

From the SIR epidemic model system, the equilibrium point value can be determined. If S(t) and I(t) respectively state the number of individuals in each subpopulation at time t, then from the above system it is obtained:

$$\frac{dS}{dt} = 0 \; and \; \frac{dI}{dt} = 0$$

So that,

$$\begin{cases} \dfrac{dS}{dt} & = \quad 861430 - 1{,}9 \times 10^{-6} SI = 0 \\ \dfrac{dI}{dt} & = \quad 1{,}9 \times 10^{-6} SI - 0{,}1 I = 0 \end{cases}$$

and if I ≠ 0, with equation 2, then $1{,}9 \times 10^{-6} S - 0{,}1 = 0$ is obtained S = 52632. Then substituted for equation 1 then $861430 - 1{,}9 \times 10^{-6} S(t)I(t) = 0$ obtained $I = 8614230$, so the equilibrium point value $(S, I) = (52632, 8614230)$.

3. 4. Analysis of the Stability of the Tuberculosis Epidemic Model Equilibrium Point

It is known that the equilibrium point of the epidemic model on the spread of tuberculosis is $(S, I) = (52632, 8614230)$. Then determine the stability analysis of theequilibrium point with the example

$$\frac{dS}{dt} = f_1\,(S, I) \;\text{ and }\; \frac{dI}{dt} = f_2\,(S, I)$$

So that,

$$\begin{cases} f_1\,(S, I) & = \quad 861430 - 1{,}9 \times 10^{-6} SI = 0 \\ f_2\,(S, I) & = \quad 1{,}9 \times 10^{-5} SI - 0{,}1 I = 0 \end{cases}$$

Obtained linearization of the equation using the jacobian matrix form

$$\mathrm{J}f\;(\mathrm{S,I}) = \begin{bmatrix} \frac{df_1(\mathrm{S,I})}{dS} & \frac{df_1(\mathrm{S,I})}{dI} \\ \frac{df_2(\mathrm{S,I})}{dS} & \frac{df_2(\mathrm{S,I})}{dI} \end{bmatrix} = \begin{bmatrix} -1{,}9 \times 10^{-6} I & -1{,}9 \times 10^{-6} S \\ 1{,}9 \times 10^{-6} I & 1{,}9 \times 10^{-6} S - 0{,}1 \end{bmatrix}$$

Then we get the equilibrium point of the jacobian matrix with $(S, I) = (52632, 8614230)$, i.e.

$$jf(2632{,}8612922) = \begin{bmatrix} -16{,}367 & -0{,}1 \\ 16{,}367 & 0 \end{bmatrix}$$

based on the jacobian matrix, the characteristic equation is obtained:

$$\lambda^2 + 16{,}367\,\lambda + 16{,}367 = 0$$

So that the eigenvalue is obtained, namely $\lambda_1 = -15.29$ and $\lambda_2 = -1.06994$ with $\lambda_1 < 0$ and $\lambda_2 < 0$ then we get the equilibrium point $(S, I) = (52632, 8614230)$ is stable. This shows that for the number of individuals who are susceptible and the number of individuals who are infected is very small, the increasing population of tuberculosis remains.

3. 5. Basic Reproduction Numbers

The equilibrium point system of equations i.e. $(S, I) = (\frac{\beta+\delta}{\alpha}, \frac{A}{\beta+\delta})$, From the equilibrium point equation, βA is obtained where the basic reproduction number is influenced by the birth rate and the transmission rate of tuberculosis.

$$R_0 = \alpha A = (1.9 \times 10^{-6})861430 = 1.6367$$

Obtained an average of 2 individuals who were infected and infecting those susceptible to tuberculosis.

3. 6. Numerical Solution

Based on the SIR model used in the previous section, we performed a simulation to predict the rate of spread of the infectious disease tuberculosis over the next 15 years as shown in Figure 2.

