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Octagonal Graceful Labeling of Some Special Graphs

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ABSTRACT

Numbers of the form $3n^2$-2n for all $n \geq 1$ are called octagonal numbers. Let G be a graph with p vertices and q edges. Let f :$V(G)\rightarrow\{0,1,2,\ldots,M_q\}$ where M_q is the q^{th} octagonal number be an injective function. Define the function f *: E(G) $\rightarrow$ $\{1,8,\ldots,M_q\}$ such that f *(uv) = $|f(u)\text{-}f(v)|$ for all edges $uv \in E(G)$. If f *($E(G)$) is a sequence of distinct consecutive octagonal numbers $\{M_1,M_2,\ldots,M_q\}$, then the function f is said to be octagonal graceful labeling and the graph which admits such a labeling is called a octagonal graceful graph. In this paper, octagonal graceful labeling of some graphs is studied.

Keywords: Octagonal graceful number, octagonal graceful labeling, octagonal graceful graphs

1. INTRODUCTION

Graphs considered in this paper are finite, undirected and simple. Let $G = (V, E)$ be a graph with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertex (edge/both) then the labeling is called a vertex (edge/total) labeling.

Rosa [1] introduced β-valuation of a graph. Golomb [8] called it as graceful labeling. Let G be a (p,q) graph. A one to one function f : V(G)$\rightarrow\{0,1,2,\ldots,q\}$ is called a graceful labeling of G if the induced edge labeling f ': E(G)$\rightarrow\{1,2,\ldots,q\}$ defined by f '(e) = $|f(\text{u})\text{-}f(\text{v})|$ for each

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e = uv of G is also one to one. The graph G possessing graceful labeling is called graceful graph. In [3], certain families of graceful graphs were constructed.

There are several types of graceful labeling and a detailed survey is found in [4]. The concept of octagonal graceful labeling was introduced by K. Kovusalya and P. Namasivayam in [5]. In this paper, octagonal graceful labeling of some other graphs is studied.

Labeled graphs are becoming an increasing useful family of mathematical models for a broad range of application like designing X-Ray crystallography, formulating a communication network addressing system, determining an optimal circuit layouts, problems in additive number theory etc. A systematic presentation of diverse applications of graph labeling is given in [6-42]. Following definitions are necessary for the present study.

Definition 1.1: Shrub St(n_1,n_2,…,n_m) is a graph obtained by connecting a vertex $v_0$to the central vertex of each of m numbers of stars.

Definition 1.2: Banana tree denoted by Bt(n_1,n_2,…,n_m) (m times n) is a graph obtained by connecting a vertex v_0 to one leaf of each of m number of stars.

Definition 1.3: A complete biparitite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges.

Definition 1.4: F-tree on n+2 vertices, denoted by FP_n, is obtained from a path P_n by attaching exactly two pendant vertices to the vertices n-1 and n of P_n.

Definition 1.5: Y-tree on n+1 vertices, denoted by Y_n, is obtained from a path P_n by attaching exactly a pendant vertex to the $(n-1)^{th}$ vertex of P_n.

Definition 1.6: Let $X_i \in N$. Then the caterpillar $S(X_1, X_2, \ldots, X_n)$ is obtained from the path $P_n$by joining $X_i$vertices to each of the i^{th} vertex of P_n ($1 \leq i \leq n$).

Definition 1.7: P_{n-1} (1,2,…,n) is a graph obtained from a path of vertices v_1,v_2,…,v_n having the path length n-1 by joining i pendant vertices at each of its i^{th} vertex.

Definition 1.8: Twig graph G is obtained from the path P_n by attaching exactly two pendant edges to each internal vertex of the path.

Definition 1.9: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 where G_1 has m vertices and n edges is defined as the graph G_1obtained by taking one copy of G_1 and m copies of G_2, and the joining by an edge the i^{th}vertex of G_1 to every vertex in the i^{th}copy of G_2.

