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Analytical investigation of heat transfer in a moving convective porous fin with temperature dependent thermal conductivity and internal heat generation

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ABSTRACT

In the present study, thermal performance of a rectangular moving porous fin with temperature dependent thermal conductivity and internal heat generation are analyzed the differential transformation method. The developed approximate analytical solutions are used to investigate the effects of thermal – geometric and thermo-physical fin parameters such as the Peclet number, thermal conductivity parameter, convection parameter, porosity parameter, Internal heat generation parameter on the dimensionless temperature distribution and heat transfer rate are discussed. From the results, it is found that increase in porosity and convective parameters, the rate of heat transfer from the fin increases and consequently improve the efficiency of the fin. Also, the values of the temperature distribution in the fin increases as the Peclet number increases. However, as thermal conductivity and the internal heat generation increase, the rate of heat transfer from the fin decreases. The analytical solution is found to be in good agreement with the direct numerical solution.

Keywords**:** Thermal analysis, Porous Fin, Convective fin, Moving fin, Differential transformation method

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1. INTRODUCTION

Extended surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface several fold. Adding a fin to an object increases the surface area of the object exposed to the surroundings and hence facilitates the rate of heat transfer. Application of fins can be found in various industrial systems such as the cooling of computer processors, air conditioning and oil carrying pipelines. In recent years, there has been extensive research on moving continuous surface. This phenomenon of a moving continuous surface occurs in a good number of Industrial applications such in Extrusion, hot rolling, glass fiber drawing and casting. Also in industrial processes, control of cooling rate of the sheets is very important to obtain desired material structure.

Several studies were performed on heat transfer using fins and moving continuous surface [1-24]. The book by Kraus, Aziz, and Welty [1] provides comprehensive coverage of the various facets of this technology. Sobamowo [2] used the Galerkin's Method of Weighted Residual for a Convective Straight Fin with Temperature-Dependent Conductivity and Internal Heat Generation to investigate the effects of thermo-geometric parameters, the coefficient of heat transfer and thermal conductivity (non-linear) parameters on the temperature distribution, heat transfer and thermal performance of the longitudinal rectangular fin. . Chiu and Chen [2] utilized the Adomian decomposition method to evaluate the efficiency and optimal length of the convective rectangular fin with variable thermal conductivity. The Adomian decomposition method was presented by Arslanturk [3] to evaluate the temperature distribution within the fins and also evaluated the fin efficiency.

Rajabi [4] studied the efficiency of straight fins with temperature-dependent thermal conductivity using the homotopy perturbation method. A series form analytical solution using the homotopy analysis method was presented for evaluating the fin efficiency of straight convective fins by Domairry and Fazeli [5]. Kulkarni and Joglekar [6] implemented a numerical technique based on residue minimization to solve the nonlinear differential equation governing the temperature distribution in straight-convective fins having temperature-dependent thermal conductivity and further evaluated the fin efficiency. In Bouaziz and Aziz [7] the efficiency of a double optimal linearization method was compared with the homotopy perturbation method, variational method, and double series regular perturbation method in evaluating the temperature distribution in straight convective-radiative fins with variable thermal conductivity. In the work of Ranjan Das [8] a simplex search method was used to evaluate the temperature field for a conductive-convective fin with variable thermal conductivity.

Aziz and Torabi [9] studied numerically the thermal performance of a convective-radiative fin with temperature dependent thermal conductivity, heat transfer coefficient, and surface emissivity. Aziz and Khani [10] presented the homotopy analysis method for the analytic solution of heat transfer in moving fins with variable thermal conductivity which is losing heat to the surroundings simultaneously through convection and radiation. A numerical study of the heat process in a continuously moving rod undergoing thermal processing of variable thermal conductivity losing heat through both convection and radiation was studied by Aziz and Lopez [11].

The differential transformation method was applied by Torabi et al. [12] for analyzing the heat transfer in moving fins with temperature-dependent thermal conductivity, losing heat through both convection and radiation. Ravi and N. Uday [13] numerically investigated the heat transfer in a continuously moving convective-radiative fin with variable thermal conductivity by using Haar wavelets. In their work, they studied the effects of the significant parameters, i.e., the thermal conductivity parameter, convection-sink temperature, radiation-sink temperature, convection-conduction parameter, radiation-conduction parameter, and Peclet number, on the temperature distribution and heat transfer characteristics of a continuously moving convective-radiative fin with variable thermal conductivity.

In finding solution to nonlinear equations, the determination of Adomian polynomials as carried out in ADM, the restrictions of HPM to weakly nonlinear problems, the lack of rigorous theories or proper guidance for choosing initial approximation, auxiliary linear operators, auxiliary functions, and auxiliary parameters in HAM, determination of Lagrangian multiplier or parameter as carried out in VIM, operational restrictions to small domains most perturbation methods necessitated the use of another relatively simple and straight forward method for the nonlinear problem.

