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Multiplicative inversions involving real zero and neverending ascending infinity in the multispatial framework of paired dual reciprocal spaces

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ABSTRACT

Inverses of complex numbers and of analytic functions are composites of mixed type for they are multiplicative inverses (i.e. reciprocals) of the modulus/magnitude combined with additive reverses of the argument/angle. Hence, the mixed inverses in the complex domain $\mathbb{C}$ are not really reciprocals and therefore their lack of truly multiplicative reciprocity was a contributing reason that spurred the – unnecessary though still ongoing – prohibition of division by zero which is the natural reciprocal of the neverending ascending real infinity. Truly reciprocal algebraic operations are presented (via multiplicative algebraic inversions) by few examples within the new multispatial framework in terms of their abstract algebraic representations subscripted by the native algebraic bases of the mutually paired dual reciprocal (even though algebraic) spaces in which the inversive operations are performed.

***Keywords*:** Multiplicative inversions, multispatial algebraic structures, dual reciprocal spaces

1. INTRODUCTION

Traditional mathematics was developed under the formerly unspoken assumption that the abstract mathematical universe resembled single space commonly identified with set. I prefer to call that unfounded assumption that is thoughtlessly accepted at its face value, the single

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space reality (SSR) paradigm because it was apparently considered as being so selfevident that it was never mentioned, and thus its validity was never questioned insofar as I can tell.

Nevertheless, many differential- and even some algebraic operations hint at the possibility that the actual mathematical reality should resemble an abstract multispatial structure if logical consistency of some previously questionable operational procedures is to be rectified. I prefer to call the acceptance of that hint the multispatial reality (MSR) paradigm.

Although set-theoretical spaces can be identified with subsets and thus understood as mere selections from the universal set in the SSR setting, in the MSR framework spaces are viewed as either algebraic or geometric or quasigeometric spatial or even multispatial structures. Therefore, when it comes to dealing with operational procedures, I prefer to talk in this paper about algebraic structures in order to avoid the quite possible complications that pop up when realistic spatial structures emerge. It is because for most algebraic operations the geometric or quasigeometric complications add unnecessary impediments at first, when only operations and not the – corresponding to them – structures are investigated. However, I have always thought that the algebraic structures are just certain simplified spatial structures whose dimensionality does not matter at first. But after investigating them I have eventually realized that operational and structural concepts are designerwise not necessarily exactly the same devices. Therefore, we should synthesize structures from operational procedures and vice versa, which shall be done elsewhere (and has already been done in some instances).

Nevertheless, for proper handling of multiplicative inversions as reciprocals of abstract objects depicted even in the simplest algebraic operational structures, a multiplicative counterpart of the – essentially additive – Borsuk-Ulam theorem is necessary. Although I have already proposed multispatial version of the original Borsuk-Ulam theorem [1], insofar as I know nothing substantial has been done thus far to develop an axiomatic basis for the multispatial framework. Therefore, the following presentation shall rely upon intuitively selfevident relationships and operationally obvious algebraic techniques.

Truly multiplicative inversions are necessary for operations performed over reciprocal structures, regardless of whether these are algebraic or geometric or quasigeometric spatial structures. However, such operations demand unrestricted division by zero, which has already been implemented via multiplication by naturally reciprocal to zero infinity [2-5].

Unambiguous division by zero, in turn, requires more finegrained differential operators offered in [6], and a new, multispatial product differentiation rule [7] that causes differential operators to act simultaneously on two paired 3D reciprocal spaces [8]. The reciprocal pairing reveals twin algebraic character (ascending and descending) of the operational neverending infinity [9], which admits operational complexification of the twin representation of infinity [10]. For discussions of traditional complexification issues in the SSR setting see [11, 12].

