



World Scientific News

An International Scientific Journal

WSN 161 (2021) 111-129

EISSN 2392-2192

Strong and Weak Interactions in Ghahramany's Integrated Nuclear Binding Energy Formula

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ABSTRACT

By modifying Ghahramany's integrated nuclear binding energy formula with strong and weak interaction features, it is possible to approximate nuclear binding energy of isotopes with one unique energy coefficient and four terms. Considering even-odd corrections, shell corrections and other microscopic corrections, it seems possible to improve the accuracy. Based on our recent work and the proposed formulae, we are very confident to say that, quark structure helps in increasing nuclear binding energy and electroweak interaction helps in reducing nuclear binding energy.

Keywords: Strong interaction, Electroweak interaction, Ghahramany's integrated nuclear binding energy formula, Nuclear drip lines

1. INTRODUCTION

Semi empirical mass formulae (SEMF) are having a crucial role in understanding nuclear structure [1-4] and constitute a minimum of 5 terms with five different energy coefficients. Three major disadvantages of SEMF are: 1) Five energy coefficients are having arbitrary origin. 2) Role of strong [5, 6] and electroweak [7-9] interactions is found to be missing. 3) Energy coefficients are having no particular relation with quarks and their structure [10, 11]. In this context, Ghahramany and team members [12-16] developed a very simple nuclear binding

energy formula based on quark-gluon plasma. For increasing its effectiveness towards isotopes, in this paper, we try to include strong and weak interaction features [17, 18] in Ghahramany's integrated nuclear binding energy formula (GINBEF). Recently, Khanna and team members [19, 20] also tried to modify GINBEF.

Even though highly intuitive, these formulae are lagging in implementing electroweak interaction in nuclear binding energy scheme associated with quark structure.

2. GHAHRAMANY'S INTEGRATED NUCLEAR MODEL

According to Ghahramany and team members, up and down quarks play a crucial role in understanding nuclear binding energy. Their unified views are:

- 1) The nuclear binding energy is of the order of 1% of the energy of the total rest mass of the constituent nucleons.
- 2) The binding energy depends upon the volume of the quark-gluon soup within the nuclei and is proportional to $3A$ where A is the mass number.
- 3) Due to the asymmetric distribution of up and down quarks and also the existence of the Coulomb force between them, binding energy depends upon terms such as $\frac{N^2 - Z^2}{Z}$.

For $A \geq 5$, different binding energy formulae proposed by Ghahramany and team members are as follows. In terms of up and down quarks,

$$BE \cong \left\{ 3A - \frac{(N^2 - Z^2) + \delta(N - Z)}{Z} - 9 \right\} \left(\frac{330 \text{ MeV}}{100} \right) \quad (1)$$

where, $\left\{ \begin{array}{l} \delta(N - Z) = 0 \text{ for } Z \neq N \\ \delta(N - Z) = 1 \text{ for } Z = N \end{array} \right.$

With reference to nucleon rest energy,

$$BE \cong \left\{ A - \frac{(N^2 - Z^2) + \delta(N - Z)}{3Z} - 3 \right\} \left(\frac{m_n c^2}{90 \text{ to } 100} \right) \text{ MeV} \quad (2)$$

where, $m_n c^2 = \text{Rest energy of nucleon}$

$\left\{ \begin{array}{l} \delta(N - Z) = 0 \text{ for } Z \neq N \\ \delta(N - Z) = 1 \text{ for } Z = N \end{array} \right.$

More advanced formula is,

$$BE \cong \left\{ A - \frac{(N^2 - Z^2) + \delta(N - Z)}{3(Z - k)} - 3 \right\} \left(\frac{A^{n+s} m_n c^2}{126(Z - k)} \right) \text{ MeV}$$

where, $m_n c^2 =$ Rest energy of nucleon

$$\left\{ \begin{array}{l} \delta(N - Z) = 0 \text{ for } Z \neq N \\ \delta(N - Z) = 1 \text{ for } Z = N \end{array} \right.$$

$$n \cong 0.87 \text{ to } 0.88, \quad k \cong \begin{cases} 2 \text{ for } Z \leq 118 \\ 0 \text{ for } Z > 118 \end{cases}$$

$$s \cong \begin{cases} 0.0003 \text{ for even } (N, Z) \\ -0.0003 \text{ for odd } (N, Z) \end{cases}$$
(3)

According to Khanna and team members [20],

$$BE \cong \left[3A - \left(\frac{(N^2 - Z^2) + \delta(N - Z)}{\sqrt{NZ}} \right) + \lambda \right] \left(\frac{m_u c^2}{100} \right) \text{ MeV}$$

where, $m_u c^2 = 330 \text{ MeV}$

$$\left\{ \begin{array}{l} \delta(N - Z) = 0 \text{ for } Z \neq N \\ \delta(N - Z) = 1 \text{ for } Z = N \end{array} \right.$$

$$\lambda \cong \begin{cases} 9 \text{ for } Z \leq 30 \\ 0.4Z \text{ for } Z = 31 \text{ to } Z = 59 \\ 0.5Z \text{ for } Z = 60 \text{ to } Z = 69 \\ 0.6Z \text{ for } Z = 70 \text{ to } Z = 79 \\ 0.7Z \text{ for } Z = 80 \text{ to } Z = 86 \\ 0.8Z \text{ for } Z = 87 \text{ to } Z = 92 \\ 0.9Z \text{ for } Z = 93 \text{ to } Z = 98 \\ Z \text{ for } Z = 99 \text{ to } Z = 103 \\ 1.1Z \text{ for } Z = 104 \text{ to } Z = 110 \\ 1.2Z \text{ for } Z = 111 \text{ to } Z = 116 \\ 1.3Z \text{ for } Z = 117 \text{ and } Z = 118 \end{cases}$$
(4)

3. UNDERSTANDING THE ROLE OF ELECTROWEAK INTERACTION IN NUCLEAR BINDING ENERGY SCHEME

With reference to strong and weak interactions in our recently published papers, we have developed a simple formula for estimating nuclear binding energy with one unique coefficient

of magnitude 10.1 MeV. In this paper, we made an attempt to include strong and weak interactions in GINBEF. It may be noted that, single variable binding energy constant and 3 simple terms play an interesting role in GINBEF. By considering strong and weak interactions, we have improved the workability of GINBEF applicable for isotopes and it needs further study with respect to $Z \approx N$.

