Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 154 (2021) 117-132 EISSN 2392-2192

Interval valued Fermatean fuzzy interior (bi) Γ – hyperideals in Γ – hypersemigroups

V. S. Subha[1] and **S. Sharmila**[2]

[1]Department of Mathematics, Dharmapuram Gnanambigai Govt. Arts College (W), Mailaduthurai, Tamil Nadu, India

[2]Department of Mathematics, Annamalai University, Chidambaram, Tamil Nadu, India

[1,2]E-mail address: dharshinisuresh2002@gmail.com , gs.sharmi30@gmail.com

ABSTRACT

Interval valued Fermatean fuzzy set is an exyension of Fermatean fuzzy set. It is a combination of interval valued fuzzy set and Fermatean fuzzy set. In this paper we propose the notion of interval valued Fermatean fuzzy et. It is a pair of intevral numbers such that the sum of the ccubic of the upper bounds should be less than or equal to one. Some basic properties based on interval valued Feramtean fuzzy set is studied. We introduce the concept of interval valued Feramtean fuzzy Γ - subsemihypergroup, interval valued Feramtean fuzzy (bi, interior) Γ - hypersemigroup. Relation between these Γ - hyperideals are also discussed with suitable examples. Finally the inverse image of an interval valued Feramtean fuzzy set is established and also proved that the inverse image of an interval valued Feramtean fuzzy (bi, interior) Γ - hyperideal is also an interval valued Feramtean fuzzy (bi, interior) Γ - hyperideal.

***Keywords*:** Γ - hypersemigroup, Interval valued Fermatean fuzzy set, Interval valued Fermatean fuzzy, Γ - subsemihypergroup, Interval valued Fermatean fuzzy interior Γ - hyperideal, Interval valued Fermatean fuzzy bi Γ - hyperideal

(Received 02 February 2021; Accepted 19 February 2021; Date of Publication 20 February 2021)

1. INTRODUCTION

Fuzzy set and interval valued fuzzy set was introduced by Zadeh. As an extension of fuzzy set, Atanassov, proposed intuitionistic fuzzy set. Yager, examined Pythagorean fuzzy set characterized by a membership grade and non membership grade such that the square sum of its membership grade and non membership grade is less than or equal to one. In 2019, Senapati et al. initiated Fermatean fuzzy sets. Fermatean fuzzy set is characterized by membership and non membership grade restricted that cube sum of its membership grade and non membership grade is less than or equal to one. Senapati et al. applied Fermatean fuzzy set in decision making problem. Manemaran studied cubic Fermatean in soft structures.

Kuroki and Hong, studied fuzzy ideals in semigroups. Hyperstructure is an algebraic structure in which the product of elements is a set while in classic structure the product of elements is an element again. Subha et al., used the terms fuzzy rough set, cubic set, interval valued Pythagorean fuzzy set as applied to algebraic structures and hyperstructures. Yaqoob generalized rough $\Gamma -$ hyperideals in $\Gamma -$ semihypergroups. Many authors studied fuzzy (bi, interior) ideals in semigroups and hyperideals in hypersemigroups.

In this paper we introducce the notion of Interval valued Fermatean fuzzy set, Interval valued Fermatean fuzzy $\Gamma -$ subsemihypergroup, Interval valued Fermatean fuzzy interio r (bi) $\Gamma -$ hyperideals in $\Gamma -$ hypersemigroup. We explained the concept with suitable examples. Inverse image of these $\Gamma -$ hyperideals are also studied.

2. PRELIMINARIES

In this section we recall some basic definitions which are used throughout this paper.

2. 1. Definition [3]

Let U and Γ be two nonempty sets. U is called $\Gamma -$ hypersemigroup if

$a\gamma b \in$ U for every $\gamma \in \Gamma$ is a hyperoperation on U and $a, b \in$ U and

$a\gamma_1(b\gamma_2 c) = (a\gamma_1 b)\gamma_2 c$ for every $a, b, c \in$ U and the hyperoperations $\gamma_1, \gamma_2 \in \Gamma$.

Let C and D be two nonempty subsets of U. Then we define

$C\Gamma D = \bigcup_{\gamma\in\Gamma} C\gamma D = \bigcup\{c\gamma d \mid c \in C, d \in D \text{ and } \gamma \in \Gamma\}$.

2. 2. Definition [10]

Let U be a $\Gamma -$hypersemigroup and $\gamma \in \Gamma$. A nonempty subset A of U is called a $\Gamma -$ subsemihypergroup of U if $A\Gamma A \subseteq A$. A subset A of a $\Gamma -$hypersemigroup U is called an interior $\Gamma -$hyperideal of U if $U\Gamma A\Gamma U \subseteq A$. A subset A of a $\Gamma -$hypersemigroup U is called an bi $\Gamma -$hyperideal of U if $A\Gamma U\Gamma A \subseteq A$.

2. 3. Definition [9]

An interval number $\bar{c} = [c^-, c^+]$ on $[0, 1]$ is a closed subinterval of $[0, 1]$, where $0 \leq c^- \leq c^+ \leq 1$.

Let $\bar{c} = [c^-, c^+]$ and $\bar{d} = [d^-, d^+]$ are two interval numbers in $D[0,1]$ where $D[0,1]$ is the family of all closed subintervals of $[0,1]$. Then we have

$\bar{c} \leq \bar{d}$ if and only if $c^- \leq d^-$ and $c^+ \leq d^+$

$\bar{c} = \bar{d}$ if and only if $c^- = d^-$ and $c^+ = d^+$

$\min\{\bar{c}, \bar{d}\} = [\min\{c^-, d^-\}, \min\{c^+, d^+\}]$

$\max\{\bar{c}, \bar{d}\} = [\max\{c^-, d^-\}, \max\{c^+, d^+\}]$

$\bar{c}' = [1 - c^+, 1 - c^-]$.

3. INTERVAL VALUED FERMATEAN FUZZY $\Gamma -$ HYPERIDEALS IN $\Gamma -$ HYPERSEMIGROUPS

In this section we define interval valued Fermatean fuzzy set. We also study the properties of interval valued Fermatean fuzzy set. Interval valued Fermatean fuzzy $\Gamma -$ hyperideal is also proposed.

