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Series Solution of Nonlinear Ordinary Differential Equations Using Single Laplace Transform Method in Mathematical Physics

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ABSTRACT

In this paper, a novel technique is created to enable the extension of the single Laplace transform method (SLTM) to solve nonlinear ordinary differential equations (ODEs) is presented. The main parts of the recommended technique are the Adomian polynomials. By developing several theorems, which include the Laplace transformation of nonlinear expressions, the Adomian polynomials are made possible. Several famous nonlinear equations including the Blasius equation, the Poisson Boltzmann equation, and numerous extra problems relating nonlinearities of many types such as exponential and sinusoidal are resolved for description. As it was revealed in the specified equations, and problems, our technique is conceptually and computationally simple. Some nonlinear examples are taken from the literature for checking the validation and convergence of the proposed technique. The suggested method is methodical, precise, then restricted to integration.

***Keywords*:** Single Laplace transform method, nonlinear ordinary differential equations, Adomian polynomials

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1. INTRODUCTION

In the early twentieth century at the earliest, the Single Laplace Transform Method (SLTM) has been received extremely well by physicists, mathematicians, engineers, and scientists at large. Certainly, the SLTM establishes the basics of many disciplines such as electrical control theory and circuit analysis. Algebraic equations to ordinary differential equations (ODEs) and partial differential equations (PDEs) in detail can be transformed by the outstanding feature of the single Laplace transform method (SLTM). In terms of effectiveness, the SLTM is believed to be perhaps second only to the Fourier transform method amongst all the integral transform methods [1-2].

In detail, the history and development of the SLTM was a review in an identical paper by Deakin. Single Laplace transform method (SLTM) has always been subject to one great imperfection that despite the significant ability it cannot be pragmatic to any type of nonlinear equations. With the help of the Adomian polynomials in this paper, we solved many nonlinear functional equations including the Blasius equation, the Poisson equation, and several other problems relating nonlinearities of many types such as exponential and sinusoidal [3]. Yet, the Single Laplace transform method (SLTM) enables the calculation of several ordinary differential equations of nonlinear terms by developing three new theorems relating to the Adomian polynomials. Also, the SLTM contrasts essentially from a current method recommended [4], which is named as the auxiliary Laplace constraint method (ALCM).

The authors avoid the integration and represents Adomian polynomials as equivalent integral equations, by adding the neutral term to the equations. In detail, the ALCM exploits the homotopy perturbation method and the SLTM by the Adomian polynomials while resembling any nonlinear terms. Lastly, it is clear that our anticipated method, that is, the Single Laplace transform method (SLTM), is not associated with the advanced order, which allows the calculation of the Single Laplace Transform (SLT) by taking advantage of the Adomian Decomposition Method (ADM) of a recognized function.

The paper is organized as follows: the Adomian Decomposition Method is defined in Section 2. The numerical Single Laplace transform method (SLTM) technique for the solution of nonlinear ordinary differential equations is presented in Section 3. In Section 4, some problem from the literature is given for the validation of SLTM by the use of Adomian polynomials. A conclusion is given in Section 5. Abbreviations is given in the last Section 6.

2. INCEPTIONS

An essential part of the nonlinear study by the Adomian decomposition method (ADM) is the Adomian polynomials. Professor George Adomian developed Adomian Decomposition Method (ADM) initially and since then he has recognized a reliable repute in various branches of the physical science, engineering, and applied science predominantly for methodically resolving nonlinear efficient equations; in this esteem, for instance, see [5-20]. In this section, we simplified and present a rapid review of the background of the Adomian polynomials and the Adomian Decomposition Method (ADM).

2. 1. Essentials of the Adomian Decomposition Method (ADM)

Let study a universal efficient equation as follows:

$$\mathrm{Lv} + \mathrm{Mv} + \mathrm{Pv} = \mathrm{f} \qquad (1)$$

Let L be selected as a simply invertible linear operator, M is a nonlinear operator, which plans a Banach space U to U, P signifies the residue linear operator, f is a confined detailed function, and v signifies the unfamiliar function. Through describing the opposite operator of L as L^{-1}, we determine that

$$\mathrm{L}^{-1}\mathrm{Lv} + \mathrm{L}^{-1}\mathrm{Mv} + \mathrm{L}^{-1}\mathrm{Pv} = \mathrm{L}^{-1}\mathrm{f}. \qquad (2)$$

By taking L as an nth-order derived operator, L^{-1} develops an n-fold addition operator. Consequently, it shows that $\mathrm{L}^{-1}\mathrm{Lv} = \mathrm{v} - \mathrm{b}$, where b occurs from additions. The ADM decays the result as $\mathrm{v} = \sum_{n=0}^{\infty} \mathrm{v}_n$. Allowing $\mathrm{v}_0 = \mathrm{L}^{-1}\mathrm{f} + \mathrm{b}$, Equation (2) will be:

$$\mathrm{v} = \mathrm{v}_0 - \mathrm{L}^{-1}\mathrm{Mv} - \mathrm{L}^{-1}\mathrm{Pv} \qquad (3)$$

Additionally, the ADM decays the nonlinearity Mu into a distinct infinite series of the Adomian polynomials as

$$\mathrm{Mv} = \sum_{n=0}^{\infty} \mathrm{A}_n, \qquad (4)$$

where the A_n are defined as [10]

$$\mathrm{A}_n(\mathrm{v}_0, \mathrm{v}_1, \dots, \mathrm{v}_n) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\mathrm{M}\sum_{i=0}^{\infty} \lambda^i \mathrm{v}_i\right]_{\lambda=0} \qquad (5)$$

Therefore, a repetition relative can be created to compute the outstanding solution mechanisms as

$$\mathrm{v}_{i+1} = -\mathrm{L}^{-1}\mathrm{A}_i - \mathrm{L}^{-1}\mathrm{Pv}_i; \qquad i \geq 0 \qquad (6)$$

It is valuable to make the situation to a number of other methods for estimation of the Adomian polynomials slightly than Equation (5) [21-22].

