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Applications of Some Well-Known Skewed Distributions to Greenhouse Gas Emissions Data

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ABSTRACT

In many problems of transportation and environmental processes and designs, fitting of a continuous probability distribution to the greenhouse gas emissions data from cars may be helpful in predicting the probability or forecasting the frequency of occurrence of the greenhouse gas emissions from burning fossil fuel for our cars, trucks, ships, trains, and planes, and planning beforehand. The objective of this paper is to study and conduct a statistical analysis of the greenhouse gas emissions data from cars. Since our data are skewed in nature, we fit the following well known skewed distributions: 3 parameter Birnbaum-Saunders (or fatigue-life), gamma, 3 parameter gamma, generalized extreme value, 3 parameter lognormal, 4 parameter Pearson 6 and Weibull distributions. We have tested the goodness of fit these distributions to a random sample of the greenhouse gas emissions data from 32 different models of cars to determine their applicability and best fit to these data based on the Kolmogorov-Smirnov, Anderson-Darling, and Chi-Squared Goodness-of-Fit Tests.

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1. INTRODUCTION

In research of many fields of pure and applied sciences such as biology, computer science, control theory, economics, engineering, genetics, hydrology, medicine, number theory, statistics, physics, psychology, reliability, risk management, etc., modeling and analyzing of real lifetime continuous type of data play very important roles. These have been extensively studied by many researchers; see, for example, Räisänen [1], Cai et al. [2], and Bojaca et al. [3], among others. In recent years, there has been a great interest in the studies of applications of probability distributions for modeling many problems of pollution and environmental processes and designs, and other applied research by many authors and researchers. For example, Spafford and MacDougall [4] studied the "quantifying of the probability distribution function of the transient climate response to cumulative CO_2 emissions". For "uncertainty analysis of Greenhouse Gas (GHG) emissions simulated by the parametric Monte Carlo simulation and nonparametric bootstrap method", see Lee et al. [5]. The interested readers are also referred to Molina-Castro and Calderon-Jimenez [6] for their investigation on "the applicability of different approaches of asymmetric distributions to address standard uncertainty estimation of emission factors in Greenhouse Gas (GHG) inventories according to the guidelines to the expression of uncertainty in measurement (GUM), specifically for the Costa Rican official database of these factors in the fuel sector".

According to Bluman [7], "when one is describing data, it is important to be able to recognize the shapes of the distribution (of data) values. The shape of a distribution also determines the appropriate statistical methods used to analyze the data. A distribution can have many shapes, and one method of analyzing a distribution is to draw a frequency histogram or frequency polygon for the distribution. The graphs of distributions can have many shapes. When the data values are evenly distributed about the mean, a distribution is said to be a symmetric distribution. For example, a normal distribution is symmetric. When the majority of the data values fall to the left of the mean, the distribution is said to be a negatively skewed distribution and when to the right of the mean, the distribution is said to be a positively skewed distribution. For example, distributions such as exponential, gamma, etc., are skewed distributions".

A random variable which can assume continuous type of data values is called a continuous random variable. According to Illowsky and Dean [8], "continuous random variables have many applications. Height, age, temperature, weight, SAT scores, IQ scores, and the length of time a computer chip lasts, are just a few. The field of reliability depends on a variety of continuous random variables. The graph of a continuous probability distribution is a curve. Probability is represented by area under the curve. The curve is called the probability density function (abbreviated as pdf). We use the symbol $f(x)$ to represent the curve. $f(x)$ is the function that corresponds to the graph; we use the density function $f(x)$ to draw the graph of the probability distribution. Area under the curve is given by a different function called the cumulative distribution function (abbreviated as cdf). The cumulative distribution function is used to evaluate probability as area".

The formal definition of a continuous random variable and corresponding distribution and probability density functions are given below; for details, see, for example, Dudewicz and Mishra [9], Stuart and Ord [10], Kapadia et al. [11], Hogg and Tanis [12], Ahsanullah et al. [13], Ramachandran and Tsokos [14], and Rohatgi and Saleh [15], among others.

Definition 1.1 (Random Variable): Let (Ω, T, P) be a probability space, where $\Omega = \{w\}$ is a set of simple events, T is a σ-algebra of events, and P is a probability measure defined on (Ω, T). Let B be an element of the Borel σ-algebra of subsets of the real line R. A random variable $X = X(w)$ is defined as a finite single-valued function $X : \Omega \rightarrow R$ such that $X^{-1}(B) = \{w : X(w) \in B\} \in T, \forall \textit{ Borel set } B \in R$. Thus, a random variable X is a real-valued function with domain Ω, that is, $X(w) \in R = \{y : -\infty < y < +\infty\}, \forall\, w \in \Omega$.

Definition 1.2 (Cumulative Distribution Function): Let $B = (-\infty, x]$ in the above definition 1.1. Then the cumulative distribution function (cdf) or distribution function (df) of the random variable $X = X(w)$ is defined by $F_X(x) = P[X \le x], \ \forall\, x \in (-\infty, +\infty)$, with the following properties:

(i) $F_X(x)$ is a non-decreasing function of x.

