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Open support of some special types of graphs under addition

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ABSTRACT

A open support of a vertex v under addition is defined by $\sum_{u\in N(v)} \deg(u)$ and it is denoted by supp(v). A open support of a graph under addition is defined by $\sum_{v\in V(G)} \mathrm{supp}(v)$ and it is denoted by supp(G). In this paper, open support of some graphs is studied.

***Keywords*:** degree of a vertex, open neighbourhood of a vertex, open support of a vertex, open support of a graph

1. INTRODUCTION

Graphs considered in this paper are finite, undirected and simple. Let G = (V,E) be a graph. The degree of a vertex is the number of edges of G incident with v and is denoted by deg (v). The minimum and maximum degrees of the vertices of G are respectively denoted by $\delta(G)$ and $\Delta(G)$. A vertex of degree 0 in G is called an isolated vertex and a vertex of degree 1 is called a pendent vertex or end vertex of G. A vertex of a graph G is said to be a full degree vertex if it is adjacent to all the other vertices of G. The neighbourhood of a vertex $v \in V(G)$ is the set $N_G(v)$ of all the vertices adjacent to v in G. For a set $S \subseteq V(G)$, the open neighbourhood $N_G(S)$ is defined to be $\cup_{v\in S} N_G(v)$ and the closed neighbourhood $N_G[S]$ is defined to be $\cup_{v\in S} N_G[v]$.

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Let u and v be (not necessarily distinct) vertices of a graph G. A u-v walk of a graph G is an alternating sequences of vertices and edges W = v_0e_1, v_1e_2, v_2, …, $v_{k-1}e_kv_k$ beginning and ending with vertices in which each $e_i = v_iv_{i+1}$. The number of edges in a walk W is called the length of the walk. The walk joining the vertices v_0 and v_k is called v_0 - v_k walk. It is also denoted by v_0, v_1, v_2 ,…, v_k. If $v_0 = v_k$ then it is called a closed walk, otherwise it is called an open walk. If all the edges of a walk are distinct then it is called trail.Open support of a vertex and open support of a graph was introduced by S. Balmurugan. et al in [5] and further studied in [6]. In this paper, open support of some graphs is studied. The following definitions and previous results are necessary for the present study [7-48].

DEFINITION 1.1 [5]: Let G = (V, E) be a graph. A open support of a vertex v under addition is defined by $\sum_{u\in N(v)} \deg(u)$ and it is denoted by supp(v).

DEFINITION 1.2 [5]: Let G = (V, E) be a graph. A open support of a graph G under addition is defined by $\sum_{v\in V(G)} \text{supp}(v)$ and it is denoted by supp(G).

DEFINITION 1.3: A path is a walk in which all the vertices as well as the edges are distinct. The path on n vertices is denoted by P_n.

DEFINITION 1.4: A closed trail whose origin and internal vertices are distinct is called a cycle.

DEFINITION 1.5: A graph in which any two distinct points are adjacent is called a complete graph. The complete graph with n points is denoted by K_n.

DEFINITION 1.6: A Bi-graph (Bipartite graph) G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 and a vertex of V_2. A bigraph G is called a complete bipartite graph if every vertex of V_1 is joined to all the vertices of V_2. The complete bipartite graph $K_{1,n}$ and $K_{n,1}$ is called a star.

DEFINITION 1.7: The Bistar $B_{m,n}$ is the graph obtained from K_2 by joining m pendant edges to on end of K_2 and n pendant edges to the other end of K_2. The edge of K_2 is called the central edge of $B_{m,n}$ and the vertices of K_2 are called the central vertices of $B_{m,n}$.

DEFINITION 1.8: The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1v_2, \dots v_n$ and $u_1u_2, \dots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

DEFINITION 1.9: The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G". Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G".

DEFINITION 1.10: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 where G_1 has m vertices and n edges is defined as the graph G obtained by taking one copy of G_1 and m copies of G_2 and joining by an edge the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

DEFINITION 1.11: A subdivision of a graph G is a graph that can be obtained from G by a sequence of edge subdivision.

Result 1.12: Let $G = P_n$ (n>1). Then supp (G) = 4n – 6.

Result 1.13: Let $G = C_n$ (n>1). Then supp (G) = 4n

Result 1.14: Let $G = K_n$ (n>1). Then supp (G) = $n (n-1)^2$.

Result 1.15: Let $G = K_{m, n}$ (m, n>1). Then supp (G) = mn(m + n).

2. RESULT

THEOREM 2.1: Let $G = D_2(P_n)$, $n \geq 2$. Then supp (G) = 16(2n – 3).

PROOF: Let $V(G) = \{u_i, v_i / 1 \leq i \leq n\}$.

$E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i+11} / 1 \leq i \leq n-1\} \cup \{u_i v_{i-1} / 2 \leq i \leq n\}$. deg u_1 = deg u_n = deg v_1 = deg v_n = 2.

deg u_j = deg v_j = 4 , $2 \leq j \leq n-1$.

$supp(u_1) = supp(v_1) = supp(u_n) = supp(v_n) = 8$. $supp(u_2) = supp(v_2) = supp(u_{n-1}) = supp(v_{n-1}) = 12$.

$supp(u_j) = supp(v_j) = 16$, $3 \leq j \leq n-2$.

$supp(G) = supp(u_1) + supp(v_1) + supp(u_n) + supp(v_n) + supp(u_2) + supp(v_2) + supp(u_{n-1}) + supp(v_{n-1}) + \sum_{j=3}^{n-2}\{supp(u_j) + supp(v_j)\} = 4(8) + 4(12) + (2n-8)(16) = 16(2n-3)$.

EXAMPLE 2.2: Consider the following graph $D_2(P_6)$.

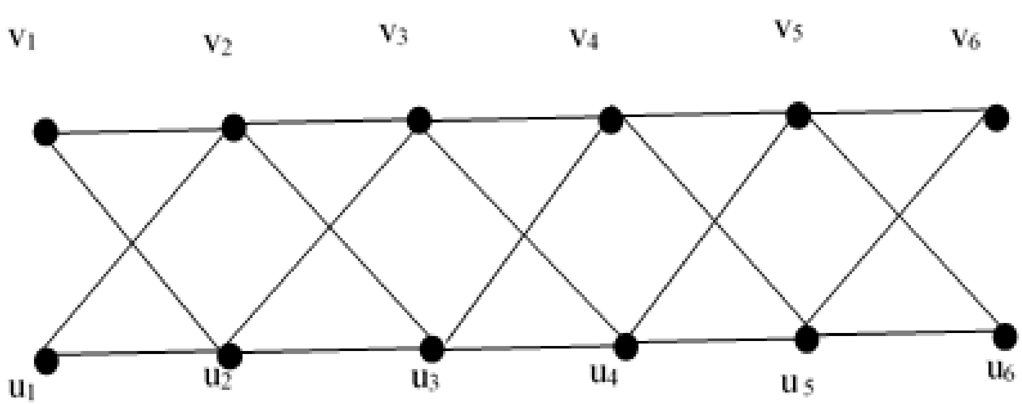

Figure 2.1

supp(v_1) = supp(v_6) = supp(u_1) = supp(u_6) = 8, supp(v_2) = supp(v_5) = supp(u_2) = supp(u_5) = 12, supp(v_3) = supp(v_4) = supp(u_3) = supp(u_4) = 16. supp(G) = 144 = 16 (2(6) – 3)

THEOREM 2.3: Let G = $D_2(C_n)$, n ≥ 3.Then supp(G) = 32n.

