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# Series Solutions of Mathematical Modeling of Environmental Problems

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**ABSTRACT**

Environmental models are mathematical expressions that describe systems by capturing relationships between systems variables. Information from models is viewed as simplified concepts of environmental issues, thereby making for easy understanding by policy makers; this way decisions on environmental issues are quickly arrived at. This review discusses the types and development of mathematical models and provided examples of some models useful in environmental analysis.

***Keywords*:** Chemometrics, Environmental pollution, Environmental modeling, Mathematical modeling

## 1. INTRODUCTION

Utmost resolution problems are no longer well-structured problems that are calm to solve by experience or instinct sustained by moderately modest calculations [1, 2]. Even the similar kind of problems that was once calm to explain and solve, has now become much more difficult due to increased globalization of the economy as well as diversity in political, environmental, and social factors. In a quest for proffering solutions Current Resolution Makers (CRMs) classically want to assimilate knowledge from numerous areas of science and practice. But however, the culture, language, and tools established to signify awareness in these key areas (e.g., economy, finance, environment management, engineering, social, and political sciences)

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are diverse in real situations, Nevertheless, models can also misinform users (e.g., Scientists, Mathematicians, Physicist, and Engineers) by providing insufficient or wrong information. Such deception cannot only result to errors in model descriptions or undependable elements of software and operation, but also from the information collected, thereby setting up confusions amongst ideal users and developers' expectations. A serious element of model-based decision support such as mathematical models, which denotes data and relations that are too difficult to be sufficiently scrutinized based merely on experience and/or perception of a RM or his/her advisors.

Models, when appropriately established and sustained, can signify not only a part of knowledge of a RM but also participate related knowledge accessible from various disciplines and sources. Furthermore, models, if appropriately scrutinized, can help the DM to extend his/her knowledge and instinct.

## 2. WHAT IS MODELING?

Modeling can be defined as the process of request of essential information or define the routine of or involvement to simulate a real system to accomplish assured goals. Models can be proficient and lucrative tools whenever it is more reasonable to work with a temporary than with the actual, frequently difficult systems. Modeling has long been an integral component in creating, consolidating, and explaining explanations of a measurements from real systems and accepting their reasons and effects [3].

Modeling is the method of creating a model; a model is an illustration of the construction and working of some system of concern. A model is related to but simpler than the system it signifies. One persistence of a model is to enable the expert to predict the outcome of variations to the system. On one hand, a model should be a close estimate to the real system and join most of its noticeable structures. On the additional hand, it should not be so difficult that it is impossible to comprehend and experiment with it. A decent model is a thoughtful adjustment between simplicity and realism. Simulation experts commend increasing the involvedness of a model iteratively. A vital issue in modeling is model cogency.

## 3. THEORETICAL AND PHYSICAL MODELS

The model used in a car for instance is something that exists in our thoughts merely. Models of this type are named as theoretical models. Theoretical models are used by each of us to explain routine problems such as a car that declines to start. They are pragmatic by scientists or engineers to modest questions or problems related to a car for example. If their problem or question is difficult enough, though, they trust on trials, and this leads us to other forms of models. To see this, let us use the simulation and modeling system to define a possible technique followed by an engineer who wants to reduce the fuel depletion of an engine: In this occasion, the problem is the discount of fuel depletion and the system is the engine. Assume that the systems scrutiny leads the engineer to the assumption that the fuel addition pump needs to be enhanced. As previously, any decision strained from such a physical model relates to the simulation step of the above outline, and the decisions need to be authenticated by data acquired from the real system, that is data acquired from real plants in this case [4].

## 4. ESSENTIALS OF MATHEMATICAL MODELS

A “system” can be assumed as a group of one or more associated objects where an “object” can be a physical unit with precise qualities or appearances. A system is considered by the detail that the modeler can explain its limitations, its qualities, and its relations with the surroundings to the point that the subsequent model can content the modelers aims. Frequently, the bigger the system, the simpler the model. A system is calling a closed system when it does not relate with the surroundings, if cooperates with the surroundings, it is called an exposed system. In secure systems, neither mass nor energy will cross the limit in divergence with exposed systems. The qualities of the system and of the surroundings that have substantial impact on the system are termed “variables”. The term variable comprises those qualities that change in value during the modeling time duration and those that remain constant during that era. Variables of the final kind are frequently stated as “factors”.

### 4. 1. Physical Models

Physical modeling comprises signifying the real system by a geometrically and vigorously related, scaled model and leading trials on it to make explanations and measurements. The results from these trials are then generalized to real systems. Dimensional study and similarity models are used in the development to confirm that model results can be generalized to the real system with buoyancy. Generally, physical modeling had been the crucial method monitored by scientists in emerging the essential concepts of natural sciences.

### 4. 2. Empirical Models

Empirical modeling (or black box modeling) is created on an inductive or databased attitude, in which previous pragmatic data are used to advance relations among variables alleged to be important in the system being considered. Statistical tools are frequently used in this method to certify legitimacy of the expectations for real system. The resulting model is measured a “black box” imitating only what variations could be estimated in the system performance due to alterations in efforts. Even yet the effectiveness value of this method is restricted to expectations, it has recognized suitable in the case of difficult systems where the basic science is not well agreed.

### 4. 3. Mathematical Models

Mathematical modeling (or systematic modeling) is established on the logical or theoretical approach. Now, essential models and ideologies leading the system along with simplifying expectations are used to develop mathematical relations among the variables recognized to be substantial. The subsequent model can be regulated using historical data from the actual system and can be certified using extra data. Estimates can then be made with predefined buoyancy. In compare to empirical models, mathematical models imitate how changes in system routine are associated to changes in efforts. The occurrence of mathematical methods to model actual systems has improved many of the limits of physical and empirical modeling. Mathematical modeling, in principle, comprises the change of the system under study from its natural environment to mathematical environment in terms of intellectual symbols and equations. The symbols have well-defined significances and can be influenced following a severe set of rules or “mathematical calculi”.

