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Best Time Series In-sample Model for Forecasting Nigeria Exchange Rate

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ABSTRACT

In this work we considered data on official Nigeria exchange rates (Naira to British Pound sterling) from January 2003 to December 2019. Four competing models ARIMA (1, 1, 1), ARIMA (2, 1, 1), ARIMA (1, 1, 0) and ARIMA (1, 1, 2) were identified for the exchange rates series. Diagnostic analysis revealed that all the competing models adequately represent the exchange rate series. However, on the basis of out-of-sample model selection and evaluation ARIMA (1, 1, 1) was selected as the optimal model with minimum information criteria for the exchange rate series. A 24 months forecast indicates that the Naira will continue to depreciate. The policy implication of our study is that the Central Bank of Nigeria (CBN), should devalue the Naira in order to not only re-establish exchange rate stability but also encourage local manufacturing and encourage foreign capital inflows.

***Keywords*:** Exchange rate, CBN, Model, Nigeria, ARIMA, Autoregressive models, Moving Average Models, Autoregressive Moving Average Model, Autoregressive Integrated Moving Average Model, Box-Jenkins Methodology

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1. INTRODUCTION

Model selection is the task of selecting a statistical model from a set of candidates' models. The objective of model selection is to discover a model that optimises a process because exact model that described a particular series is not known. Also, model selection is useful in comparing competing models for the purpose of selecting the best model that describe the series [1]. One subtle task of model selection is that numerous competing models may fit a particular series appropriately [2]. According to [3] wrong model selection results in a choice of a poor model with severe consequences. [4] is of the opinion that modelling is estimate of reality, thus model selection is to reject a model far from reality and select that which is close to reality.

In time series there two common approaches to model selection, the in-sample and the out-of-sample model selection. The in-sample model selection criteria include AIC [5], BIC [6] and Hanna and Quinn information criteria [7].The out-of-sample model selection procedure is accomplished by withholding some of the sample data from the model identification and estimation process, then use the model to make predictions for the hold-out data in order to see how accurate they are and to determine whether the statistics of their errors are similar to those that the model made within the sample of data that was fitted. The data which are not held out are used to estimate the parameters of the model. The model is then tested on data in the validation period, and forecasts are generated beyond the end of the estimation and validation periods [3, 8, 9].

Forecasting exchange rate is crucial as it has significant impact on the macroeconomic Fundamentals such as oil price, interest rate, wage, unemployment and the level of economic growth [10].

Exchange rate is the rate at which one currency exchange for another [11]. It is a variable instrument in economic management and therefore an important macro-economic indicator used in assessing the overall performance of the economy, since movement in the exchange rate has been discovered to have rippling effect on other economic variables such as interest rate, rate of inflation, etc. It is therefore important for every country as long as it opens its doors to international trade to establish a method or procedure by which its exchange rate would be determined. Exchange rate determination varies from country to country and from one period to another. For example, in a market friendly economic environment, the exchange rate is expected to respond to forces of demand and supply of foreign exchange. The reverse is the case at the other end of the spectrum when the exchange rate is administratively determined depending on the state of the economy. Other methods of exchange determination include the dual and multiple exchange regimes. In recent past, there have been frequent shift in exchange rate policies of many economies of the world as dictated by changing economic and financial conditions. Yet in many countries, efforts to achieve viable exchange rate policies are still on as their domestic and international economic environment are still unstable.

In Nigeria, exchange rate has change within the frame from regulated to deregulated regions. [12], agreed that the exchange rate of the Naira was relatively stable between 1973 and 1979 during the oil boom era and when agricultural products accounted for more than 70% of the nation's gross domestic product (GDP).

In 1986, Nigeria adopted a fixed exchange rate regime supported by exchange control regulations that engendered significant distortions in the economy prior to the introduction of the Structural Adjustment Programme (SAP). The country depends heavily on imports from various countries as most industries in Nigeria import their raw materials and massive importation of finished goods from foreign countries. This has cause adverse effect on domestic production, balance of payments position and the nation external reserves level. In addition, this has made foreign exchange rate market in fixed exchange rate period to be characterized by high demand for exchange that cannot be met with the supply of foreign exchange through the central bank of Nigeria (CBN). The inadequate supply of foreign exchange by the central bank of Nigeria promoted parallel market for foreign exchange and created uncertainty in the foreign exchange rates.

Timely forecasting of the exchange rates is able to give important information to decision makers as well as partakers in the area of the internal finance, and policy making. For the giant multinational business units, an accurate forecasting of the exchange rate is crucial since it improves their overall profitability. The importance of forecasting the exchange rates in practical aspect is that an accurate forecast can render valuable information to the investors, firms and central banks for use in allocation of assets, in hedging risk and in policy formulation.

The significance of exchange rates forecasting stems from the reality that the findings of a given financial decision made today is conditional on the exchange rate which will be prevailed in the upcoming period. For this reason forecasting exchange rate is essential for various international financial transactions, namely speculation, hedging as well as capital budgeting [13].

Several studies has been done on forecasting exchange rate, for instance (14) used a Box-Jenkis ARIMA approach to model and forecast Naira/Dollar exchange rate. Using the in-sample information criteria, ARIMA (2, 1, 1) was selected as the optimal model for forecasting Naira to Dollar exchange rate.

In his study [15], using Naira-Euro compare a seasonal autoregressive integrated moving average (0, 1, 1) x $(0, 1, 1)_7$ with ARIMA (1, 1, 1) model with a view of establishing SARIMA supremacy. His result reviewed that seasonality of order 7 was evident from the analysis of the first difference of the original series. The parameters of the moving average of the SARIMA were highly significant. Up to 51% of the variance in the dataset was explained by the model. On the other hand only 8% of the variability in the dataset was accounted by ARIMA (1, 1, 1) model. Therefore, he concluded that SARIMA model more adequately represents the dataset. [16] Used a technical approach to forecast Nigerian Naira- Dollar using seasonal ARIMA model for the period of 2004 to 2011. His result revealed that the series (exchange rate) has a negative trend between 2004 and 2007 and was stable in 2008. His good work elucidate on the seasonal difference once produced a series SDNER with slightly trend but still within desirable stationarity

In another study [17], measured the forecast of ARIMA and ARFIMA model on the application to USD/UK pounds foreign exchange. The result of their work revealed that ARFIMA model was found to be better than ARIMA model as indicated by the measurement criteria. Their persistent result revealed that ARFIMA model is more reliable and closely reflects the current economy reality in the two countries which was indicated by their forecasting tool.