Moreover, there viewed studies show that the application of differential transformation method has been limited to thermal analysis of solid fin under non-magnetic environment. Such application of differential transform method as introduced by Zhou [25] to solve nonlinear problems has fast gained ground as it appeared in many engineering and scientific research papers because of comparative advantages over the other approximate analytical methods. It solves nonlinear integral and differential equations without linearization, discretization or restrictive assumptions, perturbation and this is the main benefit of this method.

Using DTM, a closed form series solution or approximate solution can be obtained as it provides excellent approximations to the solution of non-linear equation with high accuracy. It is a more convenient method for engineering calculations compare with other approximate analytical or numerical method. It appears more appealing than the numerical solution as it helps to reduce the computation costs, simulations and task in the analysis of nonlinear problems. Several analyses have been derived using the differential transform method (DTM).

Moradi and Rafiee [26], carried out the analytical solution to convection-radiation of a continuously moving fin with temperature-dependent thermal conductivity using DTM, the differential transformation method was applied to solve simultaneous convection and radiation heat transfer problem in a continuously moving fin with temperature thermal conductivity.

They considered rectangular and exponential profiles for a moving fin. The results obtained were in good agreement with the numerical method. Dogonchi and Ganji [27] used the Differential Transformation method to investigate the temperature distribution in a moving convective-radiative fin with temperature dependent thermal conductivity, heat transfer coefficient and heat generation.

Joneidi et al. [28] have used the differential transformation method for studying the effect of temperature-dependent thermal conductivity on the fin efficiency of convective straight fins.

To the best of the authors' knowledge, the thermal analysis of heat transfer in a moving convective porous fin with temperature-dependent thermal conductivity and internal heat generation using differential transformation method has not been carried out. Therefore, in this present study, the thermal analysis of a moving porous convective fin with temperature dependent thermal conductivity and internal heat generation using differential transform method is carried out.

The developed symbolic thermal models are used to investigate the effects of thermal geometric and thermo-physical fin parameters on the thermal performance of the porous fin.