Definition 1.10: A subdivision of a graph G is a graph that can be obtained from G by a sequence of edge subdivision.

Definition 1.11: A connected, acyclic graph is called tree.

Definition 1.12: Numbers of the form $3n^2$-2n for all $n \geq 1$ are called octagonal numbers. The first few octagonal numbers are 1, 8, 21, 40, 65, 96, 133, 176,…

Definition 1.13: Let G be a graph with p vertices and q edges. Let $f:V(G)\rightarrow\{0,1,\ldots,M_q\}$ where M_q is the q^{th} octagonal number be an injective function. Define the function f^*: E(G) $\rightarrow \{1,8,\ldots,M_q\}$ such that $f^*(uv)=|f(u)-f(v)|$ for all edges $uv\in E(G)$. If $f^*(E(G))$ is a sequence of distinct consecutive octagonal numbers $\{M_1,M_2,\ldots,M_q\}$ then the function f is said to be octagonal graceful labeling and the graph which admits such a labeling is called a octagonal graceful graph.

2. RESULTS

Previous result 2.1:

(i) caterpillar $S(X_1, X_2, \ldots, X_n)$ is octagonal graceful.

Corollary 2.2: When $X_i = m$, $1 \leq i \leq n$, the graph $P_n \odot \overline{K_n}$ is octagonal graceful for all $n \geq 2$ and $m \geq 1$.

Example 2.3: Octagonal graceful labeling of $P_2 \odot \overline{K_2}$ is shown in Fig. 1.

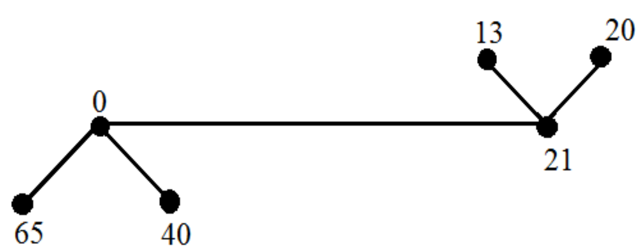

Fig. 1

Corollary 2.4: When m = 1, the graph $P_n \odot K_1$ is called a comb. Comb is octagonal graceful.

Example 2.5: Octagonal graceful labeling of $P_3 \odot K_1$ is shown Fig. 2.

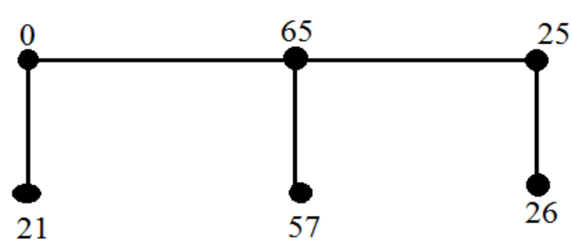

Fig. 2

Corollary 2.6: P_{n-1} $(1,2,\ldots,n)$ is octagonal graceful.

Example 2.7: Octagonal graceful labeling of P_3 (1,2,3) is shown Fig. 3.

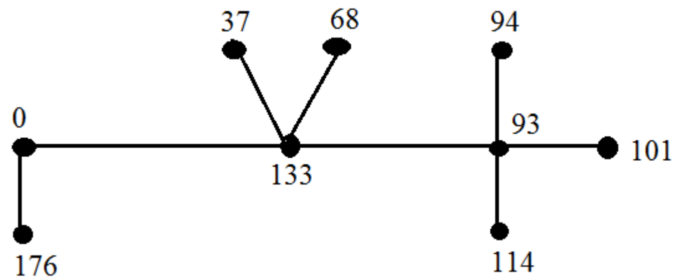

Fig. 3

Corollary 2.8: Twig graph is octagonal graceful.

Example 2.9: Octagonal graceful labeling of twig graph is obtained from the path P_4 is shown in Fig. 4.