3. 1. Definition

Let U be a universe set and $D[0,1]$ be the collection of all subsets of $[0,1]$. An interval valued Fermatean fuzzy set F is a pair of interval numbers having the form

$$F = \{(a, [\rho^-(a), \rho^+(a)], [\tau^-(a), \tau^+(a)]) : 0 \leq \rho^+(a)^3 + \tau^+(a)^3 \leq 1 \; \forall \, a \in U\}.$$

For our convenience we denote F as $F = (\tilde{\rho}, \tilde{\tau})$. Where $\tilde{\rho}, \tilde{\tau} : U \to D[0,1]$. The values $\tilde{\rho}(a)$ and $\tilde{\tau}(a)$ are membership and non membership grade of $a \in U$ respectively.

3. 2. Definition

Let $F = (\tilde{\rho}, \tilde{\tau})$ and $F_1 = (\tilde{\rho}_1, \tilde{\tau}_1)$ are two interval valued Fermatean fuzzy sets of U. Then

$F \cup F_1 = (\tilde{\rho} \vee \tilde{\rho}_1, \tilde{\tau} \wedge \tilde{\tau}_1)$ is a union of F and F_1.

$F \cap F_1 = (\tilde{\rho} \wedge \tilde{\rho}_1, \tilde{\tau} \vee \tilde{\tau}_1)$ is an intersection of F and F_1.

$F^c = (\tilde{\tau}, \tilde{\rho})$ is a complement of F.

3. 3. Definition

An interval valued Fermatean fuzzy set F is said to be an interval valued Fermatean fuzzy left Γ −hyperideal of U if

$$\tilde{\rho}(b) \leq \bigwedge_{f \in a\gamma b} \tilde{\rho}(f)$$

$\tilde{\tau}(b) \geq \bigvee_{f \in a\gamma b} \tilde{\tau}(f)$ for all $a, b \in$ U and $\gamma \in \Gamma$.

An interval valued Fermatean fuzzy set F is said to be an interval valued Fermatean fuzzy right Γ −hyperideal of U if

$$\tilde{\rho}(a) \leq \bigwedge_{f \in a\gamma b} \tilde{\rho}(f)$$

$\tilde{\tau}(a) \geq \bigvee_{f \in a\gamma b} \tilde{\tau}(f)$ for all $a, b \in$ U and $\gamma \in \Gamma$.

An interval valued Fermatean fuzzy set F is said to be an interval valued Fermatean fuzzy Γ −hyperideal of U if

$$\max\{\tilde{\rho}(a), \quad \tilde{\rho}(b)\} \leq \bigwedge_{f \in a\gamma b} \tilde{\rho}(f)$$

$\min\{\tilde{\tau}(a), \ \tilde{\tau}(b)\} \geq \bigvee_{f \in a\gamma b} \tilde{\tau}(f)$ for all $a, b \in$ U and $\gamma \in \Gamma$.

3. 4. Example

Let U $= \{a_1, a_2, a_3\}$ and $\Gamma = \{\gamma\}$ then U is a $\Gamma -$ hypersemigroup.

Table 1. Hyperoperation γ

γ	a_1	a_2	a_3
a_1	$\{a_1\}$	$\{a_1, a_2\}$	$\{a_1, a_3\}$
a_2	$\{a_1, a_2\}$	$\{a_2\}$	$\{a_2 , a_3\}$
a_3	$\{a_1, a_3\}$	$\{a_2, a_3\}$	$\{a_3\}$

Define an interval valued Fermatean fuzzy set $F = (\tilde{\rho}, \tilde{\tau})$ as:

$$\tilde{\rho}(x) = \begin{cases} [0.9, 0.98], & x = a_3 \\ [0.5, 0.72], & otherwise \end{cases}$$

and

$$\tilde{\tau}(x) = \begin{cases} [0.1, 0.35], & x = a_3 \\ [0.6, 0.85], & otherwise \end{cases}$$

By routine calculation we can say that F is an interval valued Fermatean fuzzy $\Gamma -$ hyperideal of U.

3. 5. Theorem

Let $F=(\tilde{\rho},\tilde{\tau})$ and $F_1=(\tilde{\rho}_1,\ \tilde{\tau}_1)$ are any two interval valued Fermatean fuzzy $\Gamma-$ hyperideal of U then $F\cup F_1$ is also an interval valued Fermatean fuzzy $\Gamma-$hyperideal of U.

Proof:

Let F and F_1 are any two interval valued Fermatean fuzzy $\Gamma-$hyperideal of U. Consider $a,b\ \in U$,

$$\max\{\tilde{\rho}\cup\tilde{\rho}_1(a),\tilde{\rho}\cup\tilde{\rho}_1(b)\}=\max\{\tilde{\rho}(a)\vee\tilde{\rho}_1(a),\tilde{\rho}(b)\vee\tilde{\rho}_1(b)\}$$

$$=\max\{\tilde{\rho}(a)\vee\tilde{\rho}(b),\tilde{\rho}_1(a)\vee\tilde{\rho}_1(b)\}$$

$$\leq\max\{\wedge_{f\in a\gamma b}\tilde{\rho}(f),\wedge_{f\in a\gamma b}\tilde{\rho}_1(f)\}$$

$$\leq\wedge_{f\in a\gamma b}\{\max\{\tilde{\rho}(f),\tilde{\rho}_1(f)\}\}$$

$$\leq\wedge_{f\in a\gamma b}\{\tilde{\rho}\cup\tilde{\rho}_1(f)\}\ \text{and}$$

$$\min\{\tilde{\tau}\cup\tilde{\tau}_1(a),\tilde{\tau}\cup\tilde{\tau}_1(b)\}=\min\{\tilde{\tau}(a)\wedge\tilde{\tau}_1(a),\tilde{\tau}(b)\wedge\tilde{\tau}_1(b)\}$$

$$=\min\{\tilde{\tau}(a)\wedge\tilde{\tau}(b),\tilde{\tau}_1(a)\wedge\tilde{\tau}_1(b)\}$$

$$\geq\min\{\vee_{f\in a\gamma b}\tilde{\tau}(f),\ \vee_{f\in a\gamma b}\tilde{\tau}_1(f)\}$$

$$\geq\ \vee_{f\in a\gamma b}\{\min\{\tilde{\tau}(f),\ \tilde{\tau}_1(f)\}\}$$

$$\geq\ \vee_{f\in a\gamma b}\{\tilde{\rho}\cup\tilde{\rho}_1(f)\}\ \textit{for all } a,b\ \in U \textit{ and } \gamma\in\Gamma.$$

Thus $F\cup F_1$ is an interval valued Fermatean fuzzy $\Gamma-$hyperideal of U.