In yet another study [18] in their work titled “A Suitable Model for the Forecast of Exchange Rate in Nigeria (Nigeria Naira versus US Dollar)”, used the Box-Jenkins ARIMA and ARMA method for forecasting monthly data collected from January 2000 to December 2012. The result of their analysis revealed that ARIMA (1, 1, 2) and ARMA (1, 1) are appropriate or optimal models based on the Akaike’s information criterion (AIC), Schwazt information criterion (SIC) and Hannah Quinn (HQC) information criterion.

[19] Applied ARIMA model to exchange rate (Naira- Dollar) within the period of 1982 - 2011, through Box- Jenkins methodology; an AR (1), order one was generated model and was preferred as proven through the diagnostic test of naira-Dollar exchange. Base on its potentials for better prediction and conceptual requirement

[20] Examined volatility of two exchange rates (US dollar, and Euro) vis-à-vis the Naira using GARCH models and monthly data spanned the period January 1983 and July 2011. The study compares performance of variance of the GARCH models and without the incorporation of exogenous break in-model's estimation. The finding of their study revealed that performance of the models improved by incorporating volatility breaks in the estimated models. Furthermore, all the asymmetric models fitted in the study reject existence of leverage effect in the volatility process. As an alternative, a trivariate MGARCH model could have been used to simultaneously capture the second-order movement as well as the inter-link inherent in the system of exchange rate.

[21] described an empirical study of modelling and forecasting time series data of exchange rate of Nigerian Naira to the U.S Dollar. The Box-Jenkins ARIMA methodology was used for forecasting the monthly data collected from January 1990 to December 2010. The diagnostic checking shows that ARIMA (0, 1, 1) was appropriate. A four-year forecast was made from January 2011 to December 2014, showing the Naira in steady rate against the USD In an independent study[21] applied the Box-Jenkins methodology to investigate the strength of Nigeria Naira with respect to the United States Dollar and fit an appropriate model to the date from 1972 to 2014. Their result revealed that the series was slightly stationary after first difference and sufficiently stationary after second difference, meaning the series was 1(1) or 1(2). Based on in-sample selection criteria ARIMA (0, 2, 1) was found to be appropriate model for the series. A six year forecast term was made and the values forecasted were all within the confidence limit. A precise forecast is a key component of successful economic decision making, but there is no standard technique for selecting a model that methodically outclasses the others at all horizons and with any data set. A common way to progress is to choose the best description between the candidates models based on a fitting criterion and then to generate a forecast. Comprehensively, related literatures have been reviewed in order to reveal and identify gaps in knowledge related to modelling Nigeria exchange rate. Hence, in this study, we attempt model Naira to pound exchange rate since most studies in Nigeria channelled their energy in modelling Naira to dollar Exchange rate by comparing and appraising the in-sample performance of a large number of autoregressive integrated moving average models (ARIMA) chosen according to three commonly used information criteria for model-building; AIC, BIC, and HQ on a Nigeria exchange rates (Naira to pound sterling) from 2003 to 2019. The rest of the paper is organised as follows, section two takes care of materials and methods, section three addressed results and discussion while conclusion is handled by section four.

2. MATERIALS AND METHODS

2. 1. Autoregressive (AR) Model

Autoregressive models are based on the idea that current values of the series x_t can be explained as a function of past values, $X_{t-1}, X_{t-2}, \dots, X_{t-p}$, where p determines the number of steps into the past needed to forecast the current values. An autoregressive model of order p abbreviated as $AR(p)$ can be written as:

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \cdots + \varphi_p X_{t-p} + e_t \quad (1)$$

where, x_t is the stationary series, $\varphi_{1,}\ \varphi_{2,} \dots \varphi_p$ are parameters of AR ($\varphi_p \neq 0$) unless if otherwise stated, we assumed that e_t is Gaussian white noise with mean zero and variance δ^2. The highest order of p is referred to as the order of the model.

The model in lag operator takes the following form:

$$\left(1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p\right) X_t = e_t \quad (2)$$

where, the lag (backshift) operator is defined as $\beta^p X_t = X_{t-p}, p = 0, 1, 2 \dots$

More concisely we can express the model as $\varphi(\beta) X_t = e_t$. The autoregressive operator $\varphi(B)$ is defined to be $\left(1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p\right)$

The values of φ which makes the process stationary are such that the roots of $\varphi(B) = 0$ lie outside the unit circle in the complex plane (Chatfield, 1992). If the roots of $\varphi(B)$ are larger than one in absolute values, then the process satisfying the autoregressive equation which can be represented as:

$$X_t = \sum_{j=1}^{\infty} \varphi_j X_{t-1} \quad (3)$$

The coefficient of $\varphi_j$converges to zero such that $\sum_{j=1}^{\infty} \left|\varphi_j\right| < \infty$

If some roots are "exactly" one in modulus, no stationary solution exists. The plot of the ACF of a stationary $AR(p)$ model shows a mixture of damping sine and cosine pattern and exponential decay depending on the nature of its characteristic roots.

Another characteristic feature of $AR(p)$ model is the partial autocorrelation function defined as

$$PACF(j) = Corr(X_t, X_{t-j} | X_{t-1}, X_{t-2}, \dots, X_{t-j+1}) \quad (4)$$

becomes exactly zero for values larger than p (Tsay, 2005).

2. 2. Moving Average (MA) Models

An alternative to autoregressive representation is the moving average. A series x_t is said to follow a moving average process of order q or simply $MA(q)$ process if

$$X_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_p e_{t-p} \quad (5)$$

where, $\theta_1, \theta_2, \dots, \theta_p$ are the MA parameters to be estimated $e_t, e_{t-1}, \dots, e_{t-p}$ are error terms.