**Table 1.** Typical Procedures of Mathematical Models used in solving problems in different environmental matrices [3].

| Environmental media | Concerns/ disquiets | Use of simulations |
|---|---|---|
| Atmosphere | Hazardous air pollutants, air emissions, toxic releases, acid rain; particulates, smog, CFCs, health anxieties | Concentration profiles; exposure; design and analysis of control processes and equipment; evaluation of management actions; environmental impact assessment of new projects; compliance with regulations |
| Surface water | Wastewater treatment plant discharge; industrial discharges; agricultural/urban runoff; storm water discharge; potable water source; food chain | Fate and transport of pollutants; concentration plumes; design and analysis of control processes and equipment; waste load allocations; evaluation of management actions; environmental impact assessment of new projects; compliance with regulations |
| Groundwater | Leaking underground storage tanks; leachates from landfills and agriculture; injection; potable water source | Fate and transport of pollutants; design and analysis of remedial actions; drawdowns; compliance with regulations |
| Subsurface | Land application of solid and hazardous wastes; spills; leachates from landfills; contamination of potable aquifers | Fate and transport of pollutants; concentration plumes; design and analysis of control processes; evaluation of management actions |
| Ocean | Sludge disposal; spills; outfalls; food chain | Fate and transport of pollutants; concentration plumes; design and analysis of control processes; evaluation of management actions |

Theoretical theories and process essentials are used to develop the equations that create connections among the system variables. By feeding recognized system variables as efforts, these equations or “models” can be explained to regulate a desired, unknown result. In the precomputer time, mathematical modeling could be pragmatic to model only those problems with secure form solutions: submission to difficult and dynamic systems was not realistic due to deficiency of computational tools. Presently, numerous different kinds of software authoring tools are commercially accessible for mathematical model building. They are ridiculous in with built in structures such as a library of preprogrammed mathematical functions and trials, user-friendly interfaces for data access and running, post –processing of outcomes such as plotting and animation, and high degrees of interactivity. Mathematical models can be categorized into several kinds subject on the nature of the variables, the mathematical method used, and the conduct of the system as follows:

i. Deterministic vs Probabilistic; contingent on certainty.
ii. Constant vs Distinct; contingent on endurance of variables in time province.
iii. Stationary vs Vigorous; contingent on fluctuating variables in time province.
iv. Dispersed vs Endured; contingent on varying variables in time and space provinces.
v. Linear vs Nonlinear; contingent on the variable power in leading equation.
vi. Analytical vs Numerical; contingent on secure form solution accessibility.

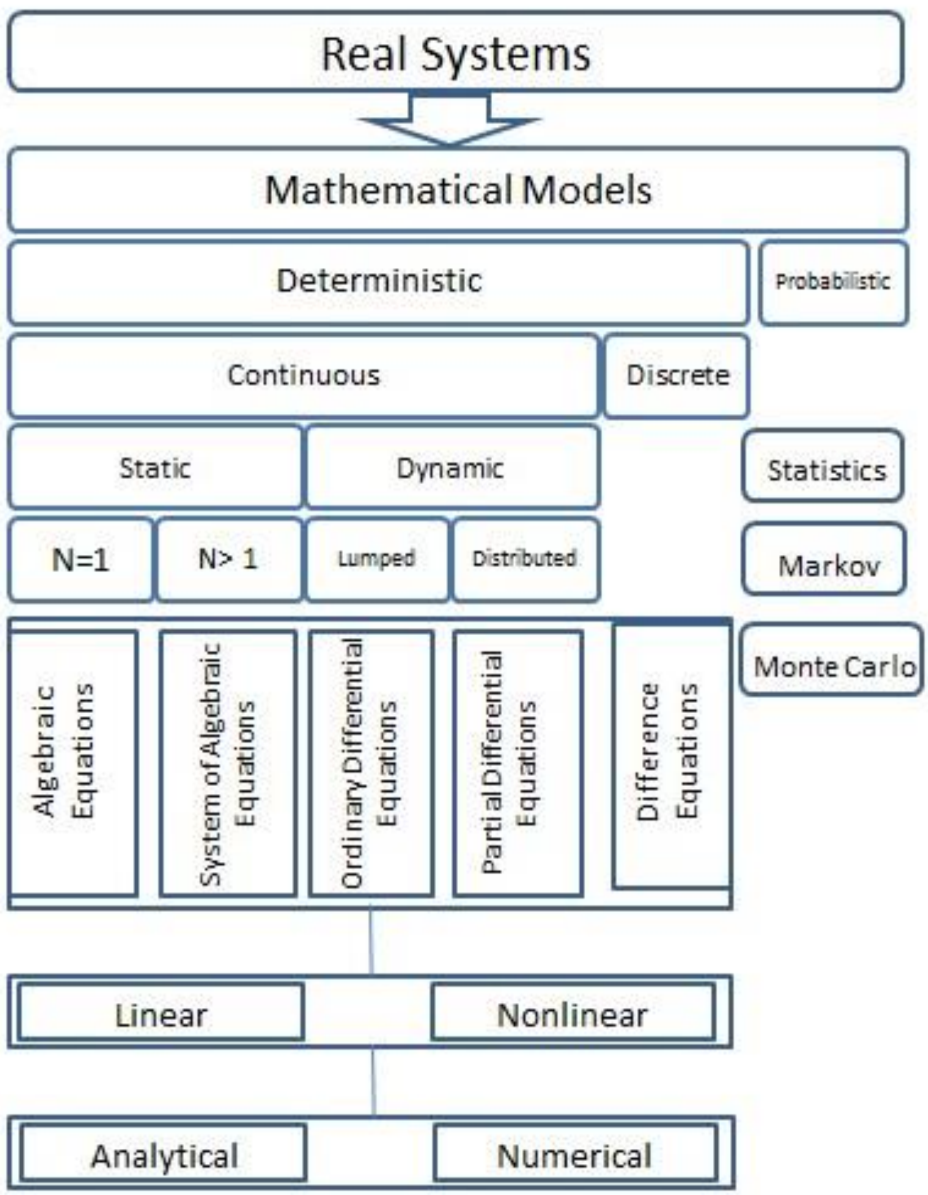


**Figure 1.** Arrangement of mathematical models (N = number of variables) [3].

Probabilistic models include statistical features, while deterministic models are built of algebraic and differential equations. In uninterrupted systems, deviations happen constantly as time progresses consistently, while in distinct models, variations happen only when distinct events occur, regardless of time path. In stationary models, the results are attained by a distinct computation of all equations while in vigorous models are acquired by repetitive computation of all equations as time permits. Endured stationary models are frequently built of algebraic equations; endured dynamic models are frequently built of ordinary differential equations; and dispersed models are frequently built of partial differential equations. When an equation comprises only one variable in each term and each variable seems only to the first power, that equation is named linear, if not, it is identified as nonlinear. Linear models content the norm of super-positioning.