3. 6. Theorem

Let $F=(\tilde{\rho},\tilde{\tau})$ and $F_1=(\tilde{\rho}_1,\ \tilde{\tau}_1)$ are any two interval valued Fermatean fuzzy $\Gamma-$hyperideals of U then $F\cap F_1$ is also an interval valued Fermatean fuzzy $\Gamma-$ hyperideal of U.

Proof:

Proof is straightforward.

3. 7. Definition

Let $F=(\tilde{\rho},\tilde{\tau})$ be any interval valued Fermatean fuzzy set of U. For any $\tilde{t}_1,\ \tilde{t}_2\in[0,1]$ we have $(\tilde{t}_1,\ \tilde{t}_2)$-level set of interval valued Fermatean fuzzy set F is defined by

$$F^{(\tilde{t}_1,\tilde{t}_2)}=(\tilde{\rho}^{\tilde{t}_1},\tilde{\tau}^{\tilde{t}_2})$$

where $\tilde{\rho}^{\tilde{t}_1}=\{x\ \in U:\tilde{\rho}(x)\geq\tilde{t}_1,\forall x\in U\}$ and

$$\tilde{\tau}^{\tilde{t}_2}=\{x\ \in U:\tilde{\tau}(x)\leq\ \tilde{t}_2,\forall x\in U\}.$$

3. 8. Theorem

Let $F=(\tilde{\rho},\tilde{\tau})$ be any interval valued Fermatean fuzzy $\Gamma-$hyperideal of U. For any

$\tilde{t}_1,\ \tilde{t}_2 \in [0,1]$ the $(\tilde{t}_1,\ \tilde{t}_2)$-level set of interval valued Fermatean fuzzy Γ −hyperideal F is a Γ −hyperideal of U.

Proof:

Let us assume that $a \in U\ b \in F^{(\tilde{t}_1,\tilde{t}_2)}$ then we have $b \in \tilde{\rho}^{\tilde{t}_1}$ and $b \in \tilde{\tau}^{\tilde{t}_2}$. Thus $\tilde{\rho}(b) \geq \tilde{t}_1$ and $\tilde{\tau}(b) \leq \tilde{t}_2$.

Since $\tilde{\rho}(b) \leq \bigwedge_{f\in a\gamma b} \tilde{\rho}(f)$ and $\tilde{\tau}(b) \geq \bigvee_{f\in a\gamma b} \tilde{\tau}(f)$

i.e., $\bigwedge_{f\in a\gamma b} \tilde{\rho}(f) \geq \tilde{t}_1$ and $\bigvee_{f\in a\gamma b} \tilde{\tau}(f) \leq \tilde{t}_2$.

i.e., $\tilde{\rho}(f) \geq \tilde{t}_1$ for $f \in a\gamma b$ and $\tilde{\tau}(f) \leq \tilde{t}_2$ for $f \in a\gamma b$

i.e., $a\gamma b \subseteq \tilde{\rho}^{\tilde{t}_1}$ and $a\gamma b \subseteq \tilde{\tau}^{\tilde{t}_2}$

i.e., $\subseteq F^{(\tilde{t}_1,\tilde{t}_2)}$. Hence $F^{(\tilde{t}_1,\tilde{t}_2)}$ is a left Γ −hyperideal of U.

Similarly we can prove that $F^{(\tilde{t}_1,\tilde{t}_2)}$ is a right Γ −hyperideal of U.

4. INTERVAL VALUED FERMATEAN FUZZY BI Γ - HYPERIDEAL

In this section we introduce the notion of interval valued Fermatean fuzzy interior $\Gamma -$ hyperideal and interval valued Fermatean fuzzy bi $\Gamma -$ hyperideal in $\Gamma -$ hypersemigroups.

4. 1. Definition

An interval valued Fermatean fuzzy set F is said to be an interval valued Fermatean fuzzy $\Gamma -$ subsemihypergroup of U if

$$\min\{\tilde{\rho}(a), \quad \tilde{\rho}(b)\} \leq \bigwedge_{f\in a\gamma b} \tilde{\rho}(f)$$

$\max\{\tilde{\tau}(a), \tilde{\tau}(b)\} \geq \bigvee_{f\in a\gamma b} \tilde{\tau}(f)$ for all $a, b \in$ U and $\gamma \in \Gamma$.

4. 2. Example

Let U $= \{a_1, a_2, a_3, a_4, a_5\}$ and $\Gamma = \{\gamma\}$ then U is a Γ −hypersemigroup.

Table 2. Hyperoperation γ

γ	a_1	a_2	a_3	a_4	a_5
a_1	$\{a_1, a_2\}$	$\{a_2, a_5\}$	$\{a_3\}$	$\{a_3, a_4\}$	$\{a_5\}$
a_2	$\{a_2, a_3\}$	$\{a_5\}$	$\{a_3\}$	$\{a_3, a_4\}$	$\{a_5\}$

a_3	$\{a_3\}$	$\{a_3\}$	$\{a_3\}$	$\{a_3\}$	$\{a_3\}$
a_4	$\{a_3, a_4\}$	$\{a_3, a_4\}$	$\{a_3\}$	$\{a_4\}$	$\{a_3, a_4\}$
a_5	$\{a_5\}$	$\{a_5\}$	$\{a_3\}$	$\{a_3, a_4\}$	$\{a_5\}$

Define an interval valued Fermatean fuzzy set $F = (\tilde{\rho}, \tilde{\tau})$ as:

$$\tilde{\rho}(x) = \begin{cases} [0.7, 0.85], & x = a_3, a_5 \\ [0.3, 0.5], & otherwise \end{cases}$$

and

$$\tilde{\tau}(x) = \begin{cases} [0.2, 0.55], & x = a_3, a_5 \\ [0.8, 0.91], & otherwise \end{cases}$$

By routine calculation we can say that F is an interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of U.