The value of q is called the order of MA model (Hamson and Robert, 2005). In order to preserve a unique representation, usually the requirement is imposed that all roots of $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q = 0$ are greater than one in absolute value. If all the roots of $\theta(B) = 0$ lie outside the unit circle, the MA process has an autoregressive representation of generally infinite order.

$$X_t = \sum_{j=1}^{\infty} \varphi x_{t-1} \quad (6)$$

with $\sum_{j=1}^{\infty}|\varphi_j| < \infty$

MA Process as an infinite autoregressive representation are said to be invertible. Moving average model is conceptually a linear regression of the current values against previous (unobserved) white noise error terms or random shocks. The random shocks at each point are assumed to come from the same distribution typically a normal distribution, with location at zero and constant scale. The distinction in the model is that these random shocks are propagated to future values of the time series. Fitting the MA estimate is more complicated than with autoregressive model because the error terms are not observable.

This means that iterative non-linear fitting procedures need to be used in place of linear least squares. MA Models also have less obvious interpretation than AR models. The characteristic feature of $MA(q)$ is that their ACF_{pj} becomes statistically insignificant after $j = q$. The property of the ACF should be reflected in the correlogram, which should cut after q. The $PACF$ converges to zero geometrically.

Sometimes the autocorrelation function (ACF) and the partial autocorrelation function (PACF) will suggest that a MA model would be a better choice and sometimes both AR and MA terms should be used in the same model (Mill, 1990).

2. 3. Autoregressive Moving Average (ARMA) Model

We now proceed with the general development of autoregressive moving average (ARMA) models for stationary time series. In most cases, it is best to develop a mixed autoregressive moving average model when building a stochastic time series. The order of an ARMA model is expressed in terms of both p and q. the model parameters relate to what happen in period t to both the past random errors that occur in the past periods. The ARMA model is defined as follows:

$$X_t = \varphi_1 X_{t-1} + \cdots + \varphi_p X_{t-p} + e_t - \theta e_{t-1} - \cdots - \theta e_{t-p} \tag{7}$$

where, the φ's are autoregressive parameters to be estimates and θ's the moving average parameters to be estimated. The e's are series of unknown errors (or residuals) which are assumed to follow the normal probability distribution. Box-Jenkins used a backshift operator to make the model easier. The backshift operator B has the effect of changing time period t to time period$t - 1$.

$BX_t = X_{t-1}, B^2 X_t = X_{t-2}$ And so on. Using the backshift notation, the above model may be rewritten as:

$$\left(1 - \varphi_1 B - \cdots - \varphi_p B^p\right) X_t = \left(1 - \theta_1 B - \cdots - \theta_q B^q\right) e_t \tag{8}$$

Equation (3.8) may be abbreviated further by writing:

$$\varphi_p(B) X_t = \theta_q(B) e_t \tag{9}$$

These formulae show that the operators $\varphi_B(B)$ and $\theta_q(B)$ are polynomials in B of orders p and q respectively. The ARMA model is stable, i.e. it has a stationary solution if all roots of $\varphi(B) = 0$ are larger than one in absolute value.

2. 4. Autoregressive Integrated Moving Average (ARIMA) Model

The general model introduced by Box and Jenkins (1970), includes autoregressive as well as the moving average parameters, and explicitly includes differencing in the formation of the model. Specifically, the three types of parameters in the model are: autoregressive parameter (p), the number of differencing passed (d), and the moving average parameter (q). In the notation introduced by Box and Jenkins, models are summarized as $ARIMA\ (p, d, q)$; so for example, a model described as ARIMA (1,1,1) means that it contains 1(one) autoregressive parameter (p) and 1 (one) moving average parameter (q) which were computed for the series after it was differenced once.

The last series for ARIMA needs to be stationary, that is, it should have a constant mean, variance and autocorrelation through the time. Therefore, usually the series need to be differenced until stationary achieved (i.e. this also requires log transformation of the data to stabilize the variance). The number of times the series is needed to be differenced to achieve is reflected in the d parameter. In order to determine the necessary level of differencing, one should examine the plot of the data and autocorrelogram. Significant changes in level (strong upward or downward changes) usually require first order non seasonal $(lag = 1)$ differencing; strong changes of slope usually require second order non-seasonal differencing. Seasonal patterns require respective seasonal differencing. If the estimated autocorrelation coefficients decline slowly at longer lags, first order differencing is usually needed. However, one should keep in mind that sometimes series may require little or no differencing and that over differencing series produces less stable coefficient estimates.

At this state which is usually called identification phase, we also need to decide how many autoregressive parameters (p) and moving average parameters (q) necessary to yield an effective but still parsimonious model of the process. Parsimonious means that it has the fewest parameters and greatest number of degrees of freedom among all models that fits the data. In practice, the numbers of the p or q parameters are rarely needed to be greater than 2 (two) (Box and Jenkins, 1976).

A process (X_t) is said to be an autoregressive integrated moving average process denoted by $ARIMA(p, d, q)$. It can be written as:

$$\varphi(B)\nabla^{\mathrm{d}} X_t = \theta(B) e_t \tag{10}$$

where, $\nabla^{\mathrm{d}} = (1 - B)^d$ with $\nabla^{\mathrm{d}} X_t$ and d^{th} consecutive differencing (Vandale, 1983).

If $E(\nabla^d X_t) = \mu$, we write the model as:

$$\mathrm{\varphi(B)\nabla^d X_t = \alpha + \theta(B) e_t} \tag{11}$$

where, α is a parameter related to the mean of the process$\{X_t\}$, by $\alpha = \mu(\varphi_1 - \varphi_2 - \ \ldots \varphi_p)$ and this process is called a white noise process, that is, a sequence of uncorrelated random variables from a fixed distribution (often Gaussian) with constant mean μ, usually assumed to be "zero" and constant variance.

If $d = 0$, it is called $ARMA(p, q)$ model, while when $d = 0$ and $q = 0$, it is referred to as autoregressive of order p model and denoted by $AR(p)$. When $p = 0$ and $d = 0$, it is called moving average of order q model and denoted by $MA(q)$.