(ii) $F_X(-\infty) = 0$, $F_X(+\infty) = 1$.

(iii) $F_X(x)$ is right continuous.

Definition 1.3 (Absolutely Continuous Distribution Function): The distribution function $F_X(x)$ of a random variable X is said to be absolutely continuous if $\exists$ a function $p_X(x) \ge 0$ such that

$$F_X(x) = \int_{-\infty}^{x} p_X(t)\,dt.$$

Definition 1.4 (Probability Density Function): The function $p_X(x)$ in the above definition 1.3 is called the probability density function (pdf) or density function or frequency function of the random variable X if it satisfies the following condition:

$$\int_{-\infty}^{\infty} p_X(x)\,dx = 1.$$

The transportation sector generates the largest share of greenhouse gas emissions. Greenhouse gas emissions from transportation primarily come from burning fossil fuel for our cars, trucks, ships, trains, and planes. A greenhouse gas is a gas that both absorbs and emits thermal radiation or heat, and when present in the atmosphere, causes a warming process called the greenhouse effect. It includes carbon dioxide, carbon monoxide, methane, nitrous oxide, ozone, etc. Many scientists believe that this is causing global warming.

According to Agency for Toxic Substances and Disease Registry (ASTDR), the greenhouse gas is one of the major types of air pollution; see www.atsdr.cdc.gov. The concentration of 'greenhouse' gases in the earth's atmosphere, resulting in a gradual increase in temperatures at the earth's surface, is an important area of research. Emissions of greenhouse gases worldwide resulting from human activities are expected to contribute to future climate changes. Furthermore, according to the Inventory of U.S. Greenhouse Gas Emissions and Sinks 1990–2017, transportation accounted for the largest portion (29%) of total U.S. GHG emissions in 2017. Over 90 percent of the fuel used for transportation is petroleum based, which includes primarily gasoline and diesel. As the greenhouse and climate change are fundamental issues of environmental sustainability and building healthy communities, the statistical analysis of the greenhouse gas emissions data from cars is therefore very crucial, and can play an important role in many studies of transportation and environmental processes and designs.

In many problems of transportation and environmental processes and designs, fitting of a probability distribution to the greenhouse gas emissions data from cars may be helpful in predicting the probability or forecasting the frequency of occurrence of the greenhouse gas emissions from burning fossil fuel for our cars, trucks, ships, trains, and planes, and planning beforehand.

In view of the above-mentioned facts, in this paper, we have studied and conducted a statistical analysis of greenhouse gas emissions data from cars by testing the goodness of fit of some well-known skewed distributions, namely, 3 parameter Birnbaum-Saunders (or fatigue-life), gamma, 3 parameter gamma, generalized extreme value, 3 parameter lognormal, 4 parameter Pearson 6 and Weibull distributions to these data. To illustrate the performance of these distributions, we have considered the greenhouse gas emissions data from 32 different models of cars as reported in Triola [16], and determine their applicability and best fit to these data based on the Kolmogorov-Smirnov, Anderson-Darling, and Chi-Squared goodness-of-fit (GOF) tests. For details on GOF tests, see, for example, Massey [17] and Stephens [18], among others.

The organization of this paper is as follows: In Section 2, we have discussed the methodology. In Section 3, we have discussed the data analysis. The results and discussions are provided in Section 4. The concluding remarks are given in Section 5.

2. METHODOLOGY

2. 1. Descriptions of Greenhouse Data

We have considered the greenhouse gas emissions data from 32 different models of cars as reported in Triola [16], which are provided in Table 1 below:

Table 1. The greenhouse gas emissions (GHG) data from 32 different models of cars, in tons per year. (Source: Triola (2010), Data Sets 16: Cars, p. 784).

Car	GHG	Car	GHG
Acura RL	8.7	Jaguar XJ8	8.6
Acura TSX	7.2	Kia Amanti	9.3

Audi A6	7.7	Kia Spectra	6.5
BMW 525i	7.7	Lexus GS300	7.4
Buick LaCrosse	7.9	Lexus LS	8.7
Cadillac STS	8.7	Lincoln Town Car	9.3
Chevrolet Impala	8.2	Mazda 3	6.5
Chevrolet Malibu	6.8	Mercedes-Benz E	7
Chrysler 300	9.3	Mercury Grand Marquis	9.3
Dodge Charger	9.3	Nissan Altima	7.1
Dodge Stratus	7.4	Pontiac G6	7.2
Ford Crown Victoria	9.3	Saturn Ion	6.7
Ford Focus	6.5	Toyota Avalon	7.2
Honda Accord	6.6	Toyota Corolla	5.5
Hyundai Elantra	6.7	Volkswagon Passat	7.3
Infiniti M35	9	Volvo S80	8.2