4. 3. Theorem

Let $F = (\tilde{\rho}, \tilde{\tau})$ and $F_1 = (\tilde{\rho}_1, \tilde{\tau}_1)$ are any two interval valued Fermatean fuzzy $\Gamma -$subsemihypergroups of U then $F \cap F_1$ is also an interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of U.

Proof:

Let F and F_1 are two interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of U. Consider $a, b \in U$,

$\min\{\tilde{\rho} \cap \tilde{\rho}_1(a), \tilde{\rho} \cap \tilde{\rho}_1(b)\} = \min\{\tilde{\rho}(a) \wedge \tilde{\rho}_1(a), \tilde{\rho}(b) \wedge \tilde{\rho}_1(b)\}$

$= \min\{\tilde{\rho}(a) \wedge \tilde{\rho}(b), \tilde{\rho}_1(a) \wedge \tilde{\rho}_1(b)\}$

$\leq \min\{\wedge_{f \in a\gamma b} \tilde{\rho}(f), \wedge_{f \in a\gamma b} \tilde{\rho}_1(f)\}$

$\leq \wedge_{f \in a\gamma b}\{\min\{\tilde{\rho}(f), \tilde{\rho}_1(f)\}\}$

$\leq \wedge_{f \in a\gamma b}\{\tilde{\rho} \cap \tilde{\rho}_1(f)\}$ and

$\max\{\tilde{\tau} \cap \tilde{\tau}_1(a), \tilde{\tau} \cap \tilde{\tau}_1(b)\} = \max\{\tilde{\tau}(a) \vee \tilde{\tau}_1(a), \tilde{\tau}(b) \vee \tilde{\tau}_1(b)\}$

$= \max\{\tilde{\tau}(a) \vee \tilde{\tau}(b), \tilde{\tau}_1(a) \vee \tilde{\tau}_1(b)\}$

$\geq \max\{\vee_{f \in a\gamma b} \tilde{\tau}(f), \vee_{f \in a\gamma b} \tilde{\tau}_1(f)\}$

$\geq \vee_{f \in a\gamma b}\{\max\{\tilde{\tau}(f), \tilde{\tau}_1(f)\}\}$

$\geq \vee_{f \in a\gamma b}\{\tilde{\tau} \cap \tilde{\tau}(f)\}$ for all $a, b \in U$ and $\gamma \in \Gamma$.

Thus $F \cap F_1$ is an interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of U.

4. 4. Theorem

Let $F = (\tilde{\rho}, \tilde{\tau})$ be any interval valued Fermatean fuzzy Γ −subsemihypergroup of U. For any $\tilde{t}_1,\ \tilde{t}_2 \in [0, 1]$ the $(\tilde{t}_1,\ \tilde{t}_2)$-level set of interval valued Fermatean fuzzy $\Gamma -$ subsemihypergroup F is a $\Gamma -$ subsemihypergroup of U.

Proof:

Let $F = (\tilde{\rho}, \tilde{\tau})$ be any interval valued Fermatean fuzzy Γ −subsemihypergroup of U. Let us assume that $a, b \in F^{(\tilde{t}_1,\tilde{t}_2)}$ then we have $a, b \in \tilde{\rho}^{\tilde{t}_1}$, $a, b \in \tilde{\tau}^{\tilde{t}_2}$ and $\gamma \in \Gamma$. Thus $\tilde{\rho}(a) \geq \tilde{t}_1$, $\tilde{\rho}(b) \geq \tilde{t}_1$ and $\tilde{\tau}(a) \leq\ \tilde{t}_2$, $\tilde{\tau}(b) \leq\ \tilde{t}_2$. Since

$\min\{\tilde{\rho}(a),\ \tilde{\rho}(b)\} \leq \bigwedge_{f\in a\gamma b} \tilde{\rho}(f)$ and $\min\{\tilde{\tau}(a), \tilde{\tau}(b)\} \geq \bigvee_{f\in a\gamma b} \tilde{\tau}(f)$.

i.e., $\bigwedge_{f\in a\gamma b} \tilde{\rho}(f) \geq \tilde{t}_1$ and $\bigvee_{f\in a\gamma b} \tilde{\tau}(f) \leq\ \tilde{t}_2$,

i.e., $\tilde{\rho}(f) \geq \tilde{t}_1$ for $f \in a\gamma b$ and $\tilde{\tau}(f) \leq\ \tilde{t}_2$ for $f \in a\gamma b$,

i.e., $a\gamma b \subseteq \tilde{\rho}^{\tilde{t}_1}$ and $a\gamma b \subseteq \tilde{\tau}^{\tilde{t}_2}$,

i.e., $\subseteq F^{(\tilde{t}_1,\tilde{t}_2)}$. Hence $F^{(\tilde{t}_1,\tilde{t}_2)}$ is a Γ −subsemihypergroup of U.

4. 5. Definition

An interval valued Fermatean fuzzy set F is said to be an interval valued Fermatean fuzzy bi Γ −hyperideal of U if

$$\min\{\tilde{\rho}(a), \quad \tilde{\rho}(b)\} \leq \bigwedge_{f\in a\gamma_1 w\gamma_2 b} \tilde{\rho}(f)$$

$\max\{\tilde{\tau}(a), \tilde{\tau}(b)\} \geq \bigvee_{f\in a\gamma_1 w\gamma_2 b} \tilde{\tau}(f)$ for all $a, b \in$ U and $\gamma_1, \gamma_2 \in\ \Gamma$.

4. 6. Example

Let U $= \{a_1, a_2, a_3\}$ and $\Gamma = \{\gamma\}$ then U is a Γ −hypersemigroup.

