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Investigation of the Solution to the Fuzzy System of Equations: Using the Fuzzy B-contraction Principle

Muhammad Shahjalal[1,*], Saikh Shahjahan Miah[2], Nirmal Kanti Mitra[3]

[1]Department of Mathematics, Faculty of Science, Bangamata Sheikh Fojilatunnesa Mujib Science and Technology University, Jamalpur-2012, Bangladesh

[2]Department of Mathematics, Faculty of Science, Mawlana Bhashani Science and Technology University, Tangail-1902, Bangladesh

[3]Department of Physical Sciences and Engineering, Independent University, Bangladesh

*E-mail address: shahjalal@bsfmstu.ac.bd

ABSTRACT

In this paper, the uniqueness of a solution to the fuzzy system of equations is proved by employing the fuzzy B-contraction Principle in fuzzy metric space. The fuzzy system of equations is transformed into an operator form and then applied to the contraction principle, which leads to a B-contraction in fuzzy Banach space. We also revisited the notion of fuzzy systems in norm space and Banach spaces, the fixed point and contraction along with the B-contrition principle. The main contribution of this study is to the analytical proof of the uniqueness of the fuzzy system of equations using the contraction principle in the fuzzy Banach space.

***Keywords*:** Fuzzy Banach space, Fuzzy Norm, fuzzy B-contraction, fuzzy system of equations

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1. INTRODUCTION

The Banach Contraction Principle is one of the most significant conclusions of analysis and regarded the beginning and major source of metric fixed point theory, which produced the most well-known result of Fixed Point Theory. It guarantees the existence and uniqueness of fixed points on specific self-maps of major spaces, as well as a contractive approach to their location. Fixed point theorems are used in physics, chemistry, engineering, biology, computer science, and other fields. The Banach contraction principle has been expanded in various ways due to its relevance. Indeed, there is a substantial body of literature devoted to the theorem's generalizations and extensions [1-14]. The concept of fuzzy sets was initially described by Zadeh [15] in 1965, and its application in pure and applied mathematics is now well acknowledged.

Kramosil and Michálek [16] proposed the concept of a fuzzy metric space in 1975, which may be regarded of as an extension of the statistical (probabilistic) metric space. Deng [17], Erceg [18], Kaleva and Seikkala [19], Kramosil and Michalek [16], and others have proposed metric spaces that have been extended to fuzzy metric spaces. Phiangsungnoena and Kumama [20] worked on fuzzy fixed point theorems for multivalued fuzzy contraction mapping. Phiangsung and et al. [21] discussed the concept of fuzzy mapping in Hausdorff metric space

Yonghong Shen et al. [22] shows that in the case of B-contraction and C-contraction maps, an existing p-convergent subsequence leads to a single fixed point.

The fuzzy system of equations (FSEs), also known as the fuzzy matrix equation, is an extension of the classical system of equations that includes fuzzy numbers for one or all coefficients, unknowns, and constant components. The solution approach of the FSEs is found to convert the fuzzy numbers into a level set that is an interval of real numbers. Different approaches of the solution of fuzzy system of equations are found in [23-30].

In this paper, fuzzy metric space along contraction mapping is reviewed and we use B-contration principle is applied to proof a unique solution of a generalized fuzzy system of equaions.

The structure of the paper is as follows: Section 2 recalls some notions and known results in t-norm, fuzzy metric spaces, and sequences in fuzzy metric space and some properties. In Section 3, recall the definitions of fuzzy normed and banach spaces, lemma and proof of a theorem. In Section 4, we discuss fixed point and contraction on fuzzy metric spaces, state important theorems on them and finally define B-contraction on fuzzy metric space. In Section 5, we apply the knowledge described in sections 2-4 to show the unique solution of a fuzzy system of equations. Section 6, provides concluding statements.

2. PRELIMINARIES

Definition 2.1. [14] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norm if * satisfies the following conditions:

(i) * is associative and commutative

(ii) * is continuous

(iii) $a * 1 = a$ for $a \in [0,1]$

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

It is clear that if we define * by a*b=ab or a*b=min(a, b) then * is a continuous t-norm.