2. PROBLEM FORMULATION

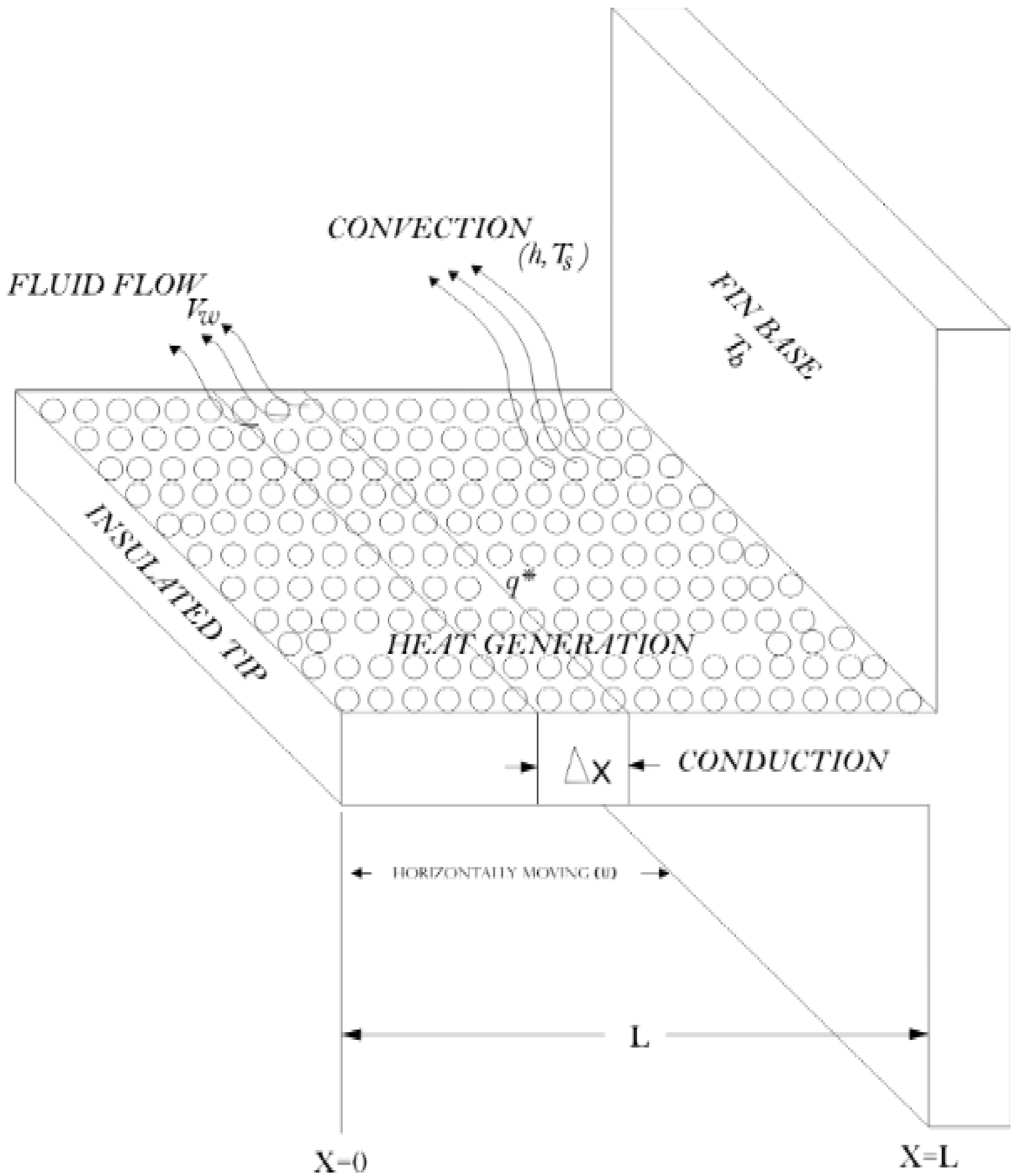

Fig. 1. Schematic of the convective longitudinal porous fin.

Consider a straight rectangular fin of constant cross sectional area, A, with length, b, width W, and thickness, t^*. The fin is porous to allow the flow of infiltrate through it. The axial coordinate, x is measured from the base of the fin. The fin is moving horizontally at a constant velocity U_X. Assuming that the porous medium is isotropic and homogeneous as well as saturated with single-phase fluid. Also, the solid matrix and fluid are at local thermal equilibrium. The coefficient of heat transfer of the surrounding medium is assumed to be constant.

Applying energy balance equation on an elemental section of the fin profile Δx and using Darcy formulation for the porous medium,

$$\begin{pmatrix} \text{rate of heat} \\ \text{Conducted into} \\ \text{the element } \Delta x \end{pmatrix} + \begin{pmatrix} \text{rate of internal} \\ \text{heat generation} \\ \text{in the element } \Delta x \end{pmatrix} - \begin{pmatrix} \text{rate of heat} \\ \text{Conducted from} \\ \text{the element } \Delta x \end{pmatrix} - \begin{pmatrix} \text{rate of heat loss} \\ \text{due to Porosity} \\ \text{of the element } \Delta x \end{pmatrix} - \begin{pmatrix} \text{rate of heat} \\ \text{Convection from} \\ \text{the element } \Delta x \end{pmatrix} - \begin{pmatrix} \text{heat Advection} \\ \text{due to motion of} \\ \text{the element } \Delta x \end{pmatrix} = 0 \tag{1}$$

$$q(x) + q^{*}(A)\Delta x - q(x+\Delta x) - \dot{m}C_p(T-T_s) - h(P.\Delta x)(T-T_s) - \rho C_p U A \Delta x \frac{dT}{dx} = 0 \tag{2}$$

As $\Delta x \to 0$, equation (3) becomes

$$-\frac{dq}{dx} - \frac{\dot{m}C_p(T-T_s)}{\Delta x} - hP(T-T_s) - \rho C_p U A \frac{dT}{dx} + q^{*}(A) = 0 \tag{3}$$

The mass flow rate of the fluid passing through the porous material can be written as

$$\dot{m} = \rho V_w \Delta x.w \tag{4}$$

From Darcy's model, we have that

$$V_w = \frac{gk\beta^{*}(T-T_s)}{\nu}. \tag{5}$$

Substituting the mass flow rate into the main equation results in

$$-\frac{dq}{dx} - \frac{\rho C_p g k \beta^{*} w (T-T_s)^2}{\nu} - hP(T-T_s) - \rho C_p U A \frac{dT}{dx} + q_o^{*} A = 0 \tag{6}$$

From Fourier law heat of conduction,

$$\frac{dq}{dx} = \frac{d}{dx}\left(-k(T)A\frac{dT}{dx}\right) \tag{7}$$

where,

$$k(T) = k_0\left(1+\lambda\left(T-T_s\right)\right) \tag{8}$$

Also,

$$q^*(T) = q_o^*\left(1+\varepsilon\left(T-T_s\right)\right) \tag{9}$$

Substituting equation (8)-(10) into equation (7), gives

$$\frac{d}{dx}\left(\left(1+\lambda\left(T-T_s\right)\right)\frac{dT}{dx}\right) - \frac{\rho C_p g k \beta^* \left(T-T_s\right)^2}{\nu k_o t} - \frac{hP\left(T-T_s\right)}{k_o A} - \frac{\rho C_p U}{k_o A}\frac{dT}{dx} + \frac{q_o^*\left(1+\varepsilon\left(T-T_s\right)\right)}{k_o A} = 0 \tag{10}$$