Table 3. Hyperoperation γ_1

γ_1	a_1	a_2	a_3
a_1	$\{a_1, a_2\}$	$\{a_1, a_2\}$	$\{ a_3\}$
a_2	$\{a_1, a_2\}$	$\{a_1, a_2\}$	$\{a_3 \}$
a_3	$\{a_3\}$	$\{ a_3\}$	$\{a_3\}$

Table 4. Hyperoperation γ_2

γ_1	a_1	a_2	a_3
a_1	$\{a_1, a_2\}$	$\{a_1, a_2\}$	$\{ a_3\}$
a_2	$\{a_1, a_2\}$	$\{a_1, a_2\}$	$\{a_3 \}$
a_3	$\{a_3\}$	$\{ a_3\}$	$\{a_3\}$

Define an interval valued Fermatean fuzzy set $F = (\tilde{\rho}, \tilde{\tau})$ as:

$$\tilde{\rho}(x) = \begin{cases} [0.83, 0.95], & x = a_1 \\ [0.32, 0.51], & otherwise \end{cases}$$

and

$$\tilde{\tau}(x) = \begin{cases} [0.27, 0.4], & x = a_1 \\ [0.75, 0.83], & otherwise \end{cases}$$

$\min\{\tilde{\rho}(a_1), \tilde{\rho}(a_3)\} = [0.32,0.51]$ and $\bigwedge_{a_1,a_3 \in a_1\gamma_1 \mathrm{w}\gamma_2 a_3}\{\tilde{\rho}(a_1), \tilde{\rho}(a_3)\} = [0.32,0.51]$.

Therefore $\min\{\tilde{\rho}(a_1), \tilde{\rho}(a_3)\} = \bigwedge_{a_1,a_3 \in a_1\gamma_1 \mathrm{w}\gamma_2 a_3}\{\tilde{\rho}(a_1), \tilde{\rho}(a_3)\}$ for all w $\in$ U.

Now $\max\{\tilde{\tau}(a_1), \tilde{\tau}(a_3)\} = [0.75, 0.83]$ and $\bigvee_{a_1 \in a_1\gamma_1 \mathrm{w}\gamma_2 a_3} \tilde{\tau}(a_1) = [0.75, 0.83]$

Therefore $\max\{\tilde{\tau}(a_1), \tilde{\tau}(a_3)\} = \bigvee_{a_1 \in a_1\gamma_1 \mathrm{w}\gamma_2 a_3} \tilde{\tau}(a_1)$ for all w $\in U$. Similarly inequalities (i) and (ii) in Definition 4.4 hold. Hence F is an interval valued Fermatean fuzzy bi Γ −hyperideal of U.

4. 7. Theorem

Let $F = (\tilde{\rho}, \tilde{\tau})$ and $F_1 = (\tilde{\rho}_1, \tilde{\tau}_1)$ be any two interval valued Fermatean fuzzy bi $\Gamma-$ hyperideal of U then $F \cap F_1$ is also an interval valued Fermatean fuzzy bi Γ −hyperideal of U.

Proof:

Let F and F_1 be any two interval valued Fermatean fuzzy bi $\Gamma-$ hyperideals of U. Consider $a, b \in U$,

$\min\{\tilde{\rho} \cap \tilde{\rho}_1(a), \tilde{\rho} \cap \tilde{\rho}_1(b) \} = \min\{\tilde{\rho}(a) \wedge \tilde{\rho}_1(a), \tilde{\rho}(b) \wedge \tilde{\rho}_1(b)\}$

$= \min\{\tilde{\rho}(a) \wedge \tilde{\rho}(b), \tilde{\rho}_1(a) \wedge \tilde{\rho}_1(b)\}$

$\leq \min\{\bigwedge_{f \in a\gamma_1 w\gamma_2 b} \tilde{\rho}(f), \bigwedge_{f \in a\gamma_1 w\gamma_2 b} \tilde{\rho}_1(f)\}$

$\leq \bigwedge_{f \in a\gamma_1 w\gamma_2 b}\{\min\{\tilde{\rho}(f), \tilde{\rho}_1(f)\}\}$

$\leq \wedge_{f\in a\gamma_1 w\gamma_2 b}\{\tilde{\rho}\cap\tilde{\rho}_1(f)\}$ and

$\max\{\tilde{\tau}\cap\tilde{\tau}_1(a),\tilde{\tau}\cap\tilde{\tau}_1(b)\} = \max\{\tilde{\tau}(a)\vee\tilde{\tau}_1(a),\tilde{\tau}(b)\vee\tilde{\tau}_1(b)\}$

$= \max\{\tilde{\tau}(a)\vee\tilde{\tau}(b),\tilde{\tau}_1(a)\vee\tilde{\tau}_1(b)\}$

$\geq \max\{\vee_{f\in a\gamma_1 w\gamma_2 b}\,\tilde{\tau}(f),\vee_{f\in a\gamma_1 w\gamma_2 b}\,\tilde{\tau}_1(f)\}$

$\geq \vee_{f\in a\gamma_1 w\gamma_2 b}\{\max\{\tilde{\tau}(f),\ \tilde{\tau}_1(f)\}\}$

$\geq \vee_{f\in a\gamma b}\{\tilde{\tau}\cap\tilde{\tau}_1(f)\}$ for all $\mathrm{a,b}\in \mathrm{U}$ and $\gamma_1,\gamma_2\in\Gamma$.

Thus $F\cap F_1$ is an interval valued Fermatean fuzzy bi $\Gamma-$ hyperideal of U.

4. 8. Theorem

Let $F=(\tilde{\rho},\tilde{\tau})$ be any interval valued Fermatean fuzzy bi $\Gamma-$hyperideal of U. For any $\tilde{t}_1,\ \tilde{t}_2\in[0,1]$, the $(\tilde{t}_1,\ \tilde{t}_2)$-level set of interval valued Fermatean fuzzy bi $\Gamma-$hyperideal F is a bi $\Gamma-$ hyperideal of U.