When all the equations in a model can be solved algebraically to yield a solution in a secure form, the model can be categorized as analytical, if not, a numerical model is mandatory to solve system of equations. These arrangements are obtainable to stress the essential of understanding input data necessities, model creation, solution techniques, and to monitor in the choice of the suitable computer software tool in modeling system. Most environmental systems can be estimated in a reasonable manner by linear and time modified explanations in an endured or dispersed routine, at least for detailed and regulated conditions. Analytical solutions are imaginable for restricted kinds of systems; while computer created mathematical modeling using numerical solutions offer solutions for problems of difficult geometry and properties.

### 4. 4. Environmental Models

Mathematical models in the environmental field can be drawn to back to the 1900s, the original work of Streeter and Phelps on disbanded oxygen being the utmost quoted. Currently, determined essentially by adjusting forces, environmental studies have to be multidisciplinary, distributing with an inclusive range of pollutants experiencing difficult biotic and abiotic processes in the soil, surface water, groundwater, and atmospheric compartments of the ecosphere. In addition, environmental studies also contain similarly varied engineered devices and processes that relate with the natural environment through pathways. Subsequently, modeling of large-scale environmental systems is frequently a difficult and challenging task. The incentive for emerging environmental models can be one or more of the subsequent [3]:

i. To advance better understanding of and collect awareness into environmental processes and their impact on the chance and transport of pollutants in the environment.
ii. To regulate short- and long-term chemical concentrations in the numerous sections of the ecosphere for use in consistency, implementation, and in the valuation of experiences, influences, and threats of present as well as suggested chemicals.
iii. To guess imminent environmental concentrations of pollutants under numerous waste loadings and/or management substitutes.
iv. To please adjusting and constitutional requirements involving to environmental emissions, discharges, transfers, and issues of controlled pollutants.
v. To use hypothesis testing linking to processes, pollution control replacements, etc.
vi. To implement in the design, operation, and optimization of reactors, methods, pollution control replacements, etc.
vii. To simulate difficult systems at real, compacted, or prolonged time horizons that may be too hazardous, too costly, or too intricate to study under actual situations.

viii. To produce data for post-processing, such as numerical study, conception study, and animation, for better understanding, communication, and dissemination of scientific information.

ix. To use environmental impact valuation of planned new events that is presently imaginary.

In environmental systems, nonlinearities and three-dimensional and sequential lags overcome. Nevertheless, all too repeatedly these system features are moved to the sidelines of scientific studies. As a concern, the occurrence of nonlinearities and three-dimensional and progressive lags knowingly eases the capacity of these studies to provide perceptions that are essential to make appropriate conclusions about the management of difficult ecological–economic systems. Novel modeling methods are essential to efficiently recognize, accumulate, and relate the info that is significant for understanding those systems, to make consensus building an integral part of the modeling process, and to guide management conclusions.

## 5. WHAT IS SIMULATION?

A simulation of a method is the procedure of a model of the method. The model can be reconfigured and tested with; frequently, this is difficult, too costly or unrealistic to do in the method it signifies. The operation of the model can be deliberate, and therefore, properties regarding the conduct of the definite method or its sub method can be concluded. In its widest intellect, simulation is a tool to estimate the performance of a system, existing or suggested, under different structures of interest and over elongated phases of real time. Simulation is used before a current method is changed or a new method built, to ease the probabilities of catastrophe to meet stipulations, to eliminate unexpected bottlenecks, to avoid under or over-utilization of resources, and to enhance system performance. For example, simulation can be used to answer enquiries like: What is the best design for a new telecommunications network? What are the related resource necessities? How will a telecommunication network achieve when the circulation load rises by 60%? How will a new transmitting algorithm disturb its routine? Which network procedure adjusts network routine? What will be the effect of a relation catastrophe? The substance of this tutorial is distinct event simulation in which the central postulation is that the method variations rapidly in reaction to assured distinct occasions. For example, in an M/M/1 queue – a sole server lines up procedure in which time amongst appearances and service time are exponential - an appearance causes the method to change instantly. On the additional hand, uninterrupted simulators, like flight simulators and weather simulators, challenge to enumerate the variations in a system constantly over time in reply to controls.

Distinct incident simulation is less complete (thicker in its smallest time unit) than uninterrupted simulation but it is much simpler to implement, and therefore, is used in a varied change of situations. Figure 2 is a representation of a simulation study. The iterative nature of the process is shown by the system under study becoming the altered system which then develops the system under study and the cycle recurrences. In a simulation study, human decision making is essential at all points, specifically, model improvement, trial design, output study, decision formulation, and making decisions to change the system under study. The only phase where human involvement is not vital is the running of the simulations, which most simulation software correspondences perform well. The significant idea is that influential simulation software is simply a hygiene aspect - its deficiency can upset a simulation study but its occurrence will not certify achievement. Proficient problem formulators and simulation modelers and experts are crucial for a prosperous simulation study. The stages elaborate in emerging a simulation model, planning a simulation trial, and acting simulation study are, Figure 3:

Stage 1. Recognize the problem.
Stage 2. Articulate the problem.
Stage 3. Gather and process actual system data.
Stage 4. Articulate and improve a model.
Stage 5. Confirm the model.
Stage 6. Document model for imminent use.
Stage 7. Choice suitable trial design.
Stage 8. Create new situations for runs.
Stage 9. Execute simulation runs.
Stage 10. Deduce and present results.
Stage 11. Applaud additional course of action.