By considering electroweak interaction, in nuclear binding energy scheme, we noticed that,

- 1) Unique binding energy coefficient is interconnected with basic nuclear potential, reciprocal of the strong coupling constant [11] and up and down quark rest masses [11]. (See section 9).
- 2) All the nucleons are not participating in nuclear binding energy.
- 3) Electroweak interaction plays a vital role in deciding the number of non-participating nucleons in the form of $\left[1 + 0.0019A\sqrt{NZ}\right]$.
- 4) Electroweak interaction decides the stable mass numbers of Z in the form of $A_s \cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2$.
- 5) With reference to pions and electro weak bosons, number 0.0016 can be expressed with a relation of the form, $\left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}}\right) \cong \left(\frac{\sqrt{134.98 \times 139.57} \text{ MeV}}{\sqrt{80379.0 \times 91187.6} \text{ MeV}}\right) \cong 0.0016032$.
- 6) Stable mass number of Z plays an interesting role in reducing the binding energy of isotopes in the form of $\frac{(A_s - A)^2}{A_s}$.
- 7) Binding energy decreases with increasing surface area and increasing radius with an effective term of the form $\left(A^{2/3}/A^{1/3}\right) \cong A^{1/3}$.

Binding energy can be approximated with a relation of the form,

$$BE \cong \left\{ A - \left[1 + 0.0019A\sqrt{NZ}\right] - A^{1/3} - \frac{(A_s - A)^2}{A_s} \right\} (10.1 \text{ MeV}) \quad (5)$$

By replacing the number 0.0019 with 0.0016, binding energy can be approximated with a relation of the form,

$$BE \cong \left\{ A - A^{1/3} - \frac{(A_s - A)^2}{A_s} \right\} (10.1 \text{ MeV}) - \left[1 + 0.0016A\sqrt{NZ}\right] (11.9 \text{ MeV}) \quad (6)$$

4. ON THE ROLE OF ELECTROWEAK INTERACTION IN INTEGRATED NUCLEAR BINDING ENERGY FORMULA

Based on the above relations, we make an attempt to include strong and weak interactions in GINBEF. For $Z \geq 3$, it can be expressed in the following way.

$$BE \cong \left\{ A - [0.00173\sqrt{ZNA A_s}] - \left[\left(0.118 \frac{(N^2 - Z^2)}{\sqrt{A A_s}} \right) + \left(\frac{(A - A_s)^2}{\sqrt{A A_s}} \right) \right] - \left[\frac{\sqrt{A}}{\sqrt{Z}} + \frac{\sqrt{A}}{2} \right] \right\} 10.3 \text{ MeV} \quad (7)$$

where $A_s \cong 2Z + 0.0064Z^2$

With reference to various semi empirical mass formulae and with trial & error we have developed this relation. Accuracy point of view, it is having many advantages compared to Ghahramany's integrated nuclear model. In this relation,

- 1) Second term can be considered as a consequence of electroweak interaction.
- 2) Third term can be considered as a consequence of Coulombic and Asymmetry.
- 3) Fourth term can be considered as a correction factor associated with neutron-proton ratio.

As a whole, accuracy depends on two numbers 0.00173 and 0.118, one energy coefficient of 10.3 MeV and selection of stable mass number of Z. With a computer program [21] and fitting the estimated data with actual binding energies, things can be improved. By considering even-odd corrections, shell corrections and other microscopic corrections, relation (7) can be reviewed and refined with a clear physical background. See Table 1 for the binding energy of isotopes of Z=50. To compare the data prepared with relations (5), (6) and (7), we have taken the following three semi empirical mass formulae as references.

$$BE_{\text{Ref1}} \cong \left\{ \begin{aligned} & \left[(A \times 15.36) \right] - \left[(A^{2/3} \times 16.32) \right] - \left[\left(\frac{Z^2}{A^{1/3}} \right) 0.6929 \right] \\ & - \left[\frac{((A/2) - Z)^2}{A} \times 90.46 \right] \pm \left(\frac{11.32}{\sqrt{A}} \right) \end{aligned} \right\} \text{ MeV} \quad (8)$$

Ref. → <https://fdocuments.in/document/nuclear-and-particle-physics.html>

$$BE_{\text{Ref2}} \cong \left(a_v * A \right) - \left(a_s * A^{2/3} \right) - \left(a_c * \frac{Z * (Z - 1)}{A^{1/3}} \right) - \left(a_a * \frac{(A - 2Z)^2}{A} \right) \pm \left(\frac{a_p}{\sqrt{A}} \right)$$

where $\left\{ \begin{aligned} & a_v \cong 15.8 \text{ MeV}; a_s \cong 18.3 \text{ MeV}; a_c \cong 0.714 \text{ MeV}; \\ & a_a \cong 23.2 \text{ MeV}; a_p \cong 12.0 \text{ MeV}; \end{aligned} \right\} \quad (9)$