2. 2. Histogram and Probability Plot

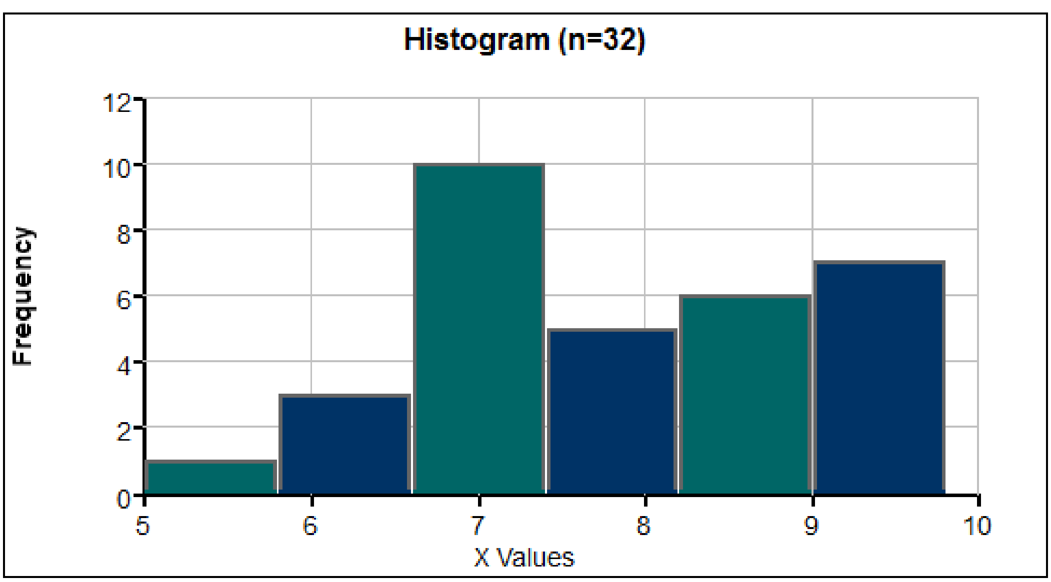

Figure 1. Histogram

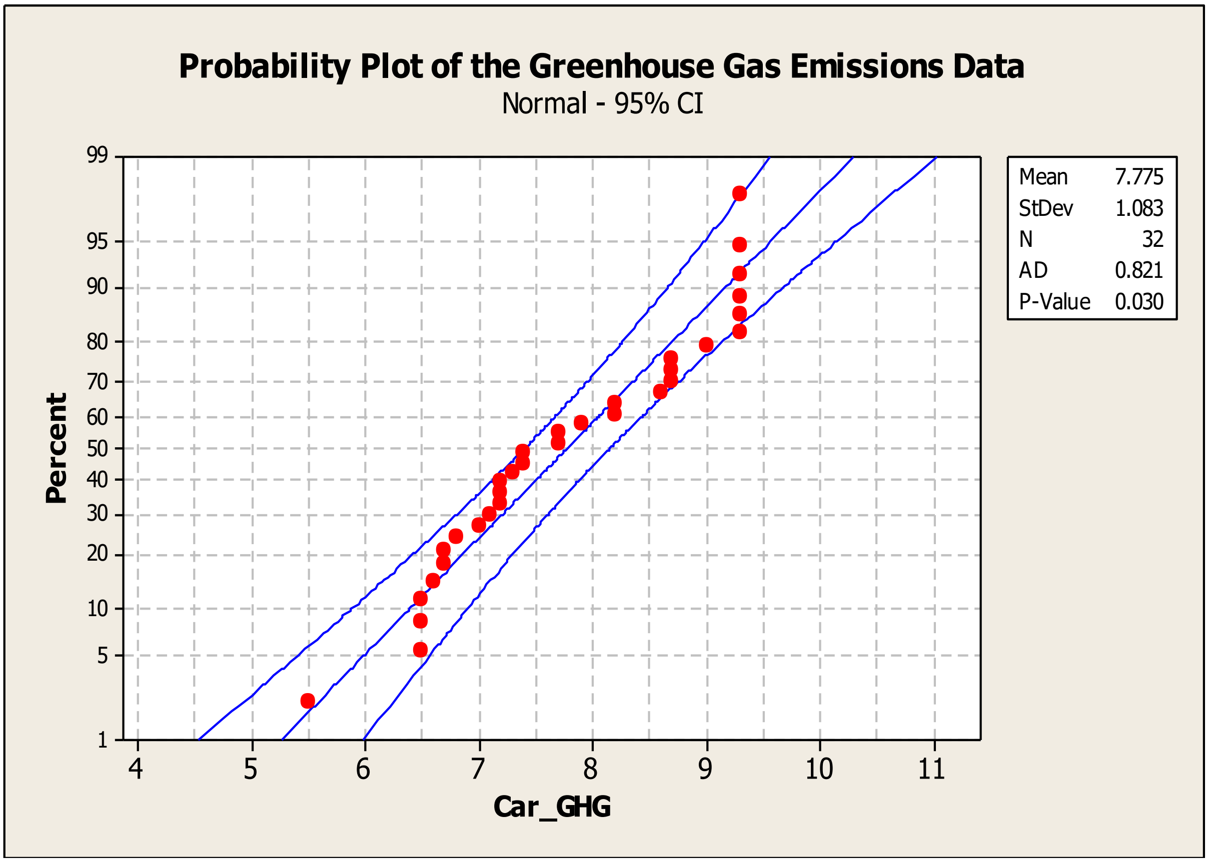

Figure 2. Probability Plot

For the above-mentioned greenhouse gas emissions data, the histogram and the probability plot of the greenhouse gas emissions data are drawn Figures 1 and 2 respectively.