Subject to the following boundary conditions;

$$T\big|_{x=0} = T_b \tag{11}$$

$$\left.\frac{dT}{dx}\right|_{x=L} = 0 \tag{12}$$

Introducing the following dimensionless variables,

$$\theta = \frac{T-T_s}{T_b-T_s} \quad X = \frac{x}{b} \quad \mathrm{M}^2 = \frac{hpb^2}{k_o A} \quad Sp = \frac{gk\beta^* b^2 (T_b - T_s)}{\nu\alpha t} = \frac{Da\,x\,Ra}{K}\left(\frac{b}{t}\right)^2$$

$$\alpha = \frac{k_o}{\rho C_p} \quad Pe = \frac{Ub}{\alpha} \quad \beta = \lambda(T_b - T_s) \quad H_t = \varepsilon(T_b - T_s), \quad G = \frac{q_o^*}{hP(T_b - T_s)}$$

where M is convection parameter that indicates the effect of surface convection of the fin; Sp is a porous parameter that indicates the effect of the permeability of the porous medium as well as buoyancy effect, β is the dimensionless thermal conductivity and Pe is the Peclet number which represent the dimensionless speed of the moving fin ($Pe = 0$ represents a stationary fin). G represents the heat generation number. H_t, represents the dimensionless internal heat generation number.

Substituting the dimensionless variables into equation (10), gives the non-dimensional governing equation

$$\frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)^2 - Sp\theta^2 - M^2\theta - Pe\frac{d\theta}{dX} + M^2 G(1+H_t\theta) = 0 \tag{13}$$

The boundary conditions appear in dimensionless form as

$$\theta(0) = 1 \tag{14a}$$

$$\frac{d\theta(1)}{dX} = 0 \tag{14b}$$

3. METHOD OF SOLUTION: DIFFERENTIAL TRANSFORM METHOD

Suppose x(t) to be an analytic function in a domain D and t = ti represents any point in D. The function x(t) is then represented by one power series whose center is located at ti. The Taylor series expansion function of x(t) is of the form [25]

$$x(t)=\sum_{k=0}^{\infty}\frac{(t-t_i)^k}{k}\left[\frac{\partial^k x(t)}{\partial t^k}\right]_{t=t_i}\quad \nabla t\in D \tag{15}$$

The particular case of Eq. (15) when ti = 0 is referred to as the Maclaurin series of x(t) and is expressed as:

$$x(t)=\sum_{k=0}^{\infty}\frac{t^k}{k}\left[\frac{\partial^k x(t)}{\partial t^k}\right]_{t=0}\quad \nabla t\in D \tag{16}$$

The differential transformation of the function x(t) is defined as follows:

$$X(k)=\sum_{k=0}^{\infty}\frac{H^k}{k}\left[\frac{\partial^k x(t)}{\partial t^k}\right]_{t=0} \tag{17}$$

where x(t) is the original function and X(k) is the transformed function. The differential spectrum of X(k) is confined within the interval t∈ [0, H], where H is a constant. The differential inverse transform of X(k) is defined as follows:

$$\mathrm{x(t)}=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k \mathrm{X(k)} \tag{18}$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function X(k) at values of argument k are referred to as discrete, i.e., X(0) is known as the zero discrete, X(1) as the first discrete, etc. The more discretes available, the more precisely it is possible to restore the unknown function. The function x(t) consists of the T-function X(k), and its value is given by the sum of the T-function with (t/H)k as its coefficient. In real applications, at the right choice of the H-constant, for larger values of argument k the discrete values of the spectrum reduce rapidly.

The function x(t) is expressed by a finite series and Eq. (18) can be written as

$$\mathrm{x(t)}=\sum_{k=0}^{n}\left(\frac{t}{H}\right)^k \mathrm{X(k)} \tag{19}$$

Table 1. Operational properties of differential transformation Method

	Original function	Transformed function
1.	$f(x)=y(x)\pm z(x)$	$F(k)=Y(k)\pm Z(k)$

2.	$f(x)=\alpha y(x)$	$F(k)=\alpha Y(k)$
3.	$f(x)=\frac{dy}{dx}$	$F(\mathrm{k})=(k+1)F(k+1)$
4.	$f(x)=\frac{d^2y}{dx^2}$	$F(k)=(k+1)(k+2)F(k+2)$
5.	$f(x)=y(x)z(x)$	$F(k,h)=\sum_{l=0}^{k}Y(\mathrm{k-l})Z(k)$
6.	$f(x)=a$ (constant)	$F(k)=a\delta(k)$ where $\delta(k)=\begin{cases}1 \ if\ k=0\\ 0\ if\ k\neq 0\end{cases}$

4. ANALYTICAL SOLUTIONS

Using the DTM, we will now attempt using it to solve the governing equation of Eq. 13