Proof:

Let $F=(\tilde{\rho},\tilde{\tau})$ be any interval valued Fermatean fuzzy bi $\Gamma-$ hyperideal of U. For any $\tilde{t}_1,\ \tilde{t}_2\in[0,1]$ the $(\tilde{t}_1,\ \tilde{t}_2)$-level set of interval valued Fermatean fuzzy bi $\Gamma-$ hyperideal F is a bi $\Gamma-$ hyperideal of U. Let us assume that $a,b\in F^{(\tilde{t}_1,\tilde{t}_2)}$ then we have $a,b\in\tilde{\rho}^{\tilde{t}_1}$, $a,b\in\tilde{\tau}^{\tilde{t}_2}$ and $\gamma_1,\gamma_2\in\Gamma$. Thus $\tilde{\rho}(a)\geq\tilde{t}_1$, $\tilde{\rho}(b)\geq\tilde{t}_1$ and $\tilde{\tau}(a)\leq\ \tilde{t}_2$, $\tilde{\tau}(b)\leq\ \tilde{t}_2$. Since

$\min\{\tilde{\rho}(a),\ \tilde{\rho}(b)\}\leq\wedge_{f\in a\gamma_1 w\gamma_2 b}\,\tilde{\rho}(f)$ and $\min\{\tilde{\tau}(a),\tilde{\tau}(b)\}\geq\vee_{f\in a\gamma_1 w\gamma_2 b}\,\tilde{\tau}(f)$.

i.e., $\wedge_{f\in a\gamma_1 w\gamma_2 b}\,\tilde{\rho}(f)\geq\tilde{t}_1$ and $\vee_{f\in a\gamma_1 w\gamma_2 b}\,\tilde{\tau}(f)\leq\ \tilde{t}_2$.

i.e., $\tilde{\rho}(f)\geq\tilde{t}_1$ for $f\in a\gamma_1 w\gamma_2 b$ and $\tilde{\tau}(f)\leq\ \tilde{t}_2$ for $f\in a\gamma_1 w\gamma_2 b$

i.e., $a\gamma_1 w\gamma_2 b\subseteq\tilde{\rho}^{\tilde{t}_1}$ and $a\gamma_1 w\gamma_2 b\subseteq\tilde{\tau}^{\tilde{t}_2}$

i.e., $\gamma_1 w\gamma_2 b\subseteq F^{(\tilde{t}_1,\tilde{t}_2)}$. Hence $F^{(\tilde{t}_1,\tilde{t}_2)}$ is a bi Γ-hyperideal of U.

5. INTERVAL VALUED FERMATEAN FUZZY BI Γ-HYPERIDEAL

5. 1. Definition

An interval valued Fermatean fuzzy set F is said to be an interval valued Fermatean fuzzy interior $\Gamma-$ hyperideal of U if

$$\tilde{\rho}(i)\leq\bigwedge_{f\in a\gamma_1 i\gamma_2 b}\tilde{\rho}(f)$$

$\tilde{\tau}(i)\geq\vee_{f\in a\gamma_1 i\gamma_2 b}\,\tilde{\tau}(f)$ for all $a,b\in\mathrm{U}$ and $\gamma_1,\gamma_2\in\Gamma$.

5. 2. Example

Let $\mathrm{U}=\{a_1,a_2,a_3\}$ and $\Gamma=\{\gamma\}$ then U is a $\Gamma-$hypersemigroup.

Table 5. Hyperoperation γ_1

γ_1	a_1	a_2	a_3
a_1	$\{a_1, a_2\}$	$\{a_1, a_2\}$	$\{a_3\}$
a_2	$\{a_1, a_2\}$	$\{a_2\}$	$\{a_3\}$
a_3	$\{a_3\}$	$\{a_3\}$	$\{a_3\}$

Table 6. Hyperoperation γ_2

γ_1	a_1	a_2	a_3
a_1	$\{a_3\}$	$\{a_3\}$	$\{a_3\}$
a_2	$\{a_1, a_3\}$	$\{a_3\}$	$\{a_3\}$
a_3	$\{a_3\}$	$\{a_3\}$	$\{a_3\}$

Define an interval valued Fermatean fuzzy set $F = (\tilde{\rho}, \tilde{\tau})$ as:

$$\tilde{\rho}(x) = \begin{cases} [0.65, 0.83], & x = a_1 \\ [0.23, 0.5], & x = a_2 \\ [0.92, 0.97], & x = a_3 \end{cases}$$

and

$$\tilde{\tau}(x) = \begin{cases} [0.4, 0.69], & x = a_1 \\ [0.7, 0.9], & x = a_2 \\ [0, 0.1], & x = a_3 \end{cases}$$

By routine calculation we can say that F is an interval valued Fermatean fuzzy interior Γ −hyperideal of U.

5. 3. Theorem

Let $F = (\tilde{\rho}, \tilde{\tau})$ be any interval valued Fermatean fuzzy interior Γ −hyperideal of U. For any $\tilde{t}_1$, $\tilde{t}_2 \in [0, 1]$ the $(\tilde{t}_1, \tilde{t}_2)$-level set of interval valued Fermatean fuzzy interior Γ− hyperideal F is an interior Γ −hyperideal of U.

Proof:

A similar proof holds as in Theorem.

5. 4. Example

Every interval valued Fermatean fuzzy $\Gamma -$ hyperideal of U is an interval valued Fermatean fuzzy $\Gamma -$ subsemihypergroup of U but converse is not true.

Let us consider Example 4.2. Then we have the Interval valued Fermatean fuzzy set $F = (\tilde{\rho}, \tilde{\tau})$ as

$$\tilde{\rho}(x) = \begin{cases} [0.7, 0.85], & x = a_3, a_5 \\ [0.3, 0.5], & otherwise \end{cases}$$

and

$$\tilde{\tau}(x) = \begin{cases} [0.2, 0.55], & x = a_3, a_5 \\ [0.8, 0.91], & otherwise \end{cases}$$

F is a interval valued Fermatean fuzzy $\Gamma -$ subsemihypergroup but not an interval valued Fermatean fuzzy $\Gamma -$ hyperideal since

$\max\{\tilde{\rho}(a_4),\ \tilde{\rho}(a_5)\} \leq \wedge_{a_3, a_5 \in a_4 \gamma a_5}\{\tilde{\rho}(a_3), \tilde{\rho}(a_4)\}$

$\max\{[0.3, 0.5], [0.7, 0.85]\} \leq \wedge\{[.7, 0.85], [0.3,0.5]\}$

$[0.7, 0.85] \nleq [0.3,0.5]$.

Hence F is not an interval valued Fermatean fuzzy $\Gamma-$hyperideal.