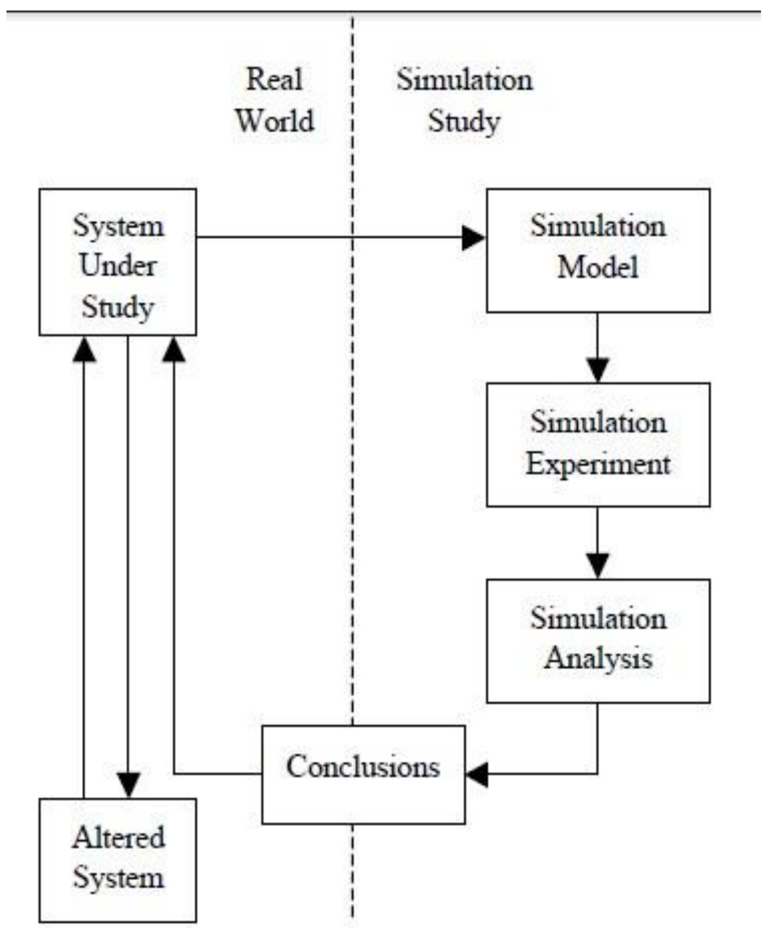


**Figure 2.** Simulation study

Though this is a rational collation of stages in a simulation study, much iteration at various sub-stages may be essential before the ideas of a simulation study are attained. Not all the stages may be promising and/or essential. On the extra hand, further stages may have to be achieved.

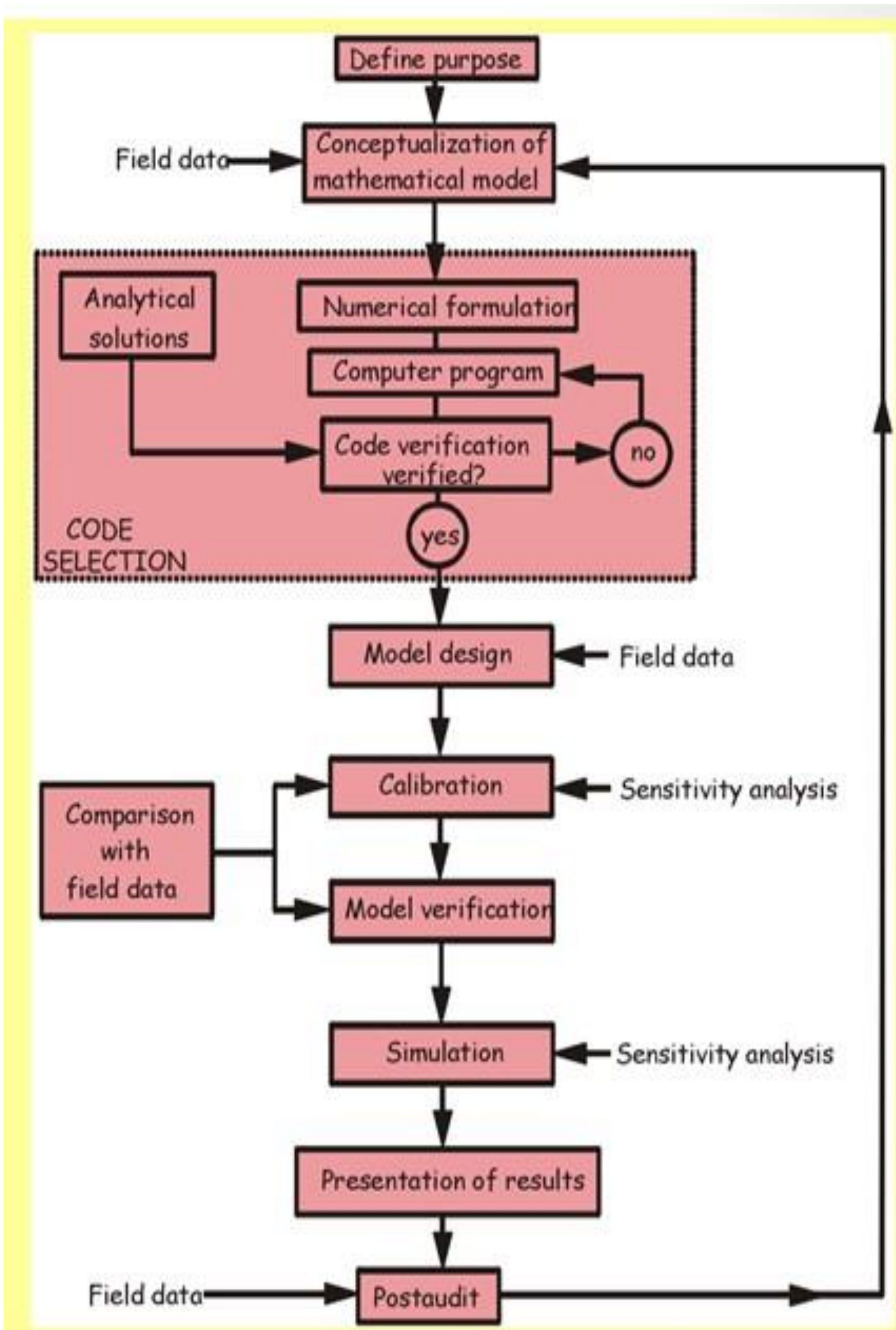


**Figure 3.** Modeling process flow chart.

## 6. ENVIRONMENTAL PROBLEMS

Actual management and development of resources and environmental methods have been of anxieties in the past eras subsequently infection and resource-scarcity problems have managed to a change of influences and responsibilities. Nevertheless, accomplishing a rational and proficient management scheme is complex since many contradictory factors have to be composed due to difficulties of the real-world problems. In resources and environmental systems, there are a number of factors that need to be deliberated by developers and decision-makers, such as social, economic, technical, legislation, institutional, and political problems, as well as environmental safety and resources protection. Furthermore, a variation of procedures and doings are related to each other, consequential in complex systems with collaborating, dynamic, nonlinear, multi-objective, multistage, multilayer, and indeterminate structures. These difficulties may be additional improved due to their connotation with economic significances if the capacities of estimated targets are disrupted. Mathematical models are known as actual tools that could help study economic, environmental, and ecological influences of another pollution-control and resources-conservation actions, and consequently utility planners or decision-makers in formulating cost-effective management rules.

