Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 156 (2021) 176-191 EISSN 2392-2192

Some results on centered triangular graceful graphs

M. Baskar[1], **P. Namasivayam**[2], **M. P. Syed Ali Nisaya**[3] and **S. Mahendran**[4]

P.G. & Research Department of Mathematics, The Madurai Diraviyam Thayumanavar Hindu College, Tirunelveli, Tamil Nadu, India

[1-4]E-mail address: baskar542000@gmail.com , vasuhe2010@gmail.com , syedalinisaya@mdthinducollege.org , mahe1999bsc@gmail.com

ABSTRACT

Let G be a graph with p vertices and q edges. The n^{th} centered triangular number is denoted by C_n, where $C_n = \frac{1}{2}(3n^2 - 3n + 2)$. A centered triangular graceful labeling of a graph G is a one-to-one function f : V (G) → $\{0,1, \ldots C_q\}$ that induces a bijection f^*: E(G) →$\{C_1, C_2, \ldots C_q\}$ of the edges of G defined by $f^*(e) = |f(u) - f(v)|$, for all e = uv ∊ E(G). The graph which admits such labeling is called a centered triangular graceful graph.

Keywords: Centered triangular numbers, centered triangular graceful labeling, centered triangular graceful graphs

1. INTRODUCTION AND DEFINITIONS

The graph considered in this paper are finite, undirected and without loops or multiple edges. Let G = (V, E) be a graph with p vertices and q edges. Undefined terms are used in the sense of Harary [1], Parthasarathy [2] and Bondy and Murthy [3]. For number theoretic terminology, we refer to [4] and [5].

Graph labeling is one of the fascinating areas of graph theory with wide ranging applications. Graph labeling was first introduced in 1960's. A graph labeling is an assignment

(Received 18 March 2021; Accepted 07 April 2021; Date of Publication 08 April 2021)

of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges / both) then the labeling is called the vertex (edge / total) labeling. Most popular graph labeling trace their origin to one introduced by Rosa [6]. Rosa called a function (labeling) f a β-valuation of a graph in the year 1966 and Golomb [7] called it as graceful labeling. There are several types of graph labeling and a detailed survey is found in [8].

The concept of a sum graph was introduced by Harary [9] in 1990 and was defined as a graph whose vertices can be labeled with distinct positive integers so that the sum of the labels on each pair of adjacent vertices is the label of some other vertex. In 1991, Harary et al. [10] defined a real sum graph. One of the earliest interesting results was due to Ellingham [11] who proved the conjecture of Harary [12].

Labeled graphs are becoming an increasing useful family of mathematical models for a broad range of application like designing X-Ray crystallography, formulating a communication network addressing system, determining an optimal circuit layouts, problems in additive number theory etc.

In [13], the concept of centered triangular sum labeling was introduced. Jeyanthi et al. [14] introduced centered triangular mean labeling. M.P. Syed Ali Nisaya [15-20] introduced centered triangular graceful labeling. In this paper, we have studied the centered triangular graceful labeling of some graphs. For more information related to graceful graphs, see [21-46].

The following definitions are necessary for present study.

Definition 1.1: A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) repectively. The number of elements of V(G) = p is called the order of G and the number of elements of E(G) = q is called the size of G. A graph of order p and size q is called a (p,q) - graph. If e = *uv* is an edges of G, we say that *u* and *v* are adjacent and that *u* and *v* are incident with e.

Definition 1.2: A connected acyclic graph is called a tree.

Definition 1.3: A Path P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \leq i \leq$ n-1.

Definition 1.4: The Fork graph, sometimes also called the chair graph, is the 5-vertex tree.

Definition 1.5: F- Tree on n+2 vertices denoted by FP_n, is obtained from a path P_n by attaching exactly two pendant vertices to the n-1 and nth vertex of P_n.

Definition 1.6: Y-tree on *n*+1 vertices, denoted by Y_n, is obtained from a path P_n by attaching exactly a pendant vertex to the (n-1)th vertex of P_n.

Definition 1.7: The complete bipartite graph $K_{1,n}$ is called a Star graph.

Definition 1.8: A graph in which any two distinct points are adjacent is called a complete graph. The complete graph with n points is denoted by K_n.

Definition 1.9: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 where G_1 has *m* vertices and *n* edges is defined as the graph G_1 obtained by taking one copy of G_1 and *m* copies of G_2, and the joining by an edge the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

Definition 1.10: A graph, which can be formed from a given graph G by breaking up each edge into exactly two segments by inserting intermediate vertices between its two ends, is called a sub division graph. It is denoted by S (G).

Definition 1.11: A caterpillar is a tree with a path P_m: $v_1, v_2, \ldots, v_m$, called spine with leaves (pendant vertices) known as feet attached to the vertices of the spine by edges known as legs. If every spine vertex v_i is attached with n_i number of leaves then the caterpillar is denoted by $S(n_1, n_2, \ldots, n_m)$.