3. METHOD

Earlier to the explanation of the SLTM approach, several theorems will be specified. To evade misperception, we entitle $L^{-1}\{y\}$ *and* $L\{y\}$ to signify its inverse and the Laplace transform throughout this article. Similarly, we will take Y to signify the Laplace transform of the purpose y, that is, $Y = Z\{Y\} = \int_0^{\infty} e^{-uy} y dy$.

3. 1. Several theorems

To place the basics of the single Laplace transform method (SLTM), information over the proximately subsequent theorem, which is due to Rach and Adomian seems crucial. Successively, we will develop Theorems 1-3.

Theorem 1: The Laplace transform of nonlinearities.
Let consider M to be an overall nonlinear operator, which records a Hilbert space H to H. lacking loss of impression, it is apparent that M_y can contain integral or differential relations. For instance,

$$\mathrm{M}_y = y^2\sqrt{dy/dx}\ or\ \mathrm{M}_y = ydy/dx + e^{dy/dx} + (dy/dx)\sin(y).$$

Also, let A_i be the Adomian polynomials, which decay the nonlinear operator M.

If that $y = \sum_{i=0}^{\infty} b_i x^i$, then it embraces factual that,

$$\mathrm{L}\{\mathrm{M}_y\} = \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_{0,} \dots \dots, b_i x^i)\} \tag{7}$$

Proof: As A_i decay M, let say

$$\mathrm{M_v} = \sum_{i=1}^{\infty} \mathrm{A}_i(v_{0,} \dots \dots, v_i) \tag{8}$$

Knowing $v = \sum_{m=0}^{\infty} V_m$

By letting $v = y, we\ have\ \sum_{i=0}^{\infty} v_i = \sum_{i=0}^{\infty} b_i x^i$

$$v_i = b_i x^i, i = 0,1, \dots \dots \dots, \infty \tag{9}$$

By replacing Equation (9) into Equation (8), we have

$$\mathrm{M}_y = \sum_{i=1}^{\infty} \mathrm{A}_i(b_{0,} \dots \dots, b_i x^i) \tag{10}$$

Taking the Laplace transform of Equation (10), we have,

$$\mathrm{L}\{\mathrm{M}_y\} = \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_{0,} \dots \dots, b_i x^i)\} \tag{11}$$

Theorem 2: The Laplace transform of nonlinear expressions of type $f(y)$. Let consider f to be a univariate function of y, and A_i be the Adomian polynomials, which decay f.

$f(y)$ signifies numerous nonlinearities met in dissimilar equations.

Problems of $f(y)$ can be y^2, $\sin(y+y^4)$, $or\ e^y - \sqrt{y} + 1/(y\sqrt[3]{y}) + \cos(y)$.

If that $y = \sum_{i=0}^{\infty} b_i x^i$, it embraces factual that

$$\mathrm{L}\{\mathrm{f(y)}\} = \sum_{i=0}^{\infty} A_i(b_0, \dots, b_i) \frac{i!}{\mathrm{S}^{i+1}} \tag{12}$$

$$f(y) = \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) x^i \tag{13}$$

Proof: The impervious to this theorem is uncertain and straight forward.
Let take the Laplace transform of Equation (13), we have

$$\mathrm{L}\{\mathrm{f(y)}\} = \mathrm{L}\left\{\sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\, x^i\right\} \tag{14}$$

Subsequently, the Laplace transform is linear, it passes over the summation in Equation (14), we have

$$\mathrm{L}\{\mathrm{f(y)}\} = \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_0, \dots, b_i) x^i\} \tag{15}$$

As the A_i is sovereign of the inconstant x, it shows that

$$\mathrm{L}\{\mathrm{f(y)}\} = \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\mathrm{L}\{x^i\} = \sum_{i=0}^{\infty} \mathrm{A}_i(b_o, \dots, b_i) \frac{i!}{\mathrm{S}^{i+1}} \tag{16}$$

Theorem 3: The Laplace transform of nonlinear terms of type $f(d^n y/dx^n)$

Let take f to be a univariate function of $d^n y/dx^n$ with $n \geq 1\ and\ \mathrm{A}_i$ be the Adomian polynomials which decompose f. Nonlinearities like $(dy/dx)^2, \sin(d^2y/dx^2)$, or $e^{dy/dx} + \sqrt{(d^3y/dx^3)\, d^2y/dx^2} - (dy/dx)^{-1}$ are the aim of this theorem, for instance.

If that $y = \sum_{i=0}^{\infty} b_i x^i$; then it embraces that,

$$\mathrm{L}\left\{\mathrm{f}\left(\frac{d^n y}{dx^n}\right)\right\} = \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \dots, e_i) \frac{i!}{S^{i+1}}, \tag{17}$$

where $b_i = (i+1)(i+2) \ldots \ldots (i+n)b_{i+n}$

Proof: To be able to use Theorem 3 for opinions relatively than y, let say k, we have to assure that k can be inscribed as an unlimited series of the inconstant x like $k = \sum_{i=0}^{\infty} e_i x^i$.
By letting $k = d^n y / dx^n$ we have,

$$\frac{d^n y}{dx^n} = \sum_{i=0}^{\infty} b_i \frac{d^n x^i}{dx^n} = \sum_{i=n}^{\infty} i(i-1)(i-2) \ldots \ldots (i-n+1) b_i x^{i-n}$$

$$= \sum_{i=0}^{\infty} (i+1)(i+2) \ldots \ldots (i+n) b_{i+n} x^i = \sum_{i=0}^{\infty} e_i x^i, \qquad (18)$$

here, it is obvious that

$$e_i = (i+1)(i+2) \ldots \ldots (i+n) b_{i+n}$$

We can take Theorem 2 to finish the proof:

$$\mathrm{L}\left\{\mathrm{f}\left(\frac{d^n y}{dx^n}\right)\right\} = \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \ldots, e_i) \frac{i!}{S^{i+1}} \qquad (19)$$

3. 2. Approach of the SLTM

One can yield the Laplace transform of a general nonlinear equation of y to take an equation in $u - domain$ which requires unidentified constants of b_i, with the help of Theorems 1-3 for the nonlinear relations and the well-known Single Laplace transform theorems for the linear parts. Relating the inverse Laplace transform, and by resolving this equation for Y, we get the solution as

$$y = \sum_{i=0}^{\infty} a_j x^i, \qquad (20)$$

here, a_j are constants involving to b_i.
On the additional hand, recall that we have implicit

$$y = \sum_{i=0}^{\infty} b_i x^i, \qquad (21)$$

as a criterion to the theorems above.