2. 5. Autocorrelation and Partial Autocorrelation Functions (ACF and PACF)

To determine a proper model for a given time series data, it is necessary to carry out the ACF and PACF analysis. These statistical measures reflect how the observations in a time series are related to each other. For modeling and forecasting purpose it is often useful to plot the ACF and PACF against consecutive time lags (Brockwell, Davis, 2003). These plots help in determining the order of AR and MA terms. Below we give their mathematical definitions:
For a time series$\{x(t), t = 0,1,2 \dots\}$, the Auto covariance at lag *k* is defined as:

$$\gamma_k = cov(X_t, X_{t+k}) = E[(X_t - \mu)(X_{t+k} - \mu)] \quad (12)$$

The Autocorrelation Coefficient at lag *k* is defined as: $\rho_k = \frac{\gamma_k}{\gamma_0}$

Here μ is the mean of the time series, i.e. $\mu = E(X_t)$. The auto covariance at lag zero i.e. γ_0 is the variance of the time series. From the definition it is clear that the autocorrelation coefficient ρ_k is dimensionless and so is independent of the scale of measurement. Also, clearly$-1 \leq \rho_k \leq 1$. Statisticians Box and Jenkins (1970) termed γ_k as the theoretical Auto covariance Function (ACVF) and ρ_k as the theoretical Autocorrelation Function (ACF).

Another measure, known as the Partial Autucorrelation Function (PACF) is used to measure the correlation between an observation *k* period ago and the current observation, after controlling for observations at intermediate lags (i.e. at lags < k). At lag 1, PACF(1) is same as ACF(1). Normally, the stochastic process governing a time series is unknown and so it is not possible to determine the actual or theoretical ACF and PACF values. Rather these values are to be estimated from the exchange rates data, i.e. the known time series at hand. The estimated ACF and PACF values from the exchange rate data are respectively termed as sample ACF and PACF. The most appropriate sample estimate for the ACVF at lag *k* is

$$C_k = \frac{1}{n}\sum_{t-1}^{n-k}(x_t - \mu) - (X_{t+k} - \mu) \quad (13)$$

Then the estimate for the sample ACF at lag *k* is given by $\mathrm{r_k} = \frac{\mathrm{c_k}}{\mathrm{c_0}}$

Here $\{X(t), t = 0,1,2 \dots\}$ is the exchange rates series of size *n* with mean μ.

As explained by Box and Jenkins, the sample ACF plot is useful in determining the type of model to fit to a time series of length *N*. Since ACF is symmetrical about lag zero, it is only required to plot the sample ACF for positive lags, from lag one onwards to a maximum lag of about N/4. The sample PACF plot helps in identifying the maximum order of AR process.

2. 6. Box-Jenkins Methodology

There are three steps in Box-Jenkins methodology. These are:

(i) Model identification
(ii) Model estimations
(iii) Model diagnosis or checking

2. 6. 1. Model identification

Model identification involves examining the given data by various methods, to determine the values of p, q and d. The values are determined by using autocorrelation function (ACF) and partial autocorrelation function (PACF). This can be done by observing the graph of the data or autocorrelation functions. For any ARIMA process, the theoretical (PACF) has non-zero partial autocorrelation at 1,2…p lags and partial autocorrelation at all lags. While the theoretical (ACF) has non-zero autocorrelation at all lags. The non-zero lags of sample PACF and ACF are tentatively accepted as the p and q parameters [18]. For a non- stationary series the data is differenced to make stationary. The number of times the series is differenced determines the order of d. Therefore for a stationary data d = 0 and ARIMA (p, d, q) can be written as ARIMA (p, q). It is important to highlight the fact that the procedure of choosing the AR and MA components is biased towards the use of personal judgment because there are no clear – cut rules on how to decide on the appropriate AR and MA components. Therefore, experience plays a pivotal role in this regard.

2. 6. 1. 1. Augmented dickey-fuller test (ADF)

The test was first introduced by Dickey and Fuller (1979) to test the presence of unit root. The regression model for the test is given as:

$$\Delta X_t = \gamma X_{t-1} + BY_{t-1} + \sigma_1 \Delta X_{t-1} + \sigma_2 \Delta X_{t-2} + \cdots + \sigma_p \Delta X_{t-p} + e_t \tag{14}$$

The hypothesis testing
Ho: $\gamma = 0$ (The series contain unit root)
Ho: $\gamma < 0$ (the series is stationary)

Test statistic$t_y = \frac{\gamma}{se(\gamma)}$, (15)
where;

ΔX_t = the difference series
X_{t-1} = the immediate past observation
$\sigma_1 \ldots \sigma_p$= the coefficient of the lagged difference term up to p.
$Y_t =$ the optimal exogenous regress which may be constant or constant trend.
γ and β are parameters to be estimated. Reject Ho if t_y is less than asymptotic critical values.

Decision rule: Reject Ho if t_y is less than asymptotic critical value

2. 6. 2. Model estimation

After an best model has been identified, the model estimation methods make it possible to estimate simultaneously all the parameters of the process, the order of integration coefficient and parameters of an ARMA structure. There many methods of estimating parameters of linear time series models but for the purpose of this study we shall consider the maximum likelihood method.

2. 6. 3. Model diagnosis or checking

The last step in Box-Jenkins methodology is model verification or model diagnosis. The conformity of white noise residual of the model fit will be judged by plotting the ACF and

PACF of the residual to see whether it does not have any pattern or we perform Ljung-Box Test on the residual. The test hypothesis:

Ho: There is no serial correlation
H1: There is serial correlation

The test statistics of the Ljung-Box

$$LB = n(n+2)\sum_{k}^{m} \frac{\rho_k^2}{n-k} \quad \text{distributed to } x_{m-e}^2 \tag{16}$$

where; n is the sample size, $m = lag\ length$ and p is the sample autocorrelation coefficient. The decision: if LB is less than critical value of x^2, then we do not reject the null hypothesis. This means that a small value of Ljung-Box statistic will be in support of no serial correlation or i.e. the errors are normally distributed. This is concern about model accuracy.