Definition 2.2. [31]**.** Let X be any arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X \times X \times]0, \infty[$. The ordered 3-tuple (X, M,*) is called a fuzzy metric space under satisfying the following conditions:

For all $x, y, z \in X$ and $s, t > 0$

(i) $M(x, y, t) > 0$

(ii) $M(x, y, t) = 1$ iff $x = y$

(iii) $M(x, y, t) = M(y, x, t)$

(iv) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$

(v) $M(x, y, .) :]0, \infty[\rightarrow]0,1]$ is continuous

Let (X, d) be a metric space and M_d be a function on $X \times X \times]0, \infty[$ defined by

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \text{for } (x, y, t) \in X \times X \times]0, \infty[$$

Then $(X, M,*)$ is a fuzzy metric space, and M is called standard fuzzy metric induced by d. This shows that every ordinary metric induces a fuzzy metric in the sense of George and Veeramani [31].

Definition 2.3. [32]. Let $(X, M,*)$ be a fuzzy metric space:

(i) A sequence (x_n) in X is said to be convergent to a point $x \in X$, if

$$\lim_{n \to \infty} M(x_n, x, t) = 1 \ \forall \ t > 0$$

(ii)A sequence (x_n) in X is called Cauchy sequence if

$$\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1 \ \forall \ t > 0 \text{ and } p > 0$$

(iii) A fuzzy metric space $(X, M,*)$ in which every Cauchy sequence is convergent, is said to be complete.

(iii) A fuzzy metric space $(X, M,*)$ is said to be compact if every sequence contains a convergent subsequence.

Definition 2.4. [2]. A sequence (x_n) in a fuzzy metric space $(X, M,*)$ is a Cauchy sequence if and only if for each $0 < \epsilon < 1$ and $t > 0$ there exist $n_0 \in N$ such that for all $n, m \geq n_0$ we have, $M(x_n, x_m, t) > 1 - \epsilon$.

3. FUZZY NORMED AND BANACH SPACES

Definition 3.1. [31]. Let X be a vector space, * is a continuous t-norm and N is a fuzzy set on $X \times]0,\infty[$, then the ordered 3-tuple (X, N,*) is said to be a fuzzy normed space under satisfying the following conditions: for each $x, y \in X$ and $t, s > 0$:

(i) $N(x,t) > 0$

(ii) $N(x,t) = 1$ iff $x = 0$

(iii) $N(\alpha x, t) = N\left(x, \frac{t}{|\alpha|}\right), \forall\, \alpha \neq 0$

(iv) $N(x,t) * N(y,s) \leq\ N(x+y, t+s)$

(v) $N(x,.):]0,\infty[\to [0,1]$ is continuous

(vi) $\lim_{t\to\infty} N(x,t)\ = 1$

Lemma 3.2. [31]. Let N be a fuzzy norm.

Then (i) $N(x,t)$ is non-decreasing with respect to t for each $x \in X$,

(ii) $N(x-y,t) = N(y-x,t)$

(iii) Let $(X, N,*)$ be a fuzzy normed space, If we define $(x,y,t) = N(x-y,t)$, then M is a fuzzy metric on X, which is called the fuzzy metric induced by the fuzzy norm N.

(iv) A fuzzy metric M which is induced by a fuzzy norm on a fuzzy norm space $(X, N,*)$ has the following properties for all $x, y, z \in\ X$ and every scalar $\alpha \neq 0$:

(a) $M(x+z, y+z, t) = M(x,y,t)$

(b) $M(\alpha x, \alpha y, t) = M\left(x, y, \frac{t}{|\alpha|}\right)$

Definition 3.3. [31]. Let $(x, \|.\|)$ be a normed space and define a*b=min (a, b) and $(x,t) = \frac{kt^n}{kt^n + m\|x\|}$, $k, m, n \in R^+$ **.** Then $(X, N,*)$ is a fuzzy normed space. In particular if $k = n = m = 1$ we have $N(x,t) = \frac{t}{t+\|x\|}$, which is called the standard fuzzy norm induced by norm $\|x\|$.