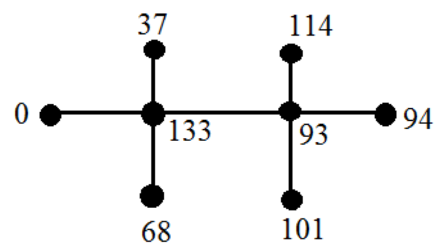

Fig. 4

Theorem 2.10: Shrub St(n_1,n_2,…,n_m) is octagonal graceful.

Proof: Let G be the graph St(n_1,n_2,…,n_m).

Let V(G) = $\{v, v_i, v_{ij} : 1 \leq i \leq m , 1 \leq j \leq n_i\}$ and E(G) = $\{vv_i , v_iv_{ij} : 1 \leq i \leq m , 1 \leq j \leq n_i\}$.

G has $m + n_1 + n_2 + \ldots + n_m + 1$ vertices and $m + n_1 + n_2 + \ldots + n_m$ edges.

Let $q = m + n_1 + n_2 + \ldots + n_m$.

Let f : V(G)$\rightarrow\{0,1,2,\ldots,M_q\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = M_{q-[n_1+n_2+\ldots+n_{i-1}+i-1]}$; $1 \leq i \leq m$.

$f(v_{ij}) = M_{q-[n_1+n_2+\ldots+n_{i-1}+i-1]} - M_{q-[n_1+n_2+\ldots+n_{i-1}+(i-1)+(j+i-1)]}$; $1 \leq i \leq m$, $1 \leq j \leq n_i$.

Let f^* be the induced edge labeling of f.

Then $f^*(vv_i) = M_{q-[n_1+n_2+\ldots+n_{i-1}+i-1]}$; $1 \leq i \leq m$.

$f^*(v_iv_{ij}) = M_{q-[n_1+n_2+\ldots+n_{i-1}+(i-1)+(j+i-1)]}$; $1 \leq i \leq m$, $1 \leq j \leq n_i$.

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers.

Hence the Shrub is octagonal graceful.

Example 2.11: Octagonl graceful labeling of St(2,3,2) is given in Fig. 5.

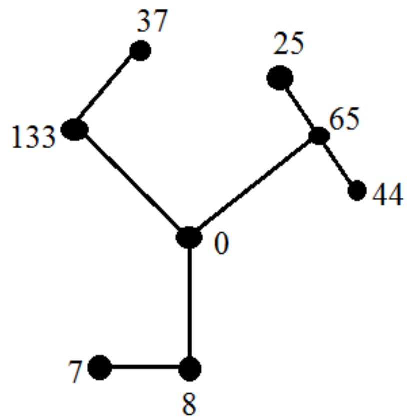

Fig. 5

Theorem 2.12: Banana tree Bt(n_1,n_2,…,n_m) is octagonal graceful.

Proof: Let G be the graph Bt(n_1,n_2,…,n_m) .

Let V(G) = {v,v_i,w_i, w_{ij} : $1 \leq i \leq m$, $1 \leq j \leq n_i - 1$} and

E(G) = { vv_i,v_iw_i ,w_iw_{ij} : $1 \leq i \leq m$, $1 \leq j \leq n_i - 1$}.

G has $m + n_1 + n_2 + \ldots + n_m + 1$ vertices and $m + n_1 + n_2 + \ldots + n_m$ edges.

Let q = $m + n_1 + n_2 + \ldots + n_m$.

Let f : V(G)→{0,1,2,…,M_q} be defined as follows

$f(v) = 0$

$f(v_i) = M_{q-i+1}$; $1 \leq i \leq m$.

$f(w_i) = f(v_i) - M_{q-m-[n_1+n_2+\ldots+n_{i-1}]}$; $1 \leq i \leq m$.

$f(w_{ij}) = f(w_i) + M_{q-m-[n_1+n_2+\ldots+n_{i-1}]-j}$; $1 \leq i \leq m$, $1 \leq j \leq n_i - 1$.

Let f^* be the induced edge labeling of f.

Then $f^*(vv_i) = M_{q-i-1}$; $1 \leq i \leq m$.