6. INVERSE IMAGE OF AN INTERVAL VALUED FERMATEAN FUZZY SET

In this section we define inverse image of an interval valued Fermatean fuzzy set and some properties.

6. 1. Definition

Let U and V be any two $\Gamma -$ hypersemigroups. By a homomorphism we mean a mapping $\Psi: U \to V$ satisfying the identity $\Psi(x\gamma y) = \Psi(x)\Psi(\gamma)\Psi(y)$ for all $x, y \in U$ and $\gamma \in \Gamma$.

6. 2. Definition

Let Ψ be a mapping from a hypersemigroup U into a hypersemigroup V. If $F = (\tilde{\rho}, \tilde{\tau})$ is an interval valued Fermatean fuzzy set in U then the image of F,

$\Psi(F) = (\Psi(\tilde{\rho}), \Psi(\tilde{\tau}))$ is an interval valued Fermatean fuzzy set in V defined by

(i) $$\Psi(\tilde{\rho})(x) = \begin{cases} \sup\limits_{y \in \Psi^{-1}(x)} \tilde{\rho}(y) \ if\ \Psi^{-1}(x) \neq \emptyset \\ [0, 0] \quad if \quad otherwise \end{cases}$$

(ii) $$\Psi(\tilde{\tau})(x) = \begin{cases} \inf\limits_{y \in \Psi^{-1}(x)} \tilde{\tau}(y) \ if\ \Psi^{-1}(x) \neq \emptyset \\ [1, 1] \quad if \quad otherwise \end{cases}$$ for all $x \in U$.

6. 3. Definition

Let Ψ be a mapping from a $\Gamma -$ hypersemigroup U to a $\Gamma -$ hypersemigroup V and $F = (\tilde{\rho}, \tilde{\tau})$ be an interval valued Fermatean fuzzy set in V. Then the inverse image of F,

$\Psi^{-1}(F) = (\Psi^{-1}(\tilde{\rho}), \Psi^{-1}(\tilde{\tau}))$ is an interval valued Fermatean fuzzy set in U and is defined by

(i) $\Psi^{-1}(\tilde{\rho})(x) = \tilde{\rho}(\Psi(\mathrm{x}))$ and

(ii) $\Psi^{-1}(\tilde{\tau})(x) = \tilde{\tau}(\Psi(\mathrm{x}))$ for all $x \in \mathrm{V}$.

6. 4. Theorem

Let U and V be two $\Gamma -$ hypersemigroups and $\Psi : U \to V$ be an onto homomorphism of $\Gamma -$ hypersemigroups. If $F = (\tilde{\rho}, \tilde{\tau})$ is an interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of V then $\Psi^{-1}(F) = (\Psi^{-1}(\tilde{\rho}), \Psi^{-1}(\tilde{\tau}))$ is also an interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of U .

Proof:

Let $F = (\tilde{\rho}, \tilde{\tau})$ be an interval valued Fermatean fuzzy interior $\Gamma -$ hyperideal of V and $a, b \in V$.

$\wedge_{f \in a\gamma b}\{\Psi^{-1}(\tilde{\rho})(f)\} = \wedge_{f \in a\gamma b}\{\tilde{\rho}(\Psi(f))\} = \wedge_{\Psi(f) \in \Psi(a\gamma b)}\{\tilde{\rho}(\Psi(f))\} =$
$\wedge_{\Psi(f) \in \Psi(a)\Psi(\gamma)\Psi(b)}\{\tilde{\rho}(\Psi(f))\} \geq \min\{\tilde{\rho}(\Psi(a)), \tilde{\rho}(\Psi(b))\}$

$\geq \min\{\Psi^{-1}(\tilde{\rho}(a)), \Psi^{-1}(\tilde{\rho}(b))\}$ and

$\vee_{f \in a\gamma b}\{\Psi^{-1}(\tilde{\tau})(f)\} = \vee_{f \in a\gamma b}\{\tilde{\tau}(\Psi(f))\}$

$= \vee_{\Psi(f) \in \Psi(a\gamma b)}\{\tilde{\tau}(\Psi(f))\}$

$= \vee_{\Psi(f) \in \Psi(a)\Psi(\gamma)\Psi(b)}\{\tilde{\tau}(\Psi(f))\}$

$\leq \max\{\tilde{\tau}(\Psi(a)), \tilde{\tau}(\Psi(b))\}$

$\leq \max\{\Psi^{-1}(\tilde{\tau}(a)), \Psi^{-1}(\tilde{\tau}(b))\}$.

Thus $\Psi^{-1}(F)$ is also an interval valued Fermatean fuzzy $\Gamma -$subsemihypergroup of U .

6. 5. Theorem

Let U and V be two $\Gamma -$ hypersemigroups and $\Psi : U \to V$ be an onto homomorphism of $\Gamma -$ hypersemigroups. If

$F = (\tilde{\rho}, \tilde{\tau})$ is an interval valued Fermatean fuzzy interior $\Gamma -$ hyperideal of U then Ψ^{-1} (F) is also an interval valued Fermatean fuzzy interior $\Gamma -$ hyperideal of V.

$F = (\tilde{\rho}, \tilde{\tau})$ is an interval valued Fermatean fuzzy bi $\Gamma -$hyperideal of U then Ψ^{-1} (F) is also an interval valued Fermatean fuzzy bi $\Gamma -$ hyperideal of V.

Proof:

Let $F = (\tilde{\rho}, \tilde{\tau})$ be an interval valued Fermatean fuzzy interior $\Gamma -$ hyperideal of V and $a, b \in V$.