## 7. FUNDAMENTALS OF ENVIRONMENTAL PROCESSES

In Nigeria, the problems of non-eco-friendly and unsustainability arising from processes occurring in the environment and its compartments has become a major conflict of interest from the Academia/Scientific studies to law enforcement sectors down to our local communities. These processes are generated directly or indirectly either from Natural or anthropogenic activities of man such as over-exploitation of resources, population expansion as well over utilization and un-even distribution of natural resources. Environmental processes occur between the different environment compartments like water air-atmosphere, soil-sediments matrices and governs the transport, distribution, reactions and fate of chemical pollutants and nutrients within these matrices, it is dependent of one another in the sense that whatever happens in one compartment has a positive or negative impact in the other. Environmental processes occurring in the environment are vast and diversified but most occurring phenomena which is of great importance are global warming, climatic change and processes within the global water cycle, these processes shoulders disproportionate burden on both social and economic alongside with cultural and political dimensions. Therefore, understanding of these processes that occurs in the environment is a useful tool in analyzing an efficient interaction between the biological existence of man and its environment.

## 8. CLIMATIC CHANGE PROCESSES

Climatic change is referred to as a long-term process that result to a change in the statistical properties of a climatic system in a specified region over an extended period of time, typically decades or century. Not all reversible temporal changes of 100-150 years are considered as climatic change, it is only a climatic change when it is pellucid and permanent [5]. The period of time taken for climatic change to occur is also very vital, because of the amount of deviation from the average weather conditions as well as its impacts in the ecosystem given that the human life is directly linked to it [6]**.** Processes that result to changes is influenced by variations/ fluctuations in natural and anthropogenic activities. The natural processes are duly as a result of biogeographical processes like astronomical and extraterrestrial factors such as eccentricity changes in the Earth's orbit, fluctuations in orbital procession and solar radiations alongside with oblique variations in the plane of eclipse. anthropogenic activities such as industrialization, urbanization, gas flaring, deforestation, burning of fossil fuels, over exploitation of land, contaminating of water sources activities which emits harmful greenhouse gases (carbon dioxide ($CO_2$), Carbon fluorocarbon (CFCs), methane ($CH_4$), Nitrous oxide ($N_2O$), Sulfuric hexafluoride ($SF_6$)) into the atmosphere and reduces rate of atmospheric carbon absorption causes climatic change [7] Investigation by [8] demonstrated that emission of greenhouse gases through incessant anthropogenic would increase the likelihood of hunger and starvation by 80 million people across Africa by 2080. However, the impact of climatic change on the ecosystem is global but the effect is more felt by developing countries like Nigeria owing to inability to combat this menace [9].

The negative impact of climatic change includes intense drought, floods, rainstorm, windstorm, landslide, tsunamis, increasing high temperature, rainfall drop, starvation, hunger, famine, desert encroachment, sea level rise, population displacement and death [10]. Present analysis conducted by [11] showed Increased desertification and redundancy/impair in the growth of plant observed in Northeastern Nigeria was directly accounted to changes in the climate, statistical analysis investigated by [10] showed that the gradual increase of 1.1 °C higher than 0.74 °C in the global average temperature from the year 1901 – 2005 approximately after 105years resulted to a massive decline of 53% in the rainfall experienced by continental regions of northeastern Nigeria accompanied with increased rainfall experienced in coastal regions like Warri and Calabar, in addition, records from this analysis depicts a notable evidence of climate change in accordance to studies carried [12]. following the increased temperature and decline/inclination in rainfall experienced by certain areas has caused the instability and high melting of ice, rise in Sea level alongside with flood and erosion is the ultimate cause of flood and displaced properties experienced in areas like Borny, warri, Lagos, porthacourt and Calabar. It is evaluated that a meter rise in Sea level can displace over 14 million Nigerians [13, 14]. Increasing temperature/rainfall declination arising due to changes in the climate is the reason why certain northern parts like Yobe, Borno, Sokoto, Jigawa and Kastina in Nigeria is liable to increased desertification, thus reducing crop production and food supply. This has prompted migration of people to areas of less desertification like Lagos thereby compounding the already existing tense population problems, this was in collaboration with the studies done by [15] who stated that most destitute who migrate due desertification problems move today nearby Urban areas to beg for alms thus, reducing social and economic values. With all these negative influences arising from climatic change processes in the Environment, it is therefore an indication that there will be hard felt Impact on man and its Environment in future years to come [25-31].

## 9. GLOBAL WARMING

Global warming is directly linked to climatic change, solely as a collaborator during the emissions of harmful greenhouse gases that has proven to be the principal delinquent for atmospheric and ocean warming throughout the globe. Global warming is described as a subtle and structured increase in the average atmospheric temperature of Earth surface and oceans over a long duration. In view of understanding the concept of global warming it is very necessary to simply understand the amount sunlight reaching the earth surface and the flow of energy with the climatic matrices. The climatic matrices comprise of different compartment such as atmosphere, ocean, cryosphere, biosphere and geosphere which determine the flow and amount of incoming radiation reaching the earth surface and ocean. Nevertheless, all unsustainable human activities leading to the emissions of greenhouse gases basically is the explicit cause of ongoing Global warming. Human activities as earlier mentioned such as industrialization, transportation, burning of fossil emits greenhouse gases into the atmosphere, also activities like water pollution and deforestation which reduces rate of carbon absorption increases the concentrations of greenhouse gases in the atmosphere which in turn depletes the ozone layer and permits large quarter of solar radiations received by the earth surface. Although advanced nations who are the most responsible for this warming finds it easier to combat this menace, but developing countries such as Nigeria is unable to abate it due lack of organization and structure.