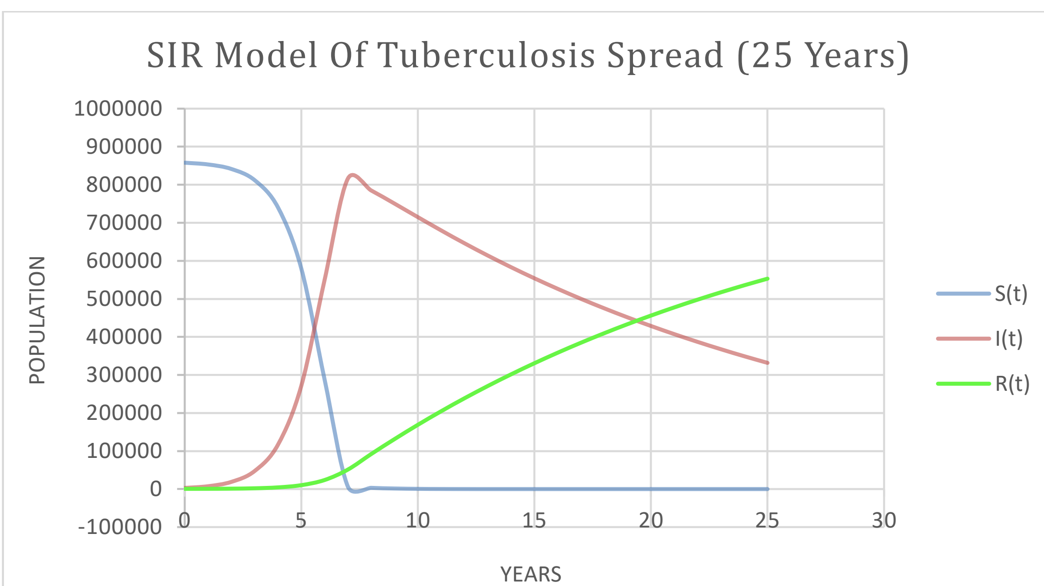

Figure 2. The spread of the tuberculosis virus in Kudus Regency in the next 25 years using the SIR model

Based on figure 2, we can see that after 25 years it can be seen that the infected population will increase even more than the susceptible population, so the disease will become endemic.

Next we do the same to get an estimate of the spread of tuberculosis for the next 50 years, as can be seen in Figure 3.

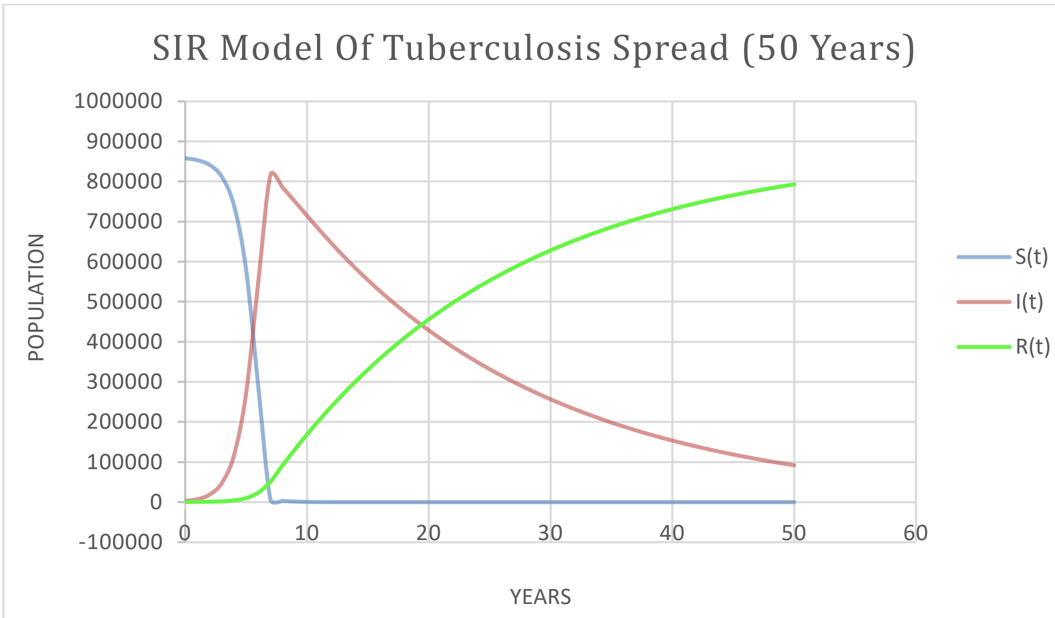

Figure 3. The spread of the tuberculosis virus in Kudus Regency in the next 50 years using the SIR model

Based on Figure 3, we can see that the number of people who are susceptible to tuberculosis will become smaller and closer to zero. Another interesting thing is the decrease in the number of people infected and the increasing number of people who have recovered. This shows that there is still hope to make tuberculosis no longer an endemic disease and even disappear in Kudus Regency.

4. CONCLUSIONS

1. Based on descriptive data analysis of the population of Kudus Regency and the number of individuals infected with Tuberculosis in 2018 and 2019, it is found that the assumptions in this study are,
   a. In the population there is a birth rate in an initial population of Kudus Regency.
   b. An increase in the number of infected individuals and individuals susceptible to contracting tuberculosis.
   c. The increase in the rate of individuals recovering from tuberculosis.
   d. Individuals who recover cannot be susceptible to contracting Tuberculosis again.

2. Based on this research, the basic reproduction number is obtained $R_0 = 1.6367$, which indicates that one infected individual can infect 28 people on average.