Taking the differential transform of Equation (13), we obtain the following recursive relation

$$\begin{aligned}&(k+1)(k+2)\theta(k+2)+\beta\left(\sum_{l=0}^{k}(k+2-l)(k+1-l)\theta(l)\theta(k+2-l)\right)\\&+\beta\left(\sum_{l=0}^{k}(\mathrm{l}+1)(k+1-l)\theta(l+1)\theta(k+1-l)\right)-Sp\left(\sum_{l=0}^{k}\theta(l)\theta(k-l)\right)\\&-M^2\theta(k)-Pe(k+1)\theta(k+1)+M^2G\delta(k)+M^2GH_t\theta(k)=0\end{aligned} \tag{20}$$

where $\theta(k)$ is the differential transform of $\theta(x)$. Taking the dimensional differential transform of the boundary condition of equation (14) we obtain the following transformations respectively,

$$\begin{aligned}&\theta(0)=1\\&\theta(1)=a\end{aligned} \tag{21}$$

the constant ‘a’ can be determined from the boundary (14) after obtaining the series solution. Substituting eqns (21) into Eq. (20), we obtain the following

$$\theta[1]=a;\quad \theta[0]=1;$$

$$\theta[2]=\frac{M^2-GM^2+aPe-a^2\beta-GM^2H_t+S_p}{2(1+\beta)};$$

$$\theta[3]=\frac{\begin{pmatrix} aM^2+M^2Pe-GM^2Pe+aPe^2-2aM^2\beta+3aGM^2\beta-4a^2Pe\beta+3a^3\beta^2 \\ -aGM^2H_t-GM^2PeH_t+2aGM^2\beta H_t+2aS_p+PeS_p-a\beta S_p \end{pmatrix}}{6(1+\beta)^2};$$

$$\theta[4]=\left(\frac{1}{12+12\beta}\left[\begin{array}{l} \dfrac{M^2\left(M^2-GM^2+aPe-a^2\beta-GM^2H_t+S_p\right)}{2(1+\beta)} \\ -\dfrac{GM^2H_t\left(M^2-GM^2+aPe-a^2\beta-GM^2H_t+S_p\right)}{2(1+\beta)}- \\ \dfrac{\beta\left(M^2-GM^2+aPe-a^2\beta-GM^2H_t+S_p\right)^2}{2(1+\beta)^2} \\ +\dfrac{Pe\begin{pmatrix} aM^2+M^2Pe-GM^2Pe+aPe^2-2aM^2\beta+3aGM^2\beta-4a^2Pe\beta \\ +3a^3\beta^2-aGM^2H_t-GM^2PeH_t+2aGM^2\beta H_t+2aS_p+PeS_p-a\beta S_p \end{pmatrix}}{2(1+\beta)^2}- \\ \dfrac{a\beta\begin{pmatrix} aM^2+M^2Pe-GM^2Pe+aPe^2-2aM^2\beta \\ +3aGM^2\beta-4a^2Pe\beta+3a^3\beta^2-aGM^2H_t \\ -GM^2PeH_t+2aGM^2\beta H_t+2aS_p+PeS_p-a\beta S_p \end{pmatrix}}{(1+\beta)^2}+ \\ S_p\left(a^2+\dfrac{M^2-GM^2+aPe-a^2\beta-GM^2H_t+S_p}{1+\beta}\right) \\ -\beta\left[\begin{array}{l} \left(\dfrac{\left(M^2-GM^2+aPe-a^2\beta-GM^2H_t+S_p\right)^2}{(1+\beta)^2}\right)+ \\ \dfrac{a\begin{pmatrix} aM^2+M^2Pe-GM^2Pe+aPe^2-2aM^2\beta \\ +3aGM^2\beta-4a^2Pe\beta+3a^3\beta^2-aGM^2H_t \\ -GM^2PeH_t+2aGM^2\beta H_t+2aS_p+PeS_p-a\beta S_p \end{pmatrix}}{(1+\beta)^2} \end{array}\right] \end{array}\right]\right)$$

Substituting Equations into Eq.(19) , we obtain the infinite series solution given by

$$\theta(X)=\theta(0)+\theta(1)X+\theta(2)X^2+\theta(3)X^3+\theta(4)X^4+\theta(5)X^5+..... \tag{22}$$

5. PARAMETERS OF ENGINEERING INTERESTS

The non-dimensional total heat flux of the fin is given by:

$$q_T = (1+\beta\theta)\frac{\partial\theta}{\partial X} \tag{23}$$

The fin efficiency

$$\eta = \frac{M^2\int_0^1 \theta dX + S_p\int_0^1 \theta^2 dX + Pe\int_0^1 \theta dX}{M^2 + S_p + Pe} \tag{24}$$

The non-dimensional total heat flux of the fin and the thermal efficiency can be found through the differential transformation method by substituting Eq. (16) into Eqs. (23) and (24) and then evaluate.