Subsequently, it is clear that an incomplete number of b_i are directly originate through the preliminary situations arranged by the example. Moreover, by comparing the factors of related powers of x in Equations (20) and (21), one can discover all b_i definitely in a recursive way. This approach can improve to be enlightened through subsequent problems.

4. RESULTS

In this part, we applied several problems to clarify the application of our technique Single Laplace transform method (SLTM). These examples are designated such that they have consistent precise logical results for the sake of simpler authentication. In the following problems, the nonlinearities elaborate are encountered in the mathematical models from different disciplines such as Physics, Mathematics, and Engineering. They are, for example, of the polynomial, sinusoidal, and exponential kinds as well as the product of the result purpose and its nth-order derivatives. One can certainly show the related path to resolve any arbitrary problem as long as the situations of Theorems 1-3, either it is suitable for the problem, are fulfilled.

Problem 1. The Blasius Equation
We have,

$$\frac{d^3y}{dx^3} + y\frac{d^2y}{dx^2} = 0 \tag{22}$$

Boundary conditions $y(0) = 0, \frac{dy(0)}{dx} = 1, \frac{dy(\infty)}{dx} = 0$

Where x denotes a three-dimensional inconstant from the corporal point of view.
Here, we say,

$$\mathrm{L}\left\{\frac{d^3y}{dx^3}\right\} + \mathrm{L}\left\{\mathrm{y}\frac{d^2y}{dx^2}\right\} = 0 \tag{23}$$

with the help of Theorem 1, equation (23) will be

$$\mathrm{S}^3\mathrm{Y} - S^2y(0) - S\frac{dy(0)}{dx} - \frac{d^2y(0)}{dx^2} + \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_{0,} \dots \dots, b_i x^i)\} = 0 \tag{24}$$

or

$$\mathrm{S}^3\mathrm{Y} - \mathrm{S} - 1 + \sum_{i=0}^{\infty} \mathrm{L}\{A_i(b_0, \dots \dots, b_i x^i)\} = 0 \tag{25}$$

Or equally,

$$\mathrm{Y} = \frac{1}{s^2} + \frac{1}{S^3} - \frac{1}{S^3}\sum_{i=0}^{\infty} \mathrm{L}\{A_i(b_0, \dots \dots, b_i x^i)\} = 0 \tag{26}$$

Significant that the initial two original situations established as $b_0 = 0\ and\ b_1 = 1$, by calculating the Adomian polynomials which decay $\mathrm{M}_y = yd^2y/dx^2$.

$$\mathrm{A}_0(b_0) = b_0\frac{d^2b_0}{dx^2} = 0,$$

$$\mathrm{A}_1(b_0, b_1x) = b_1x\frac{d^2b_0}{dx^2} + b_0\frac{d^2(b_1)x}{dx^2} = 0,$$

$$\mathrm{A}_2(b_0, b_1x, b_2x^2) = b_2x^2\frac{d^2b_0}{dx^2} + b_1x\frac{d^2(b_1x)}{dx^2} + b_0\frac{d^2(b_2x^2)}{dx^2} = 0,$$

$$\mathrm{A}_3(b_0, b_1x, b_2x^2, b_3x^3) = b_3x^3\frac{d^2b_0}{dx^2} + b_2x^2\frac{d^2(b_1x)}{dx^2} + b_1x\frac{d^2(b_2x^2)}{dx^2} + b_0\frac{d^2(b_3x^3)}{dx^2}$$
$$= 2b_1b_2x,$$

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$$\mathrm{A}_i\left(b_0, \ldots, b_ix^i\right)$$
$$= \sum_{j=0}^{i} b_{i-j}x^{i-j}\frac{d^2(b_jx^i)}{dx^2}$$
$$= \sum_{j=0}^{i} b_{i-j}c_jj(j-1)x^{i-2} = \sum_{j=2}^{i} b_{i-j}b_jj(j-1)x^{i-2}, i \geq 2 \tag{27}$$

Here,

$$\mathrm{L}\{\mathrm{A}_i\left(b_0, \ldots, b_ix^i\right)\}$$
$$= \mathrm{L}\{\sum_{j=2}^{i} b_{i-j}b_jj(j-1)x^{i-2}\}$$
$$= \sum_{j=2}^{i} b_{i-j}c_jj(j-1)\mathrm{L}\{x^{i-2}\} = \sum_{j=2}^{i} b_{i-j}b_jj(j-1)\frac{(i-2)!}{S^{i-1}}$$
$$= \frac{(i-2)!}{S^{i-1}}\sum_{j=2}^{i} b_{i-j}b_jj(j-1); i \geq 2 \tag{28}$$

By taking the previous equation into equation (26), we have

$$\mathrm{Y} = \frac{1}{S^2} + \frac{1}{S^3} - \frac{1}{S^3}\sum_{i=2}^{\infty}[\frac{(i-2)!}{S^{i-1}}\sum_{j=2}^{i} b_{i-j}b_jj(j-1)]$$
$$= \frac{1}{S^2} + \frac{1}{S^3} - \sum_{i=2}^{\infty}\left[\frac{(i-2)!}{S^{i+2}}\sum_{j=2}^{i} b_{i-j}b_jj(j-1)\right] \tag{29}$$

By taking the Laplace inversion of Equation (29), we have

$$y = x + x^2 - \sum_{i=2}^{\infty} \left[\frac{(i-2)!}{(i+1)!} x^{i+1} \sum_{j=2}^{i} b_{i-j} b_j j\,(j-1) \right] \tag{30}$$

Or in an expanded form, we have.

$$y = x + x^2 - \left(\frac{b_2}{12} x^4 + \frac{b_2^2 + 3b_3}{30} x^5 + \frac{10b_3b_5 + 3b_2b_3 + 6b_4}{60} x^6 + \cdots \right) \tag{31}$$

Equivalently,

$$y = x + x^2 - \frac{b_2}{12} x^4 - \frac{b_2^2 + 3b_3}{30} x^5 - \frac{10b_3b_5 + 3b_2b_3 + 6b_4}{60} x^6 + \cdots \tag{32}$$

Here, we know that

$$y = x + x^2 + b_3x^3 + b_4x^4 + b_5x^5 + b_6x^6 + \cdots \tag{33}$$

Then, it is deduced that

$b_3 = 0,$

$b_4 = -\frac{b_2}{12},$

$b_5 = -\frac{b_2^2 + 3b_3}{30} = -\frac{b_2^2}{30},$

.

