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Further results on centered triangular sum graphs

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ABSTRACT

Let G be a graph with p vertices and q edges. The nth centered triangular number is denoted by M_n, where $M_n = \frac{1}{2}(3n^2 - 3n + 2)$. A centered triangular sum labeling of a graph G is a one-to-one function : V (G) → N ∪ {0} that induces a bijection f^*: E(G) →$\{M_1, M_2, \dots M_q\}$ of the edges of G defined by $f^*(uv) = f(u) + f(v)$, for all e = uv ∊ E(G). The graph which admits such labeling is called a centered triangular sum graph. In this article, the centered triangular sum labeling of union of some graphs are studied.

***Keywords*:** Centered triangular numbers, centered triangular sum labeling, centered triangular sum graphs

1. INTRODUCTION AND DEFINITIONS

Graphs considered in this paper are finite, undirected and simple. Let G = (V, E) be a graph with p vertices and q edges. Undefined terms are used in the sense of Harary [11], K. R. Parthasarathy [23] and Bondy and U.S.R. Murthy [5]. For number theoretic terminology, we refer to [3] and [22]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges / both) then the labeling is called the vertex (edge / total) labeling.

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Rosa [26] introduced β-valuation of a graph in the year 1966 and Golomb [10] called it as graceful labeling. There are several types of graph labeling and a detailed survey is found in [9].

The concept of a sum graph was introduced by Harary [12] in 1990 and was defined as a graph whose vertices can be labeled with distinct positive integers so that the sum of the labels on each pair of adjacent vertices is the label of some other vertex. In 1991, Harary et al. [14] defined a real sum graph. One of the earliest interesting results was due to Ellingham [7] who proved the conjecture of Harary [12].

In [19], S. Murugesan introduced centered triangular sum labeling graphs. Jeyanthi et al. [15] introduced centered triangular mean labeling. For more information related to sum graphs, see [13], [18], [20], [21], [24-44]. The following definitions are necessary for present study.

Definition 1.1: A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) repectively. The number of elements of V(G) = p is called the order of G and the number of elements of E(G) = q is called the size of G. A graph of order p and size q is called a (p,q) - graph. If e = *uv* is an edges of G, we say that *u* and *v* are adjacent and that *u* and *v* are incident with e.

Definition 1.2: Let the graphs G_1 and G_2 have disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their union $G = G_1 \cup G_2$ is a graph with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$.Clearly $G_1 \cup G_2$ has $p_1 + p_2$ vertices and $q_1 + q_2$ edges.

Definition 1.3: A connected acyclic graph is called a tree

Definition 1.4: A Path P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \leq i \leq$ n-1

Definition 1.5: The complete bipartite graph $K_{1,n}$ is called a Star graph.

Definition 1.6: A graph, which can be formed from a given graph G by breaking up each edge into exactly two segments by inserting intermediate vertices between its two ends, is called a sub division graph. It is denoted by S (G).

Definition 1.7: The bistar $B_{m,n}$ is a graph obtained from K_2 by joining *m* pendant edges to one end of K_2 and n pendant edges to the other end of K_2.

Definition 1.8: A centered triangular number is a centered figurate number that represents a triangle with a dot in the center and all other dots surrounding the center in successive triangular layers. If the nth centered triangular number is denoted by M_n, then $M_n = \frac{1}{2}(3n^2 - 3n + 2)$.
The first few centered triangular numbers are 1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199, 235, 274,...

Definition 1.9: A Sum labeling is an injective function $f : V(G) \rightarrow N \cup \{0\}$ that induces a bijection $f^+ : E(G) \rightarrow \{1,2,\ldots q\}$ of edges G defined by $f^+(uv) = f(u) + f(v)$, for all $e = uv \in E(G)$. The graph which admits such labeling is called a sum graph.

Definition 1.10: A centered triangular sum labeling of a graph G is a one-to-one function f : V (G) → N ∪ {0} that induces a bijection f^* : E(G) →$\{M_1, M_2, \dots M_q\}$of the edges of G defined by f^* (uv) = $f(u)$ + $f(v)$, for all e = uv ∊ E(G). The graph which admits such labeling is called a centered triangular sum graph.

2. MAIN RESULTS

Theorem 2.1: The graph $P_n \cup P_m$ is a centered triangular sum for all m, n ≥ 3.

Proof: Let G be a $P_n \cup P_m$ graph for all m, n ≥ 3.