$f^*(v_iw_i) = M_{q-m-[n_1+n_2+\ldots+n_{i-1}]}$; $1 \leq i \leq m$.

$f^*(w_iw_{ij}) = M_{q-m-[n_1+n_2+\ldots+n_{i-1}]-j}$; $1 \leq i \leq m$, $1 \leq j \leq n_i - 1$.

The induced edge labels M_1,M_2,… , M_q are distinct and consecutive octagonal numbers.

Hence the Banana tree is octagonal graceful.

Example 2.13: Octagonal graceful labling of Bt(2,3) is given in Fig. 6.

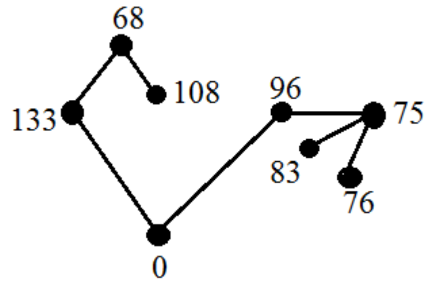

Fig. 6

Theorem 2.14: The star $K_{1,n}$ is octagonal graceful for all n.

Proof: Let V($K_{1,n}$) = {u_i : 1≤ i≤ n+1}.

Let E($K_{1,n}$) = { u_{n+1} u_i : 1≤ i≤ n}.

Define an injection f : V($K_{1,n}$)→{0,1,2,3…, M_q } by f (u_i) = M_i if 1≤ i≤ n and f(u_{n+1}) = 0.

Then f induces a bijection f_p : E($K_{1,,n}$) →{1,8,21,…, M_q }.

Hence the star $K_{1,n}$ is octagonal graceful for all n.

Example 2.15: A octagonal graceful labeling of star $K_{1,8}$ is given in Fig. 7.

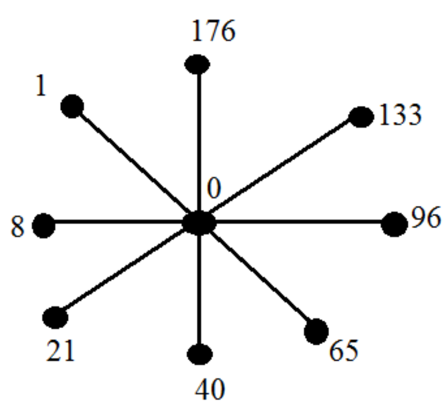

Fig. 7

Theorem 2.16: $K_{1,n} \odot K_1$ is octagonal graceful.

Proof: Let G be the graph $K_{1,n} \odot K_1$.

Let V(G) = {v, v_i, u_i ,w : $1 \le i \le n$} and E(G) = {vv_i, v_iu_i , vw: $1 \le i \le n$}.

G has $2n + 2$ vertices and $2n + 1$ edges.

Let q = $2n + 1$.

Let $f : V(G) \to \{0,1,2,\ldots,M_q\}$ be defined as follows

$f(v) = 0$

$f(v_i) = M_{q-(i-1)}$; $1 \le i \le n$

$f(w) = M_{q-n}$

$f(u_i) = f(v_i)$ - $M_{q-(i-1)}$; $1 \le i \le n$.

Let f^* be the induced edge labeling of f.

Then $f^*(vv_i) = M_{q-(i-1)}$; $1 \le i \le n$.

$f^*(vw) = M_{q-n}$.

$f^*(v_iu_i) = M_{q-(i-1)}$; $1 \le i \le n$.

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers.

Hence $K_{1,n} \odot K_1$ is octagonal graceful.

Example 2.17: Octagonal graceful labeling of $K_{1,4} \odot K_1$ is given in Fig. 8.