2. 3. Descriptive Statistics

The descriptive statistics of the greenhouse gas emissions data from 32 different models of cars are computed in Table 2.

Table 2. Descriptive Statistics.

Statistic	Value	Percentile	Value
Sample Size	32	Min	5.5
Range	3.8	5%	6.15
Mean	7.775	10%	6.5
Variance	1.1723	25% (Q1)	6.85

Std. Deviation	1.0827	50% (Median)	7.55
Coef. Of Variation	0.13926	75% (Q3)	8.7
Std. Error	0.1914	90%	9.3
Skewness	0.04408	95%	9.3
Kurtosis	2.239846	Max	9.3
Excess Kurtosis	- 1.0739	Mode	9.3

2. 4. Normality Test

We have tested the normality of the greenhouse gas emissions data by Ryan-Joiner Test (Similar to Shapiro-Wilk Test), which is given in Table 3 below.

Table 3. Ryan-Joiner Test of Normality Assessment.

Ryan-Joiner Test
Test statistic, Rp: 0.9677
Critical value for 0.05 significance level: 0.9657
Critical value for 0.01 significance level: 0.9516
Fail to reject normality with a 0.05 significance level.
Fail to reject normality with a 0.01 significance level.
Possible Outliers
Number of data values below Q1 by more than 1.5 IQR: 0
Number of data values above Q3 by more than 1.5 IQR: 0

2. 5. Descriptions of Skewed Distributions Used

Since our data are skewed in nature, we fit the following well-known skewed distributions: three parameter Birnbaum-Saunders (or fatigue-life), three parameter gamma, generalized extreme value, three parameter lognormal, four parameter Pearson 6, log-normal, and Weibull distributions. For details on these distributions, the interested readers are referred to Patel et al. [19], Johnson et al. [20], Balakrishnan and Nevzorov [21], Ahsanullah and Kirmani [22], Forbes et al. [23], and Ahsanullah [24], among others.

The probability density functions and parameters of the above-said distributions are provided in Table 4 below

Table 4. Distributions Used in Greenhouse Gas Emissions Data Analysis.

Sl. No.	Distributions	$f(x)$	Parameters
1	3 Parameter Birnbaum-Saunders (or Fatigue-Life)	Birnbaum-Saunders (3P) (or Fatigue-Life (3P)) $f(x)=\frac{1}{2\,\alpha(x-\gamma)}\left[\sqrt{\frac{x-\gamma}{\beta}}+\sqrt{\frac{\beta}{x-\gamma}}\right]$ $\times\phi\left[\frac{1}{\alpha}\left\{\sqrt{\frac{x-\gamma}{\beta}}-\sqrt{\frac{\beta}{x-\gamma}}\right\}\right],$ where $\phi(\cdot)$ denotes the standard normal pdf.	$k(>0)$: shape parameter $\alpha(>0)$: shape parameter $\beta(>0)$: scale parameter γ (real): location parameter, where $\gamma \le x < \infty$
2	Gamma	$f(x)=\frac{x^{\alpha-1}}{\beta^{\alpha}\,\Gamma(\alpha)}\exp\left(-\frac{x}{\beta}\right)$	$\alpha(>0)$: shape parameter $\beta(>0)$: scale parameter Domain: $0 \le x < \infty$
3	Gamma (3P)	$f(x)=\frac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\,\Gamma(\alpha)}$ $\times\exp\left(-\frac{(x-\gamma)}{\beta}\right)$	$\alpha(>0)$: shape parameter $\beta(>0)$: scale parameter γ (real): location parameter, Domain: $\gamma \le x < \infty$
4	Generalized Extreme Value	$f(x)=\begin{cases}\frac{1}{\sigma}\exp\left(-(1+k\,z)^{\frac{-1}{k}}\right)\times(1+k\,z)^{-1-\frac{1}{k}},\ k\neq 0\\ \frac{1}{\sigma}\exp(-z-\exp(-z)),\ k=0\end{cases}$ where $z=\frac{(x-\mu)}{\sigma}$	k (real): Shape Parameter $\sigma(>0)$: Scale Parameter μ (real): Location Parameter Domain: $1+k\frac{(x-\mu)}{\sigma}>0$, for $k\neq 0$ $-\infty < x < +\infty$, for $k=0$
5	Lognormal (3P)	$f(x)=\frac{1}{(x-\gamma)\sigma\sqrt{2\pi}}$ $\times\exp\left(-\frac{1}{2}\left(\frac{\ln(x-\gamma)-\mu}{\sigma}\right)^{2}\right)$	$\sigma(>0)$: scale parameter μ (real): location parameter γ (real): location parameter, Domain: $\gamma < x < +\infty$