Let V(G) = $\{u_i\,,\ v_j : 1 \le i \le n\,,\, 1 \le j \le m\}$ and

E(G) = $\{u_i u_{i+1}\,, v_j v_{j+1} : 1 \le i \le n-1\,, 1 \le j \le m-1\}$.

Here G has m + n vertices and m + n - 2 edges.

Let t = m + n – 2.

Define f : V(G) → {0,1, ….., M_t }as follows

$f(u_1)$ = 0.

For 2≤ $i \le n$, $f(u_i) = M_{i-1}$- $f(u_{i-1})$

$f(v_1)$ =- $f(u_n)$ – 1.

For 2≤ $j \le m$, $f(v_j) = M_{n+j-2}$- $f(v_{j-1})$.

Clearly f is injective and f induces a bijective function f^* : E(G) →{1,4,10,… M_t} as

$f^*(u_i u_{i+1})$ =M_i , 1≤ $i \le n-1$ and

$f^*(v_j v_{j+1})$ =M_{n-1+j} , 1≤ $j \le m-1$

Hence the edge labels are 1,4,… M_t.

Thus f is a centered triangular sum labeling of G.

Therefore, G = $P_n \cup P_m$ is a centered triangular sum graph.

Example 2.2: The centered triangular sum labeling of $P_3 \cup P_5$ is shown in Fig. 1.

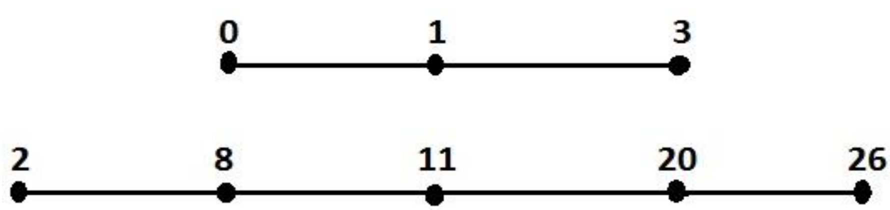

Fig. 1

Theorem 2.3: $K_{1,n} \cup B_{m,r}$ is a centered triangular sum graph for all n ≥ 3 and m, r ≥ 1.

Proof: Let G be a $K_{1,n} \cup B_{m,r}$ graph for all n ≥ 3 and m, r ≥ 1.

Let V(G) = $\{u, u_i, v, v_j, w, w_k : 1 \le i \le n, 1 \le j \le m$ and $1 \le k \le r\}$ and

E(G) = $\{uu_i, vv_j, vw, ww_k : 1 \le i \le n, 1 \le j \le m$ and $1 \le k \le r\}$

Here G has n + m + r + 3 vertices and n + m + r + 1 edges.

Let t = n + m + r + 1.

Define f : V(G) ⟶ $\{0,1,\ldots M_t\}$ as follows

$f(u) = 0$

$f(u_i) = M_i$, $1 \le i \le n$

$f(v) = f(u_{n-1}) - 1$.

$f(v_j) = M_{n+j+1} - f(v)$, $1 \le j \le m$

$f(w) = M_{n+1} - f(v)$,

$f(w_k) = M_{m+n+1+k} - f(w)$, $1 \le k \le r$

Clearly f is injective and f induces a bijective function f^* : E(G) ⟶$\{1,4,\ldots M_t\}$ as

$f^*(uu_i) = M_i$, $1 \le i \le n$

$f^*(vv_j) = M_{n+1+j}$, $1 \le j \le m$

$f^*(vw) = M_{n+1}$

$f^*(ww_k) = M_{n+m+1+k}$, $1 \le k \le r$

Hence the edge labels are 1,4,… M_t.

Thus f is a centered triangular sum labeling of G.

Therefore, G = $K_{1,n} \cup B_{m,r}$ is a centered triangular sum graph.

Example 2.4: The centered triangular sum labeling of $K_{1,3} \cup B_{3,4}$ is shown in Fig. 2.