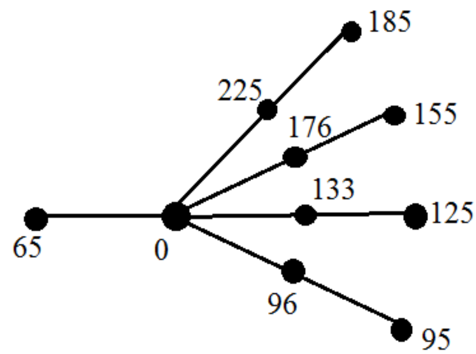

Fig. 8

Theorem 2.18: Let G be the graph obtained by identifying the leaves of $K_{1,n}$ with the central vertex of $K_{1,2}$. Then G is octagonal graceful for all $n \ge 1$.

Proof: Let G be the graph obtained by identifying the leaves of $K_{1,n}$ with the central vertex of $K_{1,2}$.

Let $V(G) = \{v, v_i, v_{ij} : 1 \le i \le n , 1 \le j \le 2\}$ and $E(G) = \{vv_i , v_iv_{ij} : 1 \le i \le n , 1 \le j \le 2\}$.

G has $3n + 1$ vertices and $3n$ edges.

Let q = $3n$.

Let $f : V(G) \to \{0,1,2,\ldots,M_q\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = M_{3(n-(i-1))}$; $1 \leq i \leq n$.

$f(v_{ij}) = f(v_i) - M_{q-(i-1)n-j}$; $1 \leq i \leq n$, $1 \leq j \leq 2$.

Let f^* be the induced edge labeling of f.

Then $f^*(vv_i) = M_{3(n-(i-1))}$; $1 \leq i \leq n$.

$f^*(v_i v_{ij}) = M_{q-(i-1)n-j}$; $1 \leq i \leq n$, $1 \leq j \leq 2$.

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers.

Hence G is octagonal graceful for all $n \geq 1$.

Example 2.19: Octagonal graceful labeling of $K_{1,3} \odot K_{1,2}$ is given in Fig. 9.

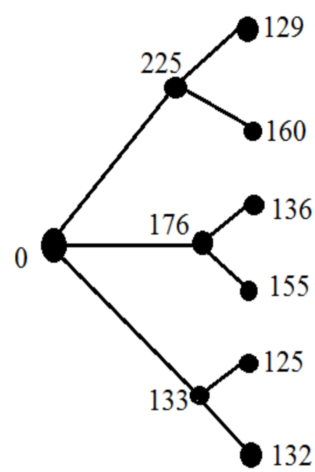

Fig. 9

Theorem 2.20: *F*-tree FP_n , $n \geq 3$ is octagonal graceful.

Proof: Let G be FP_n , $n \geq 3$.

Let V(G) = { u, v, v_i : $1 \leq i \leq n$} and E(G) = {$v_i v_{i+1}$: $1 \leq i \leq n$-1 }∪{ uv_{n-1}, vv_n}.

G has n +2 vertices and n + 1 edges.

Let $q = n + 1$.

Let f : V(G)→{0,1,2,…,M_q} be defined as follows

$f(v_1) = 0$

$f(v_i) = f(v_{i-1}) - M_{q-i+2}$ if i is odd and $2 \leq i \leq n$.

$= f(v_{i-1}) + M_{q-i+2}$ if i is even and $2 \leq i \leq n$.

$f(v) = f(v_n) - M_1$

$f(u) = f(v_{n-1}) - M_2$

Let f^* be the induced edge labeling of f.

Then $f^*(v_i v_{i+1}) = M_{q-i+1}$; $1 \le i \le n-1$.

$f^*(uv_{n-1}) = M_2$

$f^*(vv_n) = M_1$

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers..

Hence F-tree FP_n , $n \ge 3$ is octagonal graceful.

Example 2.21: Octagonal graceful labeling of FP_4 is given in Fig. 10.

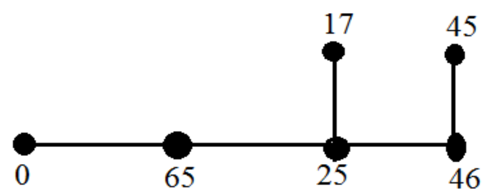

Fig. 10

Theorem 2.22: A Y-tree is octagonal graceful.