6	Pearson 6 (4 P)	$f(x)=\dfrac{\left(\dfrac{x-\gamma}{\beta}\right)^{\alpha_1-1}}{\beta B(\alpha_1,\alpha_2)\left(1+\dfrac{(x-\gamma)}{\beta}\right)^{\alpha_1+\alpha_2}}$, where $B(\alpha_1,\alpha_2)$ denotes the Beta Function	$\alpha_1(>0)$: shape parameter $\alpha_2(>0)$: shape parameter $\beta(>0)$: scale parameter γ (real): location parameter, Domain: $\gamma \le x < +\infty$
7	Weibull	$f(x)=\dfrac{\alpha}{\beta}\left(\dfrac{x}{\beta}\right)^{\alpha-1}\exp\left(-\left(\dfrac{x}{\beta}\right)^{\alpha}\right)$	$\alpha(>0)$: shape parameter $\beta(>0)$: scale parameter, and $0 \le x < +\infty$

3. DATA ANALYSIS

From Table 3 of Ryan-Joiner Test of Normality Assessment and Figure 1 (for the histogram and Probability Plot respectively), it is obvious that the shape of the greenhouse gas emissions data is skewed to the right. This is also confirmed from the skewness (0.04408) and kurtosis (2.239846) of the greenhouse gas emissions data as computed in Table 2.

Since fitting of a probability distribution to of the greenhouse gas emissions data may be helpful in predicting the probability or forecasting the frequency of occurrence of the greenhouse gas emissions, this suggests that **y**, the greenhouse gas emissions, could possibly be modeled by some skewed distributions. Since our data are skewed in nature, we fit the following well-known skewed distributions, we have tested the fitting of the 3 parameter Birnbaum-Saunders (or fatigue-life), gamma, 3 parameter gamma, generalized extreme value, 3 parameter lognormal, 4 parameter Pearson 6 and Weibull distributions based on their goodness of fit to the greenhouse gas emissions data (Table 1). For this, we have used the following goodness of fit (GOF) tests: Kolmogorov-Smirnov, Anderson-Darling, and Chi-Squared tests.

The goodness-of-fit test results are provided in Tables 5, 6 and 7, respectively. For the parameters estimated in Table 5, the probability density functions (PDF's) of the 3 parameter Birnbaum-Saunders (or fatigue-life), gamma, 3 parameter gamma, generalized extreme value, 3 parameter lognormal, 4 parameter Pearson 6 and Weibull distributions respectively have been superimposed on the histogram of the greenhouse gas emissions data, which is provided in Figure 3 below. For these distributions, we have also plotted the cumulative distribution function (CDF's), survival functions and hazard functions in Figures 4 – 6 respectively.

Table 5. Fitting Results (Estimation of the Parameters).

#	**Distributions**	**Parameters**
1	Weibull	α =7.8766, β =8.1947

2	Pearson 6 (4P)	α_1 =2567.2, α_2 =1463.1, β =18.529, γ =-24.758
3	Birnbaum-Saunders or Fatigue Life (3P)	α =0.04601, β =23.156, γ =-15.406
4	Lognormal (3P)	σ =0.0528, μ =3.0059, γ =-12.459
5	Gamma	α =51.568, β =0.15077
6	Gamma (3P)	α =62.03, β =0.13602, γ =- 0.66033
7	Gen. Extreme Value	k =- 0.23723, σ =1.0742, μ =7.3635

Table 6. Comparison Criteria and Ranking of Fitted Distributions (Based on the Chi-Square Test for Goodness-of-Fit at the Level of Significance = 0.05) (P-Value and Test Statistic Analysis)

	Model						
	Weibull (Rank 1)	Pearson 6 (4P) (Rank 2)	Birnbaum-Saunders (3P) or Fatigue Life (3P) (Rank 3)	Log-normal (3P) (Rank 4)	Gamma (Rank 5)	Gamma (3P) (Rank 6)	Gen. Extreme Value Rank 7)
Test Statistic	1.8884	2.2879	2.3025	2.3077	2.3506	2.3593	3.9375
Critical Value	7.8147	7.8147	7.8147	7.8147	7.8147	7.8147	7.8147
P-Value	0.59589	0.51485	0.51205	0.51105	0.5029	0.50125	0.41453

Table 7. Comparison Criteria and Ranking of Fitted Distributions (Based on the Kolmogorov-Smirnov and Anderson-Darling Tests of Goodness-of-Fit at the Level of Significance = 0.05).