Recall that fully operational pointlike infinity is also necessary in many theories of traditional mathematics not only for operations but for purely structural geometric purposes as well. One vivid example of that necessity is the fact that "[…] we can embed any topological space X in a compact space $X\cup\{\infty\}$, called its one-point compactification, by adding a single point ∞ […]" see [13]. Nonetheless, despite the fact that the neverending infinity is natural reciprocal of the singlevalued real zero, the operational infinity is not really a singlevalued algebraic entity but a setvalued (that is more than just multivalued) algebraic entity. That is why I have developed and proposed multispatial structures (under the MSR paradigm) endowed with quasistructural pairing of dual reciprocal spaces in particular, which allowed me to overcome the – previously considered as quite impossible even to fathom – pairing of the singlevalued real zero with its reciprocal setvalued infinity. Disregarding the conceptual incompatibility of representations of zero and infinity – and of all singlevalued magnitudes coupled with setvalued entities, in general – was the main reason why traditional operations involving zero and infinity (such as those proposed in [14, 15], for instance) generated various tacitly veiled paradoxes and such obviously treacherous (yet not disclosed therein as being such) nonsenses as 0 = 1 which have been courageously exposed and somehow eventually published by Dr. Eugenia Cheng [16].

Expanding of the algebraic/operational as well as geometric or quasigeometric spatial structures is indeed feasible and is virtually enshrined even in differential calculus [17], even though that possibility was not recognized in the traditional mathematics. Note that flawed fundamentals of tensor calculus [18] effectively inhibited investigations of multispatial structures and thus prevented research involving all the interrelated topics mentioned above.

2. INVERSES IN COMPLEX DOMAIN ARE NOT REALLY MULTIPLICATIVE INVERSE TRANSFORMATIONS EVEN THOUGH COMPLEX ANALYSIS PRETENDS THAT THEY ARE ALLEGEDLY RECIPROCAL INVERSES

In complex analysis the reciprocal transformation $w = \frac{1}{z}$ is the product of two consecutive transformations: an inversion with respect to unit circle followed by reflection in the real axis

$$z' = \frac{1}{r} e^{i\theta} \quad \& \quad w = \overline{z'} \tag{1}$$

where r is the modulus and θ the angle/argument of the complex number z. The apostrophe denotes tranformed variable and the upper bar complex conjugate as usual [19]. The authors wrote: "Instead of regarding corresponding values of z and w as being represented by points in different planes, it is convenient to think of the w plane as superposed upon the z plane. The numbers z and 1/z may then be represented by points P and Q, respectively, in the same plane." [19]. While on the subject, I shall mention that I will use the letters P and Q to denote algebraic dual reciprocal spaces that are supposed to host the two multiplicatively reciprocal – even though unspecified on purpose – variables in this presentation. Recall that the concept of reciprocal space is used in Fourier transforms [20]. Similar approach is offered by other authors: [21-35]. Inversing in algebraic setting was discussed in terms of cross ratio in [36].

Note that the mapping $z \mapsto \bar{z}$ is also called antiholomorphic reflection in the real axis [37]. For inverse function defined via derivative see [38]. Conjugate inversion was briefly introduced in [39]. The fact that point zero corresponds to a point at infinity in the complex plane w was explicitly recognized in [40-42], but the necessity of having truly multiplicative inversion for this to actually happen was conspicuously ignored. The above aproach is not wrong for theory of analytic/holomorphic functions deployed in the SSR setting, but it is not right in general.

This particular approach virtually redefines the term 'reciprocal', which is understood in mathematics as multiplicative inverse when it appears in $w = \frac{1}{z}$ whereas the product of the two consecutive transformations (1) is mixed (i.e. multiplicatively additive) combination of the superposed operations. Some authors remarked that the term 'inverse' is used in complex analysis in two senses: as the reciprocal 1/f and as the "composition inverse", i.e. an inverse for composite mappings [43]. The formerly underappreciated fact that inverse of a [complex] function f(z) should not be confused with its reciprocal 1/f has been plainly acknowledged in [44]. Although my critique does not deprecate the theory of analytic functions, it indicates that the theory is evidently incomplete and thus can conceal a part of yet unknown reality.

Speaking of complex inversion w=1/z Levinson & Redheffer wrote: "Here |w| = 1/|z| and arg w = – arg z. Hence the distance from the origin is replaced by its reciprocal, and the argument is replaced by its negative. Under translation, rotation and magnification, triangles are mapped into similar triangles, and the conformality is obvious. In the case of inversion the conformality is by no means obvious geometrically, but follows from Theorem 1.1." [45] p.263. The aforementioned theorem 1.1 states: "The mapping w = f(z) is conformal at any point where [the derivative] f '(z) exists and f '(z) ≠ 0." [45] p.260. Recall that mapping is conformal if the angle of intersection of two intersecting arcs is preserved by the mapping in both magnitude and sign – compare [45] p.260. Again, there is nothing wrong with that under the SSR paradigm, of course, but it can be wrong in general. Angle determines local spread that may not always be the same for primary functions and for their reciprocals.