PROOF: Let G = $D_2(C_n)$, n≥3. Let V(G) = {v_1, v_2,…, v_n, u_1, u_2,…, u_n}.

E(G) = { v_iv_{i+1}/1≤ $i \leq n-1$}∪{v_nv_1}∪ {u_iu_{i+1} / 1≤ $i \leq n-1$}∪{u_nu_1} ∪{u_iv_{i-1} , u_iv_{i+1}/ 2≤ $j \leq n-1$} ∪{u_1v_n , u_1v_2 , u_nv_1 , u_nv_{n-1}} .

deg v_i = deg u_i = 4 , $1 \leq i \leq n$.

supp(u_i) = $\sum_{v \in N(u_i)} \deg v$ = 4 × 4 = 16 , 1≤ $i \leq n$.

Similarly, supp(v_i) = 16 , 1≤ $i \leq n$.

supp(G) = $\sum_i supp(u_i) + \sum_i supp(v_i)$ =16n + 16n = 32n.

EXAMPLE 2.4: Consider the following graph $D_2(C_4)$

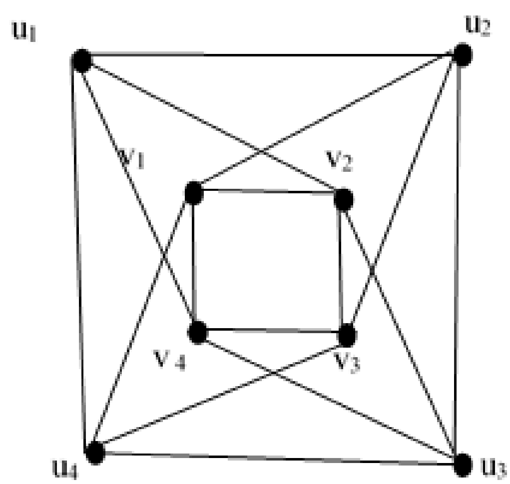

Figure 2.2

supp(u_i) = supp(v_i) = 16, *1*≤ $i \leq 4$.
supp(G) = 4[supp(u_i)] + 4[supp(v_i)] = 128

THEOREM 2.5: Let G = $D_2(K_{1,n})$, n ≥ 1. Then supp(G) = 32n.

PROOF: Let V(G)={u , v , u_i , v_i /, 1≤ $i \leq n$}.E(G) ={uu_i , vv_i ,uv_i ,vu_i / 1≤ $i \leq n$}.

deg u = deg v = 2n and deg u_i = deg v_i = 2, , 1≤ $i \leq n$.

supp(u) = $\sum_{v \in N(u)} \deg v$ = $\sum_{i=1}^{n} \deg u_i$ + $\sum_{i=1}^{n} \deg v_i$= 2n + 2n = 4n.

Similarly, supp(v) = $\sum_{i=1}^{n} \deg u_i$ + $\sum_{i=1}^{n} \deg v_i$ = 4n.

supp(u_i) = deg u + deg v = 2n + 2n = 4n.

Similarly, supp(v_i) = deg u + deg v = 2n + 2n = 4n.

supp(G) = supp(u) + supp(v) + $\sum_{i=1}^{n} \text{supp}(u_i)$ + $\sum_{i=1}^{n} \text{supp}(v_i)$ = 4n + 4n + n(4n) + n(4n)

= 8n (n+1).

EXAMPLE 2.6: Consider the following graph $D_2(K_{1,4})$

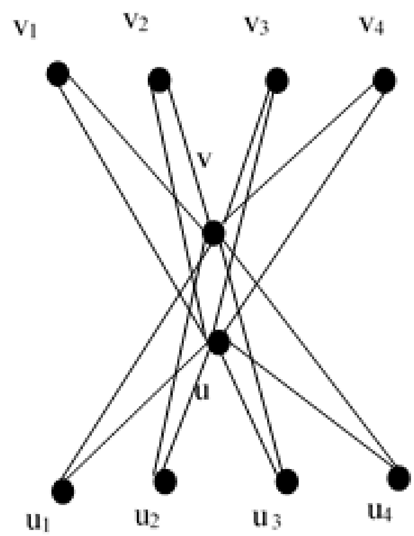

Figure 2.3

supp(u) = 8 = supp(v), supp(u_i) = supp(v_i) = 16, $1 \leq i \leq 4$
supp(G) = 160 = 8(4)(5).

THEOREM 2.7: Let G be a H graph, then supp(G) = 8n – 2.

PROOF: Let G be a H graph. Let V(G) = {v_i, u_i / $1 \leq i \leq n$}.

Case(i): Let n be a odd, n ≥ 5.

deg v_1 = deg v_n = deg u_1 = deg u_n = 1. deg v_i = deg u_i = 2, $2 \leq i \leq \frac{n+1}{2} - 1$, $\frac{n+1}{2} + 1 \leq i \leq n -$ 1. deg $v_{\frac{n+1}{2}}$ = deg $u_{\frac{n+1}{2}}$ = 3.

supp(u_1) = supp(v_1) = supp(u_n) = supp(v_n) = 2. supp(u_i) = supp(v_i) = 3 , i = 2 and n – 1.

supp(u_i) = supp(v_i) = 4, $3 \leq i \leq \frac{n+1}{2} - 2$ and $\frac{n+1}{2} + 2 \leq i \leq n-2$.

supp(u_i) = supp(v_i) = 5, $i = \frac{n+1}{2} - 1$ and $\frac{n+1}{2} + 1$. supp(u_i) = supp(v_i) = 7, $i = \frac{n+1}{2}$.

supp(G) = supp(u_1) + supp(v_1) + supp(u_n) + supp(v_n) + supp(u_2) + supp(v_2) + supp(u_{n-1}) +

supp(v_{n-1}) + $\sum_{i=3}^{\frac{n+1}{2}-2}(supp(u_i) + supp(v_i))$ + $\sum_{i=\frac{n+1}{2}+2}^{n-2}(supp(u_i) + supp(v_i))$ + supp($u_{\frac{n+1}{2}-1}$) + supp($v_{\frac{n+1}{2}-1}$) + supp($u_{\frac{n+1}{2}+1}$) + supp($v_{\frac{n+1}{2}+1}$) + supp($u_{\frac{n+1}{2}}$) + supp($v_{\frac{n+1}{2}}$) = 4(2) + 4(3) + $4\left(\frac{n-7}{2}\right)4$ + 4(5) + 2(7) = 8n – 2.

when n = 3, deg v_1 = deg v_3 = deg u_1 = deg u_3 = 1. deg v_2 = deg u_2 = 3.

supp(u_1) = supp(v_1) = supp(u_3) = supp(v_3) = 3. supp(u_2) = supp(v_2) = 5.