#	Distributions	Kolmogorov Smirnov		Anderson Darling	
		Test Statistic	Rank	Test Statistic	Rank
1	Gen. Extreme Value	0.11958	1	0.68464	1
2	Gamma	0.12683	2	0.7496	2
3	Gamma(3P)	0.12805	3	0.77877	3
4	Lognormal(3P)	0.12828	4	0.80658	4
5	Fatigue Life (3P)	0.12986	5	0.81639	5
6	Pearson 6 (4P)	0.13164	6	0.82652	6
7	Weibull	0.13904	7	0.95198	7

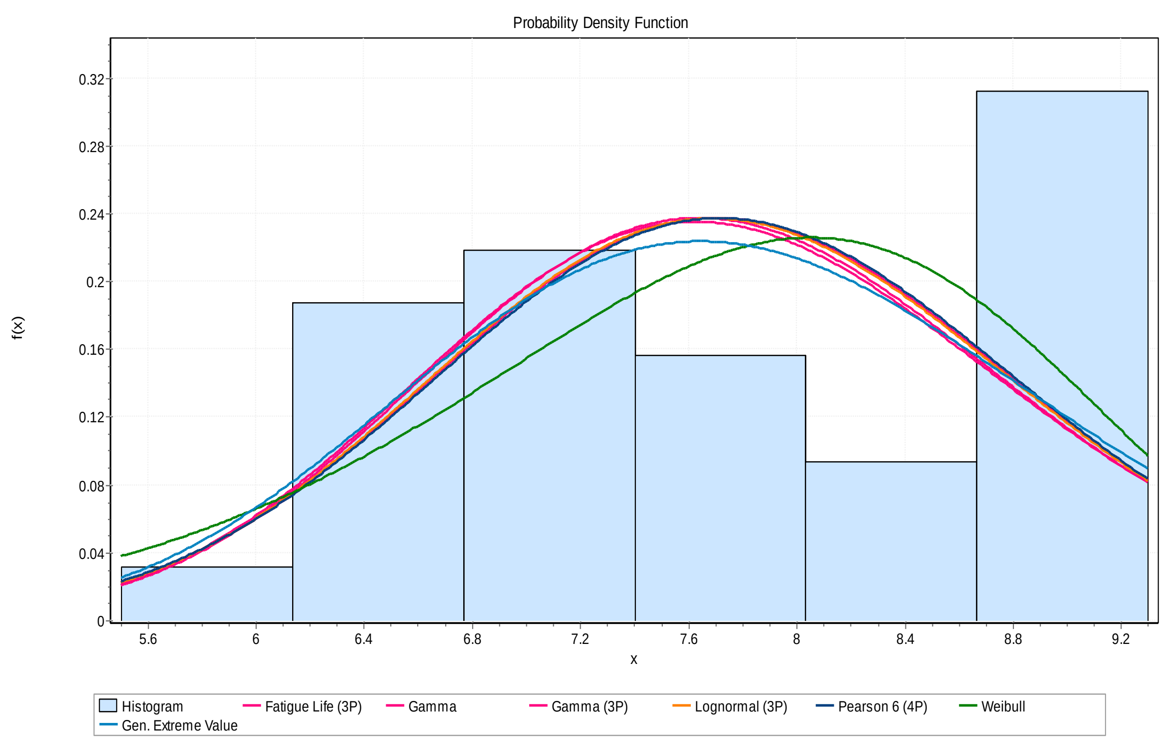

Figure 3. Fitting of PDF's to the Greenhouse Gas Emissions Data.