Ref. → https://en.wikipedia.org/wiki/Semi-empirical_mass_formula

$$BE_{Ref3} \cong (a_v * x * A) - (a_s * x * A^{2/3}) - \left(a_c * \frac{Z^2}{A^{1/3}} \right) + \left(a_{pr} * \frac{Z^2}{A} \right) \pm \left(\frac{a_p}{\sqrt{A}} \right)$$

$$\left. \begin{array}{l} x \cong \left[1 - 1.79 \left(\frac{N-Z}{A} \right)^2 \right] \\ \text{where } \left\{ \begin{array}{l} a_v \cong 15.677 \text{ MeV}; a_s \cong 18.56 \text{ MeV}; a_c \cong 0.717 \text{ MeV}; \\ a_{pr} \cong 1.211 \text{ MeV}; a_p \cong 11.0 \text{ MeV}; \end{array} \right. \end{array} \right\} \quad (10)$$

Ref. → <http://oregonstate.edu/instruct/ch374/ch418518/lecture3-1.pdf>

Table 1. Estimated nuclear binding energy of isotopes of Z=50

Z=50	N	A	Estimated stable mass number	Ref_1.BE (MeV) Relation-8	Ref_2.BE (MeV) Relation-9	Ref_3.BE (MeV) Relation-10	Est_1. BE (MeV) Relation-5	Est_2. BE (MeV) Relation-6	Est_3. BE (MeV) Relation-7
50	50	100	116	812.3	810.1	813.0	834.8	833.7	843.5
50	51	101	116	825.2	823.1	825.7	845.5	844.5	854.0
50	52	102	116	839.9	838.0	840.2	856.0	855.0	864.2
50	53	103	116	852.0	850.1	852.1	866.4	865.4	874.2
50	54	104	116	865.8	864.1	865.7	876.6	875.6	884.0
50	55	105	116	877.0	875.4	876.9	886.6	885.6	893.6
50	56	106	116	890.0	888.6	889.8	896.4	895.4	903.0
50	57	107	116	900.5	899.1	900.2	906.0	905.0	912.1
50	58	108	116	912.7	911.5	912.4	915.4	914.5	921.1
50	59	109	116	922.4	921.2	922.1	924.7	923.8	929.8
50	60	110	116	933.9	932.8	933.6	933.8	932.9	938.3
50	61	111	116	942.9	941.9	942.7	942.6	941.8	946.7
50	62	112	116	953.8	952.8	953.6	951.4	950.5	954.8
50	63	113	116	962.1	961.1	962.0	959.9	959.0	962.8
50	64	114	116	972.3	971.4	972.3	968.2	967.4	970.5
50	65	115	116	980.0	979.1	980.1	976.4	975.6	978.1
50	66	116	116	989.5	988.8	989.8	984.4	983.6	985.5
50	67	117	116	996.7	995.9	997.1	992.2	991.4	992.6
50	68	118	116	1005.6	1004.9	1006.1	999.8	999.0	999.7
50	69	119	116	1012.2	1011.4	1012.9	1007.2	1006.5	1006.5

50	70	120	116	1020.6	1019.9	1021.4	1014.5	1013.7	1013.1
50	71	121	116	1026.7	1025.9	1027.7	1021.5	1020.8	1019.6
50	72	122	116	1034.5	1033.8	1035.7	1028.4	1027.7	1025.9
50	73	123	116	1040.1	1039.3	1041.5	1035.1	1034.4	1032.0
50	74	124	116	1047.4	1046.7	1049.0	1041.6	1041.0	1038.0
50	75	125	116	1052.5	1051.6	1054.3	1048.0	1047.3	1043.8
50	76	126	116	1059.3	1058.5	1061.3	1054.1	1053.5	1049.4
50	77	127	116	1063.9	1063.1	1066.1	1060.1	1059.5	1054.8
50	78	128	116	1070.3	1069.5	1072.7	1065.9	1065.3	1060.1
50	79	129	116	1074.5	1073.5	1077.1	1071.5	1070.9	1065.3
50	80	130	116	1080.5	1079.5	1083.3	1076.9	1076.3	1070.2
50	81	131	116	1084.2	1083.2	1087.3	1082.1	1081.6	1075.1
50	82	132	116	1089.8	1088.7	1093.0	1087.2	1086.7	1079.7
50	83	133	116	1093.1	1091.9	1096.6	1092.1	1091.6	1084.2
50	84	134	116	1098.3	1097.0	1101.9	1096.8	1096.3	1088.6
50	85	135	116	1101.2	1099.9	1105.1	1101.3	1100.8	1092.8
50	86	136	116	1106.0	1104.6	1110.0	1105.6	1105.1	1096.8
50	87	137	116	1108.6	1107.1	1112.9	1109.7	1109.3	1100.7
50	88	138	116	1113.0	1111.5	1117.4	1113.7	1113.3	1104.5
50	89	139	116	1115.3	1113.6	1119.9	1117.5	1117.1	1108.1
50	90	140	116	1119.3	1117.6	1124.1	1121.1	1120.7	1111.5
50	91	141	116	1121.2	1119.4	1126.3	1124.5	1124.1	1114.9
50	92	142	116	1124.9	1123.0	1130.1	1127.7	1127.4	1118.0
50	93	143	116	1126.5	1124.5	1132.0	1130.8	1130.5	1121.1
50	94	144	116	1129.9	1127.8	1135.5	1133.7	1133.3	1124.0
50	95	145	116	1131.2	1129.0	1137.0	1136.3	1136.0	1126.7
50	96	146	116	1134.3	1131.9	1140.2	1138.8	1138.6	1129.4
50	97	147	116	1135.3	1132.8	1141.4	1141.2	1140.9	1131.9
50	98	148	116	1138.0	1135.5	1144.3	1143.3	1143.1	1134.2
50	99	149	116	1138.8	1136.1	1145.2	1145.3	1145.0	1136.4
50	100	150	116	1141.3	1138.5	1147.8	1147.0	1146.8	1138.5
50	101	151	116	1141.7	1138.8	1148.5	1148.6	1148.5	1140.5
50	102	152	116	1143.9	1140.9	1150.8	1150.1	1149.9	1142.3
50	103	153	116	1144.2	1140.9	1151.1	1151.3	1151.1	1144.0
50	104	154	116	1146.1	1142.8	1153.2	1152.3	1152.2	1145.6
50	105	155	116	1146.1	1142.5	1153.3	1153.2	1153.1	1147.0