In Nigeria emissions of GHGs has increased tremendously over the last decade. As at 2018 as shown in Fig 4 below it is obvious that carbon dioxide contributed about 81% of the total gases that trap heat detected in the atmosphere, its annual emissions grew from 0.08 in 1960 to 0.65 metric tons per capital in 2016 as shown in Fig 4, its contribution based on the analysis in the different sectors of the economy as shown in Fig 5 suggest that transportation is the major contributing factor to the vast emissions of $CO_2$ in the atmosphere followed by electricity and industries.

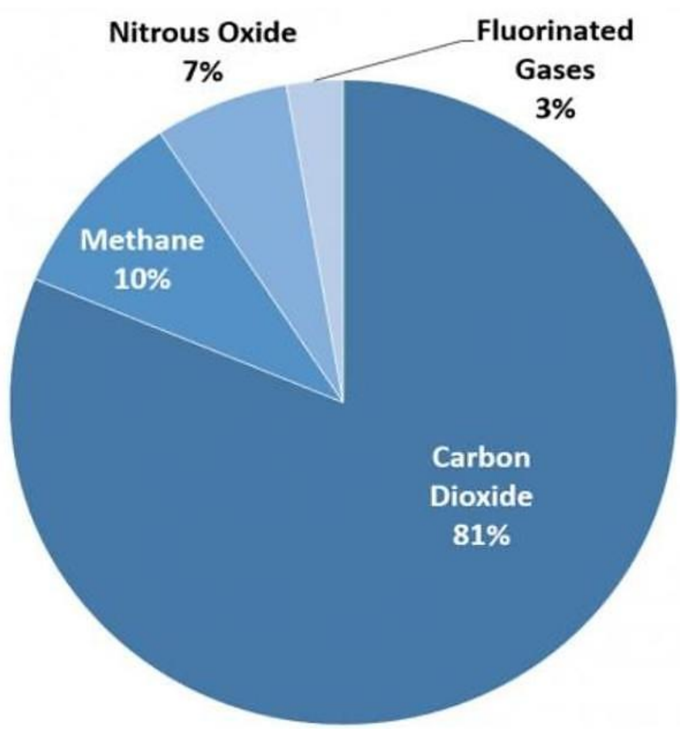


**Figure 4.** Showing global emission of anthropogenic greenhouse gases in Nigeria in 2018.

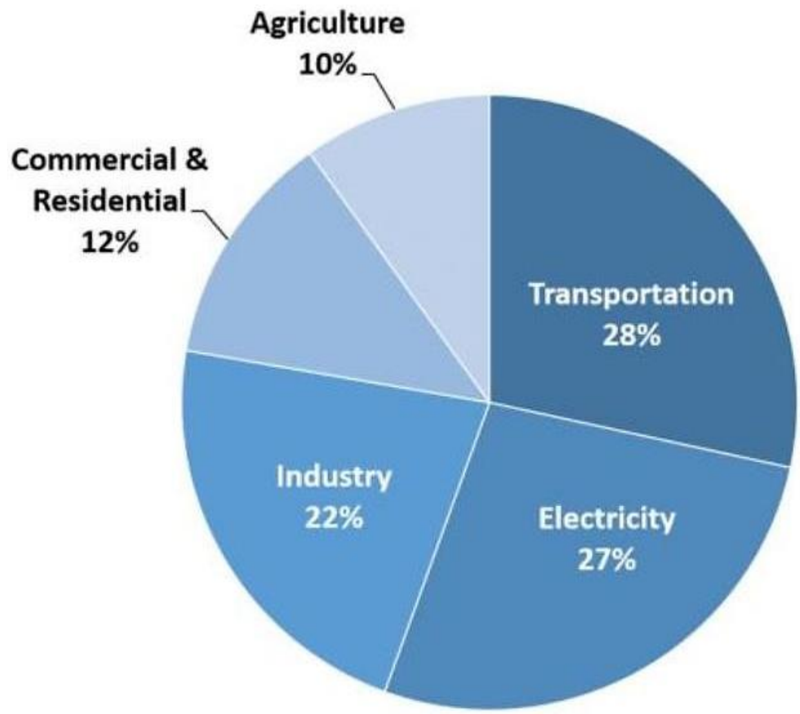


**Figure 5.** Showing global emission of anthropogenic sources of greenhouse gases in Nigeria in 2018 based on economy sectors


**Figure 6.** Show the level of production of goods contributes deposition of greenhouse gases

Truth be told it is quite obvious that in the Nigerian economy automobiles are the major contributing sources to air pollution this is because most vehicles imported are old or fairly used cars that emit a lot of carbon, also most commercial motorcyclist and vanagon bus drivers on the sake of their selfish interest to save fuel add engine oil to petrol which automatically converts to gasoline, although gasoline burns at a slower rate but it emits more carbon [16].

However due to inability for adequate power supplies the masses now opt to the use of generator sets for power which emits a lot of carbon into the atmosphere, generators couldn't even be banned following the bill passed by the federal government in 2020 due to the Nigeria's poor economy.

Nigeria in this 21$^{st}$ century has witnessed vast industrial revolution, machines with large exhaust fumes used in production of goods contributes deposition of greenhouse gases to a greater extent (Chukwu, 2011) in addition Nigeria has been recorded as the largest gas flaring that flares over 70% of methane [12]

However as earlier mentioned the same impact done by climatic change is also done by temperature increase or global warming such as the unusual deviations from normal seasonal environmental conditions for instance the August break (a month of no rainfall) observed during rainy season has ceased to exist, incessant fluctuations during harmattan period as well sea level rise. If the ongoing warming processes observed in the atmosphere and ocean continues, it may likely attain an advanced hot climatic position having great impact on man and the ecosystem.

## 10. PROFFERING SOLUTIONS TO ENVIRONMENTAL PROBLEMS USING MODELS

Processes occurring in the Environment is inter-related and often results to a multi complicated system with mutual, active, random, multi-faceted, discerning, multidimensional and undetermined features. Therefore, the need for Effective management in combating Environmental problems to bearest minimum is of great importance, to achieve a reliable and efficient management strategy is difficult, since there are so many conflicting factors that has to be balanced due to the complexity of the Environmental systems [17].

