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# Bandung City Rainfall Data Interpolation Using Two Types of Cubic Spline Interpolation

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**ABSTRACT**

This study discusses the application of two types of cubic spline, namely natural cubic spline and not-a-knotcubic spline. The applications of both types of splines are regarding rainfall data. The rainfall data studied were the monthly rainfall data for the year 2018 and 2019 in Bandung City which was obtained from the Bandung City Central Statistics Agency. Interpolation and analysis were carried out on all data using Maple software. As a result, interpolation using natural cubic spline and not-a-knotcubic spline shows a smoother and less oscillating graph than interpolation using Piecewise Linear and Polynomial Interpolation.

***Keywords*:** Interpolation, natural cubic spline, not-a-knot cubic spline

## 1. INTRODUCTION

The city of Bandung is located in Indonesia, which is located near the equator [1]. This has resulted in two seasons exist in Indonesia including Bandung, namely, the dry and the rainy season throughout the year [2]. The weather keeps changing every month. This is due to several factors such as wind speed, wind direction, humidity, temperature level (maximum, minimum, and average) [3]. Other than that, Indonesia has a unique weather pattern, particularly the characteristics of it rainfall [4]. Erratic rainfall results in fluctuations in the data studied.

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There are months that have high rainfall and some are low. Rainfall is important issue in severalsectorse.g.,ecology and agriculture [5]. Thus, rainfall data has been widely researched to be useful. Rainfall data mainly derived from three sources. There are the direct rainfall measurement from rain gauge stations, rainfall estimation from remote sensing such as satellite and rainfall estimates from weather model [6]. As for this study, rainfall data were obtained from the Central Statistics Agency of Bandung City. The data used is rainfall data every month in 2018 and 2019. This data is used because the data is the latest data that can be researched which has rainfall data from January to December.

In this study, two types of cubic spline interpolation were used, namely, natural cubic spline and not-a-knotcubic spline. Both were compared with other interpolations such as, Piecewise Linear and Polynomial Interpolation. Cubic spline interpolation is an interpolation technique to create a curve functionthrough the given pointswith smooth and stablecurves [7]. It uses third degree polynomials as the maximum degree of the interpolation function, resulting a smooth interpolation curve [8]. This is an important thing to be considered for several engineering problems [9].

Interpolation was carried out on the overall monthly rainfall data for 2018 and 2019 in Bandung City. Overall, naturalcubic spline and not-a-knotcubic spline interpolation were better than Piecewise Linear and Polynomial Interpolation.

## 2. MATERIALS AND METHODS

Spline interpolation is a method that has significant differences with other interpolation methods, such as Lagrage's and Newton's polynomials. In other methods, the obtained interpolation polynomials amount to one that interpolates all data points. Meanwhile, in spline interpolation, adjacent data points are interpolated by one polynomial so that the number of interpolation polynomials obtained is $m-1$, where $m$ represents the total number of data points [10].

As with other methods, the degree of polynomial interpolation can be determined. The hope is that the higher the degree of polynomial, the smoother the resulting curve will be and closer to the original curve. In spline interpolation, the degree of polynomials that is often used is of order three. The spline interpolation method using a third degree of polynomial is also called the cubic spline method. In the cubic spline function, each pair of adjacent points is interpolated bya cubic or lower degree of polynomials as follows:

$$S_k(x) = a_k x^3 + b_k x^2 + c_k x + d_k$$

for $k = 1, 2, ..., n$ and $n$ represents the total number of data points minus one or the number of cubic polynomials [10].

### 2. 1. Natural Cubic Spline

Natural cubic splinedefines the degree of $S$ at most three, hence it is based on cubic spline. The boundary conditions specified in the naturalcubic splineare that the second derivatives of S are zero at the boundaries,

$$S''(x_0) = S''(x_n) = 0$$

Hence, if we want to create natural cubic spline *S* interpolating function *f*, we define a number of values $x_0 < x_1 < ... < x_n$, and to meet $S''(x_0) = S''(x_n) = 0$ [10-12].

### 2. 2. Not-a-Knot Cubic Spline

Not-a-Knot cubic splinedefines the degree of *S* at most three, hence it is based on cubic spline. The conditions specified in the not-a-knotcubic splinefollow the following assumptions. Suppose that

$$x_1 < z_1 < x_2,\ x_{n-1} \le z_2 \le x_n$$

and suppose that $f(z_1)$ and $f(z_2)$ are known. Hence, the particular conditions are

$$S(z_1) = f(z_1),\ \ S(z_2) = f(z_2)$$

until a unique $s(x_1)$ is obtained [10].