My point is that traditional theory of holomorphic/analytic functions tacitly redefines significant concepts often without ever admitting that. Although anyone has the freedom to redefine the concepts they wish to alter, with the freedom must come responsibility for the prospective or just foreseeable outcomes of their clandestine redefining. However, traditional mathematics was not taking any responsibility for the terms they screwed up. If reciprocity should equate to multiplicative inverse, then it cannot also mean additive reverse at the same time, which would virtually turn it into notion of double-meaning. It is wrong even in principle. When one does this kind of pseudoscientific trickery then its outcome could conceal a part of the abstract mathematical reality and consequently also the – corresponding to it – part of physical reality, not to mention making mockery of attempts at improving the faulty mathematics by tacitly preventing rectification of the latter's flaws. Since the operation of integration, [which yields a functional, that is as if frozen function,] is also mixed inverse with respect to the operation of differentiation, [which operates on actively varying functions,] the [indefinite] integral is determined only up to a constant.

Although this mixed character of integration is not a fault, it is the main reason why traditional mathematics missed the fact that advanced/finegrained differential operators act simultaneously on functions and their reciprocals depicted in two dual reciprocal spaces [8]. That is why the former mathematics failed to recognize multispatiality. The fact that integration and differentiation are [mixed] inverses is well known – compare [46], even though the obviously mixed character of their inverse relationship's was not emphasized clearly enough.

Therefore, the distinction I am making here is not inconsequential for the prospective development of realistic mathematics.

Other authors apparently prefer to leave the whole controversy between reciprocals and complex inverses aside and choose to just define the inverse in terms of values as implied by

$$z\bar{z} = |z|^2 \quad \Rightarrow \quad z^{-1} = \frac{\bar{z}}{|z|^2} \tag{2}$$

while retaining the notation z^{-1} that tacitly alludes to multiplicative inverse which is usually understood as reciprocal [47]. This is most likely quite inadvertent allusion, but it may surely create – otherwise quite avoidable – confusion. Compare also [48-50]. Inverse mapping theorem is concisely presented in [51]; compare also [52]. For abstract invertible mappings see

[53]. Construction of inverse point in complex setting was illustrated in [54]. Other authors offer even simpler derivation of an equivalent complex inverse formula from some abstract reasonings [55, 56]; compare also [57-59]. Complex reciprocal described in terms of angle was discussed in [60]; see also [61]. While used of necessity in complex inversions, the reciprocity of 0 and ∞ in the complex domain is often justified by convention [62, 63]. Recall that Riemann removable singularity theorems defined on complex manifolds do not hold on complex spaces [64]; see also [65]; for concise account see [66, 67]. Recall that in the real domain inverse of a composition can be obtained by reversing the flow/order of components [68, 69]. For Frobenius reciprocity see [70].

Two major problems with inverses emerge wherever they are considered as reciprocals. First is that for the inverse to be unique the inverted function should be singlevalued [71], [72]. The second is that unrestricted division by zero should be permitted. It was prohibited in the former mathematics [73]. But meaningful division by zero requires presence of infinity, which is the natural reciprocal of zero. Since the North pole N of the Riemann sphere does not arise as the projection of any point in the complex plane, we used to associate with N the extended complex number ∞ shown as [setvalued] set $\{\infty\}$ and call the set $\mathbb{C}\cup\{\infty\}$ the extended complex plane, which is also known as one-point compactification in topology [74]. Note however, that the Riemann theorem fails completely in dimension two or more [75]. My most important objective in this presentation is preventing even an accidental creation of genuinely defective functions due to presence of infinity in conjunction with inverses. For traditional approach to defective functions in the context of Picard theorems see [76].