Supp(G) = 22 = 8(3) – 2.

Proof is similar when n is even

EXAMPLE 2.8: Consider the following graph H_5.

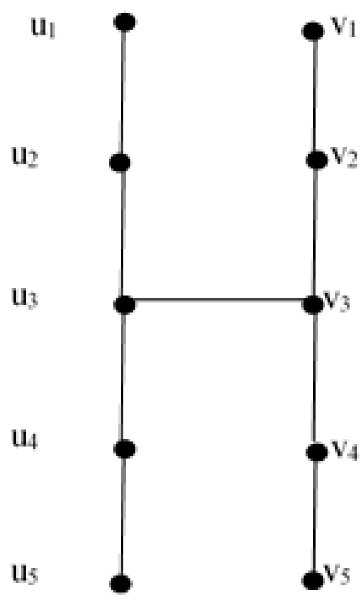

Figure 2.4

supp(u_1) = supp(v_1) = supp(u_5) = supp(v_5) = 2. supp(u_2) = supp(v_2) = supp(u_4) = supp(v_4) = 4 supp(u_3) = supp(v_3) = 7. Supp(G) = 38 = 8(5) – 2.

THEOREM 2.9: Let G be a graph obtained from $P_m \odot nK_1$, m ≥ 3, n ≥ 1 by the subdividing the edges of P_m. Then supp(u) = $mn^2 + 5mn - 4n + 8m - 10$.

PROOF: Let m≥3. Let V(G) = {v_iu_j, w_{ik} / $1 \le i \le m, 1 \le j \le m-1, 1 \le k \le n$}.

E(G) = {v_iu_i / $1 \le i \le m-1$} ∪ {u_iv_{i+1}/$1 \le i \le m-1$} ∪{$v_i w_{ik}$ / $1 \le i \le m, 1 \le i \le n$}.

deg v_1 = n+1 = deg v_m. deg v_i = n+2 i = 2,3,…,m −1.deg u_i= 2 , $1 \le i \le m-1$. deg w_{ik} = 1 , $1 \le i \le m, 1 \le k \le n$.

supp(v_1) = deg u_1+$\sum_{k=1}^{n} \deg w_{1k}$ = 2 + n. Similarly supp(v_m) = n + 2.

For each i = 2,3,…,m – 1, supp(v_i) = deg u_{i-1}+ deg u_i + $\sum_{k=1}^{n} \deg w_{ik}$ = n + 4

supp(u_1) = deg v_1 + deg v_2 = 2n + 3.

Similarly, deg u_{m-1} = 2n + 3.For each i= 2,3,…,m-2,supp(u_i) = degv_i + deg v_{i+1} = 2n + 4.

For each k =1, 2, 3,…,n, supp(w_{1k}) = deg v_1 = n+1. Similarly supp(w_{mk}) = n + 1

For each i = 2, 3,…,m – 1, k =1,2,3,…,n, supp(w_{ik}) = supp(v_i) = n + 2.

supp(G) = $\sum_{v\in V(G)} \text{supp}(v)$ = $\text{supp}(v_1) + \sum_{i=2}^{m-1} \text{supp}(v_i) + \text{supp}(v_m) + \text{supp}(u_1) + \text{supp}(u_{m-1}) + \sum_{i=2}^{m-2} \text{supp}(u_i) + \sum_{k=1}^{n} \text{supp}(w_{1k}) + \sum_{i=2}^{m-1}\sum_{k=1}^{n} \text{supp}(w_{ik}) + \sum_{k=1}^{n} \text{supp}(w_{mk})$ = n + 2 + (m – 2)(n + 4) + n + 2 + 2n + 3 + (m-3)(2n+4) + 2n + 3 + n(n + 1) + (m – 2) n (n + 2) + n(n + 1) = mn^2+ 5mn - 4n + 8m – 10.

when m = 2, supp(G) = $2n^2$ + 6n + 6.

THEOREM 2.10: Let G be a graph obtained from $C_m \odot nK_1$, m ≥ 3, n ≥ 1 by the subdividing the edges of the cycle C_m. Then supp(G) = 4mn + 10m.

PROOF: Let V(G)={ u_i,v_i ,w_{ij}/1≤ $i \le m, 1 \le i \le n$}.

E(G) = {v_iu_i/1≤ $i \le m$} ∪ {u_iv_{i+1}/1≤ $i \le m-1$} ∪ {u_nv_1}∪ { v_iw_{ij}/1≤ $i \le m, 1 \le i \le n$}

deg v_i = 4 , deg u_i = 2 , deg w_{ij} = 1 , 1≤ $i \le m, 1 \le i \le n$

For each i = 2, 3, …, m, supp(v_i) = $\sum_{v\in N(v_i)} \deg(v)$ = deg u_{i-1} +

deg u_i + $\sum_{j=1}^{n} \deg w_{ij}$ = 2 + 2 + n(1) = n + 4

supp(v_1) = $\sum_{v\in N(v_1)} \deg(v)$= deg u_1+ deg u_m+ $\sum_{j=1}^{n} \deg w_{ij}$ = n + 4.

For each i = 1,2,3,…,m , j=1,2,3,…,n, supp(w_{ij}) = deg v_i = n + 2.

For each I = 1,2,3,…,n – 1, supp(u_i) = deg v_i + deg v_{i+1}= n + 2 + n + 2 = 2n + 4.

supp(u_n) = deg v_n+ deg v_1= 2n + 4.

supp(G) = $\sum_{v\in V(G)} \text{supp}(v)$ =$\sum_{i=1}^{m} \text{supp}(v_i) + \sum_{i=1}^{m} \text{supp}(w_i) + \sum_{i=1}^{m} \text{supp}(u_i)$ = m(n + 4) + m(2n + 4) + m(n + 2) = mn + 4m + 2mn + 4m + mn + 2m. = 4mn + 10m.

THEOREM 2.11: Let G be a graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$. Then supp(G) = 2(n^2 + 3n + 3).

PROOF: Let G be a $B_{m,}$n graph. Let V(G) = {u, v, w, u_i, v_i / 1 ≤ $i \le n$}.

E(G) = {uw, wv, uu_i, vv_i / 1≤ $i \le n$}. deg w = 2, deg u = n + 1 = deg v.

supp(u) = supp(v) = n + 2. supp(w) = 2(n + 1).