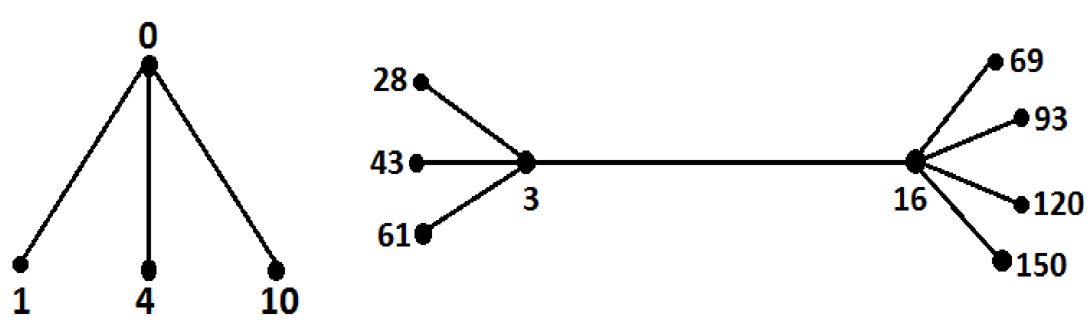

Fig. 2

Theorem 2.5: $K_{1,n} \cup K_{1,m}$ is a centered triangular sum graph for all n , m > 2.

Proof: Let G be a $K_{1,n} \cup K_{1,m}$ graph for all n , m > 2.

Let V(G) = $\{u, u_i, v, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and

E(G) = $\{uu_i, vv_j : 1 \leq i \leq n, 1 \leq j \leq m\}$

Here G has n + m + 2 vertices and n + m edges.

Let t = n + m

Define f : V(G) → $\{0,1,\ldots M_t\}$ as follows

$f(u) = 0$

$f(u_i) = M_i$, $1 \leq i \leq n$

$f(v) = f(u_{n-1}) - 1$.

$f(v_j) = M_{n+j} - f(v)$, $1 \leq j \leq m$

Clearly f is injective and f induces a bijective function f^* : E(G) →$\{1,4,\ldots M_t\}$ as

$f^*(uu_i) = M_i$, $1 \leq i \leq n$

$f^*(vv_j) = M_{n+j}$, $1 \leq j \leq m$

Hence the edge labels are 1,4,… M_t.

Thus f is a centered triangular sum labeling of G.

Therefore, G = $K_{1,n} \cup K_{1,m}$ is a centered triangular sum graph.

Example 2.6: The centered triangular sum labeling of $K_{1,3} \cup K_{1,3}$ is shown in Fig. 3.

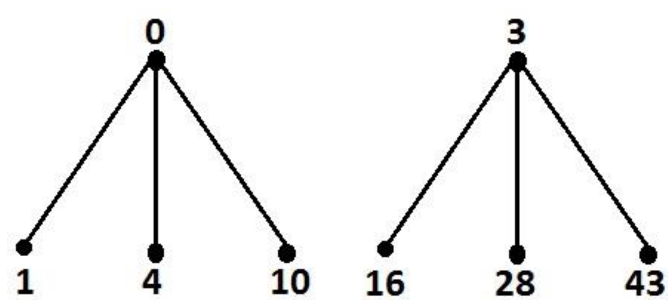

Fig. 3

Theorem 2.7: S($K_{1,n}$) ∪ $B_{r,s}$ is a centered triangular sum graph for all n> 2 and r, s > 1.

Proof: Let G be a S($K_{1,n}$) ∪ $B_{r,s}$ graph for all n> 2 and r, s > 1.

Let V(G) = $\{u, u_i, v_i, w, w_j, x, x_k : 1 \leq i \leq n, 1 \leq j \leq r$ and $1 \leq k \leq s\}$ and

E(G) = $\{uu_i, u_i v_i, ww_j, wx, xx_k : 1 \leq i \leq n, 1 \leq j \leq r$ and $1 \leq k \leq s\}$.

Here G has 2n + r + s + 3 vertices and 2n + r + s + 1 edges.

Let t = 2n + r + s + 1.

Define f: V(G) ⟶ {0,1,…M_t} as follows

$f(u) = 0$

$f(u_i) = M_i$, $1 \le i \le n$

$f(v_i) = M_{n+i} - M_i$, $1 \le i \le n$

$f(w) = f(v_{n-2}) - 2$.

$f(w_j) = M_{2n+j+1} - f(w)$, $1 \le j \le r$

$f(x) = M_{2n+1} - f(w)$,

$f(x_k) = M_{2n+r+1+k} - f(x)$, $1 \le k \le s$

Clearly f is injective and f induces a bijective function f^* : E(G) ⟶{1,4,… M_t} as

$f^*(uu_i) = M_i$, $1 \le i \le n$

$f^*(u_i v_i) = M_{n+i}$, $1 \le i \le n$

$f^*(ww_j) = M_{2n+j+1}$, $1 \le j \le r$

$f^*(wx) = M_{2n+1}$

$f^*(xx_k) = M_{2n+r+1+k}$, $1 \le k \le s$

Hence the edge labels are 1,4,… M_t.