There are many other problems related to evaluations of expressions involving infinity and inverses, especially in the cases when the inverses exhibit the power of -3 or multivalued functions – see [77], for instance. Therefore, I am not calling for a mere rectification of those pretty well known yet perhaps underestimated problems, but for an entirely new approach to realistic mathematics. We will always encounter previously unanticipated issues. That is not the point I want to make here. My point is that because it is the essentially artlike former mathematics that established the operational infrastructure for physical sciences, the abstract mathematics should be made as realistic as possible not only operationally, which is feasible and fairly easy indeed, but also structurally, which may be difficult to fathom.

But wherever we decide to go, even without knowing which direction is the right one, we must make the first decisive step by removing the huge impeding yet avoidable obstacle, which – in the algebraic/operational approach – is: the unwarranted prohibition of division by zero that precludes investigation of roles played by the operational (i.e. not the philosophical) infinity.

3. MULTISPATIAL INVERSIONS INVOLVING THE MULTIPLICATIVE NEUTRAL UNIT ELEMENT 1 IN THE ALGEBRAIC FRAMEWORK OF PAIRED DUAL RECIPROCAL SPACES

At present, the main importance of multiplicative inversions that can form reciprocal transformations consists in their prospective application to conversions of algebraic bases during transitions between various paired spatial or quasispatial structures.

If the multiplicative neutral unit element 1 shall remain unchanged during operations performed in the MSR setting, then the intuitively obvious mappings should hold true

$$\left\{1=\frac{1_p}{1_p}=\frac{1_q}{1_q}\right\}\rightarrow\left\{\frac{1_p}{1_q}=-\frac{1_q}{1_p}\right\}\leftrightarrow\left\{\frac{1_p}{1_q}=-\left(-\frac{1_p}{1_q}\right)\equiv\frac{1_p}{1_q}\right\} \tag{3}$$

which stems from the fact that that the multiplicative inversion indicated by minus sign should include an additive reversion in it as well – compare [78]. The 3D space P – assumed as the primary one – is equipped with a 3D native homogeneous algebraic basis p whereas the equidimensional dual reciprocal 3D space Q (that is paired with P) is equipped with its own, i.e. separate 3D homogeneous native algebraic basis q that is distinct and quite different from p. The two algebraic bases are assumed as multiplicatively inverse (i.e. mutually reciprocal).

The right-hand side (RHS) of the formula (3) clearly shows even in terms of mappings that the multiplicative neutral unit element 1 itself is unaffected by possible formal changes of the algebraic bases within the mutually paired algebraic spatial structures. It also confirms that the operations are logically consistent.

If so, then from the intuitively obvious implication – compare also [79] or [80]

$$\left\{\frac{1}{0}=\infty\right\}\Rightarrow\{0\cdot\infty=1\} \tag{4}$$

that is corroborated by the closely related intuition that 0!=1 [81], which perhaps could also be interpreted as attempting to redefine the neverending ascending operational infinity in the real numbers domain $\mathbb{R}$ via implicit permutations, we can assess the above pattern formula (3) for its logical consistency also in terms of inferential operators

$$1\equiv\left\{\frac{1_p}{1_p}=\frac{1_q}{1_q}\right\}\Rightarrow\left\{\frac{1_p}{1_q}=-\frac{1_q}{1_p}\right\}\Rightarrow\left\{\frac{1_p}{1_q}=-\frac{1_q}{1_p}=-\left(-\frac{1_p}{1_q}\right)\equiv\frac{1_p}{1_q}\right\} \tag{5}$$

where the logical implication sign ⇒ could sometimes be replaced by the equivalence relation sign ⇔ of course. The implication formula (5) is just logical counterpart of the formula (3) because the – essentially set-theoretical – notion of mapping is not always quite unambiguous in the multispatial framework. I am not averse in principle, to talking in terms of mappings in the SSR setting, but mappings could allow – often quite inadvertent – drawing of some nonsensical or otherwise misleading conclusions when used within the MSR framework or perhaps anywhere outside the oversimplified SSR setting, in general.

At this point I must distinguish between the two relations ⇒ and ⇔ and the corresponding to them respective mappings → and ↔, because – unlike logical relations – mappings can wreak havoc and lead to confusion when used in the MSR framework. The reason for making this distinction is that in the SSR setting mappings are presumed as being designated as direct relations between points depicted within the same single space regardless of nature of the operations or transformations through which the given mapping is implemented, whereas in the multispatial MSR framework some mappings might require intermittent passage through some other intermediate points that may be housed (or just temporarily hosted) in spaces other than those directly involved in the given interspatial operation.