$\sum_{i=1}^{n} \text{supp}(u_i) = n(n+1)$. $\sum_{i=1}^{n} \text{supp}(v_i)$ = n(n+1).

supp(G) = supp(u) + supp(v) + supp(w) + $\sum_{i=1}^{n} \text{supp}(u_i) + \sum_{i=1}^{n} \text{supp}(v_i)$ = 2(n + 2) + 2(n + 1) + 2(n(n + 1))= 2(n^2 + 3n + 3).

EXAMPLE: 2.12 Consider the following graph G obtained by the subdivision of the central edge of the bistar $B_{3,3}$.

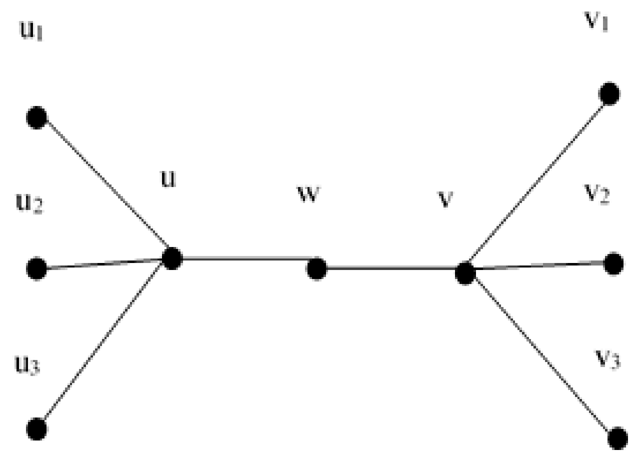

Figure 2.5

supp(G) = supp(u) + supp(v) + supp(w) + $\sum_{i=1}^{3} supp(u_i) + \sum_{i=1}^{3} supp(v_i)$
supp(G) = 5 + 5 + 8 + 12 + 12 = 42.

DEFINITION 2.13: Let G be a graph with fixed vertex v and let (P_m; G) be a graph obtained from m copies of G and the cycle P_m : $u_1,u_2,\ldots,u_m$ by joining u_i with the vertex v of i^{th} copy of G by means of an edge for $1 \le i \le n$.

THEOREM 2.14: Let G = (P_m: $K_{1,n}$) , m ≥ 2, n ≥ 1. supp(G) = $mn^2 + 3mn + 10m – 10$.

PROOF: Let V(G) = {u_i, v_j, w_{jk} / $1 \le i \le m, 1 \le j \le m, 1 \le k \le n$}.

E(G) = {u_iu_{i+1},, u_jv_j,, v_jw_{jk}/ $1 \le i \le m-1, 1 \le j \le m, 1 \le k \le n$}. deg u_1 = deg u_m = 2, deg u_i = 3, $2 \le i \le m-1$. deg v_i = n + 1, $1 \le i \le m$. deg w_{ik} = 1, $1 \le i \le m, 1 \le k \le n$.

supp(u_1) = deg v_1 + deg u_2 = n + 1 + 3 = n + 4. Similarly supp(u_m) = n + 4.

supp(u_2) = deg u_1 + deg u_3+deg v_2 = 2 + 3 + n + 1 = n + 6. Similarly supp(u_{m-1}) = n + 6.

For each i = 3,4,…, m – 2. supp(u_i) = deg u_{i-1} + deg u_{i+1} + deg v_i = 3 + 3+ n + 1 = n + 7.

supp(v_1) = deg u_1 + deg w_{11}+…+ deg w_{1n} = 2 + 1 + …+ 1 (n times) = n + 2. Similarly supp(v_m) = n + 2. For each i = 2, 3,…, m−1. supp(v_i) = deg u_i + deg w_{i1} +…+ deg w_{in} = 3 + n = n + 3. For each i = 1, 2,…,m, k = 1,2,…,n. supp(w_{ik}) = n + 1. supp(G) = supp(u_1) + supp(u_2) +$\sum_{i=3}^{m-2} supp(u_i)$ + supp(u_{m-1}) + supp(u_m) +supp(v_1) + $\sum_{i=2}^{m-1} supp(v_i)$ +supp(v_m) + $\sum_{i=1}^{m}\sum_{k=1}^{n} supp(w_{ik})$ = n + 4 + n + 6 + (m – 4)(n + 7) + n + 6 + n + 4 + n + 2 + (m – 2)(n + 3) + n + 2 + mn(n + 1) = $mn^2 + 3mn + 10m-10$.

EXAMPLE 2.15: Consider the following graph G = (P_5: K_{13}).

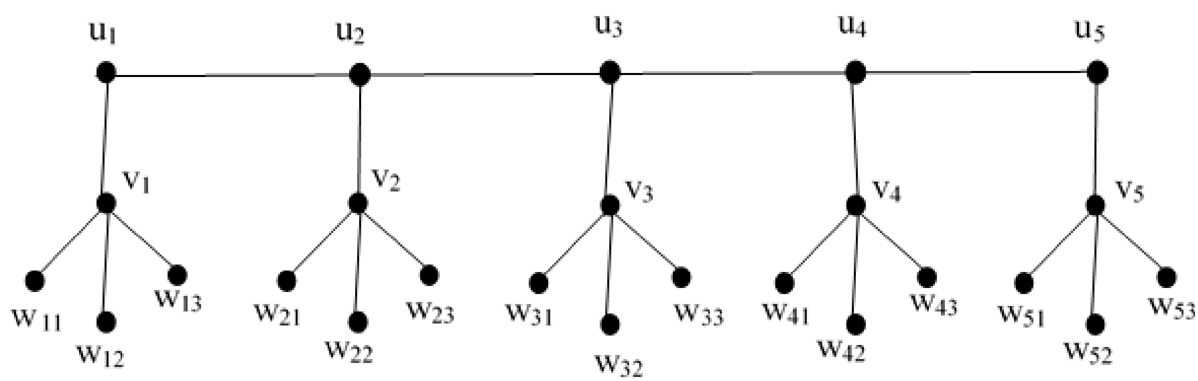

Figure 2.6

$supp(u_1) = 7 = supp(u_5)$, $supp(u_2) = supp(u_4) = 9$, $supp(u_5) = 10$

$supp(v_1) = 5 = supp(v_5)$, $supp(v_2) = supp(v_3) = supp(v_4) = 6$, $supp(w_{ik}) = 4$, $1 \le i \le 5$, $1 \le k \le 3$.

$supp(G) = 2(7) + 2(9) + 10 + 2(5) + 3(6) + 15(4) = 130 = 5(9) + 3(5)(3) + 10(5) – 10$.

DEFINITION 2.16: Let G be a graph with fixed vertex v and let $(C_m; G)$ be a graph obtained from m copies of G and the cycle $C_m : u_1,u_2,\ldots,u_m$ by joining u_i with the vertex v of i^{th} copy of G by means of an edge for $1\le i \le n$.