50	106	156	116	1147.7	1144.1	1155.0	1153.9	1153.8	1148.3
50	107	157	116	1147.5	1143.7	1154.9	1154.4	1154.3	1149.5
50	108	158	116	1148.9	1145.0	1156.4	1154.7	1154.6	1150.6
50	109	159	116	1148.4	1144.3	1156.0	1154.8	1154.8	1151.6
50	110	160	116	1149.6	1145.4	1157.2	1154.8	1154.8	1152.4
50	111	161	116	1148.9	1144.5	1156.6	1154.6	1154.6	1153.1
50	112	162	116	1149.8	1145.3	1157.6	1154.2	1154.2	1153.7
50	113	163	116	1148.9	1144.2	1156.8	1153.6	1153.6	1154.1
50	114	164	116	1149.7	1144.8	1157.5	1152.8	1152.9	1154.5

5. SECOND TERM OF RELATION (7)

We noticed that, reduction of binding energy is greatly influenced by electroweak interaction. Clearly speaking, second term plays a major role in reducing nuclear binding energy compared to other terms. This can be confirmed with relations (5) and (6). Number of non-participating nucleons seems to increase with increasing mass number. For ${}_{92}\text{U}^{238}$, number of non-participating nucleons are 48 and corresponding reduction in binding energy is 491 MeV. This is a reduction of 20% of $(238 \times 10.3 \text{ MeV}) = 2451 \text{ MeV}$.

6. FIRST PART OF THE THIRD TERM OF RELATION (7)

Considering the third term, it constitutes two sub terms. First sub term is $\left(0.118 \frac{(N^2 - Z^2)}{\sqrt{AA_s}}\right)$

. It is almost similar to the Ghahramany's proposed second term, $\frac{(N^2 - Z^2)}{3Z}$. Multiplying and dividing relation (1) by 3 and ignoring $\delta(N - Z)$, it is possible to re-express it as,

$$BE \cong \left\{ A - \frac{(N^2 - Z^2)}{3Z} - 3 \right\} (9.9 \text{ MeV}) \quad (10)$$

It may be noted that, $\frac{(N^2 - Z^2)}{3Z} \cong \frac{1}{3} \left[\frac{(N^2 - Z^2)}{Z} \right] \cong 0.333 \left[\frac{(N^2 - Z^2)}{Z} \right]$. The number $\left(\frac{1}{3}\right) \cong 0.333$

is roughly three times the strong coupling constant [14], $\alpha_s \cong 0.118$. Keeping this coincidence in view we tried to replace $\left(\frac{1}{3}\right)$ with α_s . By doing so, it is possible to bring strong interaction characteristics into nuclear binding energy scheme. It needs a serious review.

7. SECOND PART OF THE THIRD TERM OF RELATION (7)

It is quite interesting to note that, second sub term of the third term, $\left(\frac{(A - A_s)^2}{\sqrt{AA_s}}\right)$ plays a nice role in reducing the binding energy of isotopes of Z about its stable mass number. Clearly speaking, on both side of the selected stable mass number of Z, it helps in curve bending with reference to mainstream semi empirical mass formulae.

8. FOURTH TERM OF RELATION (7)

With trial-error, we have introduced the fourth term in place of the third term of the Ghahramany’s integrated nuclear model. Even though, its contribution in reducing binding energy of medium and heavy atomic nuclides is minute, for light atomic nuclides, its role is considerable. It needs a review and refinement with respect to actual binding energy.

9. UNDERSTANDING THE UNIQUE BINDING ENERGY COEFFICIENT

Unique binding energy coefficient can be understood in two possible ways. One way is based on the basic nuclear potential and reciprocal of the strong coupling constant [11, 12]. Second way is to consider the average rest energy of up and down quarks [11, 12].

Method 1. Considering nuclear potential and strong coupling constant, unique binding energy coefficient can be expressed as,

$$B_0 \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \cong (10.0 \text{ to } 10.4) \text{ MeV}$$

$$\text{where, } \begin{cases} \alpha_s \cong 0.115 \text{ to } 0.12 \\ R_0 \cong 1.20 \text{ fm} \end{cases} \tag{11}$$

Method 2. Considering up and down quark masses, unique binding energy coefficient can be expressed as,

$$B_0 \cong \left. \begin{aligned} &\frac{(2m_u c^2 + m_d c^2) + (m_u c^2 + 2m_d c^2)}{2} \\ &\cong \frac{3}{2}(m_u c^2 + m_d c^2) \cong 10.26 \text{ MeV} \\ &\text{where } \begin{cases} m_u c^2 \cong 2.16_{-0.26}^{+0.49} \text{ MeV} \\ m_d c^2 \cong 4.68_{-0.17}^{+0.48} \text{ MeV} \end{cases} \end{aligned} \right\} \tag{12}$$