Definition 1.12: The double star graph ST(*n* ,*m*) is a graph that is formed by two stars ST(*n*) and ST(*m*) via joining their centers by an edge.

Definition 1.13: A centered triangular number is a centered figurate number that represents a triangle with a dot in the center and all other dots surrounding the center in successive triangular layers. If the nth centered triangular number is denoted by C_n, then $C_n = \frac{1}{2}(3n^2 - 3n + 2)$. The first few centered triangular numbers are 1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199, 235, 274,...

Definition 1.14: A centered triangular graceful labeling of a graph G is a one-to-one function f : V (G) $\rightarrow$ { $0,1, \ldots C_q$ } that induces a bijection f^* : E(G) $\rightarrow \{C_1, C_2, \ldots C_q\}$ of the edges of G defined by $f^*(e) = |f(u) - f(v)|$, for all $e = uv \in E(G)$. The graph which admits such labeling is called a centered triangular graceful graph.

2. MAIN RESULTS

Theorem 2.1: The Fork graph is a centered triangular graceful graph

Proof: Let G be Fork graph.

Let V(G) = $\{v_i : 1 \le i \le n\}$ and

E(G) = $\{v_1 v_i \ / \ i = 2,3\} \cup \{v_3 v_i \ / \ i = 4,5\}$

Here G has 5 vertices and 4 edges.

Define f : V(G) $\rightarrow$ $\{0,1,\ldots C_4\}$ as follows

$f(v_1) = 1$,

$f(v_2) = 5$,

$f(v_3) = 0$,

$f(v_4) = 10$,

$f(v_5) = 19$,

Clearly f is injective and the edge values, the absolute difference of adjacent vertices are $\{C_1, C_{2,} \ldots\ldots C_t\}$

Thus f is a centered triangular graceful labeling of G.

Therefore, G is a centered triangular graceful graph.

Example 2.2: Centered triangular graceful labeling of Fork graph is given in Fig. 1.

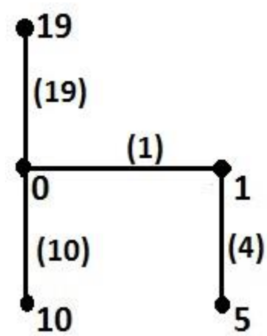

Fig. 1

Theorem 2.3: F-tree FP_n , $n \geq 3$ is a centered triangular graceful graph

Proof: Let G be a F-tree FP_n , $n \geq 3$.

Let V(G) = $\{ u, v, v_i : 1 \leq i \leq n \}$ and

E(G) = $\{v_i v_{i+1}: 1 \leq i \leq n\text{-}1\} \cup \{uv_{n-1}, vv_n\}$.

Here G has n + 2 vertices and n + 1 edges.

Let t = n + 1.

Define f : V(G) $\rightarrow \{0,1,\ldots C_t\}$ as follows

$f(v_1) = 0$,

$f(v_i) = f(v_{i-1}) - C_{t-i+2}$ if i is odd and $2 \leq i \leq n$.

$= f(v_{i-1}) + C_{t-i+2}$ if i is even and $2 \leq i \leq n$.

$f(v) = f(\mathrm{v}_n)$- 1,

$f(u) = f(\mathrm{v}_{n-1})$- 4.

Clearly f is injective and f induces a bijective function f^* : E(G) $\rightarrow \{1,4,\ldots C_t\}$ as

$f^*(v_i v_{i+1}) = C_{t-i+1}$; $1 \leq i \leq n$-1.

$f^*(uv_{n-1}) = C_2$

$f^*(vv_n) = C_1$

Hence the edge labels are $1,4,\ldots C_t$.

Thus f is a centered triangular graceful labeling of G.

Therefore, G = FP_n is a centered triangular graceful graph.

Example 2.4: Centered triangular graceful labeling of FP_6 is given in Fig. 2.

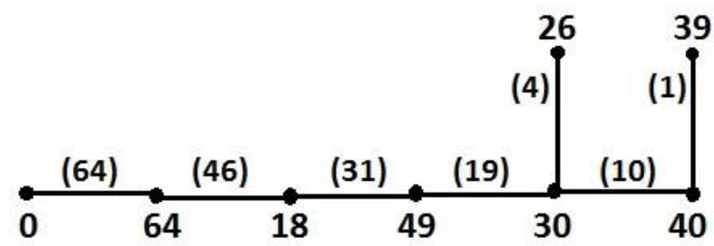

Fig. 2

Theorem 2.5: Any Y_n-tree is a centered triangular graceful graph.

Proof: Let G be the Y_n-tree.

Let V(G) = $\{v, v_i : 1 \leq i \leq n\}$ and

E(G) = $\{v_i v_{i+1}, v v_{n-1}: 1 \leq i \leq n\text{-}1\}$.