THEOREM 2.17: Let $G = (C_m : K_{1,n})$, $m \ge 3$, $n \ge 1$.Then $supp(G) = m(n^2 + 3n + 10)$.

PROOF: Let $V(G) = \{u_i, v_j, w_{jk} / 1\le i \le m, 1 \le j \le m, 1 \le k \le n\}$

$E(G) = \{v_iv_{i+1} / 1 \le i \le m - 1\} \cup \{v_iu_i / 1\le i \le m\} \cup \{u_iw_{ik} / 1 \le i \le m, 1 \le k \le n\}$.

$\deg v_i = 3$, $\deg u_i = n + 1$, $\deg w_{ik} = 1$, $1 \le i \le m, 1 \le k \le n$.

For each $i = 1,2,\ldots ,m – 1$, $supp(v_i) = \deg v_{i-1}+\deg v_{i+1}+\deg u_i = 3+3+n+1 = n+7$.

Similarly, $supp(v_m) = n + 7$.

For each $i = 1,2,\ldots,m$, $supp(u_i) = \deg v_i + \deg w_{i1}+\ldots+\deg w_{in} = 3 + 1 + \ldots+ 1$ (n times) $= n + 3$.

For each $i = 1,2,\ldots,m$, $k = 1,2,\ldots, n$. $supp(w_{ik}) = \deg u_i = n + 1$.

$supp(G) =\sum_{i=1}^{m} supp(u_i) +\sum_{i=1}^{m} supp(v_i) +\sum_{i=1}^{m}\sum_{k=1}^{n} supp(w_{ik}) = m (supp(u_i)) + m(supp(v_i)) + mn(supp(w_{ik})) = m(n+7) + m(n+3) + mn(n+1) = m(n^2+3n+10)$.

EXAMPLE 2.18: Consider the following graph $G = (C_4 : K_{1,3})$

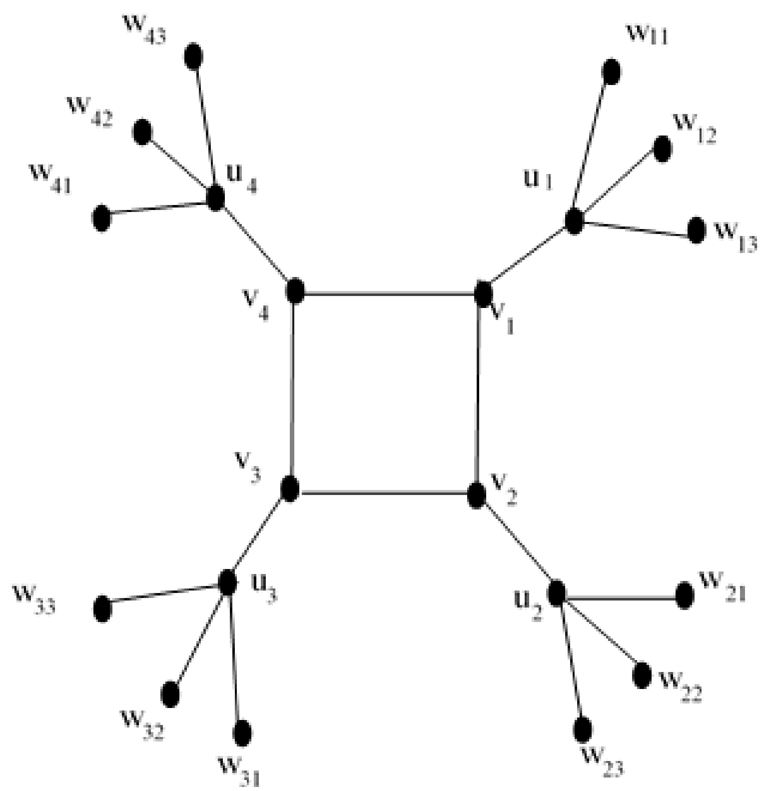

Figure 2.7

supp(v_i) = 10, $1 \leq i \leq 4$, supp(u_i) = 6, $1 \leq i \leq 4$, supp(w_{ik}) = 4, $1 \leq i \leq 4, 1 \leq k \leq 3$.

supp(G) = $\sum_{i=1}^{4} supp(v_i) + \sum_{i=1}^{4} supp(u_i) + \sum_{i=1}^{4}\sum_{k=1}^{3} supp(w_{ik})$

= 4(10) + 4(6) + 4(3)(4) = 112 = $4(3^2 + (3\times3) + 10)$.

DEFINITION 2.19: Let G be a graph with fixed vertex v and let (Km; G) be a graph obtained from m copies of G and the complete graph K_m : $u_1, u_2, \ldots, u_m$ by joining u_i with the vertex v of i^{th} copy of G by means of an edge for $1 \leq i \leq n$.

THEOREM 2.20: Let G = (K_m : $K_{1,n}$), m ≥ 4, n ≥ 1. Then supp(G) = $m(m^2 + n^2 + 3n + 1)$.

PROOF: Let V(G) = { v_i , u_i, w_{ik} / $1 \leq i, j \leq m, 1 \leq k \leq n$}.

E(G) = {v_iv_j / $1 \leq i, j \leq m, i \neq j$} ∪ {$v_iu_i$, u_iw_{ik} / $1 \leq i \leq m, 1 \leq k \leq n$}.

deg v_i = m, deg u_i = n + 1, $1 \leq i \leq m$, deg w_{ik} = 1, $1 \leq i \leq m, 1 \leq k \leq n$.

For each i = 1,2,…,m. supp(v_i) = $\sum_{\substack{j=1 \\ j\neq i}}^{m} \deg(v_j)$ + deg u_i = (m – 1)(m) + n + 1 = $m^2 - m + n + 1$.

For each i = 1,2,…,m. supp(v_i) = deg v_i + $\sum_{k=1}^{n} \deg(w_{ik})$ = m + n(1) = m + n.

For each i = 1,2,…,m, k = 1,2,…,n. supp(w_{ik}) = deg u_i = n + 1.

supp(G) = $\sum_{i=1}^{m} \text{supp}(u_i)$ + $\sum_{i=1}^{m} \text{supp}(v_i)$ + $\sum_{i=1}^{m}\sum_{k=1}^{n} supp(w_{ik})$ = m $(m^2 - m + n + 1)$ + $m(m + n) + mn(n + 1) = m^3 + mn + m - m^2 + m^2 + mn + mn^2 + mn = m(m^2 + n^2 + 3n + 1)$.