Combining relations (11) and (12),

$$\alpha_s \cong \frac{2e^2}{12\pi\epsilon_0 R_0 (m_u + m_d) c^2} \quad (13)$$

$$R_0 \cong \left(\frac{1}{\alpha_s} \right) \frac{2e^2}{12\pi\epsilon_0 (m_u + m_d) c^2} \quad (14)$$

10. DISCUSSION

- 1) Relation (7) is absolutely different from the currently believed major nuclear binding energy terms like surface energy and coulombic energy. But it is in line with Ghahramany's integrated nuclear binding energy relations (GINBER).
- 2) It is true that, at first look, relation (7) seems to be quite complicated. Even then, compared to relations (1) to (4), it is helping in estimating nuclear binding energy to a very good approximation with prefixed parameters applicable for the whole range of atomic nuclides. In this context, we have developed the following much simple workable relation applicable for $Z = (3 \text{ to } 120)$ and we are working on it.

$$BE \cong \left\{ A - \left[\frac{1}{2} k (Z^2 + A^2) \right] - \frac{(A_s - A)^2}{A_s} - 3 \right\} (B_0 \cong 9.85 \text{ MeV})$$

$$\text{where } \left\{ \begin{array}{l} k \cong \frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}} \cong 0.0016 \\ A_s \cong 2Z + k (2Z)^2 \cong 2Z + 0.0064Z^2 \\ B_0 \cong \left(\frac{1}{\alpha_s} \right) \frac{e^2}{4\pi\epsilon_0 R_0} \cong (9.6 \text{ to } 10.0) \text{ MeV} \\ \text{where, } \left\{ \begin{array}{l} \alpha_s \cong 0.115 \text{ to } 0.12 \\ R_0 \cong 1.25 \text{ fm} \end{array} \right. \end{array} \right. \quad (15)$$

- 3) Even though minor error is persisting in estimating the binding energy, starting from $Z = 3$ to 120 , based on the trend of binding energy fitting, it is very clear to say that, our proposed relation (7) is having a very deep interconnection with nuclear binding energy scheme and quarks & strong coupling constant.
- 4) Understanding nuclear binding energy with single energy coefficient is a challenging task and it needs a lot of physical basis. We are working in this direction.
- 5) We believe that, by considering other microscopic binding energy terms like, shell effect, even-odd effect and congruence effect, there is a possibility for improving the accuracy.
- 6) By considering the second term, number of non-participating nucleons can be estimated. With further study, current nuclear models [22,23] can be studied in a unified manner.

- 7) Considering Z as a characteristic representation of range of bound isotopes of Z and considering $k \cong 0.0016$ as a characteristic electroweak coefficient, lower [3] and upper mass limits of Z can be expressed as, $A_{low} \cong 2Z + 2kZ^2$ and $A_{up} \cong 3Z + 2kZ^2$ respectively. Mean stable mass number can be approximated with, $A_{mean} \cong 2Z + 4kZ^2 \cong 2Z + 0.0064Z^2$.
- 8) Independent of nucleon separation energies, approximately, lower and upper mass limits of proton drip lines [24] can be understood with, $(A_p)_{low} \cong (2Z + kZ^2) - \ln\left(\frac{1}{k}\right) \cong (2Z + kZ^2) - 6.44$ and $(A_p)_{up} \cong (2Z + 2kZ^2) \cong 2Z(1 + kZ)$. Geometric mass limit of proton drip lines can be expressed as, $(A_p)_{gm} \cong \sqrt{(A_p)_{low}(A_p)_{up}}$. Similarly neutron drip lines can be approximated with $A_N \cong A_{up} \cong 3Z + 2kZ^2$. See Table 2. Considering even-odd corrections and shell effects, to some extent, errors in estimation can be minimized. We are working in this direction.

Table 2. Estimated drip lines and mass limits of $Z = 8$ to 121

Z	$(A_p)_{low}$	$(A_p)_{gm}$	$(A_p)_{up}$ A_{low}	A_{mean}	A_N A_{up}	Z	$(A_p)_{low}$	$(A_p)_{gm}$	$(A_p)_{up}$ A_{low}	A_{mean}	A_N A_{up}
8	9.7	11.7	16.2	16.4	24.2	65	130.3	136.6	143.5	157.0	208.5
9	11.7	14.0	18.3	18.5	27.3	66	132.5	138.9	145.9	159.9	211.9
10	13.7	16.2	20.3	20.6	30.3	67	134.7	141.2	148.4	162.7	215.4
11	15.8	18.4	22.4	22.8	33.4	68	137.0	143.6	150.8	165.6	218.8
12	17.8	20.5	24.5	24.9	36.5	69	139.2	145.9	153.2	168.5	222.2
13	19.8	22.6	26.5	27.1	39.5	70	141.4	148.2	155.7	171.4	225.7
14	21.9	24.7	28.6	29.3	42.6	71	143.6	150.5	158.1	174.3	229.1
15	23.9	26.8	30.7	31.4	45.7	72	145.9	152.9	160.6	177.2	232.6
16	26.0	28.9	32.8	33.6	48.8	73	148.1	155.2	163.1	180.1	236.1
17	28.0	31.1	34.9	35.8	51.9	74	150.3	157.6	165.5	183.0	239.5
18	30.1	33.2	37.0	38.1	55.0	75	152.6	159.9	168.0	186.0	243.0
19	32.1	35.3	39.2	40.3	58.2	76	154.8	162.3	170.5	189.0	246.5
20	34.2	37.4	41.3	42.6	61.3	77	157.0	164.6	173.0	191.9	250.0
21	36.3	39.5	43.4	44.8	64.4	78	159.3	167.0	175.5	194.9	253.5
22	38.3	41.6	45.5	47.1	67.5	79	161.5	169.4	178.0	197.9	257.0
23	40.4	43.7	47.7	49.4	70.7	80	163.8	171.8	180.5	201.0	260.5
24	42.5	45.9	49.8	51.7	73.8	81	166.1	174.1	183.0	204.0	264.0
25	44.6	48.0	52.0	54.0	77.0	82	168.3	176.5	185.5	207.0	267.5