G has n +1 vertices and n edges.

Let t = n.

Define f : V(G) → $\{0,1,\ldots C_t\}$ as follows

$f(v_1) = 0$

$f(v_i) = f(v_{i-1}) - C_{t-i+2}$ if i is odd and $2 \leq i \leq n$.

$\quad = f(v_{i-1}) + C_{t-i+2}$ if i is even and $2 \leq i \leq n$.

$f(v) = f(v_{n-1}) - 1$

Clearly f is injective and f induces a bijective function f^* : E(G) →$\{1,4,\ldots C_t\}$ as

$f^*(v_i v_{i+1}) = C_{t-i+1}$; $1 \leq i \leq n\text{-}1$.

$f^*(v v_{n-1}) = C_1$

Hence the edge labels are 1,4,… C_t.

Thus f is a centered triangular graceful labeling of G.

Therefore, G = Y_n is a centered triangular graceful graph.

Example 2.6: Centered triangular graceful labeling of Y_6 is given in Fig. 3.

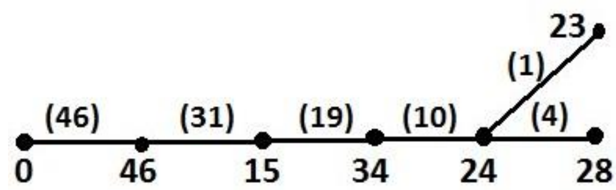

Fig. 3

Theorem 2.7: Let G be the graph obtained by identifying a pendant vertex of P_m with a leaf of $K_{1,n}$. Then G is centered triangular graceful for all m ≥ 2 and n ≥ 1.

Proof: Let G be the graph obtained by identifying the pendant vertex v_1 of P_m with a leaf u_n of $K_{1,n}$.

Let V(G) = $\{u, u_i, v_j : 1 \le i \le n\text{-}1, 1 \le j \le m\}$ and

E(G) = $\{uu_i, uv_1, v_jv_{j+1}: 1 \le i \le n - 1, 1 \le j \le m - 1\}$.

Here G has m + n vertices and m + n – 1 edges.

Let t = m + n - 1.

Define f : V(G) ⟶ $\{0,1,\ldots C_t\}$ as follows

$f(u) = 0$

$f(u_i) = C_{t-(i-1)}$; $1 \le i \le n$-1

$f(v_1) = f(u_n) = C_m$

$f(v_j) = f(v_{j-1}) + C_{n-(j-2)}$ if j is odd $2 \le j \le m$.

$\quad = f(v_{j-1}) - C_{n-(j-2)}$ if j is even $2 \le j \le m$.

Clearly f is injective and f induces a bijective function f^* : E(G) ⟶$\{1,4,\ldots C_t\}$ as

$f^*(uu_i) = C_{t-(i-1)}$; $1 \le i \le n$-1.

$f^*(uv_1) = f^*(u_n) = C_m$

$f^*(v_jv_{j+1}) = C_{m-j}$;$1 \le j \le m$-1.

Hence the edge labels are 1,4,… C_t.

Thus f is a centered triangular graceful labeling of G.

Therefore, G is a centered triangular graceful graph.

Example 2.8: Centered triangular graceful labeling of identifying a pendant vertex of P_5 with a leaf of $K_{1,6}$ is given in Fig. 4.

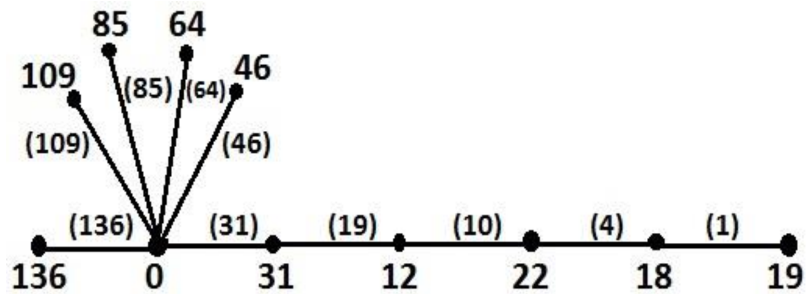

Fig. 4

Theorem 2.9: The graph obtained from $P_n \odot K_1$ by subdividing the edges of the pah P_n is centered triangular graceful for all $n \geq 2$.

Proof: Let G be the graph obtained from $P_n \odot K_1$ by subdividing the edges of the pah P_n.

Let V(G) = $\{v_i, u_i, w_j : 1 \leq i \leq n , 1 \leq j \leq n\text{-}1\}$ and

E(G) = $\{v_i w_i , v_j u_j, w_k v_{k+1} : 1 \leq i \leq n\text{-}1 , 1 \leq j \leq n , 1 \leq k \leq n\text{-}1\}$.

Here G has 3n - 1 vertices and 3n - 2 edges.

Let t = 3n – 2.