References

[1] Adamu, H. A., Muhammad, M., M.Jingi, A. & Usman, M. A., 2019. Mathematical Modelling Using Improved SIR Model With More Realistic Assumptions. *International Journal of Engineering and Applied Sciences,* 6(1), 64-69

[2] Affandi, P., 2020. Optimal Control For Dysentery Epidemic Model With Treatment. *International Journal of Scientific & Technology Research,* 9(3), Pp. 6709-6713

[3] Agyeman, A. A. & Ofori-Asenso, R., 2017. Tuberculosis - An Overview. *Journal of Public Health and Emergency,* 1(7), Pp. 1-11

[4] Ansumana, R. Et Al., 2017. Impact Of Infectious Disease Epidemics On Tuberculosis Diagnostic, Management, And Prevention Services: Experiences And Lessons From The 2014–2015 Ebola Virus Disease Outbreak In West Africa. *International Journal of Infectious Diseases,* Pp. 101-104

[5] A, S. Et Al., 2019. Tuberculosis: A Review of Current Trends. *Epidemiology International Journal,* 3(2), Pp. 1-13

[6] Castillo-Chavez, C. & Song, B., 2004. Dynamical Models of Tuberculosis and Their. *Mathematical Biosciences and Engineering,* 1(2), Pp. 361-404

[7] Chakaya, J. Et Al., 2021. Global Tuberculosis Report 2020 – Reflections On The Global TB Burden. *International Journal of Infectious Diseases,* Pp. 1-6

[8] Fudolig, M. & Howard, R., 2020. The Local Stability Of A Modified Multi-Strain SIR Model For Emerging Viral Strains. *PLOS ONE*, Pp. 1-27

[9] Iskandar, T. Et Al., 2017. Mathematical Model Of Tuberculosis Epidemic With Recovery Time Delay. *AIP Conference Proceedings.* 1913, 020021-1–020021-5; https://doi.org/10.1063/1.5016655

[10] Khan, M. A. Et Al., 2019. Modeling The Transmission Dynamics Of Tuberculosis In Khyber Pakhtunkhwa Pakistan. *Advances in Mechanical Engineering*, 11(6), Pp. 1-13

[11] Maheswari, U. K. Et Al., 2018. A Study On Epidemic Model. *International Journal for Scientific Research & Development*, 5(12), Pp. 97-99

[12] Mettle, F. O., Affi, P. O. & Twumasi, C., 2020. Modelling The Transmission Dynamics Of Tuberculosis In The Ashanti Region Of Ghana. *Interdisciplinary Perspectives on Infectious Diseases,* Pp. 1-16

[13] Miggiano, R., Rizzi, M. & Ferraris, D. M., 2021. Mycobacterium Tuberculosis Pathogenesis, Infectionprevention And Treatment. *Pathogens,* 9(385), Pp. 1-4

[14] Naresh, R., Sharma, D. & Tripathi, A., 2009. Modelling The Effect Of Tuberculosis On The Spread Of HIV Infection In A Population With Density-Dependent Birth And Death Rate. *Mathematical and Computer Modelling,* 50(7-8), Pp. 1154-1166

[15] Nayeem, J. & Sultana , I., 2019. Mathematical Analysis Of The Transmission Dynamics Of Tuberculosis. *American Journal of Computational Mathematics,* Volume 9, Pp. 158-173

[16] Nicho, J., 2010 . The SIR Epidemiology Model In Predicting Herd Immunity. *Undergraduate Journal of Mathematical Modeling: One + Two,* Volume 2.

[17] Nugraha, T. & Larasati, R., 2019. A Recent Update Of The Diagnostic Methods For Tuberculosis And Their Applicability In Indonesia: A Narrative Review. *Medical Journal of Indonesia,* 28(3), Pp. 284-291

[18] S. A. Egbetade, I.A. Salawu & P.A. Fasanmade , 2018. Local Stability Of Equilibrium Points Of A SIR Mathematical Model Of Infectious Diseases. *World Journal of Research and Review,* 6(3), Pp. 79-81

[19] Side, S., 2015. A Susceptible-Infected-Recovered Model And Simulation For Transmission Of Tuberculosis. *Advanced Science Letters,* Volume 21, Pp. 137-139.

[20] Side, S. & Noorani, M. S. M., 2010. A SEIR Model For Transmission Of Tuberculosis. *International Journal of Modern Physics: Conference Series,* 1(1), Pp. 1-10.

[21] Side, S., Sanusi, W., Aidid, M. K. & Sidjara, S., 2016. Global Stability Of SIR And SEIR Model For Tuberculosis Disease Transmission With Lyapunov Function Method. *Asian Journal of Applied Sciences,* 9(3), Pp. 87-96.

[22] Surya, A. Et Al., 2017. Quality Tuberculosis Care In Indonesia: Using. *The Journal of Infectious Diseases*, Pp. S725-S732.

[23] Ucakan, Y., Gulen, S. & Koklu, K., 2021. Analysing Of Tuberculosis In Turkey Through SIR, SEIR And BSEIR Mathematical Models. *Mathematical and Computer Modelling of Dynamical Systems,* 27(1), Pp. 179-202.

[24] Zammarchi, L., Bartalessi, F. & Bartoloni, A., 2014. Tuberculosis In Tropical Areas And Immigrants. *Mediterranean Journal of Hematology and Infectious Diseases*, 6(1).