26	46.6	50.1	54.2	56.3	80.2	83	170.6	178.9	188.0	210.1	271.0
27	48.7	52.2	56.3	58.7	83.3	84	172.8	181.3	190.6	213.2	274.6
28	50.8	54.4	58.5	61.0	86.5	85	175.1	183.7	193.1	216.2	278.1
29	52.9	56.5	60.7	63.4	89.7	86	177.4	186.1	195.7	219.3	281.7
30	55.0	58.7	62.9	65.8	92.9	87	179.7	188.5	198.2	222.4	285.2
31	57.1	60.8	65.1	68.2	96.1	88	182.0	190.9	200.8	225.6	288.8
32	59.2	63.0	67.3	70.6	99.3	89	184.2	193.4	203.3	228.7	292.3
33	61.3	65.1	69.5	73.0	102.5	90	186.5	195.8	205.9	231.8	295.9
34	63.4	67.3	71.7	75.4	105.7	91	188.8	198.2	208.5	235.0	299.5
35	65.5	69.5	73.9	77.8	108.9	92	191.1	200.6	211.1	238.2	303.1
36	67.6	71.6	76.1	80.3	112.1	93	193.4	203.1	213.7	241.4	306.7
37	69.8	73.8	78.4	82.8	115.4	94	195.7	205.5	216.3	244.6	310.3
38	71.9	76.0	80.6	85.2	118.6	95	198.0	208.0	218.9	247.8	313.9
39	74.0	78.2	82.9	87.7	121.9	96	200.3	210.4	221.5	251.0	317.5
40	76.1	80.4	85.1	90.2	125.1	97	202.6	212.9	224.1	254.2	321.1
41	78.2	82.6	87.4	92.8	128.4	98	204.9	215.3	226.7	257.5	324.7
42	80.4	84.8	89.6	95.3	131.6	99	207.2	217.8	229.4	260.7	328.4
43	82.5	87.0	91.9	97.8	134.9	100	209.6	220.3	232.0	264.0	332.0
44	84.7	89.2	94.2	100.4	138.2	101	211.9	222.7	234.6	267.3	335.6
45	86.8	91.4	96.5	103.0	141.5	102	214.2	225.2	237.3	270.6	339.3
46	88.9	93.6	98.8	105.5	144.8	103	216.5	227.7	239.9	273.9	342.9
47	91.1	95.8	101.1	108.1	148.1	104	218.9	230.2	242.6	277.2	346.6
48	93.2	98.1	103.4	110.7	151.4	105	221.2	232.7	245.3	280.6	350.3
49	95.4	100.3	105.7	113.4	154.7	106	223.5	235.2	248.0	283.9	354.0
50	97.6	102.5	108.0	116.0	158.0	107	225.9	237.7	250.6	287.3	357.6
51	99.7	104.8	110.3	118.6	161.3	108	228.2	240.2	253.3	290.6	361.3
52	101.9	107.0	112.7	121.3	164.7	109	230.6	242.7	256.0	294.0	365.0
53	104.1	109.3	115.0	124.0	168.0	110	232.9	245.2	258.7	297.4	368.7
54	106.2	111.5	117.3	126.7	171.3	111	235.3	247.7	261.4	300.9	372.4
55	108.4	113.8	119.7	129.4	174.7	112	237.6	250.3	264.1	304.3	376.1
56	110.6	116.0	122.0	132.1	178.0	113	240.0	252.8	266.9	307.7	379.9

57	112.8	118.3	124.4	134.8	181.4	114	242.4	255.3	269.6	311.2	383.6
58	114.9	120.6	126.8	137.5	184.8	115	244.7	257.9	272.3	314.6	387.3
59	117.1	122.9	129.1	140.3	188.1	116	247.1	260.4	275.1	318.1	391.1
60	119.3	125.1	131.5	143.0	191.5	117	249.5	262.9	277.8	321.6	394.8
61	121.5	127.4	133.9	145.8	194.9	118	251.8	265.5	280.6	325.1	398.6
62	123.7	129.7	136.3	148.6	198.3	119	254.2	268.1	283.3	328.6	402.3
63	125.9	132.0	138.7	151.4	201.7	120	256.6	270.6	286.1	332.2	406.1
64	128.1	134.3	141.1	154.2	205.1	121	259.0	273.2	288.9	335.7	409.9

- 9) Considering Ghahramany’s quark concepts of nuclear binding energy and our proposal of reduction of nuclear binding energy with electroweak interaction, there is a wide scope for studying low energy nuclear physics and high energy nuclear physics in a unified approach. In this context, relations (5), (6), (7) and (15) can be given some encouragement.
- 10) Proposed relation (15) is very simple to understand. Considering the second term of relation (15), it is very clear to say that, via electroweak interaction, squared proton number and squared mass number play a major role reducing the nuclear binding energy.
- 11) Compared to current models of Nuclear Density Functional Theories [25] and holographic quantum chromodynamic mean field nuclear potentials [26], qualitatively and quantitatively, our proposal is very simple to understand and very easy to follow. It can be proved with relations (5) and (15). To confirm the effectiveness of our proposal, readers are encouraged to see Table 3 prepared with relations (8), (10) and (15).