Define f : V(G) → $\{0,1,\dots C_t\}$ as follows

$f(v_1) = 0$

$f(v_i) = f(w_{i-1}) - C_{t-1-(2(i-2))}$; $2 \leq i \leq n$

$f(w_j) = f(v_j) + C_{t-2(j-1)}$; $1 \leq j \leq n\text{-}1$

$f(u_i) = f(v_i) + C_{n-i+1}$; $1 \leq i \leq n$.

Clearly f is injective and f induces a bijective function f^* : E(G) →$\{1,4,\dots C_t\}$ as

$f^*(v_i w_i) = C_{t-2(i-1)}$; $1 \leq i \leq n\text{-}1$.

$f^*(v_j u_j) = C_{n-j+1}$; $1 \leq j \leq n$.

$f^*(w_k v_{k+1}) = C_{t-2k+1}$; $1 \leq k \leq n\text{-}1$.

Hence the edge labels are $1,4,\dots C_t$.

Thus f is a centered triangular graceful labeling of G.

Therefore, G is a centered triangular graceful graph.

Example 2.10: Centered triangular graceful labeling of $P_3 \odot K_1$ by subdividing the edges of the pah P_3 is given in Fig. 5.

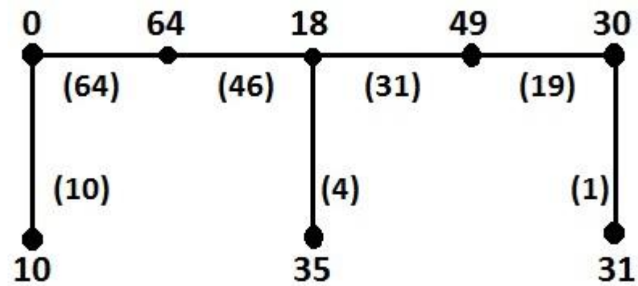

Fig. 5

Theorem 2.11: Let G be the graph obtained by identifying the leaves of $K_{1,n}$ with the central vertex of $K_{1,2}$. Then G centered triangular graceful for all n ≥ 1.

Proof: Let G be the graph obtained by identifying the leaves of $K_{1,n}$ with the central vertex of $K_{1,2}$.

Let V(G) = { v, v_i, v_{ij} : $1 \leq i \leq n$, $1 \leq j \leq 2$ } and

E(G) = {vv_i ,v_iv_{ij}: $1 \leq i \leq n$, $1 \leq j \leq 2$ }.

Here G has 3n +1 vertices and 3n edges.

Let t = 3n.

Define f : V(G) → {0,1,… C_t} as follows

$f(v) = 0$

$f(v_i) = C_{3(n-(i-1))}$; $1 \leq i \leq n$.

$f(v_{ij}) = f(v_i) - C_{t-(i-1)n-j}$; $1 \leq i \leq n$, $1 \leq j \leq 2$.

Clearly f is injective and f induces a bijective function f^* : E(G) →{1,4,… C_t} as

$f^*(vv_i) = C_{3(n-(i-1))}$; $1 \leq i \leq n$.

$f^*(v_iv_{ij}) = C_{t-(i-1)n-j}$; $1 \leq i \leq n$, $1 \leq j \leq 2$.

Hence the edge labels are 1,4,… C_t.

Thus f is a centered triangular graceful labeling of G.

Therefore, G is a centered triangular graceful graph for all $n \geq 1$.

Example 2.12: Centered triangular graceful labeling of identifying the leaves of $K_{1,2}$ with the central vertex of $K_{1,2}$ is given in Fig. 6.

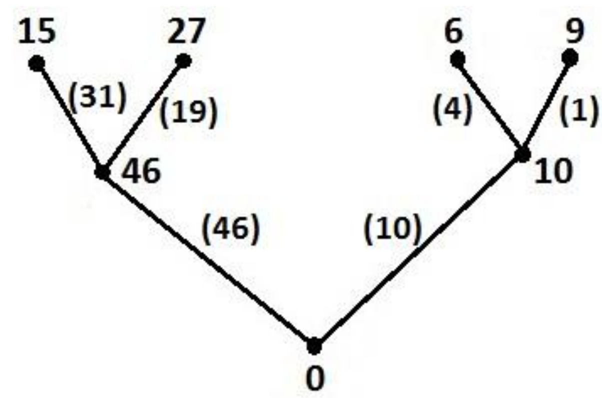

Fig. 6

Theorem 2.13: $K_{1,n} \odot K_1$ is centered triangular graceful graph.

Proof: Let G be the graph $K_{1,n} \odot K_1$.

Let V(G) = $\{v, v_i, u_i, w : 1 \leq i \leq n\}$ and

E(G) = $\{vv_i, v_iu_i, vw: 1 \leq i \leq n\}$.

Here G has 2n + 2 vertices and 2n + 1 edges.

Let t = 2n + 1.