EXAMPLE 2.21: Consider the following graph G = (K_5: $K_{1,2}$)

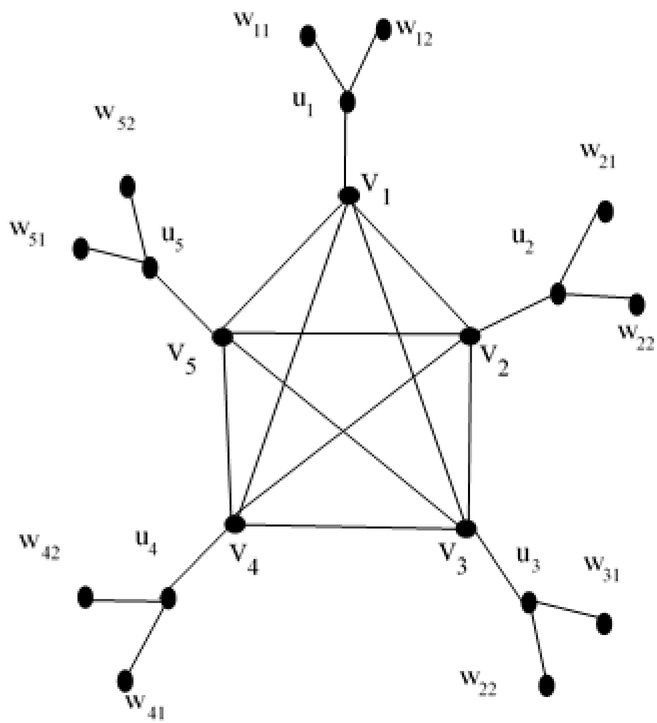

Figure 2.8

supp(v_i) = 23, $1 \leq i \leq 5$, supp(u_i) = 7, $1 \leq i \leq 5$, supp(w_{ik}) = 3, $1 \leq i \leq 5, 1 \leq k \leq 2$.

supp(G) = $\sum_{i=1}^{5} supp(v_i) + \sum_{i=1}^{5} supp(u_i) + \sum_{i=1}^{5}\sum_{k=1}^{2} supp(w_{ik})$ = 5(23) + 5(7) + 5(2)(3) = 115 + 35 + 30 = 180 = $5(5^2 + 2^2 + 3(2) + 1)$.

DEFINITION 2.22: Let G be a graph with fixed vertex v and let (Wm; G) be a graph obtained from m copies of G and the Wheel W_m : $u_1, u_2, \ldots, u_m$ by joining u_i with the vertex v of i^{th} copy of G by means of an edge for $1 \leq i \leq n$.

THEOREM 2.23: Let G = (W_m: $K_{1,n}$), m ≥ 3, n ≥ 1.Then supp(G) = $m(5m + n^2 + 3n - 3)$.

PROOF: Let V(G) = {v, v_i, u_i, w_{ik} / $1 \leq i \leq m, 1 \leq k \leq n\}$.

E(G) = {vv_i, v_iv_{i+1}/ $1 \leq i \leq m\} \cup$ {v_iu_i, u_iw_{ik} / $1 \leq i \leq m, 1 \leq k \leq n\}$.

deg v = m, deg v_i = m – 1, deg u_i = n+1, $1 \leq i \leq m$, deg w_{ik} = $1 \leq i \leq m, 1 \leq k \leq n$.

For each i = 1,2,…,m. supp(v) = $\sum_{i=1}^{m} \deg v_i = m^2 - m$.

For each i = 1,2,…,m, supp(v_i) = deg v + deg v_{i+1} + deg v_{i-1} + deg u_i = 3m + n – 1.

For each i = 1,2,…,m. $supp(u_i) = \deg v_i + \sum_{k=1}^{n} \deg (w_{ik}) = m + n - 1$.

For each i = 1,2,…,m, k = 1,2,…,n. $supp(w_{ik}) = \deg u_i = n + 1$.

$supp(G) = supp(v) + \sum_{i=1}^{m} supp(u_i) + \sum_{i=1}^{m} supp(v_i) + \sum_{i=1}^{m}\sum_{k=1}^{n} supp(w_{ik}) = m^2 - m + m(3m + n - 1) + m(m + n - 1) + mn(n + 1) = 5m^2 + mn^2 + 3mn - 3m = m(5m + n^2 + 3n - 3)$

EXAMPLE 2.24: Consider the following graph $G = (W_5: K_{1,2})$

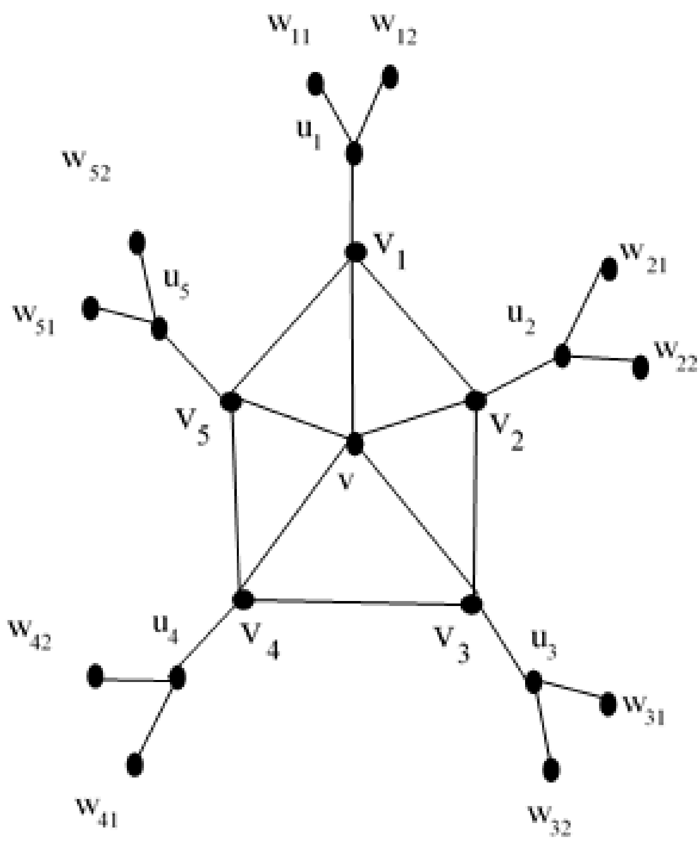

Figure 2.9

$supp(v) = 20$, $supp(v_i) = 16, 1 \leq i \leq 5$, $supp(u_i) = 6, 1 \leq i \leq 5$, $supp(w_{ik}) = 3, 1 \leq i \leq 5, 1 \leq k \leq 2$. $supp(G) = supp(v) + \sum_{i=1}^{5} supp(v_i) + \sum_{i=1}^{5} supp(u_i) + \sum_{i=1}^{5}\sum_{k=1}^{2} supp(w_{ik}) = 20 + 5(16) + 5(6) + 5(2)(3) = 160 = 5[5(5) + 2^2 + 3(2) - 3]$

DEFINITION 2.25: Let G be a graph with fixed vertex v and let $(K_{1,m}; G)$ be a graph obtained from m copies of G and the Star $K_{1,m} : u_1, u_2, \dots, u_m$ by joining u_i with the vertex v of i^{th} copy of G by means of an edge for $1 \leq i \leq n$.