Definition 3.4. [33]**.** Let $(X, N,*)$ be a fuzzy normed space. We define open ball $B(x,r,t)$ and the close ball $B[x,r,t]$ with center $x \in\ X$ and radius $0 < r < 1, t > 0$ as follows:

$$B(x,r,t) = \{y \in X: N(x-y,t) > 1 - r\}$$

$$B[x,r,t] = \{y \in X: N(x-y,t) \geq 1 - r\}$$

Theorem 3.1. [33]. Let $(X, M,*)$ be a fuzzy metric space induced by the fuzzy norm N and $a * b = min(a, b)$. Let $T_i: X \to X$ be a function with at least one fixed point x_i for each $i = 1,2, \cdots$, and $T_0: X \to X$ be a fuzzy contraction mapping with fixed pint x_0 .

If the sequence (T_i) converges uniformly to T_0, then the sequence (x_i) converges to x_0

Proof: $M(T_i(x), T_0(x), t) = M(x_i, x_0, t) \geq N\left(x_i - x_0, \frac{t}{2}\right) * N\left(0, \frac{t}{2}\right) \to 1$ as $i \to \infty$

Again if: If $x_n \to x$ and $y_n \to y$ then as $n \to \infty$,

$$M(x_n + y_n, x + y, t) = N((x_n + y_n) - (x + y), t) \geq N\left((x_n - x), \frac{t}{2}\right) * N\left((y_n - y), \frac{t}{2}\right) \to 1$$

Definition 3.5. [31]. The fuzzy normed space $(X, N,*)$ is said to be a fuzzy Banach space whenever X is complete with respect to the fuzzy metric induced by the fuzzy norm.

4. FIXED POINT AND CONTRACTION ON FUZZY METRIC SPACE

In classical topology there are notions of fixed point and contraction mapping. In this section we shall present the notions in fuzzy context. In this section, we shall discuss the same concept in fuzzy context.

Definition 4.1. [9]. Suppose X is any nonempty set in R and a self-mapping $T: X \to X$ is called a fixed point if $T(x) = x$ for $x \in X$.

Definition 4.2. Let $X \subset \mathbb{R}$ be a nonempty set and I^X be the set of all fuzzy numbers (fuzzy set) of X. Suppose $x \subset I^X$ be a fuzzy number and $T: x \to x$ be a self-mapping. Then for any $n \in \mathbb{N}$, $T(x_n) \subset I^X$ is called a fixed pint of T if $T(x_n) = (x_n)$.

Definition 4.3. [32]. Let $(X, M,*)$ be a fuzzy metric space. A self-mapping $f: X \to X$ is called a fuzzy contraction mapping on $(X, M,*)$ if $(x, y, t) \geq 1 - r \; \forall \; 0 < r < 1$, then $M(f(x), f(y), t) \geq 1 - r_0$ for each $x, y \in X$ for some $r_0 < r < 1$ and $t > 0$.

The contraction indicates that the degree of nearness between f(x) and f(y) is greater than that of x and y.

For some, $0 < r_0 < r < 1$ implies to $1 - r_0 > 1 - r > 0$

We obtained $M(f(x), f(y), t) \geq 1 - r_0 > 1 - r$

Therefore $M(f(x), f(y), t) > M(x, y, t)$.

Proposition 4.4. Every fuzzy contraction mapping on $(X, M,*)$ is continuous.

Proof: Let a self-mapping $f: X \to X$ be a fuzzy contraction mapping on fuzzy metric space $(X, M,*)$. Therefore, for $x_1, x_2 \in X$ given $r \in]0,1[$, $t > 0$ we can find $r_0 \in]0,1[$, $kt > 0$ such that $1 - r = (1 - r_0) * (1 - r_0)$.