.

.

Then,

$$y = x + x^2 - \frac{b_2}{12} x^4 - \frac{b_2^2}{30} x^5 + \frac{b_2}{240} x^6 + \cdots\cdots, \tag{34}$$

Problem 2. Let consider the following problem:

$$\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 + y^2 = 1 - \cos(x) \tag{35}$$

Boundary conditions $y(0) = 0, \frac{dy}{dx}(0) = 1$

The Laplace transform of Equation (35), will be

$$S^2Y - Sy(0) - \frac{dy}{dx}(0) + \mathrm{L}\left\{(\frac{\mathrm{dy}}{\mathrm{dx}})^2\right\} + \mathrm{L}\{y^2\} = \frac{1}{S} - \frac{S}{1+S^2} \tag{36}$$

Applying Theorem 2 and 3 to equation (36) insight of the two stated original situations, we have.

$$\mathrm{Y} = \frac{1}{S^2} + \frac{1}{S^3} - \frac{1}{S^2}\frac{S}{1+S^2} - \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \dots, e_i)\frac{i!}{S^{i+3}} - \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\frac{i!}{S^{i+3}} \tag{37}$$

where $e_i = (i+1)b_i$ and A_i are the Adomian polynomials disintegrating the quadratic nonlinear operator $\mathrm{M}_v = v^2$.

Consistently,

$$\mathrm{Y} = \frac{1}{S^2} + \frac{1}{S^3} - \left(\frac{1}{S^2}\frac{S}{1+S^2}\right) - \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \dots, e_i)\frac{i!}{S^{i+3}} - \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\frac{i!}{S^{i+3}} \tag{38}$$

Or

$$\mathrm{Y} = \frac{1}{S^2}\frac{S}{1+S^2} + \frac{1}{S^3} + \frac{1}{S^2} - \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \dots, e_i)\frac{i!}{S^{i+3}} - \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\frac{i!}{S^{i+3}} \tag{39}$$

Equation (39) inversion Laplace, will be

$$y = x\cos(x) + \frac{x^2}{2!} + x - (e_0^2 + b_0^2)\frac{x^2}{2!} - (2e_0e_1 + 2b_0b_1)\frac{x^3}{3!} - (2e_0e_2 + e_1^2 + 2b_0b_2 + b_1^2)\frac{2!\,x^4}{4!} - (2e_0e_3 + 2e_1e_2 + 2b_0b_3 + 2b_1b_2)\frac{3!\,x^5}{5!} + \cdots\cdots, \tag{40}$$

Or equally,

$$y = \left(x\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i+1)!}x^{2i+1}\right) + \frac{x^2}{2!} + x - (b_0^2 + e_0^2)\frac{x^2}{2!} - (2e_0e_1 + 2b_0b_2)\frac{x^3}{3!} - (2e_0e_2 + e_1^2 + 2b_0b_2 + b_1^2)\frac{2!\,x^4}{4!} - (2e_0e_3 + 2e_1e_2 + 2b_0b_3 + 2b_1b_2)\frac{3!\,x^5}{5!} + \cdots\cdots \tag{41}$$

Accepting $y = \sum_{i=0}^{\infty} b_i x^i$, the two given initial conditions fixes $b_0 = 0\ and\ b_1 = 0$.
Therefore,

$$y = \left(x \sum_{i=0}^{\infty} \frac{(-i)^i}{(2i+1)!} x^{2+1} \right) + \frac{x^2}{2!} + x - (1^2 + 0)\frac{x^2}{2!} - (2 \times 1 \times 2b_2 + 2 \times 0 \times 1)\frac{x^3}{3!}$$
$$- (2b_1 \times 3b_3 + 4b_2^2 + 2 \times 0 \times b_2 + 1^2)\frac{2!\,x^4}{4!}$$
$$- (2b_1 \times 4b_4 + 2 \times 2b_2 \times 3b_3 + 2b_0 b_3 + 2b_1 b_2)\frac{3!\,x^5}{5!}$$
$$+ \cdots \cdots \cdots \qquad (42)$$

By associating Equation (42) and $y = \sum_{i=0}^{\infty} b_i x^i$, one can simply locate that $b_{2i} = 0$ by comparing the constants of related powers of x. This will, in turn, nil all the relations in Equation (42) excluding for the summation and the x.
Thus,

$$y = \left(x \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!} x^{2i+1} \right) + 0 = x\cos(x) \qquad (43)$$

Problem 3. Let consider the following nonlinear ODE

$$\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 + y^2 = 1 + \sin(x) \qquad (44)$$

Boundary conditions $y(0) = 0, \frac{dy}{dx}(0) = 1$

The Laplace transform of Equation (44), will be

$$S^2Y - Sy(0) - \frac{dy}{dx}(0) + \mathrm{L}\{(\frac{\mathrm{dy}}{\mathrm{dx}})^2\} + \mathrm{L}\{y^2\} = \frac{1}{S} + \frac{1}{1+S^2} \qquad (45)$$

Applying Theorems 2, and 3 to Equation (45) insight of the two stated original situations, we have.