The Box-Jenkins forecast method is schematically shown in Fig. 1:

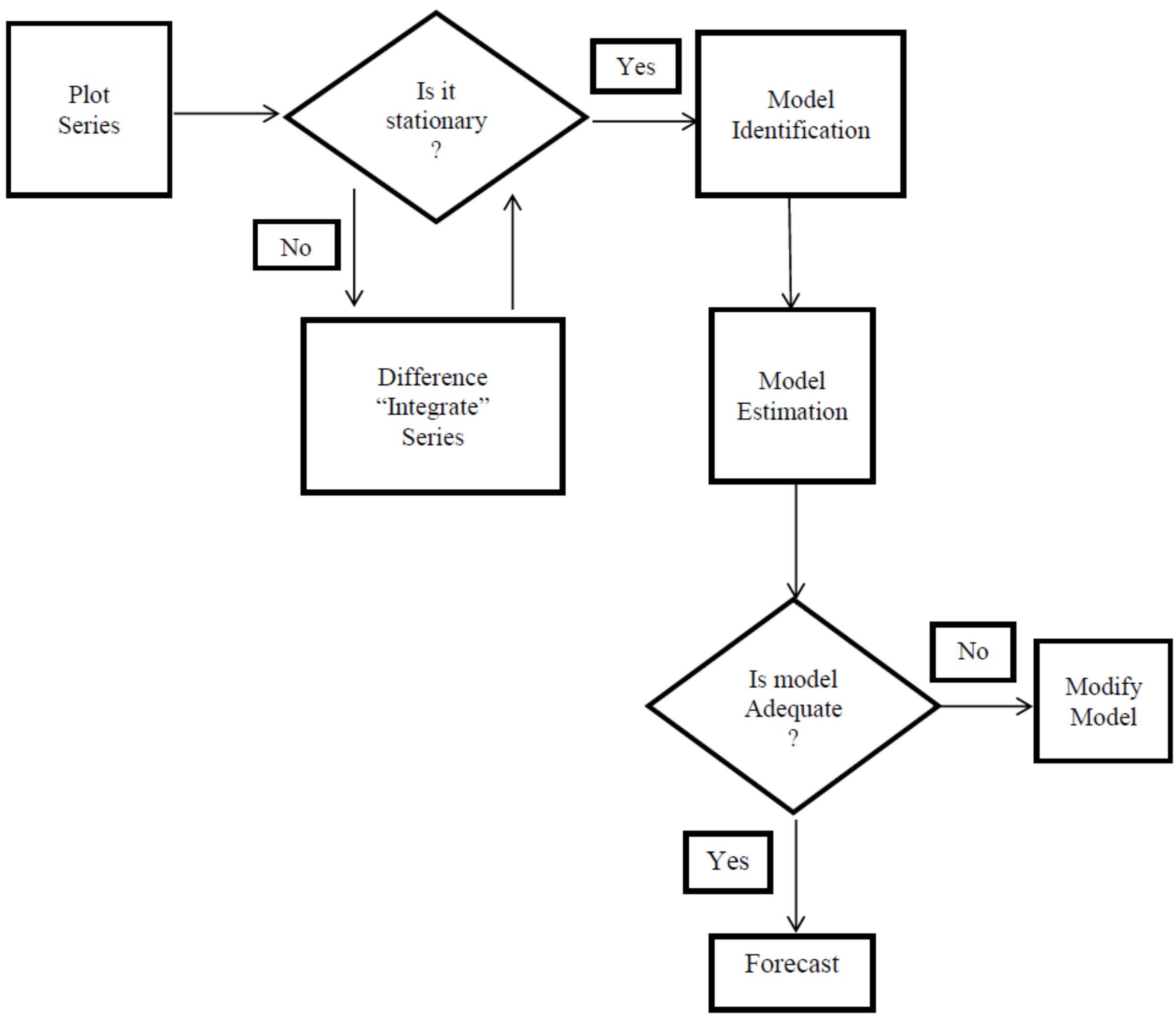

Figure 1. The Box-Jenkins methodology for optimal model selection

2. 7. In-sample Information Criteria

Given several competing models, we agree upon a final model which is one standard method to use a model selection criteria Alkaike's Information criteria (AIC), Schwartz Information criteria and the Hannan Quinn criteria (HQC) which endeavours to choose a model that sufficiently describes the data but in the most parsimonious way as possible or minimizing the number of parameters . For example AR (4) model hardly outclass AR (3) model by a certain predefined quantity or criteria, than AR (3), the most parsimonious model is selected. In general, the model chosen is the one that minimizes the respective criteria scores

There are several information criteria available to determine the order, p, of an AR process and the order, q, of MA (q) process; all of them are likelihood based. For this work, we shall consider Akaike information criterion (AIC), Bayesian information criterion (BIC) and Hannan Quinn criteria (HQ).

The idea of AIC [5] is to select the model that minimises the negative likelihood penalised by the number of parameters. The AIC is specify in equation (17) below

$$AIC = -2logP(L) + 2P, \tag{17}$$

where L refers to the likelihood under the fitted model and p is the number of parameters in the model. Specifically, AIC is aimed at finding the best approximating model to the unknown true data generating process and its applications [5]

Unlike Akaike Information Criteria, BIC is derived within a Bayesian framework as an estimate of the Bayes factor for two competing models [6]. BIC is defined as

$$BIC = -2logP(L) + Plog(n), \tag{18}$$

Superficially, BIC differs from AIC only in the second term which now depends on sample size n. Models that minimize the Bayesian Information Criteria are selected. From a Bayesian perspective, BIC is designed to find the most probable model given the data.

The Hanna – Quinn criteria originally proposed by Hanna and Quinn was derived from the law of iteration logarithm, it is another dimension consistent model and only differs from AIC and BIC with respect to the penalty term. The HQ criteria to be minimized is

$$HQ(n) = \log(\sigma^2) + \frac{2nlog(T)}{T} \tag{19}$$

where; n is the dimensionality of the model, σ^2 is the maximum likelihood estimate of the white noise variance and T is the sample

3. RESULTS AND DISCUSSION

In this section, we shall use the monthly official exchange rate in Nigeria to identify and estimate ARIMA model that adequately represents the series and use some diagnostic tests to evaluate the model. The data set is from Nigeria official exchange rate for the Naira to Pound sterling from January 2003 to December 2019. Gretl and E-views are statistical software's used for data analysis.