Table 3. Estimated nuclear binding energy of isotopes of Z = 92

Z = 92	N	A	Estimated stable mass number	Est. BE (MeV) Relation (15)	Ref 1.BE (MeV) Relation (8)	%Error Relations (8) & (15)	Ref 2.BE (MeV) Relation (10)	%Error Relations (10) & (15)
92	92	184	238	1328.7	1268.0	-4.8	1273.7	-4.3
92	93	185	238	1340.1	1282.4	-4.5	1287.9	-4.1
92	94	186	238	1351.3	1298.1	-4.1	1303.5	-3.7
92	95	187	238	1362.5	1312.0	-3.9	1317.2	-3.4
92	96	188	238	1373.6	1327.3	-3.5	1332.3	-3.1
92	97	189	238	1384.6	1340.6	-3.3	1345.5	-2.9
92	98	190	238	1395.4	1355.4	-3.0	1360.1	-2.6
92	99	191	238	1406.2	1368.3	-2.8	1372.9	-2.4
92	100	192	238	1416.9	1382.7	-2.5	1387.1	-2.1

92	101	193	238	1427.5	1395.1	-2.3	1399.4	-2.0
92	102	194	238	1438.0	1409.0	-2.1	1413.2	-1.8
92	103	195	238	1448.3	1421.0	-1.9	1425.1	-1.6
92	104	196	238	1458.6	1434.5	-1.7	1438.4	-1.4
92	105	197	238	1468.8	1446.1	-1.6	1449.9	-1.3
92	106	198	238	1478.9	1459.1	-1.4	1462.7	-1.1
92	107	199	238	1488.9	1470.3	-1.3	1473.8	-1.0
92	108	200	238	1498.8	1482.9	-1.1	1486.3	-0.8
92	109	201	238	1508.6	1493.7	-1.0	1497.0	-0.8
92	110	202	238	1518.3	1505.9	-0.8	1509.1	-0.6
92	111	203	238	1527.9	1516.3	-0.8	1519.4	-0.6
92	112	204	238	1537.4	1528.1	-0.6	1531.1	-0.4
92	113	205	238	1546.8	1538.1	-0.6	1541.0	-0.4
92	114	206	238	1556.1	1549.6	-0.4	1552.3	-0.2
92	115	207	238	1565.3	1559.2	-0.4	1561.9	-0.2
92	116	208	238	1574.4	1570.3	-0.3	1572.9	-0.1
92	117	209	238	1583.4	1579.6	-0.2	1582.1	-0.1
92	118	210	238	1592.3	1590.3	-0.1	1592.7	0.0
92	119	211	238	1601.1	1599.3	-0.1	1601.6	0.0
92	120	212	238	1609.8	1609.7	0.0	1611.8	0.1
92	121	213	238	1618.4	1618.3	0.0	1620.4	0.1
92	122	214	238	1626.9	1628.3	0.1	1630.3	0.2
92	123	215	238	1635.4	1636.7	0.1	1638.5	0.2
92	124	216	238	1643.7	1646.4	0.2	1648.0	0.3
92	125	217	238	1651.9	1654.4	0.1	1656.0	0.2
92	126	218	238	1660.0	1663.7	0.2	1665.2	0.3
92	127	219	238	1668.0	1671.4	0.2	1672.8	0.3
92	128	220	238	1676.0	1680.5	0.3	1681.7	0.3
92	129	221	238	1683.8	1687.9	0.2	1689.0	0.3
92	130	222	238	1691.5	1696.7	0.3	1697.6	0.4
92	131	223	238	1699.1	1703.8	0.3	1704.6	0.3
92	132	224	238	1706.7	1712.3	0.3	1713.0	0.4
92	133	225	238	1714.1	1719.1	0.3	1719.7	0.3
92	134	226	238	1721.4	1727.3	0.3	1727.7	0.4
92	135	227	238	1728.7	1733.8	0.3	1734.1	0.3
92	136	228	238	1735.8	1741.7	0.3	1741.9	0.3

92	137	229	238	1742.8	1748.0	0.3	1748.0	0.3
92	138	230	238	1749.8	1755.7	0.3	1755.5	0.3
92	139	231	238	1756.6	1761.7	0.3	1761.4	0.3
92	140	232	238	1763.3	1769.0	0.3	1768.6	0.3
92	141	233	238	1770.0	1774.8	0.3	1774.2	0.2
92	142	234	238	1776.5	1781.9	0.3	1781.1	0.3
92	143	235	238	1783.0	1787.4	0.3	1786.5	0.2
92	144	236	238	1789.3	1794.3	0.3	1793.2	0.2
92	145	237	238	1795.6	1799.6	0.2	1798.3	0.2
92	146	238	238	1801.7	1806.2	0.2	1804.7	0.2
92	147	239	238	1807.8	1811.3	0.2	1809.6	0.1
92	148	240	238	1813.7	1817.6	0.2	1815.8	0.1
92	149	241	238	1819.6	1822.5	0.2	1820.4	0.0
92	150	242	238	1825.3	1828.6	0.2	1826.3	0.1
92	151	243	238	1831.0	1833.2	0.1	1830.7	0.0
92	152	244	238	1836.5	1839.1	0.1	1836.4	0.0
92	153	245	238	1842.0	1843.5	0.1	1840.6	-0.1
92	154	246	238	1847.3	1849.2	0.1	1846.1	-0.1
92	155	247	238	1852.6	1853.3	0.0	1850.0	-0.1
92	156	248	238	1857.8	1858.8	0.1	1855.2	-0.1
92	157	249	238	1862.8	1862.8	0.0	1859.0	-0.2
92	158	250	238	1867.8	1868.1	0.0	1864.0	-0.2
92	159	251	238	1872.7	1871.8	0.0	1867.5	-0.3
92	160	252	238	1877.4	1876.9	0.0	1872.3	-0.3
92	161	253	238	1882.1	1880.4	-0.1	1875.6	-0.3
92	162	254	238	1886.7	1885.3	-0.1	1880.2	-0.3
92	163	255	238	1891.2	1888.6	-0.1	1883.3	-0.4
92	164	256	238	1895.5	1893.3	-0.1	1887.7	-0.4
92	165	257	238	1899.8	1896.5	-0.2	1890.6	-0.5
92	166	258	238	1904.0	1900.9	-0.2	1894.8	-0.5
92	167	259	238	1908.1	1903.9	-0.2	1897.5	-0.6
92	168	260	238	1912.0	1908.2	-0.2	1901.5	-0.6
92	169	261	238	1915.9	1911.0	-0.3	1904.0	-0.6
92	170	262	238	1919.7	1915.1	-0.2	1907.8	-0.6
92	171	263	238	1923.4	1917.7	-0.3	1910.1	-0.7
92	172	264	238	1927.0	1921.7	-0.3	1913.7	-0.7