Thus f is a centered triangular sum labeling of G.

Therefore, G = S($K_{1,n}$) ∪ $B_{r,s}$ is a centered triangular sum graph.

Example 2.8: The centered triangular sum labeling of S($K_{1,3}$) ∪ $B_{2,3}$ is shown in Fig. 4.

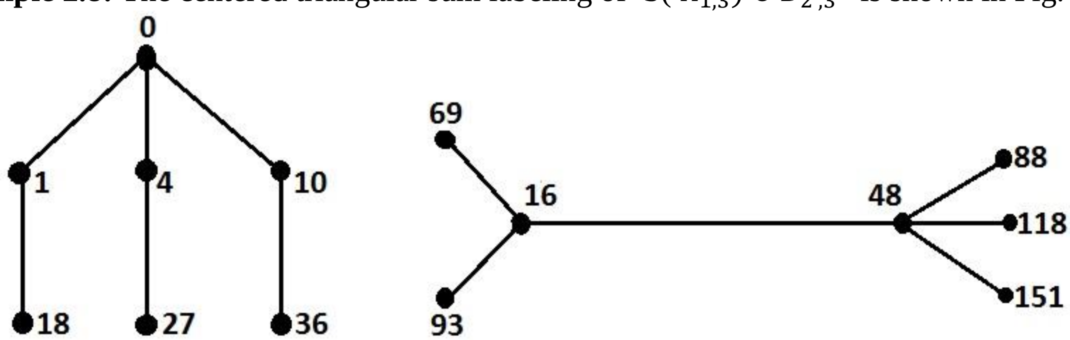

Fig. 4

Theorem 2.9: S($K_{1,n}$) ∪ S($K_{1,m}$) is a centered triangular sum graph for all n, m > 2

Proof: Let G be a S($K_{1,n}$) ∪ S($K_{1,m}$) graph for all n, m > 2.

Let V(G) = {u, u_i, v_i, w, w_j, x_j: $1 \le i \le n$, $1 \le j \le m$} and

E(G) = $\{uu_i, u_i v_i, ww_j, w_j x_j : 1 \leq i \leq n, 1 \leq j \leq m\}$.

Here G has 2n + 2m + 2 vertices and 2n +2m edges.

Let t = 2n + 2m.

Define f: V(G) → $\{0,1,\ldots M_t\}$ as follows

$f(u) = 0$

$f(u_i) = M_i$, $1 \leq i \leq n$

$f(v_i) = M_{n+i} - M_i$, $1 \leq i \leq n$

$f(w) = f(v_{n-2}) - 2$.

$f(w_j) = M_{2n+j} - f(w)$, $1 \leq j \leq m$

$f(x_j) = M_{2n+m+j} - f(w_j)$, $1 \leq j \leq m$

Clearly f is injective and f induces a bijective function f^* : E(G) →$\{1,4,\ldots M_t\}$ as

$f^*(uu_i) = M_i$, $1 \leq i \leq n$

$f^*(u_i v_i) = M_{n+i}$, $1 \leq i \leq n$

$f^*(ww_j) = M_{2n+j}$, $1 \leq j \leq m$

$f^*(w_j x_j) = M_{2n+m+j}$, $1 \leq j \leq m$

Hence the edge labels are $1,4,\ldots M_t$.

Thus f is a centered triangular sum labeling of G.

Therefore, G = S($K_{1,n}$) ∪ S($K_{1,m}$) is a centered triangular sum graph.

Example 2.10: The centered triangular sum labeling of S($K_{1,4}$) ∪ S($K_{1,5}$) is shown in Fig. 5.

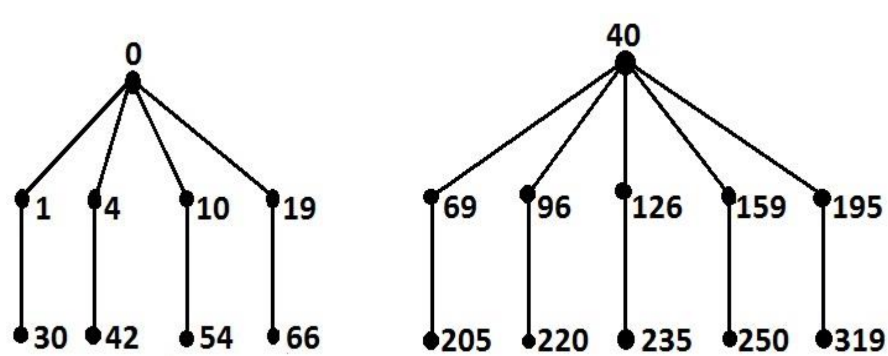

Fig. 5.