3. 1. Stationarity Test

The time plot of Naira to Dollar exchange rate from Figure 2 indicates that the Naira to pound series is not stationary at level form simply because the mean of the series is varying over time and there is no stability in the variance over the time considered. Evidence for Figure 3 suggest that the series became stationary at the first difference because the series fluctuate around a common mean

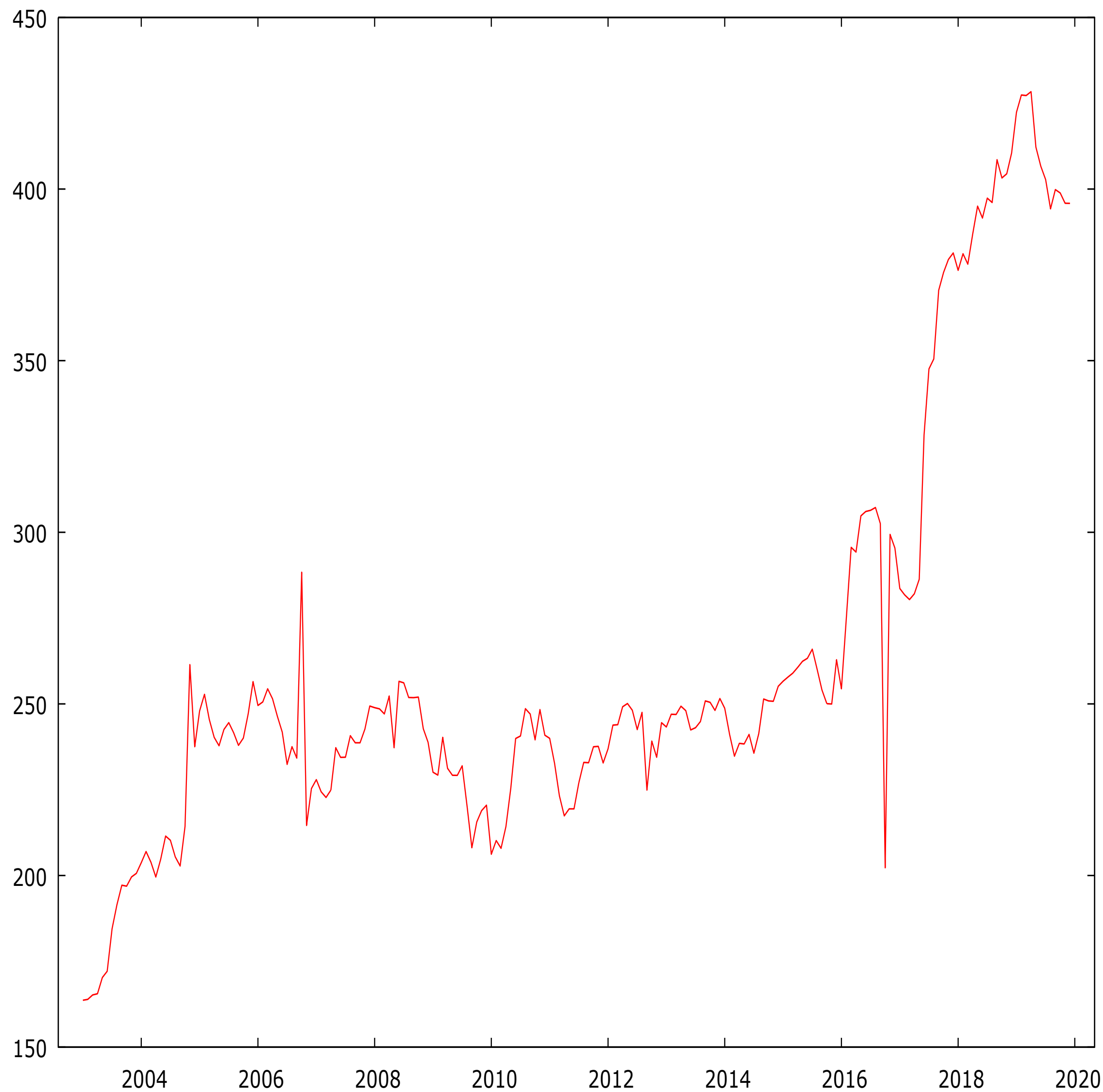

Figure 2. Time plot of Naira to Pond Sterling Exchange Rate

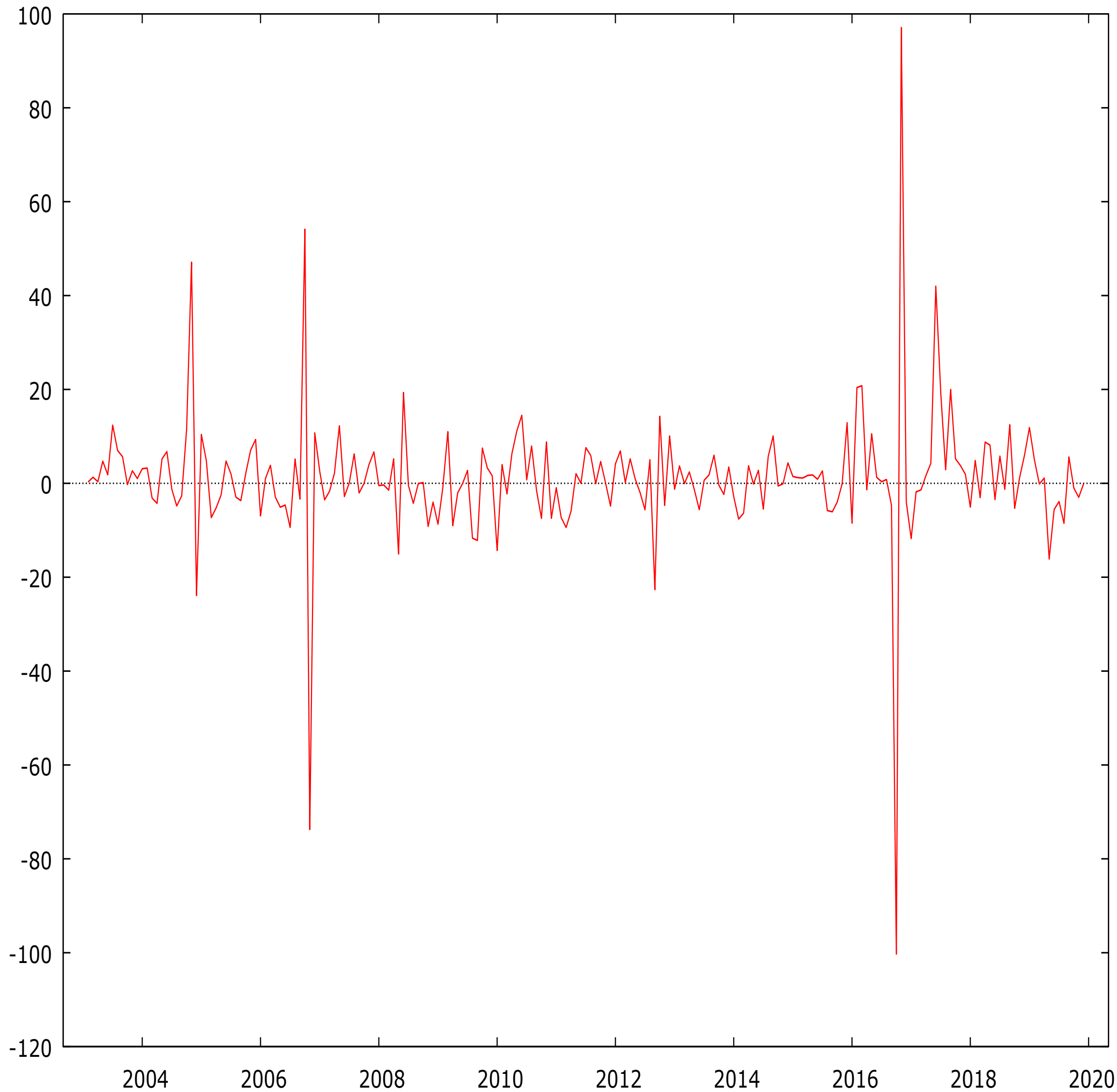

Figure 3. Time plot of Naira to Pond Sterling Exchange Rate at First Difference.

Table 1. ADF Test Result at Level.

Intercept but no trend				Intercept and trend		
variables	Test statistics	Critical value	p-value	Test statistics	Critical value	p- value
POUNDS	-0.690578	-2.875680	0.8455	-0.627799	-3.432226	07787

Table 2. ADF Test Result at First Difference.

Intercept but no trend				Intercept and trend		
variables	Test statistics	Critical value	p-value	Test statistics	Critical value	p- value
DPOUNDS	-20.49243	-2875680	0.0000	-20.45471	-3.432226	0.0000

To further test for stationarity of the data, we applied ADF test. Evidence from the analysis from Table 1 revealed that at level form, the series was not stationary because at each assumption; intercept, intercept and trend, each ADF test statistic were less than the corresponding value of level of significance (5%). But at first difference the series became stationary given that the ADF statistic at various assumptions was greater than the corresponding level of significance, Table 2.

3. 2. In-sample Model Selection