6. RESULTS AND DISCUSSION

The developed models are simulated and the the effects of the thermogeometric parameter M, the thermal conductivity β , the porosity factor Sp, the peclet number Pe, the internal heat generation Ht, and generation parameter G, are investigated on the temperature distribution of the porous fin. Before furnishing outcomes obtained from the present study, it is required to validate with a numerical scheme. Hence, a numeric scheme based on the finite difference method has been used for the validation purpose.

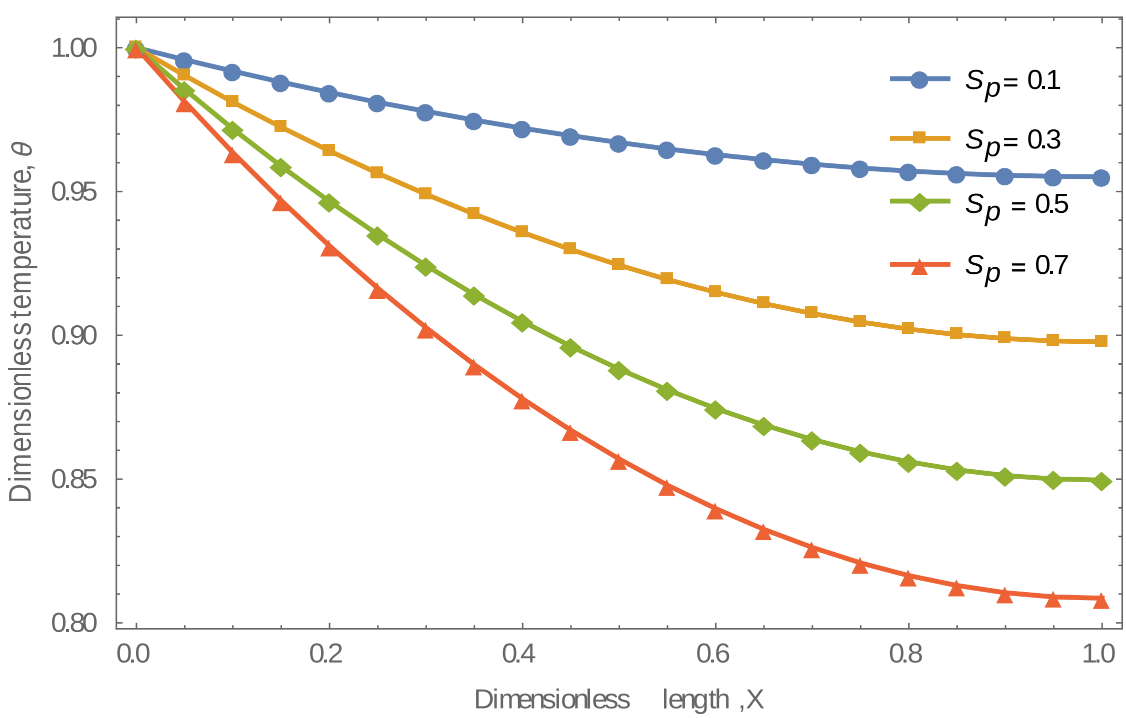

Fig. 2. Dimensionless temperature distribution in the fin parameters for varying porosity parameters when $\beta = 0.2; M = 0.3; \mathrm{Pe} = 0.5; G = 0.4; H_t = 0.6$