Theorem 2.11: S($K_{1,n}$) ∪ $K_{1,m}$ is a centered triangular sum graph for all n, m > 2

Proof: Let G be a S($K_{1,n}$) ∪ $K_{1,m}$ graph for all n, m > 2.

Let V(G) = $\{u, u_i, v_i, w, w_j : 1 \le i \le n, 1 \le j \le m\}$ and

E(G) = $\{uu_i, u_i v_i, ww_j : 1 \le i \le n, 1 \le j \le m\}$.

Here G has 2n + m + 2 vertices and 2n + m edges.

Let t = 2n + m.

Define f: V(G) ⟶ $\{0,1,\ldots M_t\}$ as follows

$f(u) = 0$

$f(u_i) = M_i$, $1 \le i \le n$

$f(v_i) = M_{n+i} - M_i$, $1 \le i \le n$

$f(w) = f(v_{n-2}) - 2$.

$f(w_j) = M_{2n+j}$ - $f(w)$, $1 \le j \le m$

Clearly f is injective and f induces a bijective function f^* : E(G) ⟶$\{1,4,\ldots M_t\}$ as

$f^*(uu_i) = M_i$, $1 \le i \le n$

$f^*(u_i v_i) = M_{n+i}$, $1 \le i \le n$

$f^*(ww_j) = M_{2n+j}$, $1 \le j \le m$

Hence the edge labels are $1,4,\ldots M_t$.

Thus f is a centered triangular sum labeling of G.

Therefore, G = S($K_{1,n}$) ∪ $K_{1,m}$ is a centered triangular sum graph.

Example 2.12: The centered triangular sum labeling of S($K_{1,4}$) ∪ $K_{1,5}$ is shown in Fig. 6.

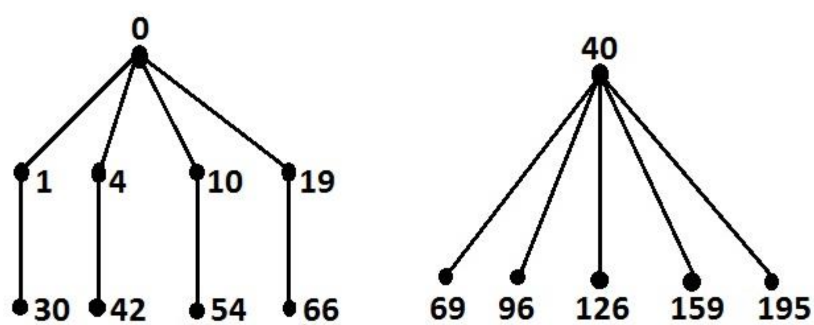

Fig. 6

Theorem 2.13: The graph $P_n \cup K_{1,m}$ is a centered triangular sum for all m, n ≥ 3.

Proof: Let G be a $P_n \cup k_{1,m}$ graph for all m, n ≥ 3.

Let V(G) = $\{u_i, w, v_j : 1 \le i \le n, 1 \le j \le m\}$ and

E(G) = $\{u_i u_{i+1}, wv_j : 1 \leq i \leq n-1, 1 \leq j \leq m\}$.

Here G has n + m + 1 vertices and n + m - 1 edges.

Let t = n + m - 1.

Define f: V(G) → {0,1, ….., M_t }as follows

$f(u_1) = 0$

For $2 \leq i \leq n$, $f(u_i) = M_{i-1} - f(u_{i-1})$ and

$f(w) = -f(u_n) - 1$.

$f(v_j) = M_{n+j-1} - f(w)$, $1 \leq j \leq m$

Clearly f is injective and f induces a bijective function f^*: E(G) →{1,4,10,… M_t} as

$f^*(u_i u_{i+1}) = M_i$, $1 \leq i \leq n-1$ and

$f^*(wv_j) = M_{n+j-1}$, $1 \leq j \leq m$

Hence the edge labels are 1,4,… M_t.

Thus f is a centered triangular sum labeling of G.