$$\mathrm{Y} = \frac{1}{S^2} + \frac{1}{S^3} - \frac{1}{S^2}\frac{1}{1+S^2} - \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \dots, e_i)\frac{i!}{S^{i+3}} - \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\frac{i!}{S^{i+3}} \qquad (46)$$

where $e_i = (i+1)b_{i+1}$ and A_i are the Adomian polynomials disintegrating the quadratic nonlinear operator $\mathrm{M}_v = v^2$.
Consistently,

$$\mathrm{Y} = \frac{1}{S^2} + \frac{1}{S^3} - \left(\frac{1}{S^2}\frac{1}{1+S^2}\right) - \sum_{i=0}^{\infty} \mathrm{A}_i(e_0, \dots, e_i)\frac{i!}{S^{i+3}} - \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\frac{i!}{S^{i+3}} \qquad (47)$$

or

$$\mathrm{Y}=\frac{1}{S^2}\frac{1}{1+S^2}-\frac{1}{S^3}+\frac{1}{S^2}-\sum_{i=0}^{\infty}\mathrm{A}_i(e_0,\dots,e_i)\frac{i!}{S^{i+3}}$$

Equation (48) inversion Laplace, will be

$$\begin{aligned}y=x\sin(x)-\frac{x^2}{2!}+x-(e_0^2-b_0^2)\frac{x^2}{2!}-(2e_0e_1+2b_0b_1)\frac{x^3}{3!}\\-(2e_0e_2+e_1^2+2b_0b_2+b_1^2)\frac{2!\,x^4}{4!}\\-(2e_0e_3+2e_1e_2+2b_0b_3+2b_1b_2)\frac{3!\,x^5}{5!}\\+\cdots\cdots\end{aligned}\qquad(49)$$

Or equally,

$$\begin{aligned}y=\left(x\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i+1)!}x^{2i+1}\right)-\frac{x^2}{2!}+x-(e_0^2+b_0^2)\frac{x^2}{2!}-(2e_0e_1+2b_0b_1)\frac{x^3}{3!}\\-(2e_0e_2+e_1^2+2b_0b_2+b_1^2)\frac{2!\,x^4}{4!}\\-(2e_0e_3+2e_1e_2+2b_0b_3+2b_1b_2)\frac{3!\,x^5}{5!}\\+\cdots\cdots,\end{aligned}\qquad(50)$$

Using $y=\sum_{i=0}^{\infty}b_ix^i$, the two given original situations as $b_0=0\ and\ b_1=0$.

Thus,

$$\begin{aligned}y=\left(x\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i+1)!}x^{2i+1}\right)-\frac{x^2}{2!}+x-(1^2+0)\frac{x^2}{2!}-(2\times1\times2b_2+2\times0\times1)\frac{x^3}{3!}\\-(2b_1\times3b_3+4b_2^2+2\times0\times b_2+1^2)\frac{2!\,x^4}{4!}\\-(2b_1\times4b_4+2\times2b_2\times3b_3+2b_0b_3+2b_1b_2)\frac{3!\,x^5}{5!}+\cdots\cdots,\end{aligned}\qquad(51)$$

Relating Equation (51) and $y=\sum_{i=0}^{\infty}b_ix^i$, one can simply take $b_{2i}=0$ by comparing the constants of related powers of x. This will, in turn, zero all the relations in Equation (51) excluding for the summation and x.

Thus,

$$y=\left(x\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i+1)!}x^{2i+1}\right)-0=x\sin(x)\qquad(52)$$

Problem 4. Consider the following equation

$$\frac{dv}{dt} - 1 = v^2(t) \tag{53}$$

Subject to the condition $v(0) = 0$

The Laplace transform of Equation (53), will be

$$SY - \mathrm{v}(0) - \frac{1}{s} = L\{v^2\} \tag{54}$$

From Theorem 1, Equation (54) will be

$$SY = \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+1}} + \frac{1}{S}$$

$$\mathrm{Y} = \frac{1}{S} \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+1}} + \frac{1}{S^2}$$

$$\mathrm{Y} = \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+2}} + \frac{1}{S^2} \tag{55}$$

Equation (55) inversion Laplace, will be

$$\mathrm{y} = \big(1 + \mathrm{A}_0(b_0)\big)x + \mathrm{A}_1(b_0, b_1)\frac{x^2}{2!} + \mathrm{A}_2(b_0, b_1, b_2)\frac{2!\,x^3}{3!} + \mathrm{A}_3(b_0, b_1, b_2, b_3)\frac{3!\,x^4}{4!} + \cdots, \tag{56}$$

we can assume that

$$\mathrm{y} = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + \cdots, \tag{57}$$

Accordingly, from the original state $v(0) = 0$ it shows that $b_0 = 0$

Comparing the constants of equal powers of x in Equations (56) and (57), we will have

$$b_1 = 1 + \mathrm{A}_0(b_0) = 1 + b_0^2 = 1,$$

$$b_2 = \frac{\mathrm{A}_1(b_0, b_1)}{2!} = \frac{2b_0 b_1}{2!} = \frac{2 \times 0 \times 1}{2} = 0,$$

$$b_3 = \frac{2!\,\mathrm{A}_2(b_0, b_1, b_2)}{3!} = \frac{2!}{3!}(2b_0 b_2 + b_1^2) = \frac{2!}{3!}(2 \times 0 \times 0 + 1^2) = \frac{1}{3},$$

$$b_4 = \frac{3!\,\mathrm{A}_3(b_0, b_1, b_2, b_3)}{4!} = \frac{3!}{4!}(2b_0 b_3 + 2b_1, b_2) = \frac{2!}{3!}\left(2 \times 0 \times \frac{1}{3} + 2 \times 1 \times 0\right) = 0,$$

$$b_5 = \frac{4!\,A_4(b_0, b_1, b_2, b_3, b_4)}{5!} = \frac{4!}{5!}(b_2^2 + 2b_1, b_3 + 2b_0 b_4) = \frac{4!}{5!}\left(0^2 + 2 \times 1 \times \frac{1}{3} + \times 0 \times 0\right)$$
$$= \frac{2}{15},$$

$$b_6 = \frac{5!\,A_5(b_0, b_1, b_2, b_3, b_4, b_5)}{6!} = \frac{5!}{6!}(2b_2, b_3 + 2b_0, b_5 + 2b_1, b_4)$$
$$= \frac{5!}{6!}\left(2 \times 0 \times \frac{1}{3} + 2 \times 0 \times \frac{2}{15} + 2 \times 1 \times 0\right) = 0,$$

$$b_7 = \frac{6!\,A_6(b_0, b_1, b_2, b_3, b_4, b_5, b_6)}{7!} = \frac{6!}{7!}(2b_0 b_6 + 2b_1 b_5 + 2b_2 b_4 + b_3^2)$$
$$= \frac{6!}{7!}(2 \times 1 \times 0 + 2 \times 1 \times \frac{2}{15} + 2 \times 0 \times 0 + (\frac{1}{3})^2) = \frac{17}{315},$$