THEOREM 2.26: Let $G = (K_{1,m}: K_{1,n})$, $m \geq 4$, $n \geq 1$. Then $supp(G) = m^2 + mn^2 + n^2 + 3mn + 7m + 3n + 2$.

PROOF: Let V(G) = {v, u, v_i , u_i, w_{ik}/1≤ $i \leq m, 1 \leq k \leq n$}.

E(G) = {vu, vv_i /≤ $i \leq m$} ∪ {v_iu_i, uw_{ik}, u_iw_{ik} /1 ≤ $i \leq m, 1 \leq k \leq n$}. deg v = m + 1,

deg u = n+1,deg v_i = 2, deg u_i = n+1, 1 ≤ $i \leq m$, deg w_{ik} = 1, 1 ≤ $i \leq m, 1 \leq k \leq n$.

For each i = 1,2,…,m. supp(v) = deg u + $\sum_{i=1}^{m} \deg\ (v_i)$ = n + 1+ 2m.

For each i = 1,2,…,m, supp(u) = deg v+$\sum_{k=1}^{n} \deg\ (w_{ik})$ = m + 1 + n(1) = m + n + 1.

For each i = 1,2,…,m, supp(v_i) = deg v + deg u_i = m + n + 2.

For each i = 1,2,…,m. supp(u_i) = deg v_i+= $\sum_{k=1}^{n} \deg\ (w_{ik})$ = 2+n(1) = 2+n.

For each i = 1,2,…,m, k = 1,2,…,n, supp(w_{ik}) = deg u_i = n + 1.

supp(G) = $supp(v) + supp(u) + \sum_{i=1}^{m} supp(u_i)$+$\sum_{i=1}^{m} supp(v_i)$+$\sum_{i=1}^{m+1} \sum_{k=1}^{n} supp(w_{ik})$ = 2m+n+1+m+n+1+m(m+n+2)+m(2+n)+n(m+1)(n+1) = 2m+n+1+m+n+1+m^2+ mn+2m+ 2m+ mn+ mn^2+mn+n^2+n = m^2+mn^2+n^2+3mn+7m+3n+2.

EXAMPLE 2.27: Consider the following graph G,

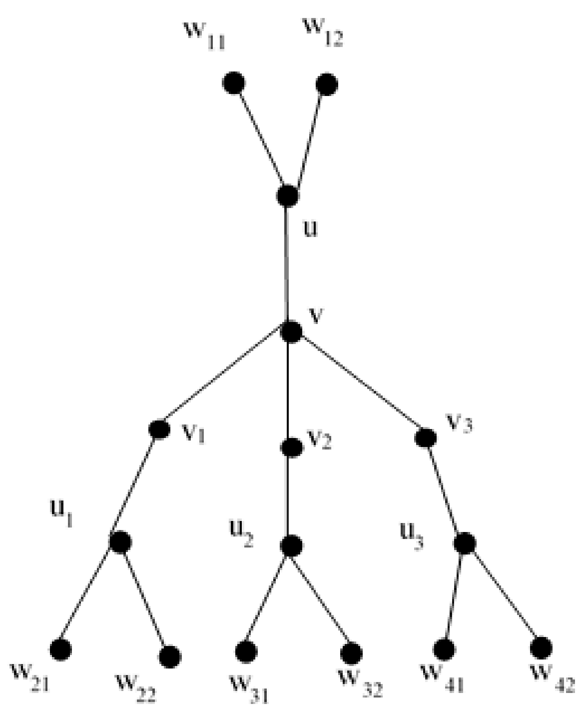

Figure 2.10

supp(u) = 6, supp(v) = 9, supp(v_i) = 7, 1≤ $i \leq 3$, supp(u_i) = 4, 1 ≤ $i \leq 3$,

supp(w_{ik}) = 3, 1 ≤ $i \leq 3, 1 \leq k \leq 2$.

supp(G) = supp(v) + supp(u) + $\sum_{i=1}^{3} supp(v_i) + \sum_{i=1}^{3} supp(u_i)$+ $\sum_{i=1}^{4} \sum_{k=1}^{2} supp(w_{ik})$ = 9 + 6 + 3(7) + 3(4) + 4(2)3 = 72 = 3^2+ 3(2^2) + 2^2+ 3(3)(2) + 7(3) + 3(2) + 2.

3. CONCLUSION

In this paper, the authors studied the open support of some graphs under addition. Similar study can be extended to closed support of graphs under addition, open support of some graphs under multiplication, closed support of graphs under multiplication.

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References

[1] M. Annapoopathi & N, Meena. Strong Efficient Edge Domination Number of Some Subdivision Graphs. *Advances in Mathematics: Scientific Journal,* 8, No. 3, 607-616, 2019

[2] M. Annapoopathi & N. Meena. Strong Efficient Edge Domination Number of Some Corona Related Graphs. *Journal of Emerging Technologies and Innovative Research,* Volume 6, Issue 2, 778-787, 2019

[3] M. Annapoopathi & N. Meena. Strong Efficient Edge Domination Number of Some Corona Related Graphs. *International Journal of Mathematical Trends and Technology,* Vol. 59, Number 3, 201-208

[4] Aram H., Sheikholeslami, S.M. Favaron, O. Domination subdivision number of trees. *Discrete Math.* 309, 622–628, 2009

[5] R. Balakrishnan, K. Ranganathan. A Textbook of Graph Theory, Springer, 2011.

[6] S. Balamurugan, M. Anitha, P. Aristotle, C.Karnan. A Note on Open Support of a Graph Under Addition I, *International Journal of Mathematics Trends and Technology,* Volume 65, Issue 5, 110-114, 2019

[7] S. Balamurugan, M. Anitha, P. Aristotle, C. Karnan. A Note on Open Support of a Graph under Addition II. *International Journal of Mathematics Trends and Technology,* Volume 65, Issue 5, 115-119, 2019

[8] S. Balamurugan, M. Anitha, C. Karnan. Closed Support of a Graph Under Addition I. *International Journal of Mathematics Trends and Technology,* Volume 65, Issue 5, 115-119, 2019

[9] S. Balamurugan, M. Anitha, C. Karnan, P. Palanikumar. Closed Support of a Graph under Addition II. *International Journal of Mathematics Trends and Technology,* Volume 65, Issue 5, 123-128 , 2019

[10] Benecke, S., Mynhardt, C.M. Trees with domination subdivision number one. *Australas. J. Combin.* 42, 201-209, 2008

[11] J.A. Bondy, U.S.R. Murthy, Graph theory with Applications, North-Holland, 1982.