Therefore, relations may not always be the same as mappings, even though the absurd oversimplification is thoughtlessly tolerated in the former SSR setting. Perhaps that is why the SSR paradigm was formerly unspoken and thus its actual as well as possible ambiguities were never questioned in the past.

Although the pattern formula (3) seems admissible in the set-theoretical SSR setting, the equivalent mapping ↔ cannot be replaced with the logical equivalence relation ⇔ in (5) because multispatial framework is conceptually far richer than the singlespatial one. In other words: there is not foolproof one-to-one relationship between the forward and backward evaluation of conversions from and to paired dual reciprocal spaces, not at this present stage. That is: no single way of associating two paired dual reciprocal spaces exists yet. This is not just a problem with nonuniqueness of the conversions but rather an issue with possible hierarchical structures of pairing of dual reciprocal spaces, i.e. a structural issue, regardless of whether it is geometric or quasigeometric. Since this paper deals with operational issues alone, the discussion of structural aspects of the pairings shall be discussed elsewhere.

As the one-to-one relationship is essential for devising abstract proofs in traditional mathematics, the multispatial framework apparently demands the development of slightly different tools for validating operational procedures. This is not a critique of the sacrosanct one-to-one relationship upon which mathematical homomorphisms and homeomorphisms are devised, but I want to underscore the fact that under the MSR paradigm more emphasis should be put on the richness of abstract structures as well as on the necessity to realize that while some algebraic operational procedures might be invented indeed, the – corresponding to them – geometric or quasigeometric structures should be discovered, not just invented at whim. For the actual physical reality already exists, and its still unknown features have been revealed in some formerly unanticipated experimental results, which the traditional mathematics left unreconciled or just unexplained. Consequently, this realization disallows making existential postulates and thus supports abstract synthetic reasonings instead, which demand conceptual designing of spatial structures so as to fit the – corresponding to them – operational procedures and vice versa. The latter feat is the essence of my new synthetic approach to mathematics and to mathematized sciences in general.

The evaluation (5) can be considered as a proof of consistency of a simple derivation of the proposed conversion (of various representations of the neutral unit element 1) that takes place during inversion of algebraic reciprocal bases throughout abstract interspatial transitions. It is not a proof of validity of the interspatial transitions, but it surely demonstrates their consistency. The pattern formula (5) implies also complexification of the multiplicative neutral unit element 1 for we can easily obtain also the following algebraic evaluation

$$\left\{\frac{1_p}{1_q} = -\frac{1_q}{1_p}\right\} \Leftrightarrow \left\{\left(1_p\right)^2 = -\left(1_q\right)^2\right\} \Rightarrow \left\{1_p = i1_q\right\} \Rightarrow \left\{\frac{1_p}{i} = 1_q\right\} \tag{6}$$

during interspatial transition from the primary space P|p equipped with its homogeneous native algebraic basis p to the equidimensional dual reciprocal space Q|q equipped with its own, distinct native homogeneous algebraic basis q. Here i denotes the imaginary unit, as usual. Notice that the imaginary unit virtually acts also as an algebraic operator there. Note that nothing has been postulated there, but the displayed above relationships have been discovered through evaluation of certain already known and operationally uncontested algebraic facts. I am purposely neglecting possible differences stemming from left and right operations due to the fact that the bases are assumed to be orthogonal and homogeneous within the paired 3D algebraic/operational spatial structures, and the operations symbolizing transitions between the algebraic spaces are undeniably interspatial.

Making the operational notation more compact I can restate the formula (6) as follows:

$$\left\{\left\{\frac{1_p}{1_q} \coloneqq 1_{pq}\right\} \Rightarrow \left\{1_{qp} \coloneqq \frac{1_q}{1_p}\right\}\right\} \Leftrightarrow \{1_{pq} = -1_{qp}\} \Rightarrow \{1_{pq} = i^2 1_{qp}\} \Rightarrow \left\{\frac{1_{pq}}{i} = i1_{qp}\right\} \tag{7}$$

which shows that the minus sign that indicates reversal of direction of the interspatial transit involves of necessity also conversion of the basis in which the neutral unit element 1 is being represented within the given space. The double index sequence pq indicates interspatial transition from the primary space P that is equipped with native homogeneous algebraic basis p to the dual reciprocal space Q that is equipped with its own native homogeneous algebraic basis q, of course. Although the sequence is not absolutely necessary in general, it is advisable for grasping the topics presented in this section. One can see, however, that the preliminary double index notation virtually changes the conceptual perspective from which the interspatial transit, and the accompanying it conversion of algebraic bases, are viewed.