It should be noted that, even if an ARIMA model has been correctly identified and give good result, this does not mean that it is the only model that can be considered, various model should be identified and tested. With regard to Naira to pounds exchange rate four tentative ARIMA models were identified namely, ARIMA (1, 1, 1), ARIMA (2, 1, 1), ARIMA (1, 1, 0) and ARIMA (1, 1, 2). The four competing models were estimated using the maximum likelihood. All the parameters of the models were found to significant at 5% level. Based on in-sample information criteria (AIC, BIC and H&G), ARIMA (1, 1, 1.) was found to be the optimal model with minimum information criteria (refer to Table 3).

Table 3. ARIMA Models for Naira to Pounds Exchange Rate.

Model	Parameter	Estimate	s.e	Z-ratio	p-value	In-sample Criteria		
						AIC	BIC	H&Q
ARIMA(1,1,1)	φ_1 θ_1	−0.3505 −0.6000	0.0656 0.0163	−5.339 −3.673	<0.0001 <0.0001	1631.9	163 7.1	1639.3
ARIMA(2,1,2)	φ_1 φ_2 θ_1	−0.3922 −0.1156 −1.0000	0.0698 0.0697 0.0186	−5.616 −1.660 −53.80	<0.0001 <0.0069 <0.0001	1633.1	1649.7	1639.9
ARIMA(1,1,0)	φ_1	−0.6357	0.0538	−11.81	<0.0001	1749.1	1759.1	1753.3
ARIMA(1,1,2)	φ_1 θ_1 θ_2	−0.1318 −1.2571 0.2571	0.1607 0.1543 0.1533	−0.820 −8.144 1.677	0.0124 <0.0001 0.0236	1633.6	1650.2	1640.3

3. 3. Model Diagnostic Checking or Evaluation

We use diagnostic test of the model residuals to check if the model has adequately fitted the series. First we plot the ACF and the PACF of the standardized residuals to see if there exists serial correlation. Next we perform the Ljung- Box test for the model to check if there exists serial correlation in the residual. The ACF and PACF plots of the residuals from ARIMA (1, 1,1), revealed that all correlations are within the threshold limits indicating that the residuals are not correlated. This can be seen in Figure 4. A Ljung-Box test for ARIMA (1,1,1) model returns p-values (0.8163) greater than the critical value at 5%, this suggests that the residuals are white noise and that the models are adequate. This can be seen in Table 4. From Table 5 all the coefficients of ARIMA (1, 1, 1) are within 95% confidence interval which also confirmed the adequacy of the model