92	173	265	238	1930.5	1924.1	-0.3	1915.8	-0.8
92	174	266	238	1933.9	1927.9	-0.3	1919.2	-0.8
92	175	267	238	1937.1	1930.2	-0.4	1921.2	-0.8
92	176	268	238	1940.3	1933.7	-0.3	1924.4	-0.8
92	177	269	238	1943.4	1935.9	-0.4	1926.2	-0.9
92	178	270	238	1946.4	1939.3	-0.4	1929.3	-0.9
92	179	271	238	1949.3	1941.2	-0.4	1930.9	-1.0
92	180	272	238	1952.1	1944.5	-0.4	1933.8	-0.9
92	181	273	238	1954.8	1946.3	-0.4	1935.2	-1.0
92	182	274	238	1957.4	1949.4	-0.4	1937.9	-1.0
92	183	275	238	1959.9	1951.0	-0.5	1939.2	-1.1
92	184	276	238	1962.3	1953.9	-0.4	1941.7	-1.1
92	185	277	238	1964.6	1955.4	-0.5	1942.8	-1.1
92	186	278	238	1966.8	1958.2	-0.4	1945.2	-1.1
92	187	279	238	1969.0	1959.6	-0.5	1946.2	-1.2
92	188	280	238	1971.0	1962.2	-0.4	1948.4	-1.2
92	189	281	238	1972.9	1963.4	-0.5	1949.2	-1.2
92	190	282	238	1974.7	1965.9	-0.4	1951.2	-1.2
92	191	283	238	1976.4	1966.9	-0.5	1951.8	-1.3
92	192	284	238	1978.0	1969.3	-0.4	1953.7	-1.2
92	193	285	238	1979.5	1970.2	-0.5	1954.2	-1.3
92	194	286	238	1981.0	1972.4	-0.4	1956.0	-1.3
92	195	287	238	1982.3	1973.2	-0.5	1956.3	-1.3
92	196	288	238	1983.5	1975.2	-0.4	1957.9	-1.3
92	197	289	238	1984.6	1975.9	-0.4	1958.1	-1.4
92	198	290	238	1985.6	1977.8	-0.4	1959.5	-1.3
92	199	291	238	1986.6	1978.3	-0.4	1959.6	-1.4
92	200	292	238	1987.4	1980.1	-0.4	1960.9	-1.4
92	201	293	238	1988.1	1980.5	-0.4	1960.8	-1.4
92	202	294	238	1988.8	1982.2	-0.3	1962.0	-1.4
92	203	295	238	1989.3	1982.5	-0.3	1961.8	-1.4
92	204	296	238	1989.7	1984.0	-0.3	1962.8	-1.4
92	205	297	238	1990.1	1984.1	-0.3	1962.4	-1.4
92	206	298	238	1990.3	1985.5	-0.2	1963.3	-1.4
92	207	299	238	1990.4	1985.6	-0.2	1962.8	-1.4
92	208	300	238	1990.5	1986.9	-0.2	1963.5	-1.4

92	209	301	238	1990.4	1986.8	-0.2	1962.9	-1.4
92	210	302	238	1990.3	1987.9	-0.1	1963.5	-1.4
92	211	303	238	1990.0	1987.7	-0.1	1962.8	-1.4
92	212	304	238	1989.6	1988.8	0.0	1963.3	-1.3

11. CONCLUSION

Ghahramany's integrated nuclear binding energy formula can be modified to include strong and electroweak interaction features with a single energy coefficient. Based on relations (5), (6), (7) and (15) and data presented in Table 1 and Table 3, we are very confident to say that, quark structure and electroweak interaction play a vital role in fixing nuclear binding energy and it is absolutely possible to understand nuclear binding energy with one unique energy coefficient. In this context, to explore the ground reality, we appeal the science community to look into these kinds of unified and simplified nuclear binding energy models.

Data Availability statement

Energy coefficients of relations (8), (9) and (10) can be obtained from the following URL's.

- 1) Relation-8: <https://fddocuments.in/document/nuclear-and-particle-physics.html>
- 2) Relation-9: https://en.wikipedia.org/wiki/Semi-empirical_mass_formula
- 3) Relation-10: <http://oregonstate.edu/instruct/ch374/ch418518/lecture3-1.pdf>

Acknowledgements

Authors are very much thankful to Dr. N. Ghahramany and team and Dr. K.M. Khanna and team for their intuitive and heuristic contributions in this most advanced field of nuclear research. Author Seshavatharam is indebted to Professors Shri M. Nagaphani Sarma, Chairman, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

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