⋮
⋮
⋮

Hence, the exact solution of equation (53) will be

$$y = t + \frac{1}{3}t^3 + \frac{2}{15}t^5 + \frac{17}{315}t^7 + \cdots\cdots, \qquad (58)$$

Which has the closed-form result of $y = \tan(y)$.

Problem 5. The Poisson Boltzmann Equation

$$\frac{d^2y}{dx^2} + \frac{\alpha}{x}\frac{dy}{dx} = e^y \qquad (59)$$

Boundary conditions $y(0) = 0, \frac{dy}{dx}(0) = 1$

Taking the Laplace transform of Equation (59), we have

$$L\left\{\frac{d^2y}{dx^2}\right\} + L\left\{\frac{\alpha}{x}\frac{dy}{dx}\right\} = L\{e^y\} \qquad (60)$$

From equation (60), the Laplace transform will be

$$S^2Y - sy(0) + L\left\{\frac{\alpha}{x}\frac{dy}{dx}\right\} = L\{e^y\} \qquad (61)$$

By Theorem 1, Equation (61) is converted to,

$$S^2Y - sy(0) + \alpha \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_0, \dots, b_i x^i)\} = \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+1}} \tag{62}$$

α is a function of x $i.e. f(\alpha)$

Solving Equation (62) for Y, we obtain

$$S^2Y = \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+1}} - \alpha \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_0, \dots, b_i x^i)\}$$

Divide through by S^2, we have

$$\frac{S^2Y}{S^2} = \frac{1}{S^2}\left[\sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{l+1}} - \alpha \sum_{I=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_0, \dots, b_i x^i)\}\right]$$

$$\therefore \mathrm{Y} = \frac{1}{S^2}\left[\sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+1}} - \alpha \sum_{i=0}^{\infty} \mathrm{L}\{\mathrm{A}_i(b_0, \dots, b_i x^i)\}\right] \tag{63}$$

From Equation (63), substantial that the initial dual original situations established as $b_0 = 0$ and $b_i = 1$, we start calculating the Adomian polynomials which decay $\mathrm{M}_y = y\, d^2y/dx^2$.

$$\mathrm{A}_0(b_0) = b_0 \frac{d^2 b_0}{dx^2} = 0,$$

$$\mathrm{A}_1(b_0, b_1 x) = b_1 x \frac{d^2 b_0}{dx^2} + b_0 \frac{d^2(b_1 x)}{dx^2} = 0,$$

$$\mathrm{A}_2(b_0, b_1 x, b_2 x^2) = b_2 x^2 \frac{d^2 b_0}{dx^2} + b_1 x \frac{d^2(b_1 x)}{dx^2} + b_0 \frac{d^2(b_2 x^2)}{dx^2} = 0,$$

$$\mathrm{A}_3(b_0, b_1 x, b_2 x^2, b_3 x^3) = b_3 x^3 \frac{d^2 b_0}{dx^2} + b_2 x^2 \frac{d^2(b_1 x)}{dx^2} + b_1 x \frac{d^2(b_1 x^2)}{dx^2} + b_0 \frac{d^2(b_3 x^3)}{dx^2}$$
$$= 2b_1 b_2$$

$$\vdots$$

$$\mathrm{A}_i(b_0, \dots, b_1 x^i) = \sum_{j=0}^{i} b_{i-j} x^i \frac{d^2(b_j x^i)}{dx^2} = \sum_{j=0}^{i} b_{i-j} b_j j(j-1) x^{i-2} = \sum_{j=2}^{i} b_{i-j} b_j\, j\,(j-1) x^{i-2};\ i \geq 2.$$

So,

$$\mathrm{L}\{\mathrm{A}_i(b_0,\dots,b_i x^i)\} = \mathrm{L}\left\{\sum_{j=2}^{i} b_{i-j}b_j j(j-1)x^{i-2}\right\} = \sum_{j=2}^{i} b_{i-j}b_j j\,(j-1)\mathrm{L}\{x^{i-2}\}$$

$$= \sum_{j=2}^{i} b_{i-j}b_j j\,(j-1)\frac{(i-2)!}{S^{i-1}} = \frac{(i-2)!}{S^{i-1}}\sum_{j=2}^{i} b_{i-j}b_j j(j-1);\, i \geq 2 \qquad (64)$$

By taking the previous equation into Equation (62), we have

$$\mathrm{Y} = \frac{1}{S^2}\left[\sum_{i=0}^{\infty}\mathrm{A}_i\,(b_0,\dots,b_i)\frac{i!}{S^{i+1}} - \alpha\sum_{i=2}^{\infty}\left(\frac{(i-2)!}{S^{i-i}}\sum_{j=2}^{i} b_{i-j}b_j j\,(j-1)\right)\right]$$

$$\mathrm{Y} = \frac{1}{S^2}\sum_{i=0}^{\infty}\mathrm{A}_i\,(b_0,\dots,b_i)\frac{i!}{S^{i+1}} - \alpha\frac{1}{S^2}\sum_{i=2}^{\infty}\left[\frac{(i-2)!}{S^{i-1}}\sum_{j=2}^{i} b_{i-j}b_j j\,(j-1)\right] \qquad (65)$$

A_i is the Adomian polynomials decomposing the exponential nonlinearity.
Hence, equation (65), will be

$$\mathrm{Y} = \sum_{i=0}^{\infty}\mathrm{A}_i\,(b_0,\dots,b_i)\frac{i!}{S^{i+3}} - \alpha\sum_{i=2}^{\infty}\left[\frac{(i-2)!}{S^{i+1}}\sum_{j=2}^{i} b_{i-j}b_j j\,(j-1)\right] \qquad (66)$$