Define f : V(G) → $\{0,1,\dots C_t\}$ as follows

$f(v) = 0$

$f(v_i) = C_{t-(i-1)}$; $1 \leq i \leq n$

$f(w) = C_{t-n}$

$f(u_i) = f(v_i) - C_{n-(i-1)}$; $1 \leq i \leq n$.

Clearly f is injective and f induces a bijective function f^* : E(G) →$\{1,4,\dots C_t\}$ as

$f^*(vv_i) = C_{t-(i-1)}$; $1 \leq i \leq n$.

$f^*(vw) = C_{t-n}$.

$f^*(v_iu_i) = C_{n-(i-1)}$; $1 \leq i \leq n$.

Hence the edge labels are $1,4,\dots C_t$.

Thus f is a centered triangular graceful labeling of G.

Therefore, G = $K_{1,n} \odot K_1$ is a centered triangular graceful graph.

Example 2.14: Centered triangular graceful labeling of $K_{1,4} \odot K_1$ is given in Fig. 7.

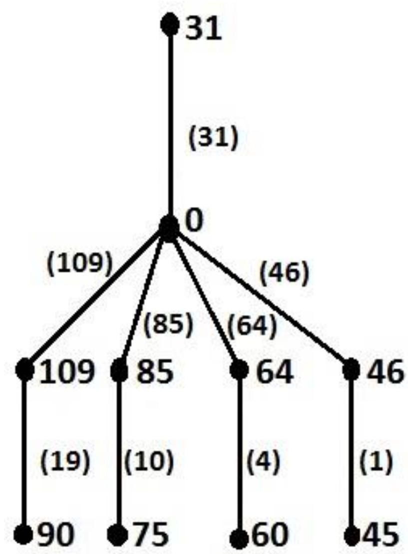

Fig. 7

Theorem 2.15: The caterpillar $S(n_1 , n_2 , \ldots , n_m)$ is a centered triangular graceful graph.

Proof: Let G be the graph $S(n_1 , n_2 , \ldots , n_m)$.

Let V(G) = $\{v_i , v_{ij} : 1 \leq i \leq m$ and $1 \leq j \leq n_m\}$ and

E(G) = $\{v_i v_{i+1}: 1 \leq i \leq m\text{-}1 \} \cup \{ v_i v_{ij} : 1 \leq i \leq m$ and $1 \leq j \leq n_m\}$.

Here G has $n_1 + n_2 + \cdots + n_m + m$ vertices and $n_1 + n_2 + \cdots + n_m + m - 1$ edges.

Let $t = n_1 + n_2 + \cdots + n_m + m - 1$.

Define f: V(G) → $\{0,1,\ldots C_t\}$ as follows

$f(v_1) = 0$

$f(v_{1j}) = C_{t-(j-1)}$; $1 \leq j \leq n_1$

For $2 \leq i \leq m$,

$f(v_i) = f(v_{i-1})$ - $C_{t-n_1- n_2-\cdots-n_{i-1}-(i-2)}$ if i is odd

$= f(v_{i-1}) + C_{t-n_1- n_2-\cdots-n_{i-1}-(i-2)}$ if i is even

$f(v_{ij}) = f(v_{i-1})$ - $C_{t-n_1- n_2-\cdots-n_{i-1}-(i-2)-i}$, $1 \leq j \leq n_m$

Clearly the vertex labels are distinct and the resulting edge labels are of the form $\{C_1 , C_2 , \ldots C_t\}$

Thus f is a centered triangular graceful labeling of G.

Therefore, G = $S(n_1 , n_2 , \ldots , n_m)$ is a centered triangular graceful graph.

Example 2.16: Centered triangular graceful labeling of $S(3 , 4 ,5,6)$ is given in Fig. 8.

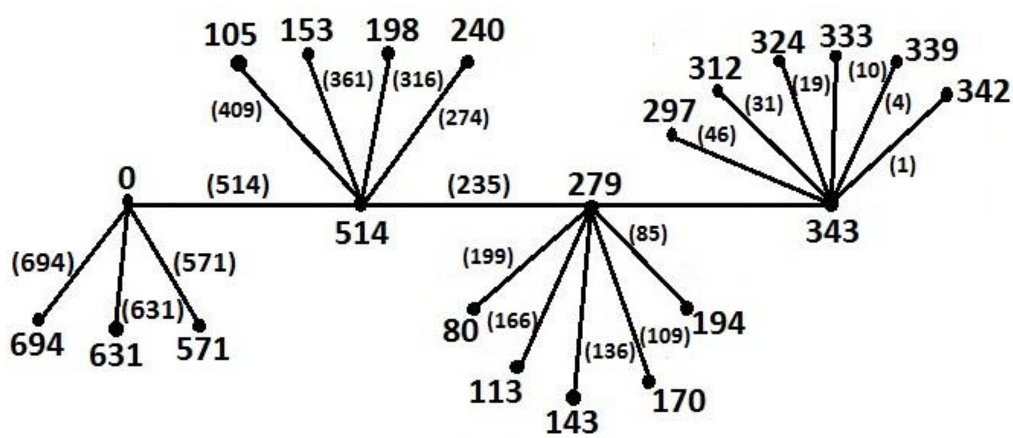

Fig. 8

Corollary 2.16.1: The double star ST(n, m) is a centered triangular graceful graph

Proof: Let G be a double star ST(n, m) graph.