The use of modelling provides a platform for facilitating discussion of new and emerging technologies for supporting decisions pertaining environmental management and also focuses on exposition of innovative methods for solving problems in modelling different environmental problems, modelling is an integral component in organization, synthesis, rationalization and observations of measurements from real life situations as well as understanding their causes and effects.

Models allows you to predict the effect of changes in the system. An important issue in modelling is model validity, model validation requires simulating the model under already known input variable and thus comparing the output with fixed system output and only a mathematical model can carry out simulation study.

However, a lot of model approaches such physical, empirical, environmental and natural models has been implemented but Mathematical models are recognized as feasible tool that proffers strategic solutions to most environmental and ecological impacts of global warming and climatic change. Mathematical models are able to reflect various combination of these complexities by representing data and relations that are too complex to be adequately analyzed solely on reality.

## 11. MATHEMATICAL MODELS

The mathematical model or mechanistic approach of environmental systems refers to the mathematical expressions like symbols and equations used to describe factors that results to changes or transformation in environmental Processes with time, space and Condition control. It is of great significance to study the parameters of the established mathematical models in environmental processes, by making necessary adjustments, so that the various compartments can be developed in a more conducive system required by man. Environmental processes are usually based on theoretical deductive approach. These Basic theories and laws that governs the atmosphere are used in deriving of mathematical relationship between fluctuating gases in the atmosphere. mathematical models also reflect on how changes in the average weather condition are related to changes in the factors causing global warming and climatic changes [8].

## 12. STEPS IN DEVELOPING MATHEMATICAL MODELS FOR ENVIRONMENTAL QUALITY

The establishment of mathematical models for Environmental quality reflects totally on the laws governing the emissions of organic pollutants into the atmosphere, quite a handful of successful mathematical modelling approach have been used to analyze Environmental quality.

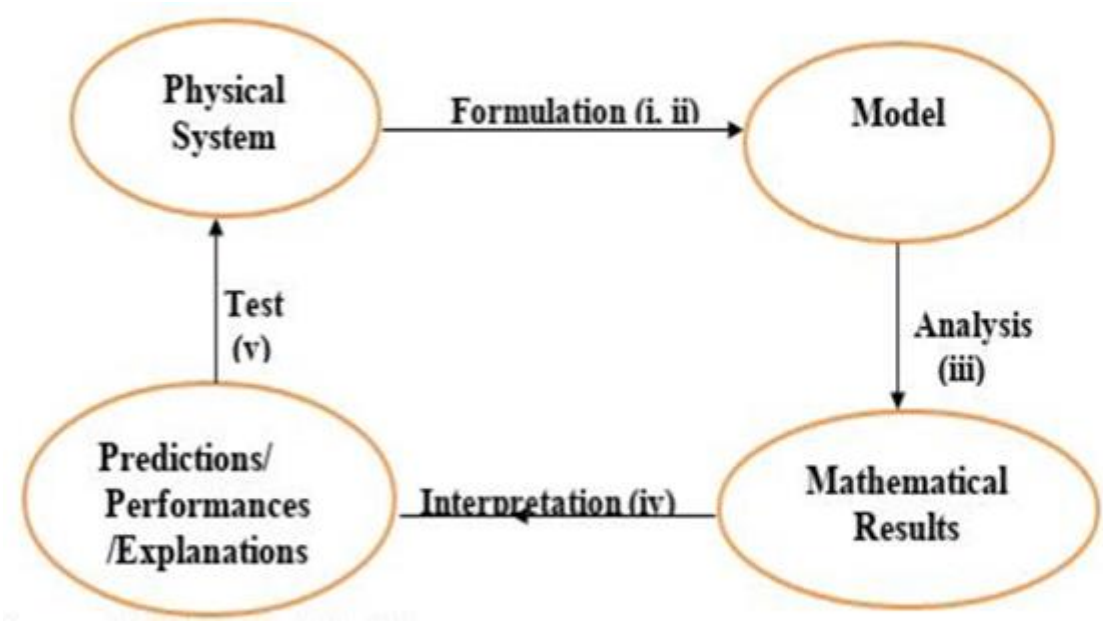


**Figure 7.** Process of Mathematical Modeling [18}

It is established accordingly in consensual procedure, although the reason for establishment May differ depending on the time and parameters of establishment, however the general procedure for developing a mathematical model is: 1. Draw a picture 2. Identify the mechanisms (i.e., how the process works). Make a list of assumptions. 3. Identify the pertinent variables and properties and specify a nomenclature. 4. correct and develop the appropriate differential equation(s) by performing mass, energy, and or momentum balances as appropriate and using the constitutive relationships (e.g., Newton's law of viscosity (relating stress to velocity), Fourier's law of heat conduction (relating heat flux to temperature), Fick's law of diffusion (relating species mass flux to concentration)). Add new assumptions to the list of assumptions. 5. Calibrate by Identifying restricting conditions (i.e., boundary and initial conditions) required to solve the differential equation(s). Add new assumptions to the list of assumptions. 6. non-dimensionalize the independent and dependent variables of the problem if possible. 7. Write down what you expect the solution to look like (e.g., draw a graph). 8. Solve the problem if possible.

If too complicated (or not solvable with the methods at hand), simplify and solve. Add all simplifying assumptions to the list of assumptions. 9. Check the solution to confirm that it gives sensible results. Confirm that: the solution satisfies the differential equation, the solution goes to the correct result at the boundaries and in limiting cases of time (e.g., at t = 0, short time, long time and t Æ infinity), a plot of the solution is sensible. 10. How does the solution behave compared with what you expected? If it is different, does the solution make sense?

If not (e.g., it predicts negative numbers when only positive numbers are possible), then re-examine what you did in Steps 2-9.

## 13. SQM SPACE OF MATHEMATICAL MODELS

Mathematical model description can be seen in the literature but their differences in definition are usually explained according to the author's scientific interests [3]. A mathematical model is said to be stationary when one of the systems parameters or its state variable rely on time and stationary otherwise. But when one of the systems parameters or state variables rely on a space variable and lumped otherwise, the mathematical model is said to be distributed [18]. The fact you understand where exactly you are in the space of mathematical models is where the practical use of a classification of mathematical models lies on [19].

Practical use of a classification of a mathematical model lies also on the fact that you understand the types the types of models that might be applicable to your problem.