Now, $M(x_1, x_2, t) > 1 - r_0 \Rightarrow M(f(x_1), f(x_2), kt) > 1 - s > 1 - r_0$, where $s \in]0, r_0[$

Let $x_3 \in X$.

Then $M(f(x_1), f(x_3), kt) > M\left(f(x_1), f(x_2), \frac{k}{2}t\right) * M\left(f(x_2), f(x_3), \frac{k}{2}t\right)$
$> (1 - r_0) * (1 - r_0) > 1 - r, r_0 \in]0,1[$

Which implies that f is continuous.

Definition 4.5. [34]. Let on $(X, M,*)$ be a fuzzy metric space. For $t > 0$, the self-mapping $f: X \to X$ is said to be t-uniformly continuous if for each $\epsilon \in]0,1[$ there exist $r \in]0,1[$ such that $M(x_1, x_2, t) \geq 1 - r \Rightarrow M(f(x_1), f(x_2), t) \geq 1 - \epsilon$, for each $x_1, x_2 \in X$.

Theorem 4.1. [6, 31]. Every fuzzy contraction mapping on a complete metric space has a unique fixed point.

Proof: The near proof is found in [35]. Let $f: X \to X$ be a contraction mapping on a complete fuzzy metric space $(X, M,*)$. Let x_1, x_2 be two distinct fixed points of X.

Then, $f(x_1) = x_1$ and $f(x_2) = x_2 \Rightarrow f^n(x_1) = x_1$ and $f^n(x_2) = x_2$

Now $M(x_1, x_2, t) = M(f^n(x_1), f^n(x_2), t) \geq 1 - \frac{r}{k^n} > M(x_1, x_2, t)(= 1 - r)$;

where $k > 1$, which is a contradiction. This implies $x_1 = x_2$.

Definition 4.6. [9, 14]. Let $(X, M,*)$ be a fuzzy metric space. A self-mapping $T: X \to X$ is called B-contraction of T on X if there is a positive real number $k \in]0,1[$ such that $M(T(x), T(y), kt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$. The number k is called a contraction constant of the self-mapping T.

5. APPLICATION TO FUZZY SYSTEM OF EQUATIONS

Definition 5.1. Let X be any nonempty set and $x \in I^X$ be a fuzzy number under X. A self-mapping $f: x \to x$ is called contraction on x if there is a positive real number $k \in]0,1[$ such that for $t > 0$, $M\left(f(x_i), f(x_j), kt\right) \geq M(x_i, x_j, t)$ for all $i, j \in \mathbb{N}$ and $x_i, x_j \subset I^X$.

Definition 5.2. [31]. Let X be any nonempty set and I^X be set of fuzzy numbers under X. Again let * be a continuous t-norm and N is a fuzzy set on $I^X \times]0, \infty[$, then the ordered 3-tuple (I^X,N,*) is said to be a fuzzy normed space under satisfying the following conditions: for each $x, y \in I^X$ and $t, s > 0$:

Then

(i) $N(x, t)$ is non-decreasing with respect to t for each $x \in I^X$,

(ii) $N(x \ominus y, t) = 1$ iff $x = y$

(iii) $N(x \ominus y, t) = N(y \ominus x, t)$

(iv) $N(\alpha \circ x, t) = N\left(x, \frac{t}{|\alpha|}\right), \forall \alpha \neq 0$

(v) $N(x, t) * N(y, s) \leq N(x \oplus y, t \oplus s)$

(vi) Let $(X, N,*)$ be a fuzzy normed space, If we define $M(x,y,t) = N(x \ominus y, t)$, then M is a fuzzy metric on X, which is called the fuzzy metric induced by the fuzzy norm N.