Let V(G) = $\{u, u_i, v, v_j : 1 \le i \le n, 1 \le j \le m\}$ and

E(G) = $\{uv, uu_i, vv_j : 1 \le i \le n, 1 \le j \le m\}$.

Here G has n + m + 2 vertices and n + m + 1 edges.

Let t = n + m + 1.

Define f: V(G) ⟶ $\{0,1,\dots C_t\}$ as follows

$f(u) = C_1$

$f(u_i) = C_{i+1} + 1, 1 \le i \le n$

$f(v) = 0$,

$f(v_j) = C_{n+1+j}, 1 \le j \le m$

Clearly f is injective and f induces a bijective function f^*: E(G) ⟶$\{1,4,\dots C_t\}$ as

$f^*(uv) = C_1$,

$f^*(uu_i) = C_{i+1}, 1 \le i \le n$

$f^*(vv_j) = C_{n+1+j}, 1 \le j \le m$

Hence the edge labels are $1,4,\dots C_t$.

Thus f is a centered triangular graceful labeling of G.

Therefore, G = ST(n, m) is a centered triangular graceful graph.

Example 2.16.2: Centered triangular graceful labeling of ST(5,4) is given in Fig. 9.

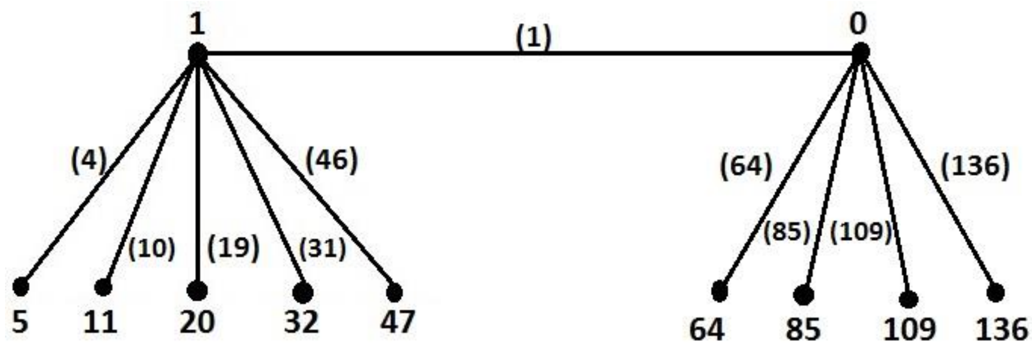

Fig. 9

Corollary 2.16.3: $\mathrm{ST}(n,0,m)$ is a centered triangular graceful graph

Proof: Let G = $\mathrm{ST}(n,0,m)$ be a double star with one vertex joining the end vertices of stars.

Let V(G) = $\{u\,, v, w, u_i\,, w_j : 1 \leq i \leq n\,,\, 1 \leq j \leq m\}$ and

E(G) = $\{uu_i, uv, vw, ww_j\;: 1 \leq i \leq n\,, 1 \leq j \leq m\}$.

Here G has n + m + 3 vertices and n + m + 2 edges.

Let t = n + m + 2.

Define f : V(G) ⟶ $\{0,1,\ldots C_t\}$ as follows

$f(u) = C_1$,

$f(u_i) = C_{i+1} + 1\,,\, 1 \leq i \leq n$

$f(v) = 0$,

$f(w) = C_t$,

$f(w_j) = C_t - C_{n+1+j},\;\; 1 \leq j \leq m$

Clearly f is injective and f induces a bijective function f^* : E(G) ⟶$\{1,4,\ldots C_t\}$ as

$f^*(uu_i) = C_{i+1},\, 1 \leq i \leq n$

$f^*(uv) = C_1$,

$f^*(vw) = C_t$,

$f^*(ww_j) = C_{n+1+j}\,,\, 1 \leq j \leq m$

Hence the edge labels are $1,4,\ldots C_t$.

Thus f is a centered triangular graceful labeling of G.

Therefore, G = $\mathrm{ST}(n,0,m)$ is a centered triangular graceful graph.

Example 2.16.4: Centered triangular graceful labeling of ST(5,0,4) is given in Fig. 10.