The space of mathematical models consists of a system (S), question (Q), and a set of mathematical statement (M). It is normal to classify mathematical models in SQM space according to its definition [20].

At systems (S) we have; psychological systems, social systems, economic systems, biological systems, chemical systems, mechanical systems, and electrical systems. At question (Q) we have; speculation, prediction, analysis, control and design [21]. At set of mathematics, we have; algebraic equations (AEs) and differential equations (DEs). The black box at the extreme of the spectrum describes psychological system and social system. Due to the complexity of psychological system and social system, only vague phenomenological models can be developed [22].

However, other systems such as mechanical systems, electrical systems etc. are well understood as such they are placed at the white box end of the spectrum. Models which can be used to give more or less reliable predictions based on data are at the black box end of the spectrum while at the white box end of the spectrum, models are designed, tested, and optimized on a computer system in order to realize physically [23].

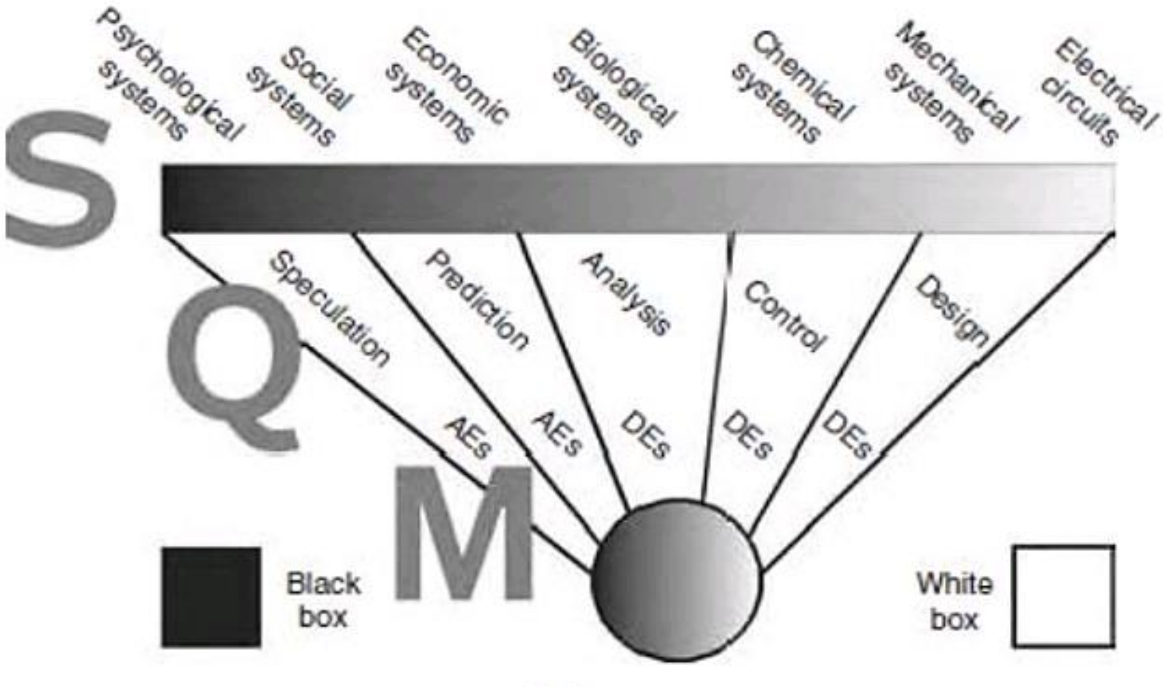


**Figure 8.** SQM space of mathematical models

## 14. GROUNDWATER FLOW AND SOLUTE TRANSPORT MODELING

Finite differences, grid discretization, triangulation, and finite elements always overwhelm people who are new to groundwater modeling [24]. When one gains a fundamental understanding of the terminologies and the procedures in groundwater modeling, question which one often face is what numerical method will be useful for one's project? But you should know that the answer is based on what software or method is always available but not always on what you should use. However, two main arithmetical methods which are finite difference method and finite element method should be considered. However, the groundwater flow has a general governing equation which represents the three dimensional unconfined, transient, heterogeneous and anisotropic groundwater flows [25]. The general governing equation is given as;

$$\frac{\partial}{\partial x}\left(Kx\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Ky\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(Kz\frac{\partial h}{\partial z}\right)=Ss\frac{\partial h}{\partial t}-R^{*} \quad (1)$$

where:

$Kx, Ky\ and\ Kz$ are hydraulic conductivity tensor;

$h$: Hydraulic head

$Ss$: Storage coefficient

$R$: Source or sink

Contaminant transport in groundwater was postulated in 1987, where Bear offered the fundamental equations. Advective transport and dispersive transport were the two components which Bear presented for hydrodynamic dispersion of the contaminant concentration.

The outcome of microscopic variation of velocity is an expression of dispersive flux which is a microscopic flux.

$$\partial \frac{(nc^k)}{\partial t} = \frac{\partial}{\partial x_i}\left(nD_{ij}\frac{\partial c^k}{\partial x_j}\right) - \frac{\partial}{\partial x_i}(nv_{si}c^k) + q_s c^k s + \sum R_n \tag{2}$$

where:

$C^k$ = Dissolved concentration of species$k$, $kgm^{-3}$

$n$ = Porosity of the subsurface medium, dimensionless

$t$ = Time, $s$

$x_i$ = Distance along the respective Cartesian coordinate axis, $m$

$D_{ij}$ = Hydrodynamic dispersion coefficient tensor, $m^2 s^{-1}$

$v_{si}$ = Seepage or linear pore water velocity, $ms^{-1}$

$q_s$ = Volumetric flow rate per unit volume of aquifer representing fluid sources (positive) and sinks (negative), $s^{-1}$

Seepage or linear pore water velocity relates to specific discharge or Darcy flux through the relationship $v_{si} = {q_i}/{n}$

$C^k s$ = Concentration of the source or sink flux for species $k, kgm^{-3}$

Note; dispersion in porous medium is the dispersal of contaminants over a superior region than would be predicted only from the normal groundwater velocity vectors.

## 15. CONCLUSION

Overall, the use of models to solve environmental problems is very important as it simplifies the problem and proffer information that could help in decision making. This study has provided some important models that can be used in solving environmental problems ranging from groundwater to land use.

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