Proof: Let G be the Y-tree.

Let $V(G) = \{v, v_i : 1 \le i \le n\}$ and $E(G) = \{v_i v_{i+1}, vv_{n-1} : 1 \le i \le n-1\}$.

G has $n+1$ vertices and n edges.

Let $q = n$.

Let $f : V(G) \to \{0,1,2,\ldots,M_q\}$ be defined as follows

$f(v_1) = 0$

$f(v_i) = f(v_{i-1}) - M_{q-i+2}$ if i is odd and $2 \le i \le n$.

$\quad = f(v_{i-1}) + M_{q-i+2}$ if i is even and $2 \le i \le n$.

$f(v) = f(v_{n-1}) - M_1$

Let f^* be the induced edge labeling of f.

Then $f^*(v_i v_{i+1}) = M_{q-i+1}$; $1 \le i \le n-1$.

$f^*(vv_{n-1}) = M_1$

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers.

Hence the Y-tree is octagonal graceful.

Example 2.23: Octagonal graceful labeling of Y_4 is given in Fig. 11.

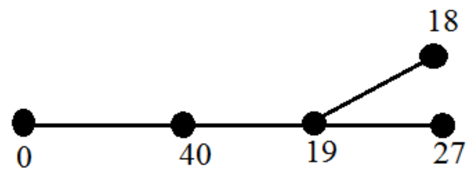

Fig. 11

Theorem 2.24: Let G be the graph obtained by identifying a pendant vertex of P_m with a leaf of $K_{1,n}$. Then G is octagonal graceful for all m ≥ 2and n ≥ 1.

Proof: Let G be the graph obtained by identifying the pendant vertex v_1 of P_m with a leaf u_n of $K_{1,n}$.

Let V(G) = {u,u_i,v_j : $1 \leq i \leq n$-1, $1 \leq j \leq m$ } and $E(G)$ = {uu_i,uv_1,$v_j v_{j+1}$: $1 \leq i \leq n$-1, $1 \leq j \leq m$-1 }.

G has m + n vertices and m + n – 1 edges.

Let q = m + n - 1.

Let f : V(G)→{0,1,2,…,M_q} be defined as follows

$f(u) = 0$

$f(u_i) = M_{q-(i-1)}$;$1 \leq i \leq n$

$f(v_1) = M_m$

$f(v_j) = f(v_{j-1}) + M_{n-(j-2)}$ if j is odd $2 \leq j \leq m$.

$= f(v_{j-1}) - M_{n-(j-2)}$ if j is even $2 \leq j \leq m$.

Let f^* be the induced edge labeling of f.

Then $f^*(uu_i) = M_{q-(i-1)}$; $1 \leq i \leq n$-1.

$f^*(uv_1) = M_m$

$f^*(v_j v_{j+1}) = M_{m-j}$;$1 \leq j \leq m$-1.

The induced edge labels M_1,M_2,… , M_q are distinct and consecutive octagonal numbers.

Hence G is octagonal graceful for all m ≥ 2and n ≥ 1.

Example 2.25: Octagonal graceful labeling graph obtained by identifying a pendant vertex of P_3 with a leaf of $K_{1,5}$ is given in Fig. 12.

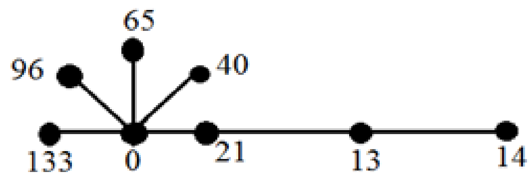

Fig. 12

Theorem 2.26: The graph obtained by subdividing the edges of the star $K_{1,n}$ is octagonal graceful for all $n \geq 1$.

Proof: Let G be the graph obtained by subdividing the edges of the star $K_{1,n}$ for all $n \geq 1$.

Let $V(G) = \{u, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{uv_i, v_iu_i : 1 \leq i \leq n\}$.