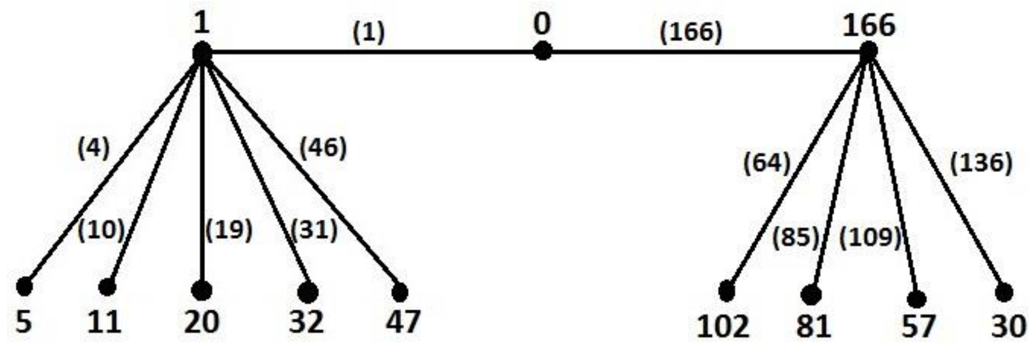

Fig. 10

3. CONCLUSIONS

In this paper, we have studied the centered triangular graceful labeling of some tree related graphs. This work contributes several new results to the theory of graph labeling. The centered triangular graceful can be verified for many other graphs. Also some more centered triangular graceful labeling can be investigated.

ACKNOWLEDGEMENT

Authors are thankful to the anonymous reviewer for the valuable comments and suggestions that improve the quality of this paper.

References

[1] F. Harary, Graph Theory, Adision – Wesley, Reading Mass, 1969.

[2] K. R. Parthasarathy. Basic Graph Theory, Tata Mcgraw Hill Publishing Company Limited (1994).

[3] J. A. Bondy and U. S. R. Murty. Graph Theory with Applications, Macmillan Press, London (1976).

[4] M. Apostal, Introduction to Analytic Number Theory, Narosa Publishing House, Second Edition, 1991.

[5] I. Niven and Herbert S. Zuckerman. An Introduction to the Theory of Numbers, Wiley Eastern Limited, Third Edition, 1991.

[6] A. Rosa, On certain valuation of the vertices of a graph, Thoery of Graphs (Proc. Internat. Symposium, Rome, 1966), Gordon and Breach, N. Y and Dunod Paris (1967), 349-355.

[7] S.W. Golomb, How to number a Graph, Graph Theory and Computing, R.C. Read, Academic Press, New York (1972).

[8] J.A.Gallian, A Dynamic Survey of Graph Labeling. *The Electronic Journal of Combinatorics*, 22 (2019) #DS6

[9] F. Harary, Sum Graphs and Difference Graphs. *Congr. Numer.* 72 (1990) 101-108

[10] F. Harary, I. R. Hentzel and D. Jacobs Digitizing Sum Graphs over the Reals, *Caribbean J. Math. Comput. Sci.* 1 (1991) 1-4

[11] M. Ellingham, Sum Graphs from Trees. *Ars Combinatoria*, 35 (1993) 335-349.

[12] F. Harary, Graph Theory, Adision – Wesley, Reading Mass, 1969.

[13] S. Murugesan, D. Jayaraman and J. Shiama. Centered triangular sum labeling of graphs, *International Journal of Applied Information Systems*, 5(7) (2013) 1-4

[14] P. Jeyanthi, R. Kalaiyarasi and D. Ramya, Centered Triangular Mean Graphs. *International J. Math. Combin.* Vol. 1 (2019) 126-133

[15] M. Basker, P. Namasivayam, M. P. Syed Ali Nisaya. Some Results on Centered Triangular Sum Graphs. *World Scientific News* 155 (2021) 113-128

[16] D. S. T. Ramesh and M. P. Syed Ali Nisaya, Some Important Results on Pentagonal Graceful Graphs. *International Journal of Applied Mathematical Sciences,* 7(1), (2014), 71-77

[17] M. P. Syed Ali Nisaya and D. S. T. Ramesh, Pentagonal graceful labeling of Caterpillar Graphs. *International Journal of Engineering Devlopment and Research* 6(4) (2018) 150-154

[18] D. S. T. Ramesh and M. P. Syed Ali Nisaya, Some More Polygonal Graceful Labeling of Path. *International Journal of Imaging Science and Engineering,* 6(1), (2014), 901-905

[19] M. Vanu Esakki, M. P. Syed Ali Nisaya. Two Modulo Three Sum Graphs. *World Scientific News* 145 (2020) 274-285

[20] M. Vanu Esakki, M. P. Syed Ali Nisaya. Some Results on Two Modulo Three Sum Graphs. *Journal of Xidian University*, 14(9) (2020) 1090-1099

[21] K. Kovusalya and P. Namasivayam, Some Results on Octagonal Graceful Graphs. *World Scientific News* 156 (2021) 1-12

[22] Frank Werner, Graph Theoretic Problems and their New Applications. *Mathematics*, S 445, (2020) 1-4