From the intuitively clear implication (4) and the operational consequence of the logical tautology (5) we can obtain also the following expression

$$\left\{\frac{1_p}{1_q} = -\frac{1_q}{1_p}\right\} \Rightarrow \left\{\frac{1_p}{0_q \cdot \infty_q} = -\frac{0_q \cdot \infty_q}{1_p}\right\} \Rightarrow \left\{\frac{1_p}{0_q \cdot \infty_q} \cdot \infty_q = -\frac{0_q \cdot \infty_q}{1_p} \cdot \infty_q\right\} \Rightarrow \left\{\frac{1_p}{0_q} = -\frac{\infty_q}{0_p \cdot \infty_p}\right\} \Rightarrow$$

$$\left\{1_p = -\frac{\infty_q}{0_p \cdot \infty_p} \cdot 0_q\right\} \Rightarrow \left\{1_p = -\frac{\infty_q}{0_p \cdot \infty_p} \cdot 0_q\right\} \Rightarrow \left\{1_p = \frac{\infty_q}{0_p \cdot \infty_p} \cdot \frac{1}{\infty_q}\right\} \Rightarrow \left\{1_p = -\frac{1}{1_p}\right\} \tag{8}$$

whose final inference is due to successive elimination of any explicit references to the dual reciprocal basis q (and consequently also to the dual reciprocal space Q that is denominated by the reciprocal algebraic basis q) and thus can be summarized in the pattern formula:

$$\left\{1_p = -\frac{\infty_q}{0_p \cdot \infty_p} \cdot 0_q\right\} \Rightarrow \left\{1_p = \frac{\infty_q}{0_p \cdot \infty_p} \cdot \frac{1}{\infty_q}\right\} \Rightarrow \left\{1_p = -\frac{1}{1_p}\right\} \tag{9}$$

and then reevaluating the chain of inferences (9) in yet another way (with both algebraic spaces P and Q involved) in conjunction with the formula (6) we eventually obtain the following suggestive and presumably inspirational expression:

$$\{1_p = -\frac{\infty_q}{0_p \cdot \infty_p} \cdot 0_q\} \Rightarrow \{1_p = -\frac{0_q \cdot \infty_q}{0_p \cdot \infty_p}\} \Rightarrow \{1_p = -\frac{1_q}{1_p}\} \Rightarrow \{1_p = -i\frac{i1_q}{1_p}\} \Rightarrow$$

$$\left\{1_p = -i\frac{1_{p(q)}}{1_p}\right\} \Rightarrow \left\{1_p = \frac{1_{p(q)}}{i}\right\} \Rightarrow \{i1_p = 1_{p(q)}\} \tag{10}$$

which indicates that if the qualified by subscripted/index neutral unit element reemerges in the given primary algebraic space P via conversion of the basis q from the dual reciprocal space Q then it virtually becomes an imaginary algebraic operator as the result of an intermittent interspatial inversion. This feat too suggests possibility of hierarchical pairings. Recall that the conversion is necessary because even though the basis q is native to the reciprocal space Q, it is definitely foreign basis with respect to the primary space P and thus any meaningful operations performed on objects depicted within the space P demand effecting conversion from the objects' representations in the basis q to their representations depicted in the basis p that is native to the space P.

Although the pattern formulas obtained thus far could be applied also to other spaces immersed in more abstract or more general hyperspatial structures, at present I prefer to use the qualifying phrase “algebraic space” just in order to avoid certain explanatory interjections pertaining to the geometric and/or quasigeometric spatial or quasispatial structures, whose concepts are – of necessity – far richer than those of operationally valid algebraic spaces, and thus could add some unnecessary extra confusion to the topics at hand. Doing this I am not pursuing only simplification of the subject matter but am also trying to avoid such generalizations that are unnecessary at this stage of theoretical development of the topics discussed here and therefore, they shall be further elaborated in proper context elsewhere.