[12] M. Dettlaff, M. Lemaniskaa, J. Topp. P. Zylinski. Coronas and Domination Subdivision Number of a Graph. *Bull. Malays. Math. Sci. Soc.* 41: 1717-1724, 2018

[13] Favaron, O., Haynes, T.W., Hedetniemi, S.T. Domination subdivision numbers in graphs. *Util. Math.* 66, 195-209 2004

[14] Fei Huang, Shanshan Guo, and Jinjiang Yuan, A Short note on Open Neighbourhood Conflict- Free Colorings of Graphs. *SIAM Journal on Discrete Mathematics,* Vol. 34, No. 3, 2009-2015, 2020

[15] R. Guruviswanathan & V. Swaminathan, Support Strong Dom Chromatic Number in Graphs. *International Journal of Pure and Applied Mathematics,* Volume 109, No. 9, 150-158, 2016

[16] F. Harary. Graph Theory, Addsion Wesley Reading, Mass, (1972).

[17] P.P. Kale and N.V. Deshpande. Corrections to the Neighborhood number of a graph. *Indian J. Pure Appl. Math.* 19(9), 927-929, September, 1985

[18] V. R. Kulli and Sigarkanti. Further results on the Neighborhood number of a graph. *Indian J. Pure Appl. Math.* 23(8); 575-577, August 1992

[19] V.Lavanya, D.S.T.Ramesh, & N.Meena, Bidomination in Corono Related Graphs. *Journal of Computer and Mathematical Sciences,* Vol. 10(11), 1650-1653, November 2019

[20] M. Basker, P. Namasivayam, M. P. Syed Ali Nisaya. Some Results on Centered Triangular Sum Graphs. *World Scientific News* 155 (2021) 113-128

[21] K. Murugan, A .Subramanian. Skolem difference mean labeling of H-graphs. *International Jurnal of Mathematics and Soft Computing* Vol. 1 , No:1, 115-129, 2011

[22] K. Murugan and A. Subramanian, Labeling of Subdivided Graphs. *American Journal of Mathematics and Sciences* Vol. 1, No: 1, 143-149 , 2012

[23] M. Prema and K. Murugan, Oblong sum labeling of some graphs. *World Scientific News* 98 (2018) 12-22

[24] K. Murugan. Square Graceful Labeling of Some Graphs. *International Journal of Innovative Research in Science, Engineering and Technology*, Vol. 4, Issue 2, February 2015

[25] S. K. Patel and N. P. Shrimali, Neighborhood-prime labeling. *International Journal of Mathematics and Soft Computing,* Vol. 5, No. 2 (2015) 135-143

[26] S. K. Patel and N. P. Shrimali, Neighborhood-prime labeling of some union graphs. *International Journal of Mathematics and Soft Computing,* Vol. 6, No. 1 (2016) 39-47

[27] R. Sakthi Sankari and M.P. Syed Ali Nisaya, Second order triangular graceful graphs. *World Scientific News* 155, 140-154, 2021

[28] E. Sampathkumar and Prabha S. Neeralagi. The Neighbourhood number of a graph. *Indian J. Pure Appl. Math.* 16(2), 126-132, February 1985.

[29] M. Shikhi and V. Anil Kumar, Common neighbor polynomial of graphs. *Far East Journal of Mathematical Sciences,* Volume 102, Number 6, 2017, Pages 1201-1221

[30] Soner, N. D. and B. Chaluvaraju, The paired-neighborhood number of a graph. *Proc. Jangjeon Mathematical Society, South Korea*, 8(1), 113-121, 2005

[31] Soner, N. D. and B. Chaluvaraju, Maximal edge neighborhood in graphs. *Indian Journal of Mathematics* 46(2- 3), 283-291, 2004

[32] P. Gnanachandra, M. Lellis Thivagar, Separation axioms by virtue of soft semi*-open sets. *World Scientific News* 145 (2020) 74-84

[33] R. Sakthi Sankari and M. P. Syed Ali Nisaya, Seond Order Triangular Gracful Graphs. *World Scientific News* 155 (2021) 140-154

[34] Kostochka, Alexandr, Yager, Derrek, Yu, Gexin 2020. Disjoint Chorded Cycles in Graphs with High Ore-Degree. *Discrete Mathematics and Applications Vol*. 165, p. 259. https://doi.org/10.1007/978-3-030-55857-4_11

[35] Costalonga, J.P., Kingan, Robert J., Kingan, Sandra R. 2021. Constructing Minimally 3-Connected Graphs. *Algorithms*, Vol. 14, Issue. 1, p. 9. https://doi.org/10.3390/a14010009

[36] G. Muthumanickavel, K. Murugan, Oblong Sum Labeling of Union of Some Graphs. *World Scientific News* 145 (2020) 85-94

[37] K. R. Parthasarathy, Basic Graph Theory, Tata Mcgraw Hill Publishing Company Limited, 1994

[38] M. Vanu Esakki, M. P. Syed Ali Nisaya, Two Modulo Three Sum Graphs. *World Scientific News* 145 (2020) 274-285

[39] M. Vanu Esakki, M. P. Syed Ali Nisaya, Some Results on Two Modulo Three Sum Graphs. *Journal of Xidian University*, 14(9) (2020) 1090-1099

[40] N.Vedavathi, Dharmaiya Gurram, Applications on Graph Theory. *International Journal of Engineering Research and Technology*, Vol. 2, Issue 1, (2013) 1-4

[41] N. Meena, M. Madhan Vignesh. Strong Efficient Co-Bondage Number of Some Graphs. *World Scientific News* 145 (2020) 234-244

[42] Frank Werner.Graph Theoretic Problems and their New Applications. *Mathematics*, S 445, (2020) 1-4

[43] Xiaojing Yang, Junfeng Du, Liming Xiong. Forbidden subgraphs for supereulerian and Hamiltoniangraphs. *Discrete Applied Mathematics* Volume 288, 15 January 2021, Pages 192-200. https://doi.org/10.1016/j.dam.2020.08.034

[44] A. Fathima Banu, S. Chelliah, M. P. Syed Ali Nisaya, Even Vertex Tetrahedral Mean Graphs. *World Scientific News* 156 (2021) 26-39

[45] R. Sakthi Sankari, M. P. Syed Ali Nisaya, Higher order triangular graceful labeling of some graphs. *World Scientific News* 156 (2021) 40-61

[46] M. Baskar, P. Namasivayam, M. P. Syed Ali Nisaya, Further results on centered triangular sum graphs. *World Scientific News* 156 (2021) 13-25

[47] Chiba, Shuya, Yamashita, Tomoki, 2018. Degree Conditions for the Existence of Vertex-Disjoint Cycles and Paths: A Survey. *Graphs and Combinatorics*, Vol. 34, Issue. 1, p. 1. https://doi.org/10.1007/s00373-017-1873-5

[48] Molla, Theodore, Santana, Michael, Yeager, Elyse 2020. Disjoint cycles and chorded cycles in a graph with given minimum degree. *Discrete Mathematics*, Vol. 343, Issue. 6, p. 111837. https://doi.org/10.1016/j.disc.2020.111837