$\wedge_{f\in a\gamma_1 i\gamma_2 b}\{\Psi^{-1}(\tilde{\rho})(f)\} = \wedge_{f\in a\gamma_1 i\gamma_2 b}\{\tilde{\rho}(\Psi(f))\}$

$= \wedge_{\Psi(f)\in\Psi(\mathrm{a})\Psi(\gamma_1)\Psi(\mathrm{i})\Psi(\gamma_2)\Psi(\mathrm{b})}\{\tilde{\rho}(\Psi(f))\}$

$\geq \tilde{\rho}(\Psi(i))$

$\geq \Psi^{-1}(\tilde{\rho}(i))$ and

$\vee_{f\in a\gamma_1 i\gamma_2 b}\{\Psi^{-1}(\tilde{\tau})(f)\} = \vee_{f\in a\gamma_1 i\gamma_2 b}\{\tilde{\tau}(\Psi(f))\}$

$= \vee_{\Psi(f)\in\Psi(\mathrm{a})\Psi(\gamma_1)\Psi(\mathrm{i})\Psi(\gamma_2)\Psi(\mathrm{b})}\{\tilde{\tau}(\Psi(f))\}$

$= \vee_{\Psi(f)\in\Psi(\mathrm{a})\Psi(\gamma_1)\Psi(\mathrm{i})\Psi(\gamma_2)\Psi(\mathrm{b})}\{\tilde{\tau}(\Psi(f))\}$

$\leq \tilde{\tau}(\Psi(i))$

$\leq \Psi^{-1}(\tilde{\tau}(i))$ for all $a, b \in V$ and $\gamma \in \Gamma$.

Thus $\Psi^{-1}(F)$ is an interval valued Fermatean fuzzy interior Γ −hyperideal of U .

Similarly we can prove that $\Psi^{-1}(F)$ is an interval valued Fermatean fuzzy bi Γ −hyperideal of U.

7. CONCLUSIONS

In this paper we have introduced a new concept, interval valued Fermatean fuzzy set and defined some operations. We have also defined interval valued Fermatean fuzzy $\Gamma-$ subsemihypergroup, interval valued Fermatean fuzzy $\Gamma-$ hyperideal and interval valued Fermatean fuzzy interior (bi) $\Gamma-$ hyperideals. Inverse image of an interval valued Fermatean fuzzy set is also studied and we have established that inverse image of an interval valued Fermatean fuzzy set. These results would be helpful to apply some other fuzzy sets in algebraic hyperstructures.

References

[1] K. T. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 31 (1989) 343-349

[2] S. M. Anvariyeh, S. Miravakili and B. Davvaz. On Γ-hyperideals in Γ – hypersemigroups. *Carpathian Journal of Mathematics*, 26(1) (2010) 11-23

[3] Basar and M. Y. Abbasi. On ordered bi-Γ-ideals in ordered Γ-semigroups, *Journal of Discrete Mathematical Sciences and Cryptography*, 20(3) (2017) 645-652

[4] Davvaz, Fuzzy hyperideals in semihypergroups. *Italian Journal of Pure and Applied Mathematics*, 8 (2000) 67-74

[5] Hong, S. M. Jun, Y. B. and Meng, Fuzzy interior ideals in semigroups. *Indian J. Pure Appl. Math.* 26(9) (1995) 859-863

[6] N. kuroki, On Fuzzy ideals and fuzzy bi-ideals in semigroups. *Fuzzy Sets and Systems*, 5 (1981) 203-215

[7] S. V. Manemaran and R. Nagarajan, Novel Cubic Fermatean Fuzzy Soft Ideal Structures. *International Journal of Innovative Technology and Exploring Engineering* 9(2), (2019) 4915-4920. DOI: 10.35940/ijitee.D9929.129219

[8] Pibaljommee and B. Davvaz, Characterizations of (fuzzy) bi-hyperideals in ordered semihypergroups. *Journal of Intelligent and Fuzzy Systems*, 28(5) (2015) 2141-2148

[9] T. Senapati and R. R. Yager, Some new operations over Fermatean fuzzy numbers and application of Fermatean fuzzy WPM in multiple criteria decision making. *Informatica*, 30(2) (2019) 391-412

[10] Senapati, T., Yager, R.R. Fermatean fuzzy sets. *J Ambient Intell Human Comput* 11, 663–674 (2020). https://doi.org/10.1007/s12652-019-01377-0

[11] T. Senapati and R. R. Yager, Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods. *Engineering Applications of Artificial Intelligence*, 85 (2019) 112-121

[12] Somsak Lekkoksung, Nareupanat Lekkoksung. On interval valued intuitionistic fuzzy hyperideals of ordered semihypergroups. *Korean J. Math.* Vol 28, No 4 (2020) pp.753-774. DOI: https://doi.org/10.11568/kjm.2020.28.4.753

[13] S. K. Sardar and S. K. Majumder, On Fuzzy Ideals in Γ-Semigroups. *International Journal of Algebra*, 3(16) (2009) 775-784

[14] S. K. Sardar and S. K. Majumder. A study on fuzzy interior ideals in $\Gamma-$semigroups. *Computers and Mathematics with Applications*, 60(1) (2010), 90-94.

[15] V. S. Subha and S. Sharmila, A characterization of semihypergroups in terms of cubic interior hyperideals. *Bulletin of the International Mathematical Virtual Institute*, 11(2) (2021) 307-317

[16] V. S. Subha, N. Thillaigovindan and S. Sharmila, Fuzzy rough prime and semi-prime ideals in semigroups. *AIP Conference Proceedings*, 2177, 020093 (2019)

[17] V. S. Subha and S. Sharmila, A short note on some of the fuzzy rough hyper-ideals in semihyper-groups. *Malaya Journal of Matematik*, 8(2) (2020) 522-526

[18] Shahzadi G, Akram M. Hypergraphs Based on Pythagorean Fuzzy Soft Model. *Mathematical and Computational Applications* 2019, 24(4): 100. https://doi.org/10.3390/mca24040100

[19] N. Yaqoob and S. Haq, Generalized Rough Γ-Hyperideals in Γ-Semihypergroups. *Journal of Applied Mathematics*, Volume 2014, Article ID 658252, 6 pages

[20] N. Yaqoob, M. Aslam and S. Haq Rough Fuzzy Hyperideals in Ternary Semihypergroup. *Advances in Fuzzy Systems*, Volume 2012, Article ID 595687, 9 pages

[21] R. R. Yager, Pythagorean fuzzy subsets. *IFSA World Congress and NAFIPS Annual Meeting IEEE* (2013). 57-61.

[22] L. A. Zadeh, Fuzzy sets. *Information Control*, 8 (1965) 338-353

[23] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I. *Information Science*, 8 (1975) 199-249