G has 2n + 1 vertices and $2n$ edges.

Let $q = 2n$.

Let $f : V(G) \rightarrow \{0,1,2,\ldots,M_q\}$ be defined as follows

$f(u) = 0$

$f(v_i) = M_{q-i+1}$; $1 \leq i \leq n$

$f(u_i) = f(v_i) - M_{n-i+1}$; $1 \leq i \leq n$

Let f^* be the induced edge labeling of f.

Then $f^*(uv_i) = M_{q-i+1}$; $1 \leq i \leq n$.

$f^*(v_iu_i) = M_{n-i+1}$; $1 \leq i \leq n$.

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers.

Hence the graph G is octagonal graceful for all $n \geq 1$.

Example 2.27: Octagonal graceful labeling of the graph obtained by subdividing the edges of the star $K_{1,5}$ is shown in Fig. 13.

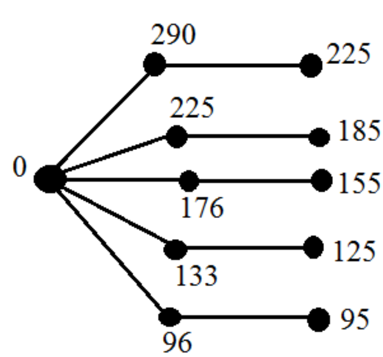

Fig. 13

Theorem 2.28: The graph obtained from $P_n \odot K_1$ by subdividing the edges of the pah P_n is octagonal graceful for all $n \geq 2$.

Proof: Let G be the graph obtained from $P_n \odot K_1$ by subdividing the edges of the pah P_n.

Let V(G) = $\{v_i, u_i, w_j : 1 \leq i \leq n, 1 \leq j \leq n\text{-}1\}$ and

E(G) = $\{v_i w_i, v_j u_j, w_k w_{k+1} : 1 \leq i \leq n\text{-}1, 1 \leq j \leq n, 1 \leq k \leq n\text{-}1\}$.

G has 3n - 1 vertices and $3n$ - 2 edges.

Let q = $3n - 2$.

Let f : V(G) → $\{0,1,2,\ldots,M_q\}$ be defined as follows

$f(v_1) = 0$

$f(v_i) = f(w_{i-1}) - M_{q-1-(2(i-2))}$; $2 \leq i \leq n$

$f(w_j) = f(v_j) + M_{q-2(j-1)}$; $1 \leq j \leq n\text{-}1$

$f(u_i) = f(v_i) + M_{n-i+1}$; $1 \leq i \leq n$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_i w_i) = M_{q-2(i-1)}$; $1 \leq i \leq n\text{-}1$.

$f^*(v_j u_j) = M_{n-j+1}$; $1 \leq j \leq n$.

$f^*(w_k w_{k+1}) = M_{q-2k+1}$; $1 \leq k \leq n\text{-}1$.

The induced edge labels $M_1, M_2, \ldots, M_q$ are distinct and consecutive octagonal numbers.

Hence the graph G is octagonal graceful for all $n \geq 2$.

Example 2.29: Octagonal graceful labeling of $P_3 \odot K_1$ by subdividing the edges of the path P_3 is shown in Fig. 14.

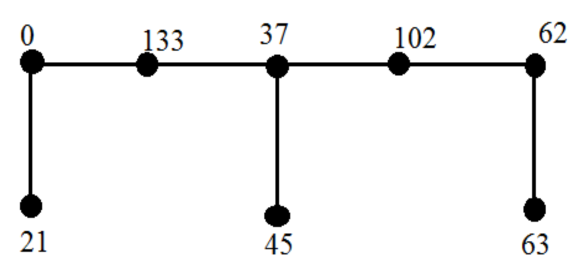

Fig. 14

3. CONCLUSION

In this paper, the authors studied the octagonal graceful labeling of some graphs. Similar study can be extended for other graphs. The octagonal graceful can be verifed for many other graphs. Also some more octagonal graceful labrling can be investigated.

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