[23] P. Lalitha, M. Gayathri, L. Tamilselvi and A.V. Arun. Application of Graceful Labeling in Dental Arch. *Drug Innovation Today*, Vol. 11, Issue 3, (2019) 637-638

[24] V. Lavanya, D.S.T. Ramesh and N. Meena, Bi-Domination in Corona Related Graphs. *Journal of Computer and Mathematical Sciences,* Vol. 10 (11), November (2019), 1650-1653

[25] S. Mahendran and K. Murugan. Pentagonal Graceful Labeling of Some Graphs. *World Scientific News* 155 (2021) 98-112

[26] Monika K, Murugan K. Fibonacci Sum Labeling of Tree Related Graphs. *Adalya Journal*, Vol. 8, Issue 9, (2019) 320-323. DOI:16.10089.AJ.2019.V8I9.285311.6154

[27] Muhammed Imran, Adnan Aslam, Sohail Zafar and Waqar Nazeer, Further Results on Edge Irregularity Strength of Graphs. *Idonesian Journal of Combinnatorics* 1(2), (2017) 82-97

[28] G. Muppidathi Sundari & K. Murugan, Extra Skolem Difference Mean Labeling of Some Graphs. *World Scientific News* 145 (2020) 210-221

[29] K. Murugan and A. Subramanian, Labeling of Subdivided Graphs. *American Journal of Mathematics and Sciences* Vol. 1, No. 1, January (2012), 143-149

[30] K. Murugan and A. Subramanian, Skolem Difference Mean Graphs. *Mapana J Sci* 11, 4, (2012) 109-120

[31] K. Murugan, Square Graceful Labeling of Some Graphs. *International Journal of Innovative Research in Science, Engineering and Technology*, Vol. 4, Issue 2, (2015) 511-520

[32] R. Sivaraman, Graceful Graphs and its Applications. *International Journal of Current Research*, Vol. 8, Issue 11, (November 2016) 41062-41067

[33] K. Thajeswari, S. Kirupa. Application of Graceful Graph in MPLS. *International Journal for Scientific Research and Development*, Vol. 6, Issue 06, (2018) 196-198

[34] T. Tharmaraj, P. B. Sarasija, Square Graceful Graphs. *International Journal of Mathematics and Soft Computing*, Vol. 4, No. 1, (2014) 129-137

[35] N. Vedavathi, Dharmaiya Gurram, Applications on Graph Theory. *International Journal of Engineering Research and Technology*, Vol. 2, Issue 1, (2013) 1-4

[36] P. Gnanachandra, M. Lellis Thivagar, Separation axioms by virtue of soft semi*-open sets. *World Scientific News* 145 (2020) 74-84

[37] Kostochka, Alexandr, Yager, Derrek, Yu, Gexin 2020. Disjoint Chorded Cycles in Graphs with High Ore-Degree. *Discrete Mathematics and Applications* Vol. 165, p. 259. https://doi.org/10.1007/978-3-030-55857-4_11

[38] Costalonga, J.P., Kingan, Robert J., Kingan, Sandra R. 2021. Constructing Minimally 3-Connected Graphs. *Algorithms*, Vol. 14, Issue. 1, p. 9. https://doi.org/10.3390/a14010009

[39] G. Muthumanickavel, K. Murugan, Oblong Sum Labeling of Union of Some Graphs. *World Scientific News* 145 (2020) 85-94

[40] N. Vedavathi, Dharmaiya Gurram, Applications on Graph Theory. *International Journal of Engineering Research and Technology*, Vol. 2, Issue 1, (2013) 1-4

[41] N. Meena, M. Madhan Vignesh. Strong Efficient Co-Bondage Number of Some Graphs. *World Scientific News* 145 (2020) 234-244

[42] Frank Werner.Graph Theoretic Problems and their New Applications. *Mathematics*, S 445, (2020) 1-4

[43] Xiaojing Yang, Junfeng Du, Liming Xiong. Forbidden subgraphs for supereulerian and Hamiltoniangraphs. *Discrete Applied Mathematics* Volume 288, 15 January 2021, Pages 192-200. https://doi.org/10.1016/j.dam.2020.08.034

[44] M. Prema and K. Murugan, Oblong sum labeling of some graphs. *World Scientific News* 98 (2018) 12-22

[45] Chiba, Shuya, Yamashita, Tomoki, 2018. Degree Conditions for the Existence of Vertex-Disjoint Cycles and Paths: A Survey. *Graphs and Combinatorics,* Vol. 34, Issue. 1, p. 1. https://doi.org/10.1007/s00373-017-1873-5

[46] Molla, Theodore, Santana, Michael, Yeager, Elyse 2020. Disjoint cycles and chorded cycles in a graph with given minimum degree. *Discrete Mathematics,* Vol. 343, Issue. 6, p. 111837. https://doi.org/10.1016/j.disc.2020.111837