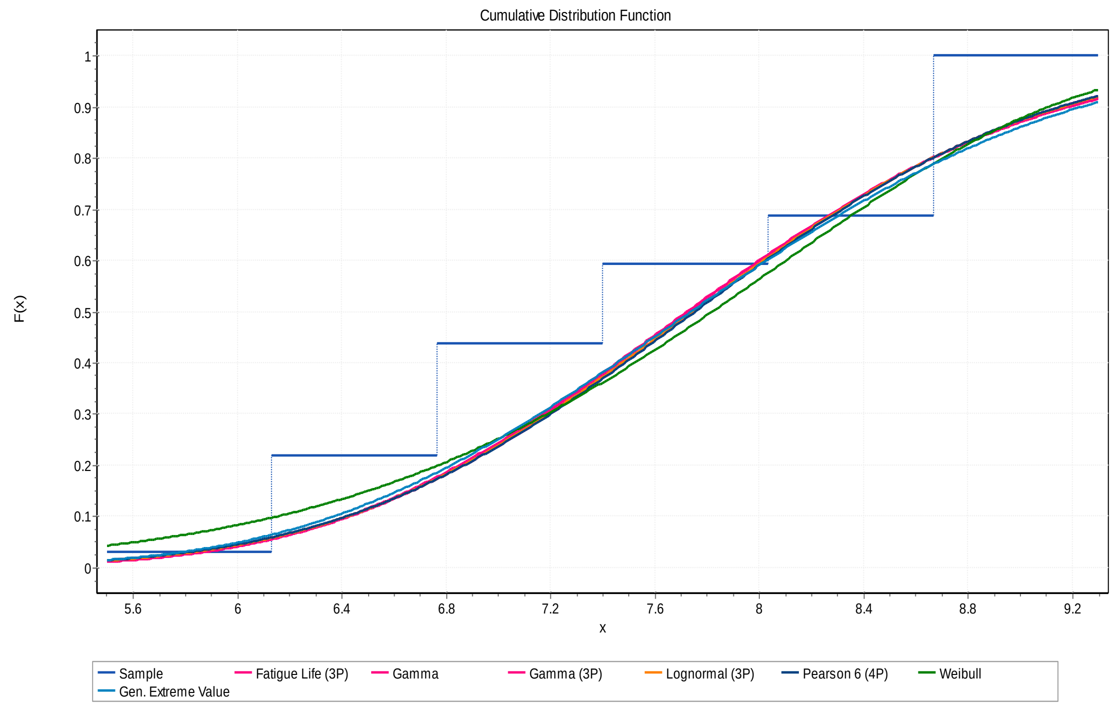

Figure 4. Fitting of CDF's to the Greenhouse Gas Emissions Data

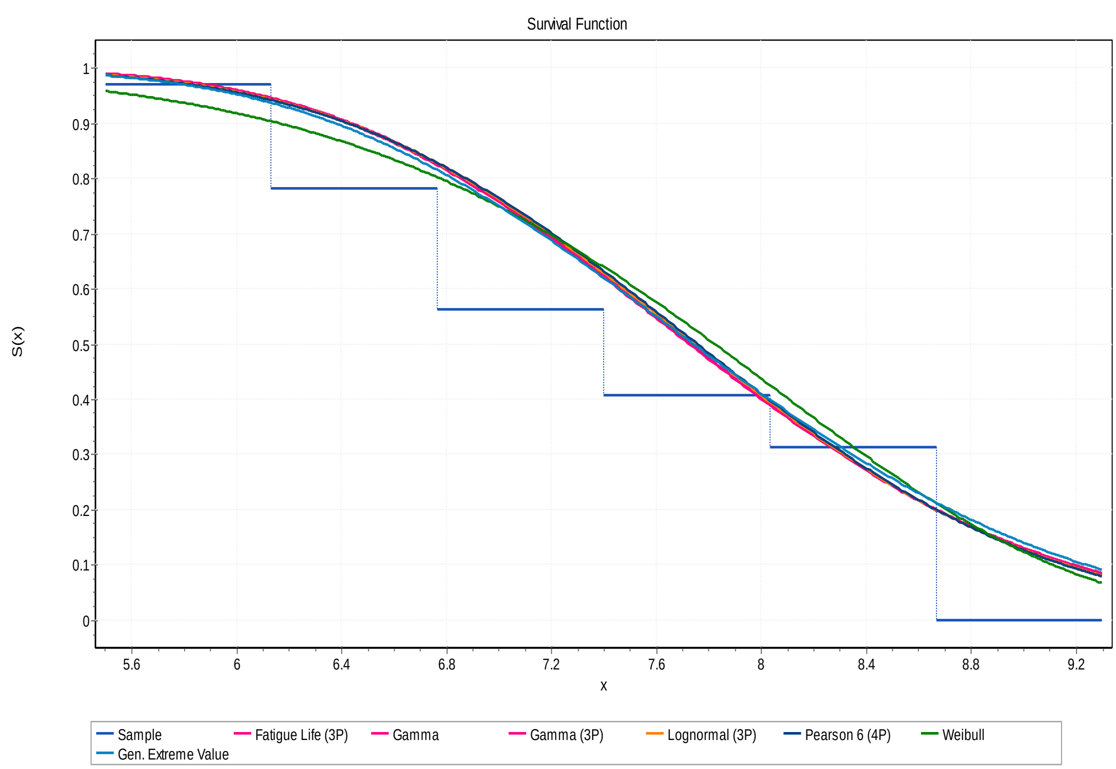

Figure 5. Survival Functions surve for the Greenhouse Gas Emissions Data.