The two results (9) and (10) indicate that reciprocal conversions of algebraic bases involve genuinely interspatial transit. Yet because the imaginary unit evidently plays also the more abstract role of an algebraic operator, we should examine if that role too is consistent with our operational patterns for alternating interspatial transitions.

From the results of evaluations obtained in this section we can also infer that

$$\left\{\frac{1_p}{1_q} = -\frac{1_q}{1_p}\right\} \Rightarrow \left\{1_p = -i\,\frac{1_{p(q)}}{1_p}\right\} \Rightarrow \left\{\,1_p = i^2 \cdot \frac{1}{1_p} = i\,\frac{i}{1_p} = \frac{i}{\frac{1_p}{i}}\right\} \Leftrightarrow \left\{\frac{1_p}{i} = \frac{1}{\frac{1_p}{i}} = \frac{i}{1_p}\right\}' \tag{11}$$

where the RHS of the equivalence (11) seems to imply that the multiplicative neutral unit element 1 expressed in the native algebraic basis p of the given primary space P is its own inverse just as one might have suspected.

From the pattern formula (11) we see that the action of interspatial neutral multiplicative unit (defined in the primary space P) can be equated to that of the imaginary unit in its extra role as an algebraic operator

$$1_p = i \equiv \sqrt{(-1)} \tag{12}$$

in its purely algebraic yet definitely operational sense. This fairly straightforward conclusion could also be drawn directly from the formula (8), of course, after performing few simple algebraic rearrangements.

From the formula (8) we can also derive the value of the reciprocal neutral element unit (defined in the dual reciprocal space Q) as follows:

$$\left\{\frac{1_p}{1_q} = -\frac{1_q}{1_p}\right\} \Rightarrow \left\{\frac{1_p}{1_q \cdot \infty_q} = -\frac{0_q}{1_p}\right\} \Rightarrow \left\{\frac{i}{1_q \cdot \infty_q} = 0_q\right\} \Rightarrow \left\{1_q^2 = i\right\} \Rightarrow \left\{1_q = \sqrt{(i)}\,\right\} \tag{13}$$

which result seems to confirm also slightly different kind of reciprocity of the operational spatial representations depicted in the paired dual reciprocal spaces P|p and Q|q equipped with their homogeneous native algebraic bases p and q, respectively. This pattern formula suggests the possibility of multipronged operational hierarchy that is not unique in general.

For from expressions evaluated in the formulas (10), (9) and (7) we can see that

$$\left\{\left\{i1_p = 1_{p(q)}\right\} \& \left\{1_{pq} = i^2 1_{qp}\right\}\right\} \Rightarrow \left\{\left|\left\{1_{p(q)q(p)} = i^4 1_{q(p)p(q)}\right\}\right| \equiv |1|\right\} \tag{14}$$

because it must actually take four suboperations (i.e. two inversions between reciprocal spaces as well as two subsequent conversions of algebraic bases) to go the full circle during transit between twin paired dual reciprocal spaces before we can arrive back at the beginning of the algebraic operational structure. However, since $1^4=i^4$ the evaluation is not unique and thus should be somehow restricted once evaluated in much richer geometric structural terms, which shall be done elsewhere. Compare also similar issue with complex logarithm requiring restriction due to its inherent nonuniqueness [82].

If the lack of uniqueness does not sound an alarm in my ears it is because operationally similar spatial and especially quasispatial heterogenous structures seem to come in almost identical pairs too. I have called the 4D twin of the usual 4D spacetime a timespace in [83] and started exploring the – partly overlapping – twin heterogenous structures in [84]. The fact that the 4D spacetime seemingly comes in two apparently quite distinct flavors is also known to some mathematicians working in the SSR setting [85].