(vii) A fuzzy metric M which is induced by a fuzzy norm on a fuzzy norm space $(X, N,*)$ has the following properties for all $x, y, z \in X$ and every scalar $\alpha \neq 0$:

(a) $M(x \oplus z, y \oplus z, t) = M(x,y,t)$

(b) $M(\alpha \circ x, \alpha \circ y, t) = M\left(x, y, \frac{t}{|\alpha|}\right)$

Proposition 5.3. Let (I^X, M, t) the elements of I^X are set of all fuzzy numbers under $X \subset \mathbb{R}$. Again let $T: I^X \to I^X$ be defined by $T(x) = x$. Then the three algebraic operations $\circ$, $\oplus$ and $\ominus$ on fuzzy numbers (for different algebraic operations on fuzzy number we refer to [36]) following hold:

Consider the fuzzy numbers $y, y', x, x', b, \alpha \in I^X$ such that they are composed by

$$y = \alpha \circ x \oplus b \text{ and } y' = \alpha \circ x' \oplus b \text{ then } M(T(y), T(y'), t) = M(y, y', t)$$

$$\Rightarrow M(y, y', t) = M(\alpha \circ x \oplus b, \alpha \circ x' \oplus b, t) \geq M\left(\alpha \circ x, \alpha \circ x', \frac{t}{2}\right) * M\left(b, b, \frac{t}{2}\right)$$

(1) Again consider the fuzzy numbers $x_i, x_i', x_j, x_j', b_i, \alpha_{ij} \in I^X$ if x_i and x_i' are composed by $x_i = \alpha_{ij} \circ x_j \oplus b_i$ and $x_i' = \alpha_{ij} \circ x_j' \oplus b_i$,

and $\sup_{1<i,j<n} (\alpha_{ij}) \leq k < 1$

then $M(Tx_i, Tx_i', kt) \geq M(x_i, x_j, t) \; \forall \, (x_i, x_j) \in I^X, k \in]0,1[$

Since $T(x) = x, T(x') = x'$ and $T(x_i) = x_i, T(x_i') = x_i'$ and

$$M(Tx_i, Tx_i', kt) \geq M(x_i, x_j, t) \; \forall \, (x_i, x_j) \in I^X, k \in]0,1[$$

From Definition 5.2 we get, $M(x_i, x_j, t) = N(x_i \ominus x_i', t)$

This implies $M(\alpha_{ij} \circ x_j \oplus b_i, \alpha_{ij} \circ x_j' \oplus b_i, t) = N((\alpha_{ij} \circ x_j \oplus b_i) \ominus (\alpha_{ij} \circ x_j' \oplus b_i), t)$

$$\Rightarrow N((\alpha_{ij} \circ x_j \oplus b_i) \ominus (\alpha_{ij} \circ x_j' \oplus b_i), t) \geq N\left(\alpha_{ij} \circ x_j, \alpha_{ij} \circ x_j', \frac{t}{2}\right) * N\left(b_i, b_i, \frac{t}{2}\right)$$

$$\Rightarrow N\left(\alpha_{ij} \circ x_j, \alpha_{ij} \circ x_j', \frac{t}{2}\right) = N\left(\alpha_{ij} \circ x_j \ominus \alpha_{ij} \circ x_j', \frac{t}{2}\right) * 1 = N\left(\alpha_{ij} \circ (x_j \ominus x_j'), \frac{t}{2}\right) * 1$$

We have $\min\left\{N\left(\alpha_{ij} \circ (x_j \ominus x_j'), \frac{t}{2}\right) * 1 : \forall \, x_j, x_j', \alpha_{ij} \in I^X\right\} \geq 1 - r$

Therefore $(x_i, x_j, t) > 1 - r$. This implies $M(Tx_i, Tx_i', kt) \geq M(x_i, x_j, t)$

5. 1. Solution of Fuzzy System of Equations

A system of fuzzy equations is represented by $A \circ x = b$, where A is a $m \times n$ matrix.

The elements of A is denoted by a_{ij}, where $i = 1, \cdots, m; j = 1, \cdots, n$. and b are fuzzy numbers.

Let $x = (x_1, x_2 \cdots x_n)$ and $b = (b_1, b_2, \cdots, b_n)$. The matrix form of the system of equation is denoted by using generalized $\circ$, $\oplus$ and $\ominus$ operations.