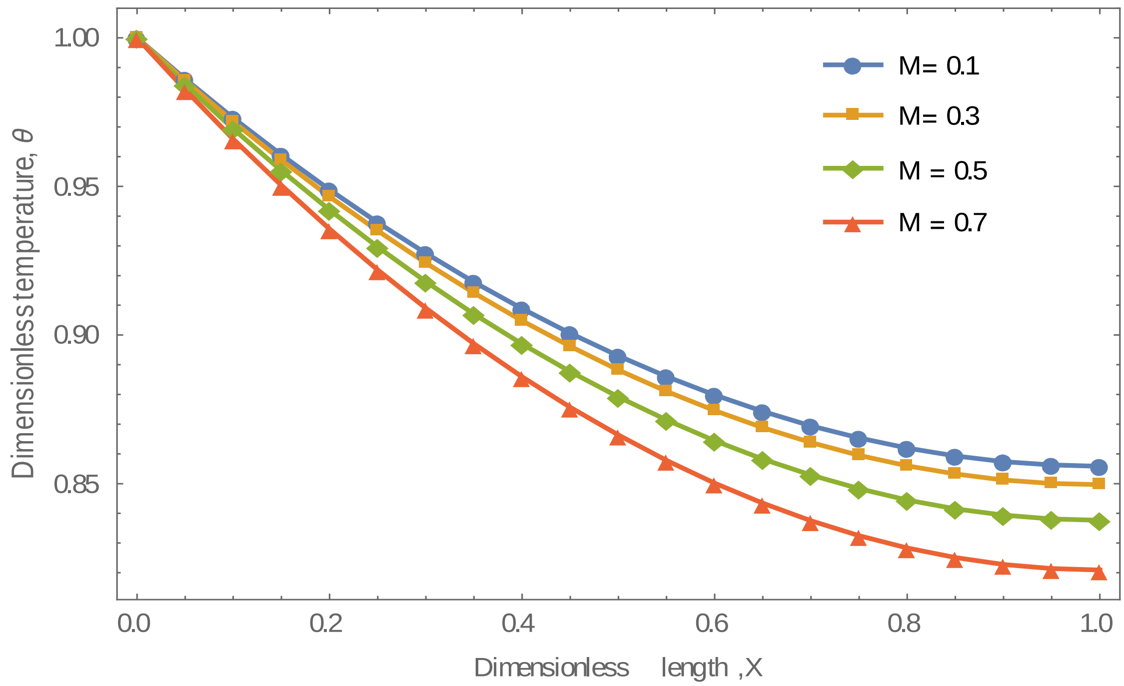

Fig. 3. Dimensionless temperature distribution in the fin parameters for varying thermo geometric parameters when $\beta = 0.2; S_p = 0.5; \mathrm{Pe} = 0.5; G = 0.4; H_t = 0.6$

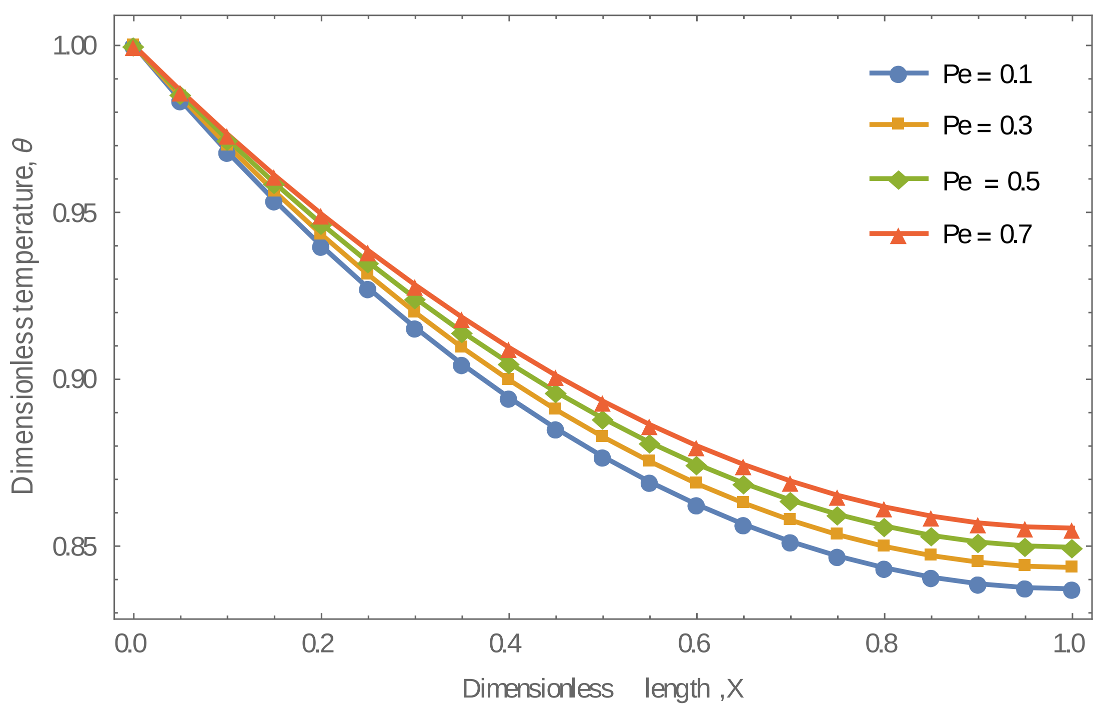

Fig. 4. Dimensionless temperature distribution in the fin parameters for varying peclet number when $\beta = 0.2; M = 0.3; S_p = 0.5; G = 0.4; H_t = 0.6$