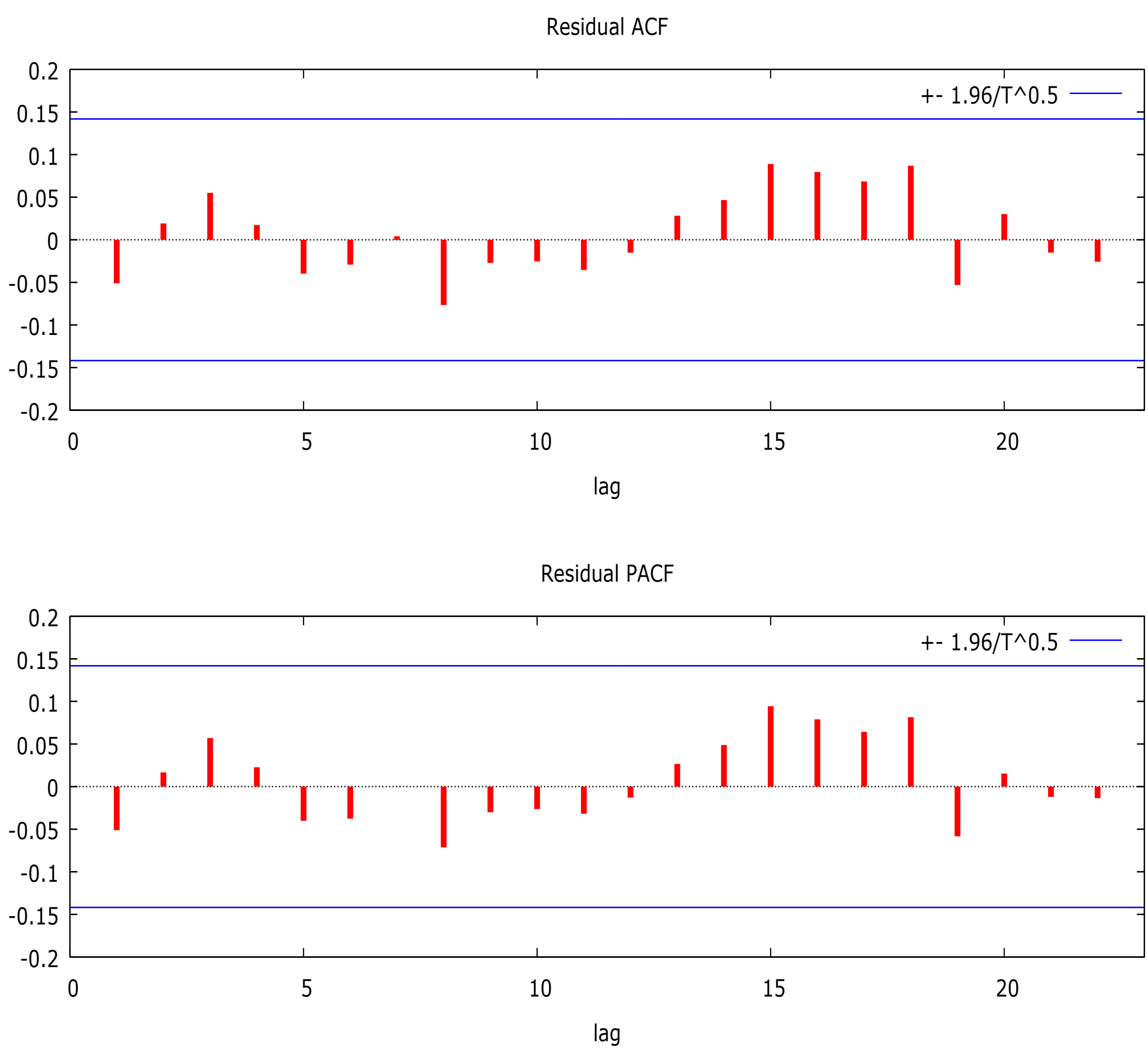

Figure 4. ACF and PACF of Residuals of ARIMA (1, 1, 1) of Naira to Pound Exchange

Table 4. Ljung-Box Model Diagnosis.

Model	Test statistic	p- value
ARIMA (1, 1, 1,)	5.98943	0.8162

Table 5. Confidence Interval for ARIMA (1, 1, 1).

Variable	Coefficient	95 Confidence interval
φ_1	-0.350454	(-0.479111, -0.221797)
θ_1	−0.6000	(-1.03202, -0.967982)

3. 4. Forecas

After diagnosing and certifying that ARIMA (1, 1, 1) model is adequate, next we make forecast. A forecast for Naira to pound exchange rate for the period of 24 months revealed that the Naira is likely to continue depreciating [see Table 6].

Table 6. Forecast of Naira to Pounds Exchange Rate for 24 months.

Period	**Forecast**	**s.e**	**95% interval**
2020:01	397.935	13.2301	372.005 - 423.866
2020:02	403.966	15.4799	368.626 - 429.306
2020:03	407.130	17.7532	365.334 - 434.926
2020:04	407.277	19.7306	362.606 - 439.949
2020:05	408.427	21.5313	360.226 - 444.628
2020:06	409.576	23.1921	358.121 - 449.032
2020:07	411.726	24.7418	356.233 - 453.219
2020:08	412.875	26.1999	354.524 - 457.226
2020:09	413.024	27.5810	352.967 - 461.082
2020:10	415.174	28.8963	351.538 - 464.809

2020:11	420.323	30.1542	350.222 - 468.424
2020:12	421.473	31.3617	349.005 - 471.940
2021:01	423.622	32.5243	347.875 - 475.369
2021:02	424.771	33.6469	346.825 - 478.718
2021:03	425.921	34.7332	345.845 - 481.996
2021:04	428.070	35.7865	346.074 - 488.365
2021:05	429.219	36.8097	344.074 - 478.365
2021:06	431.369	37.8052	343.272 - 491.466
2021:07	433.518	38.7751	342.520 - 494.516
2021:08	435.668	39.7214	341.815 - 497.520
2021:09	436.817	40.6457	341.153 - 500.481
2021:10	438.966	41.5494	340.531 - 503.402
2021:11	439.116	42.4338	339.947 - 506.285
2021:12	441.265	43.3002	339.398 - 509.132

4. CONCLUSION

Box- Jenkins methodology was used to estimate ARIMA model that best fits the monthly official exchange rate in Nigeria. The dataset was monthly exchange rate for the Naira to pound sterling from January 2003 to December 2019.

The modelling was in three stages, the first was identification stage where the dataset (Naira to British Pound sterling exchange rate) was Non-stationary at level form based on observed pattern of the time plot and the result of ADF test. It was found that the dataset became stationary after the first difference. Four competing models, ARIMA (1, 1,1), ARIMA (2, 1, 1) ARIMA (1, 1, 0) and ARIMA (1, 1, 2) were identified and fitted tentatively for Naira to British Pound sterling exchange rate.

The second stage was model estimation where the parameters of the identified models for the dataset were estimated using maximum likelihood. All the tentative models were found significant. However, based on in-sample information criteria ARIMA (1, 1, 1,) was found to be the optimal model with minimum information criteria (AIC, BIC and H&N). Finally the last stage was model diagnosis, here errors derived from all the identified model (ARIMA (1, 1, 1)) were normally distributed, random (no time dependence) and no presence of serial correlation.

A twenty four months forecast indicate that the Naira is likely to depreciate during the forecasted period. While Nigeria is an import – dependent country, we still suggest devaluation of the Naira in order to restore exchange rate stability in Nigeria. While this policy position would make the importation of commodities into Nigeria more expensive and difficult, the good part of it is that it would encourage local manufacturing and the much needed inflow of foreign capital.

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