Therefore, G = $P_n \cup K_{1,m}$ is a centered triangular sum graph.

Example 2.14: The centered triangular sum labeling of $P_5 \cup K_{1,5}$ is shown in Fig. 7.

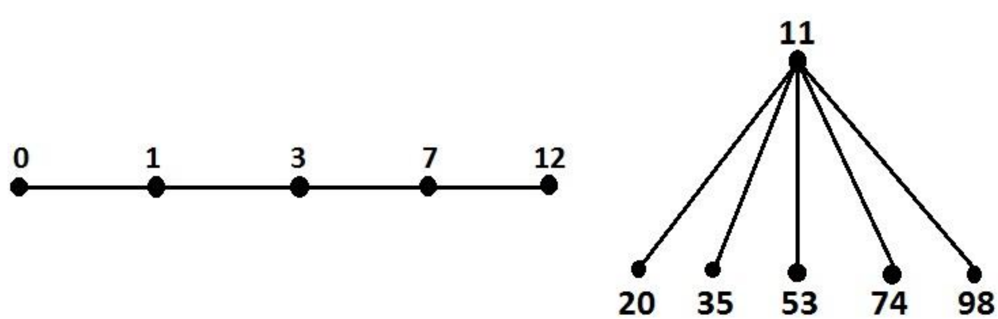

Fig. 7

Theorem 2.15: The graph $P_n \cup S(K_{1,m})$ is a centered triangular sum for all m, n ≥ 3.

Proof: Let G be a $P_n \cup S(k_{1,m})$ graph for all m, n ≥ 3.

Let V(G) = $\{ u_i, w, v_j, x_j : 1 \leq i \leq n, 1 \leq j \leq m \}$ and

E(G) = $\{u_i u_{i+1}, wv_j, v_j x_j : 1 \leq i \leq n-1, 1 \leq j \leq m\}$.

Here G has n + 2m + 1 vertices and n + 2m - 1 edges.

Let t = n + 2m - 1.

Define f: V(G) → {0,1, ….., M_t } as follows

$f(u_1) = 0$

For $2 \leq i \leq n$, $f(u_i) = M_{i-1} - f(u_{i-1})$ and

$f(w) = - f(u_n) - 1$.

$f(v_j) = M_{n+j-1} - f(w)$, $1 \leq j \leq m$.

$f(x_j) = M_{n+m+j-1} - f(v_j)$, $1 \leq j \leq m$.

Clearly f is injective and f induces a bijective function f^* : E(G) →{1,4,10,… M_t} as

$f^*(u_i u_{i+1}) = M_i$, $1 \leq i \leq n-1$ and

$f^*(w v_j) = M_{n+j-1}$, $1 \leq j \leq m$

$f^*(v_j x_j) = M_{n+m+j-1}$, $1 \leq j \leq m$

Hence the edge labels are 1,4,… M_t.

Thus f is a centered triangular sum labeling of G.

Therefore, G = $P_n \cup S(K_{1,m})$ is a centered triangular sum graph.

Example 2.16: The centered triangular sum labeling of $P_5 \cup S(K_{1,3})$ is shown in Fig. 8.

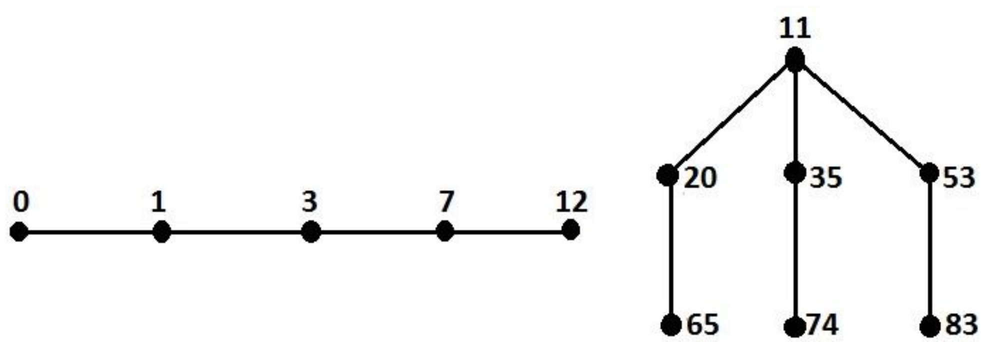

Fig. 8

3. CONCLUSION

In this paper, we have studied the centered triangular sum labeling of some union graphs. This work contributes several new results to the theory of graph labeling.

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