To the best of my knowledge, the startling realization has not prompted them to abandon the antiquated SSR paradigm. Having analyzed some conceptually unsettling consequences of the – routinely suppressed in the past, as either being inconvenient or just inconsistent with the mathematical ideas developed under auspices of the formerly unspoken SSR paradigm – works of Abel, Galois and Lagrange (AGL), I have encapsulated the AGL results in the MSR paradigm. Nevertheless, the unquestionably valid AGL results apparently stirred an envious resentment of gatekeepers of some scientific journals, presumably because the ignored AGL results effectively conveyed the embarrassing message that the traditional mathematics is full of operational as well as structural nonsenses that were not even addressed, not to mention rectified. This present paper pursues just one algebraic venue leading towards their prospective rectification. Although the preliminary notational conventions used above are useful for this algebraic operational presentation, their foreseeable extension onto geometric and/or quasigeometric spatial structures demands much more elaborate notation, which shall be pursued elsewhere. Perhaps a more inspirational – and presumably theoretically more advanced – presentation of the topic of multiplicative interspatial inversions within the multispatial framework could be developed once appropriate multispatial axiomatics based upon the multiplicative version [1] of the essentially additive original Borsuk-Ulam theorem is established. Axiomatics is not always wrong. But it should not be concocted right up front and it should not replace the discovery phase of meanings of the notions included in the theory to be axiomatized.

4. CONCLUSIONS

It has been shown that traditional mathematics often circumvented controversial issues and tacitly redefined the inconvenient meanings of some previouly misunderstood concepts without realizing that consequences of such redefinitions could damage not only the abstract mathematical reasonings but also the mathematical infrastructure of physical sciences.

The redefinitions revealed in analytic functions theory are quite unwarranted and often repel the most honest aspiring adepts of mathematized sciences. This presentation was restricted on purpose to the complex domain alone in order to set the stage for future geometric and quasigeometric considerations, for which this presentation is just an operational prerequisite. It was demonstrated that in traditional theories of analytic/holomorphic functions the notion of inverses of complex numbers and of analytic functions are composites of mixed type for they are multiplicative inverses (i.e. reciprocals) of the modulus/magnitude combined with additive reverses of the argument/angle. Hence, the mixed (partly multiplicative and partly additive) inverses in the complex domain $\mathbb{C}$ are not really multiplicative reciprocals and therefore their – tacitly concealed – lack of truly multiplicative reciprocity spurred the – unnecessary though still ongoing – prohibition of division by zero, which is the natural reciprocal of the neverending ascending infinity if the infinity is understood as fully operational algebraic entity. Besides, the misidentification of complex inverses mistakenly equated with reciprocals is not only unnecessary but also conceptually harmful and it can be easily avoided within the multispatial framework of mathematical reality.

Multiplicative inversions have been presented by hands-on examples within the new multispatial framework in terms of their abstract algebraic representations subscripted/indexed by the native algebraic bases of the paired dual reciprocal (yet still essentially algebraic/operational) spaces in which the inversive operations are performed.

As a byproduct of this presentation, operations involving both the real zero and the neverending ascending real infinity have been shown as feasible and operationally unambiguous. The feasibility of quite unrestricted algebraic operations favors the quest for repeal of the infamous prohibition of division by zero that had been instituted by decree in the traditional mathematics, presumably in order to cover up the objectionable operational absurd exemplified in this paper as well as the conceptual nonsenses, some of which have also been exposed above. Truly realistic mathematics that can be safely applied in physical and other mathematized sciences must not be ruled by insidious decrees nor should it tolerate tacitly interjected deceptive redefinitions of common terms, for such redefinitions generated various formerly undisclosed nonsenses.

Theory of analytic functions has numerous successful applications in conceptually confined areas, but also some controversial claims as well as previously unidentified mistakes. Making mistakes is not abnormal. But preventing attempts that aim for rectification of the mistakes (by tacit suppression of any mention of the mistakes' presence) borders on inexcusable intellectual fraud, to say the least. Presence of unresolved problems and even suspected mistakes should be openly advertised as such rather than be covered up or defended by deceptive misrepresentations.

The general conclusion is that the tacitly perpetrated and thoughtlessly perpetuated mathematical screwups indicate debilitating incompleteness of mathematical theories and thus hint at presence of yet unknown part of the abstract mathematical (and consequently also physical) reality. Instead of defending the indefensible claims of mathematics, we should create an open to comments and independently maintained, database of mathematics and mathematized scientific topics an thus stop endorsing the idiotic presentations that tend to humiliate alert honest students who cannot voice their doubts for fear of departmental reprisals and tacitly beckoned institutional retaliation.

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