### 2. 3. Linear Interpolation

Consider the construction of polynomial through two given data points. This form of interpolation is known as linear interpolation. Given two data points $(x_0, y_0)$ and $(x_1, y_1)$ with $x_0 \ne x_1$ Linear interpolation is a straight line which connects the given points [10, 13, 14].

$$P_1(x) = \frac{(x_1 - x)y_0 + (x - x_0)y_1}{x_1 - x_0}$$

The graph of this function is the straight line determined by $(x_0, y_0)$ and $(x_1, y_1)$. The function interpolates the data $(x_i, y_i)$, or equivalently written as

$$P_1(x_i) = y_i, i = 0.1$$

### 2. 4. Quadratic Interpolation

Most data arise from graphs that are curved rather than straight. To better approximate such behavior, we can use a polynomialfunction witha degree greater than 1. Given three data points $(x_0, y_0), (x_1, y_1)$, and $(x_2, y_2)$, with $x_0, x_i, x_2$ being distinct numbers. A quadratic polynomial passing through these points is constructed as follows:

$$P_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

with

$$L_0(x) = \frac{(x-x_1)+(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$L_1(x) = \frac{(x-x_0)+(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$L_2(x) = \frac{(x-x_0)+(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

Formula (3) is also known as Lagrange's formula for quadratic interpolation; and the polynomials $L_0, L_1$, and $L_2$ are Lagrange interpolation basis functions [10].

Each polynomial $L_i(x)$ has a polynomial degree of 2. In addition,

$$L_1(x_j) = 0, \ \ j \neq i$$

$$L_1(x_j) = 1,$$

for $0 \leq i, j \leq 2$. These two statements are combined into the statement

$$L_i(x_j) = \delta_{ij}, 0 \leq i, j \leq 2$$

where $\delta_{ij}$ is Kronecker delta function

$$\delta_{ij} \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$$

With the use of these properties, we can easily show that $P_2(x)$ interpolates the given data, which could be expressed with

$$P_2(x_i) = y_i, i = 0,1,2.$$

The shape of the graph is used as a means to find out which interpolation method is better. By using the 2018 and 2019 Bandung City rainfall data, each interpolation method gives the corresponding interpolation graph whose graphs are shown following Table 1 and Table 2.

**Table 1.** Bandung City Rainfall Data for 2018.

| Month | Rainfall (mm) |
|---|---|
| 1 | 191.00 |

| | |
|---|---|
| 2 | 239.30 |
| 3 | 292.00 |
| 4 | 297. 60 |
| 5 | 123.90 |
| 6 | 33.40 |
| 7 | 0.30 |
| 8 | 38.90 |
| 9 | 40.80 |
| 10 | 124.80 |
| 11 | 483.20 |
| 12 | 323.50 |

**Table 2.** Bandung City Rainfall Data for 2019.

| Month | Rainfall (mm) |
|---|---|
| 1 | 231.60 |
| 2 | 269.10 |
| 3 | 222.70 |
| 4 | 298.90 |
| 5 | 245.70 |
| 6 | 26.50 |
| 7 | 13.40 |
| 8 | 0.20 |
| 9 | 55.00 |
| 10 | 84.20 |
| 11 | 270.70 |
| 12 | 313.50 |