From Equation (66), the Laplace transform inversion from both sides will be

$$y = \sum_{i=0}^{\infty}\frac{\mathrm{A}_i(b_0,\dots,b_i)}{(i+2)(i+1)}x^{i+2} - \alpha\sum_{i=2}^{\infty}\left[\frac{(i-2)!}{(i+1)!}x^i\sum_{j=2}^{i} b_{i-j}b_j j\,(j-1)\right] \qquad (67)$$

Or in an expanded form, we will have

$$y = \frac{1}{2}x^2 + \frac{b_1}{6}x^3 + \frac{(b_2 + (b_1^2/2))}{12}x^4 + \frac{(b_3 + b_1 b_2 + (1/6)b_1^3)}{20}x^5$$
$$+ \frac{(b_4 + b_1 b_3 + (1/2)b_2^2 + (1/2)b_1^2 b_2 + (1/24)b_1^4)}{30}x^6 + \cdots$$
$$- \alpha\sum_{i=2}^{\infty}[\frac{(i-2)!}{(i+1)!}x^i\sum_{j=2}^{i} b_{i-j}b_j j(j-1)] \qquad (68)$$

Through the postulation of Theorem 1, we identify that

$$y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + \cdots - \alpha \sum_{i=2}^{\infty}\left[\frac{(i-2)!}{(i+1)!}x^i \sum_{j=2}^{i} b_{i-j} b_j j(j-1)\right] \tag{69}$$

Hence, we will compare the alike powers of x in Equations (68) and (69) to methodical obtain

$$b_o = 0$$

$$b_1 = 0$$

$$b_2 = \frac{1}{2!}$$

$$b_3 = \frac{b_1}{3!}$$

$$b_4 = \frac{(b_2 + (b_1^2/2))}{4!}$$

$$b_5 = \frac{(b_3 + b_1 b_2 + (1/6)b_1^3}{5!}$$

$$b_6 = \frac{(b_4 + b_1 b_3 + (1/2)b_2^2 + (1/2)b_1^2 b_2 + (1/24)b_1^4)}{6!}$$

This accomplishes that the curtailed estimated result of equation (59) will be

$$y = \frac{1}{2!}x^2 + \frac{1/2}{4!}x^4 + \frac{1/8}{6!}x^6 + 0(x^7) + \cdots - \alpha \sum_{i=2}^{\infty}\left[\frac{(i-2)!}{(i+1)!}x^i \sum_{j=2}^{i} b_{i-j} b_j j(j-1)\right] \tag{70}$$

Problem 6. Let us consider the following problem

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + sinx = fcosx \tag{71}$$

Initial conditions $y(0) = 0, \frac{dy(0)}{dx} = 1$

Equation (71) the Laplace transform will be

$$\mathrm{L}\left\{\frac{d^2y}{dx^2}\right\} + \mathrm{L}\left\{a\frac{dy}{dx}\right\} + \mathrm{L}\{sinx\} = \mathrm{L}\{fcosx\} \tag{72}$$

The Laplace transform of equation (72), will be

$$S^2\text{Y} - \text{sy}(0) + \text{L}\left\{a\frac{dy}{dx}\right\} + \frac{1}{1+S^2} = \frac{S}{1+S^2} \tag{73}$$

By theorem 1, 2, and 3, equation (73), will be

$$S^2\text{Y} = \frac{S}{1+S^2} - \frac{1}{1+S^2} - \sum_{i=0}^{\infty} \text{L}\{\text{A}_i(b_0, \dots, b_i x^i)\}$$

$$\therefore \quad \text{Y} = \frac{1}{S^2}\left[\frac{S}{1+S^2} - \frac{1}{1+S^2} - \sum_{i=0}^{\infty} \text{L}\{\text{A}_i(b_0, \dots, b_i x^i)\}\right] \tag{74}$$

Equation (74) can also be as:

$$\text{Y} = \frac{1}{S^2}\frac{S}{1+S^2} - \frac{1}{S^2}\frac{1}{1+S^2} - \frac{1}{S^2}\sum_{i=0}^{\infty} \text{L}\{\text{A}_i(b_0, \dots, b_i x^i)\} \tag{75}$$

Significant that the first dual original situations established as $b_0 = 0\ and\ b_1 = 1$, we start calculating the Adomian polynomials which decay $\text{M}_y = y\, d^2y/dx^2$.

$$\text{A}_0(b_0) = b_0\frac{d^2 b_0}{dx^2} = 0,$$

$$\text{A}_1(b_0, b_1 x) = b_1 x\frac{d^2 b_0}{dx^2} + b_0\frac{d^2(b_1 x)}{dx^2} = 0,$$

$$\text{A}_2(b_0, b_1 x, b_2 x^2) = b_2 x^2\frac{d^2 b_0}{dx^2} + b_1 x\frac{d^2(b_1 x)}{dx^2} + b_0\frac{d^2(b_2 x^2)}{dx^2} = 0,$$

$$\text{A}_3(b_0, b_1 x, b_2 x^2, b_3 x^3) = b_3 x^3\frac{d^2 b_0}{dx^2} + b_2 x^2\frac{d^2(b_1 x)}{dx^2} + b_1 x\frac{d^2(b_2 x^2)}{dx^2} + b_0\frac{d^2(b_3 x^3)}{dx^2}$$
$$= 2b_1 b_2 x,$$

.
.
.

$$\text{A}_i(b_0, \dots, b_i x^i) = \sum_{j=0}^{i} b_{i-j} x^{i-j}\frac{d^2(b_j x^j)}{dx^2} = \sum_{j=0}^{i} b_{i-j} b_j j(j-1) x^{i-2}$$
$$= \sum_{j=0}^{i} b_{i-j} b_j j\,(j-1) x^{i-2};\ i \geq 2. \tag{76}$$