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \circ (x_1, x_2 \cdots x_n)^t = (b_1, b_2, \cdots, b_n)^t$$

Therefore, the i-th equation of the given fuzzy system of equations is written by

$$a_{i1} \circ x_1 \oplus a_{i2} \circ x_2 \oplus \cdots \oplus a_{in} \circ x_n = b_i \quad (1)$$

This equation is written in terms of index i, j as follows

$$a_{ij} \circ x_i = b_i \text{ for } i = 1, \cdots, m; j = 1, \cdots, n. \quad (2)$$

$$\Rightarrow x_i = x_i \ominus (a_{ij} \circ x_i) \oplus b_i \text{ for } i = 1, \cdots, m; j = 1, \cdots, n.$$

$$\Rightarrow x_i = \left(1 \ominus (a_{ij})\right) \circ x_i \oplus b_i \text{ for } i = 1, \cdots, m; j = 1, \cdots, n. \quad (3)$$

Introducing δ_{ij} where: $\delta_{ij} = \begin{cases} 1 \; if \; i = j \\ 0 \; if \; i \neq j \end{cases}$

Equation (3) implies to

$$x_i = (\delta_{ij} \ominus a_{ij}) \circ x_i \oplus b_i \text{ for } i = 1, \cdots, m; j = 1, \cdots, n. \quad (4)$$

Define $\alpha_{ij} = \delta_{ij} \ominus a_{ij}$ and $sup|\alpha_{ij}| = |\delta_{ij} \ominus a_{ij}| \leq k < 1$; for $i = 1, \cdots, m; j = 1, \cdots, n$

$$x_i = \alpha_{ij} \circ x_j \oplus b_i \text{ for } i = 1, \cdots, m; j = 1, \cdots, n. \quad (5)$$

Write the equation (5) in the operator form such that $Tx = x$ where T is a self-mapping on $(I^X, M, *)$.

For the fuzzy number space x, x', x_i and x_i': we have

$$M(Tx, Tx', kt) \geq M(x, x', t)$$

$$M(Tx_i, Tx_i', kt) \geq M(x_i, x_i', t)$$

We have $M(x, x', t) = N(x \ominus x', t)$

$$\Rightarrow M(x_i, x_i', t) = N(x_i \ominus x_i', t) = N\,(\alpha_{ij} \circ x_i \oplus b_i \ominus \alpha_{ij} \circ x_i' \ominus b_i, t)$$

$$\Rightarrow N\,(\alpha_{ij} \circ x_i \oplus b_i \ominus \alpha_{ij} \circ x_i' \ominus b_i, t) \geq N\left(\alpha_{ij} \circ x_i \ominus \alpha_{ij} \circ x_i', \tfrac{t}{2}\right) * N\left(b_i \ominus b_i, \tfrac{t}{2}\right)$$

$$= N\left(\alpha_{ij} \circ (x_i \ominus x_i'), \frac{t}{2}\right) * 1$$

$$= \min(1 - r, 1)$$

Hence: $M(x_i, x_i', t) \geq 1 - r$

Therefore $M(Tx_i, Tx_i', kt) \geq M(x_i, x_i', t)$

This shows that T is a B-contraction mapping into itself on fuzzy Banach space. There is a unique fixed point T as a result of the B-contraction principle, which leads to a solution of the fuzzy system of equations.

6. CONCLUSION

In the present work, some features of fuzzy norm space, Banach space, fixed point theorem and contraction principles under the fuzzy metric space are revisited. The study considered a generalized fuzzy system of equations under three operations. The fuzzy system of equations consisted of the fuzzy numbers of the coefficient matrix and constant matrix. In this study, fuzzy numbers are considered as a general type. They can be taken as triangular, trapezoidal, or Gaussian fuzzy numbers. The type of fuzzy numbers depends on the consideration of membership functions. The generalized fuzzy system of equations is transformed into an operator form which is defined by a self-mapping called contraction mapping. This contraction mapping is followed by the B-contraction. As the self-contraction mapping has a fixed point, the B-contraction leads to a unique solution to the fuzzy system of equations.

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