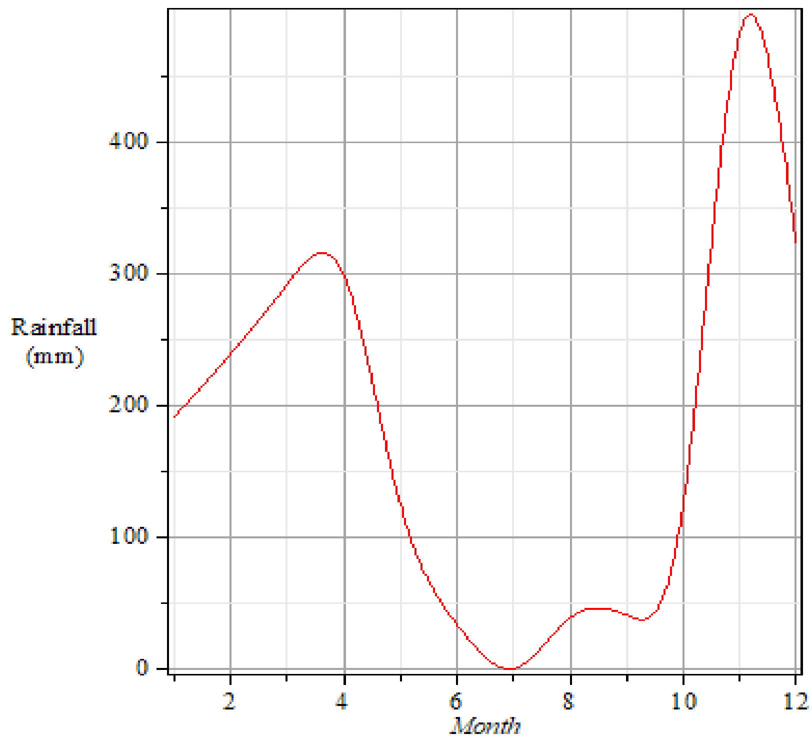


**Figure 1.** Natural Cubic Spline Interpolation (2018)

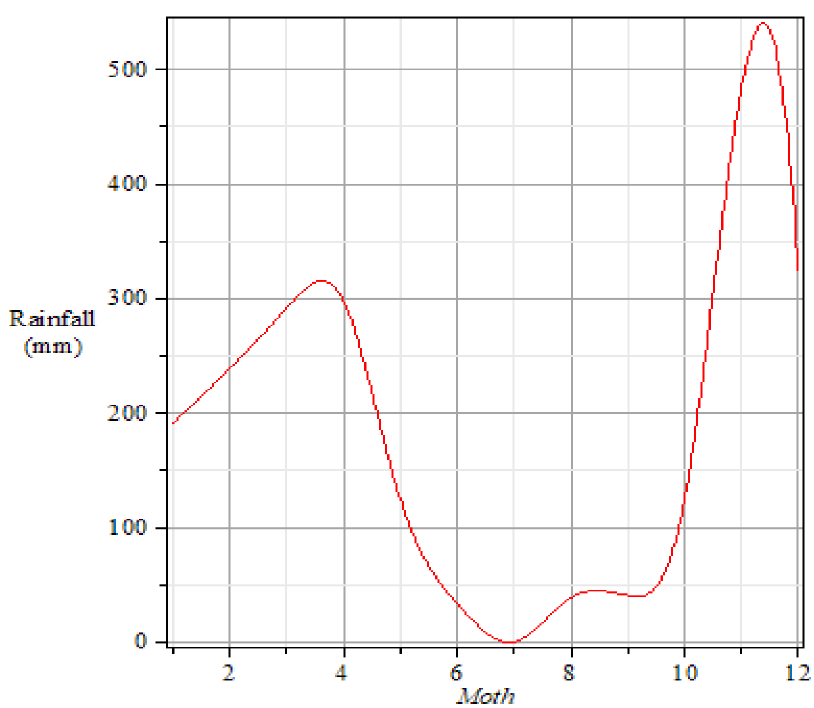


**Figure 2.** Not-a-Knot Cubic Spline Interpolation (2018)

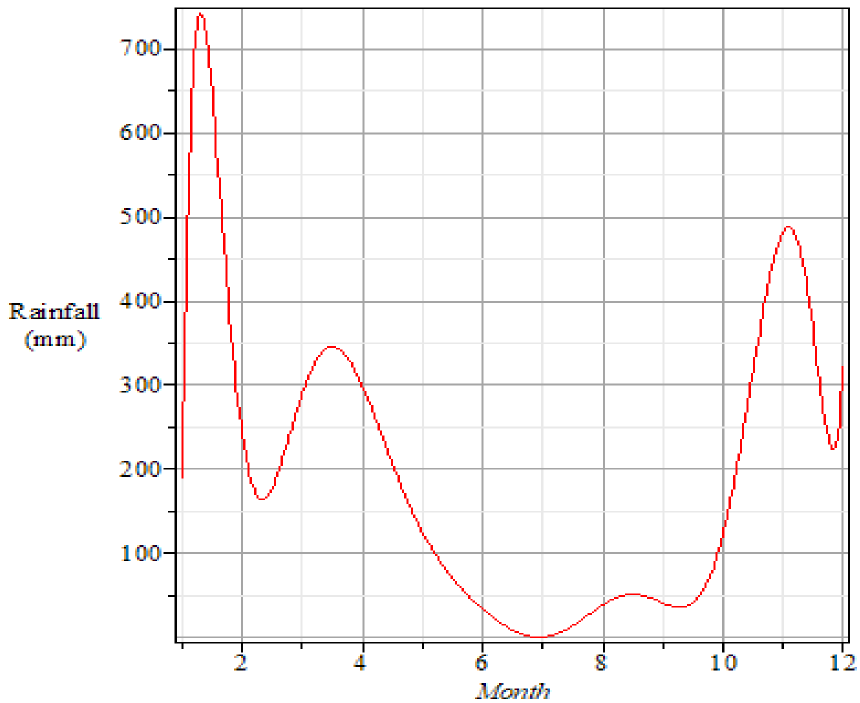


**Figure 3.** Polynomial Interpolation (2018)

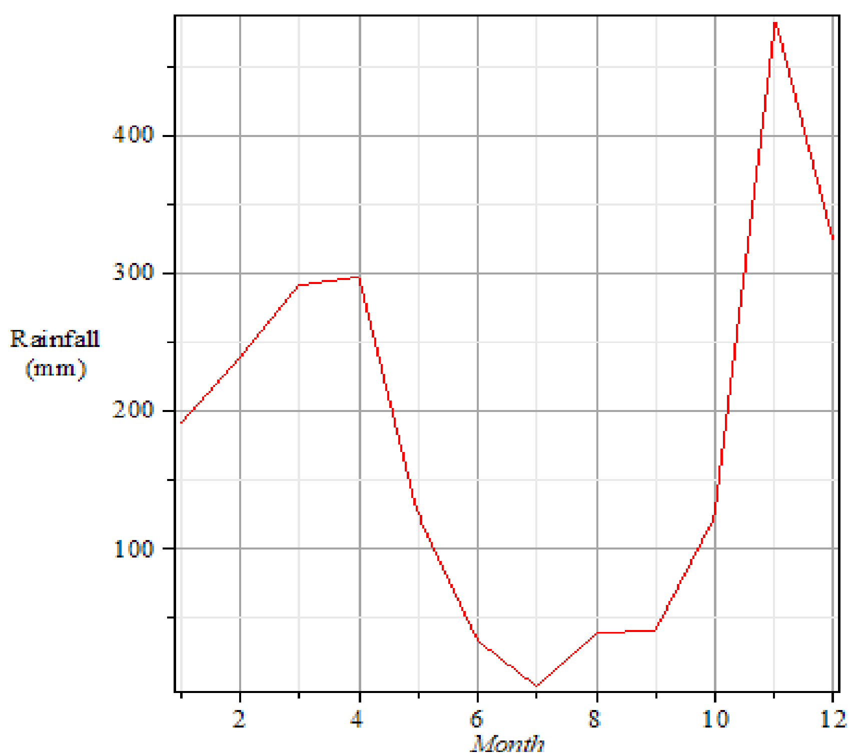


**Figure 4.** Piecewise Linear Interpolation (2018)

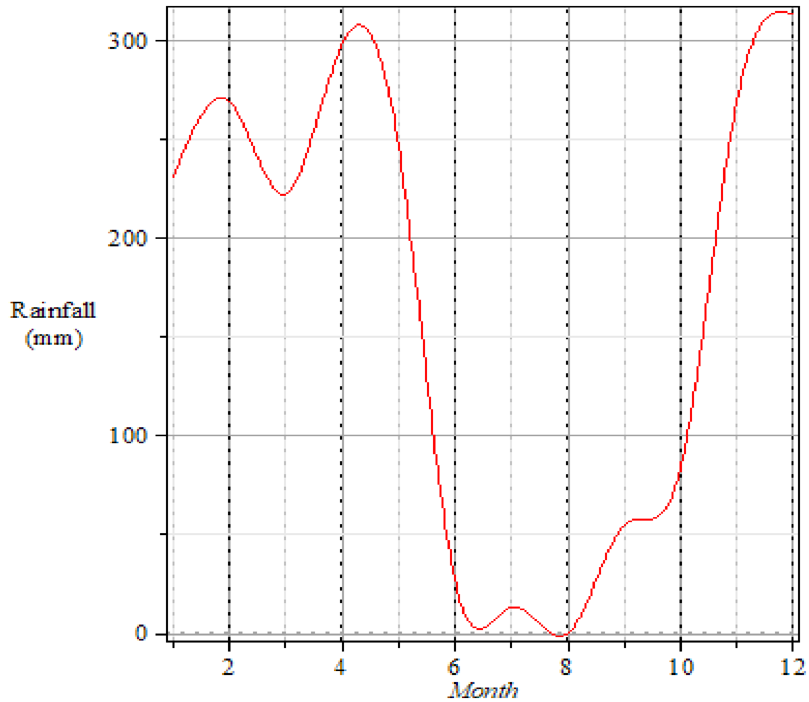


**Figure 5.** Natural Cubic Spline Interpolation (2019)

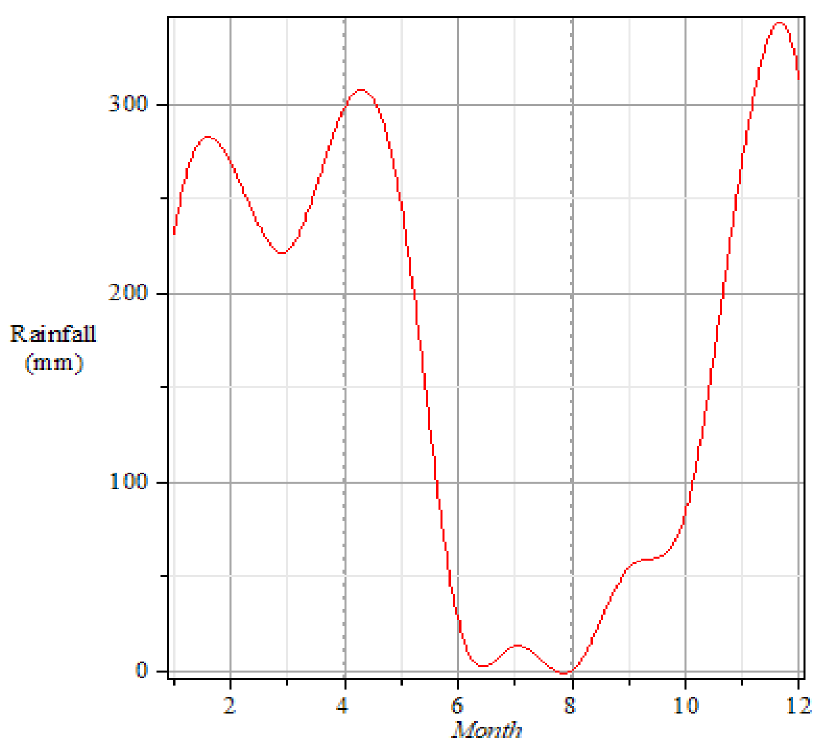


**Figure 6.** Not-a-Knot Cubic Spline Interpolation (2019)

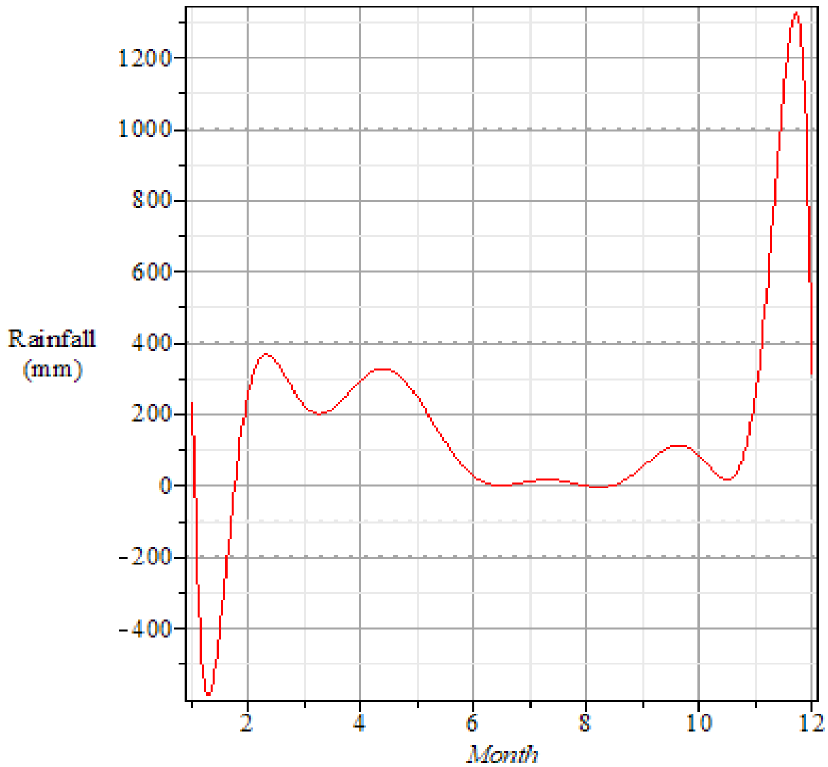


**Figure 7.** Polynomial Interpolation (2019)

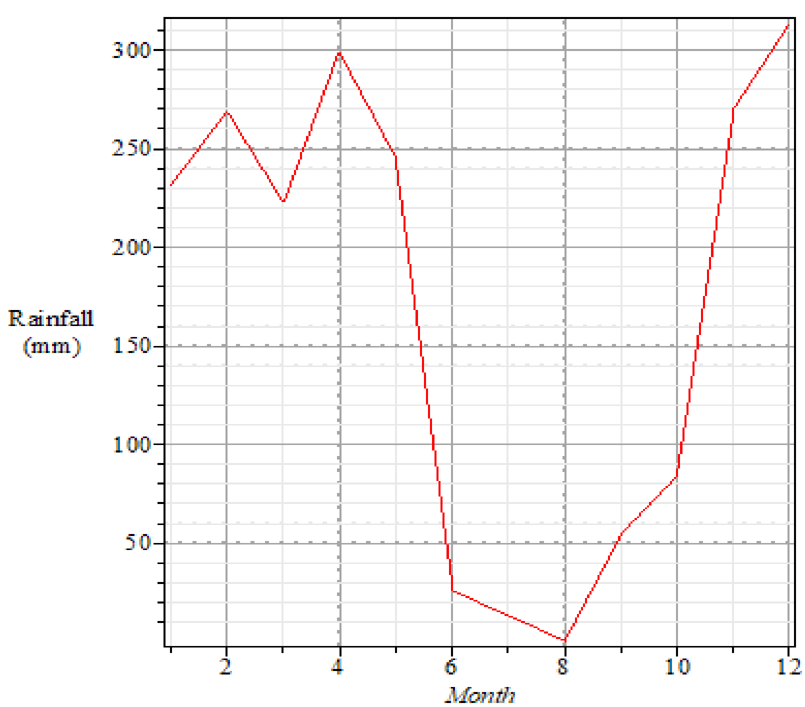