Hence,

$$\mathrm{L}\{\mathrm{A}_i(b_0, \dots, b_i x^i)\} = \mathrm{L}\left\{\sum_{j=2}^{i} b_{i-j} b_j j(j-1)x^{i-2}\right\} = \sum_{j=2}^{i} b_{i-j} b_j j\,(j-1)\mathrm{L}\{x^{i-2}\}$$

$$= \sum_{j=2}^{i} b_{i-j} b_j j\,(j-1)\frac{(i-2)!}{S^{i-1}}$$

$$= \frac{(i-2)!}{S^{i-1}} \sum_{j=2}^{i} b_{i-j} b_j j(j-1); i \geq 2 \quad (77)$$

Substituting the preceding equation into equation (75), we have

$$\mathrm{Y} = \frac{1}{S^2}\frac{S}{1+S^2} - \frac{1}{S^2}\frac{1}{1+S^2} - \frac{1}{S^2}\sum_{i=2}^{\infty}\left[\frac{(i-2)!}{S^{i-1}}\sum_{j=2}^{i} b_{i-j} b_j j(j-1)\right]$$

$$\mathrm{Y} = \frac{1}{S^2}\frac{S}{1+S^2} - \frac{1}{S^2}\frac{1}{1+S^2} - \sum_{i=2}^{\infty}\left[\frac{(i-2)!}{S^{i+1}}\sum_{j=2}^{i} b_{i-j} b_j j(j-1)\right] \quad (78)$$

Upon the Laplace inversion of equation (78), we obtain the exact solution as

$$y = xcos(x) - xsin(x) - \sum_{i=2}^{\infty}\left[\frac{(i-2)!}{S^{i+1}}\sum_{j=2}^{i} b_{i-j} b_j j(j-1)\right] \quad (79)$$

Problem 7. Consider the following equation

$$\frac{d^2y}{dx^2} + \pi^2 e^y = 0 \quad (80)$$

$$y'(0) = \alpha, y(1) = 0$$

The uncertainty we describe the Laplace transform of $y(x)$ as $\mathrm{Y(S)} = \mathrm{L}\{y(x)\} = \int_0^{+\infty} e^{-sx} f(x)dx$ over the prolonged province of $[0, +\infty]$ we will have

$$S^2\mathrm{Y} - \mathrm{Sy}(0) - y'(0) + \pi^2\mathrm{L}\{e^y\} = 0 \quad (81)$$

By theorem 1, equation (81) will be

$$S^2\mathrm{Y} - \alpha + \pi^2 \sum_{i=0}^{\infty} \mathrm{A}_i(b_0, \dots, b_i)\frac{i!}{S^{i+1}} = 0, \quad (82)$$

where $\alpha = y'(0)\ and\ \mathrm{A}_i$ are the Adomian polynomials disintegrating the exponential nonlinearity. Through reorganizing equation (82), we can write that

$$\mathrm{Y} = \frac{\alpha}{S^2} - \pi^2 \frac{1}{S^2} \sum_{i=0}^{+\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+1}} \tag{83}$$

Hence,

$$\mathrm{Y} = \frac{\alpha}{S^2} - \pi^2 \sum_{i=0}^{+\infty} \mathrm{A}_i(b_0, \dots, b_i) \frac{i!}{S^{i+3}} \tag{84}$$

From Equation (84), the Laplace transform inversion from both sides will be

$$y = \alpha x - \pi^2 \sum_{i=0}^{+\infty} \frac{\mathrm{A}_i(b_0, \dots, b_i)}{(i+2)(i+1)} x^{i+2} \tag{85}$$

or equivalently

$$\begin{aligned} y = \alpha x - \pi^2 \frac{e^{b_0}}{2} x^2 - \pi^2 \frac{b_1 e^{b_0}}{6} x^3 - \pi^2 \frac{\left(b_2 + \left(b_1^2/_2\right)\right) e^{b_0}}{12} x^4 \\ - \pi^2 \frac{(b_3 + b_1 b_2 + (1/6) b_1^3) e^{b_0}}{20} x^5 \\ - \pi^2 \frac{(b_4 + b_1 b_3 + (1/2) b_2^2 + (1/2) b_1^2 b_2 + (1/2) b_1^2 b_2 + (1/24) b_1^4) e^{b_0}}{30} x^6 \\ + \cdots, \end{aligned} \tag{86}$$

From equation (86) $e^{b_0} = 1$

Through the postulation of Theorem 1, we have that

$$y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + \cdots\cdots, \tag{87}$$

By equating the similar powers of x in equations (86) and (87), we will obtain

$b_0 = 0,$

$b_1 = \alpha,$

$b_2 = \frac{\pi^2}{2!},$

$b_3 = \frac{\pi^2 \alpha}{3!},$

$$b_4 = \frac{\pi^4 + \pi^2\alpha^2}{4!},$$

$$b_5 = \frac{4\pi^4\alpha + \pi^2\alpha^3}{5!},$$

$$b_6 = \frac{11\pi^4\alpha^2 + \pi^2\alpha^4 + 4\pi^6}{6!},$$

The exact solution of equation (80) will be

$$y = \alpha x - \frac{\pi^2}{2!}x^2 - \frac{\pi^2\alpha}{3!}x^3 - \frac{\pi^4+\pi^2\alpha^2}{4!}x^4 - \frac{4\pi^4\alpha+\pi^2\alpha^3}{5!}x^5 - \frac{11\pi^4\alpha^2+\pi^2\alpha^4+4\pi^6}{6!}x^6 - 0(x^7) \quad (88)$$

5. CONCLUSIONS

In this paper, a novel technique is created to enable the extension of the single Laplace transform method (SLTM) to solve nonlinear ordinary differential equations (ODEs). The main parts of the recommended technique are the Adomian polynomials. As it was shown in the given illustrative problems, our technique is conceptually and computationally simple as well as it is devoid of integration. This contribution may be considered as the missing section that completes the single Laplace transform method (SLTM) in Mathematical Physics.

Abbreviations

ADM: Adomian Decomposition Method
SLTM: Single Laplace Transform Method
ALCM: Auxiliary Laplace Constraint Method
ODEs: Ordinary Differential Equations
PDEs: Partial Differential Equations

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