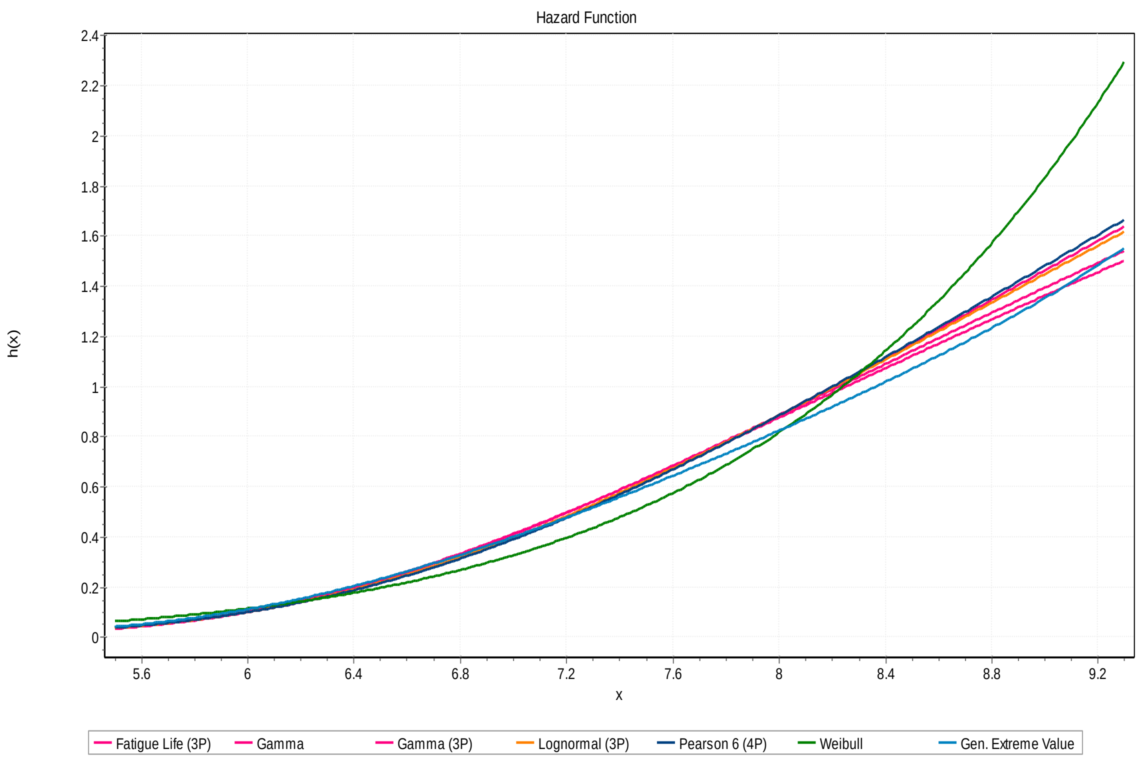

Figure 6. Hazard Functions of Distributions for the Greenhouse Gas Emissions Data

4. RESULTS AND DISCUSSIONS

From Table 4 of Ryan-Joiner Test of Normality Assessment and Figures 1 and 2 (for the histogram and the probability plot respectively), it is obvious that the shape of the greenhouse gas emissions data is skewed to the right. This is also confirmed from the skewness (0.04408) and kurtosis (2.239846) of the greenhouse gas emissions data as computed in Table 2. Based on the Chi-Squared test for goodness-of-fit, using the P-values and test statistics analysis, as provided in Table 6, Weibull distribution was found to be the best fit (Rank 1) for the greenhouse gas emissions data, followed by Pearson 6 (4P) (Rank 2), Birnbaum-Saunders (3P) or Fatigue Life (3P) (Rank 3), Lognormal (3P) (Rank 4), Gamma (Rank 5), Gamma (3P) (Rank 6) and Gen. Extreme Value (Rank 7). From the Kolmogorov-Smirnov and Anderson-Darling GOF tests as provided in Table 7, we observed that the Gen. Extreme Value distribution is the best fit amongst the seven continuous probability distributions to the greenhouse gas emissions data, since it has the lowest test statistic.

5. CONCLUDING REMARKS

As the greenhouse and climate change are fundamental issues of environmental sustainability and building healthy communities, the statistical analysis of the greenhouse gas emissions data from cars is therefore very crucial, and can play an important role in many studies of transportation and environmental processes and designs. Fitting of a probability distribution to the greenhouse gas emissions data from cars may be helpful in predicting the probability or forecasting the frequency of occurrence of the greenhouse gas emissions from cars, and planning beforehand. Motivated by the importance of such studies, in this paper, the goodness of fit of 3 parameter Birnbaum-Saunders (or fatigue-life), gamma, 3 parameter gamma, generalized extreme value, 3 parameter lognormal, 4 parameter Pearson 6 and Weibull distributions to a random sample of the greenhouse gas emissions data from 32 different models of cars to determine their applicability and best fit to these data based on the Kolmogorov-Smirnov, Anderson-Darling, and Chi-Squared Goodness-of-Fit Tests. Based on the Chi-Squared test for goodness-of-fit, using the P-values and test statistics analysis, Weibull distribution was found to be the best fit (Rank 1) for the greenhouse gas emissions data, followed by Pearson 6 (4P) (Rank 2), Birnbaum-Saunders (3P) or Fatigue Life (3P) (Rank 3), Lognormal (3P) (Rank 4), Gamma (Rank 5), Gamma (3P) (Rank 6) and Gen. Extreme Value (Rank 7). Gen. Extreme Value distribution is the best fit amongst the seven continuous probability distributions to the greenhouse gas emissions data, since it has the lowest test statistic. It is hoped that this study will be helpful in many problems of transportation and environmental processes and designs, and other applied research.

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