**Figure 8.** Piecewise Linear Interpolation (2019)

Natural cubic spline, not-a-knotcubic spline, Polynomial Interpolation, and Piecewise Linear Interpolation were performed using Maple software to obtain the following interpolation graph. As seen in Figure 1, Figure 2, Figure 5, and Figure 6, all four natural cubic spline interpolation yields a smooth and less-oscillating curves compared to polynomial interpolation in Figure 3 and Figure 7. This is in line with the maximum order of polynomial used in constructing the function of cubic spline interpolation, which limited to order $n = 3$ for every subintervals. The low-order of polynomial in cubic splines at each interval guarantee that the oscillation will be more anticipated when compared to polynomial interpolation, in which the order is not limited [15]. The higher order used in the functions, the more likely the curves are going to oscillate. The piecewise linear interpolation in Figure 4 and Figure 8 might be less-oscillating like cubic spline interpolations. Nevertheless, the curve of piecewise linear interpolation is not as smooth as cubic spline.

## 3. CONCLUSIONS

In this study, four types of interpolation were carried out on the rainfall data for Bandung City in 2018 and 2019. The interpolations are naturalcubic spline, not-a-knotcubic spline, Polynomial Interpolation, and Piecewise Linear. Figures 1, 2, 5, and 6 show the results of interpolation carried out using naturalcubic spline and not-a-knotcubic spline interpolations respectively, which show smooth and non-oscillating lines. Meanwhile, Figure 3 and 7 shows the result of interpolation performed using Polynomial Interpolation which shows an oscillating curve and Figure 4 and 8 shows the result of interpolation performed using Piecewise Linear which shows a curve that is not smooth because it has many angles. Hence, it can be concluded that in this study interpolation using naturalcubic spline and not-a-knotcubic splineare better than interpolation using Polynomial Interpolation and Piecewise Linear.

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