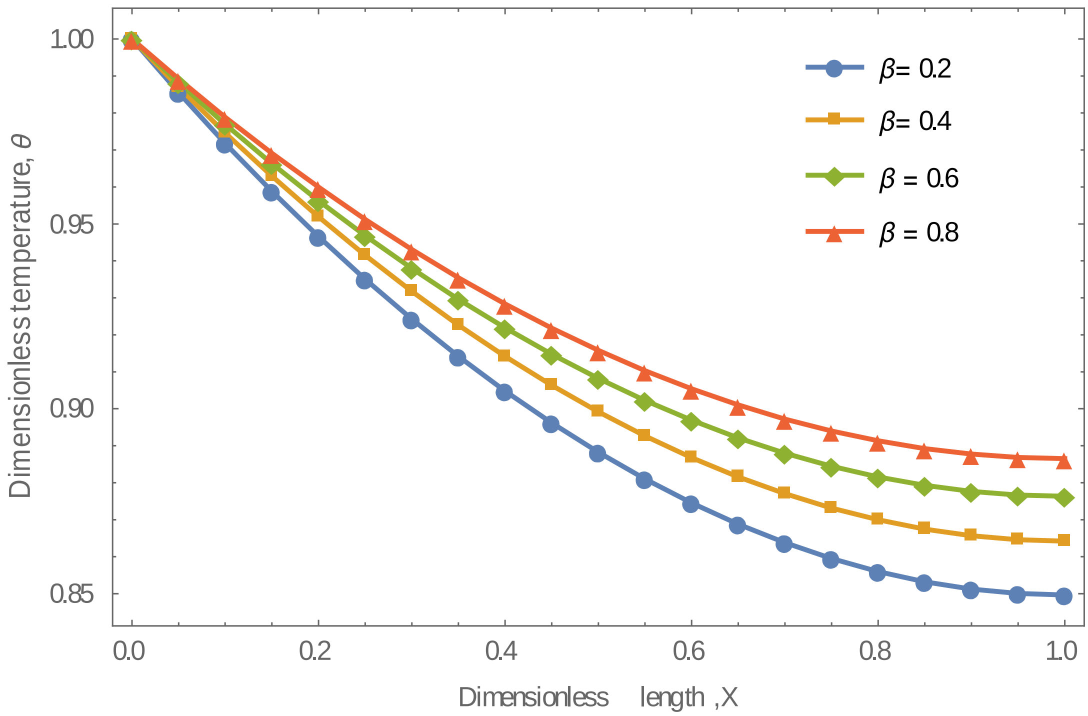

Fig. 5. Dimensionless temperature distribution in the fin parameters for varying thermal conductivity parameter when $S_p = 0.5$; $\mathrm{Pe} = 0.5$; $G = 0.4$; $H_t = 0.6$; $M = 0.3$

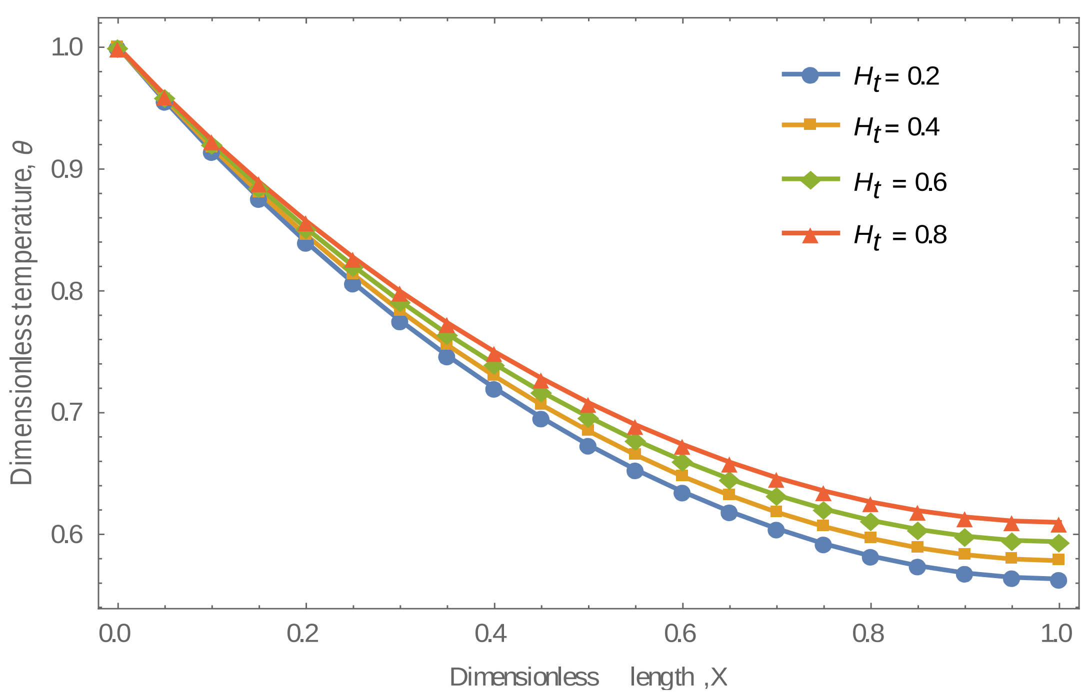

Fig. 6. Dimensionless temperature distribution in the fin parameters for varying internal heat generation parameter when $\beta = 2$; $M = 2$; $S_p = 5$; $\mathrm{Pe} = 2$; $G = 0.4$

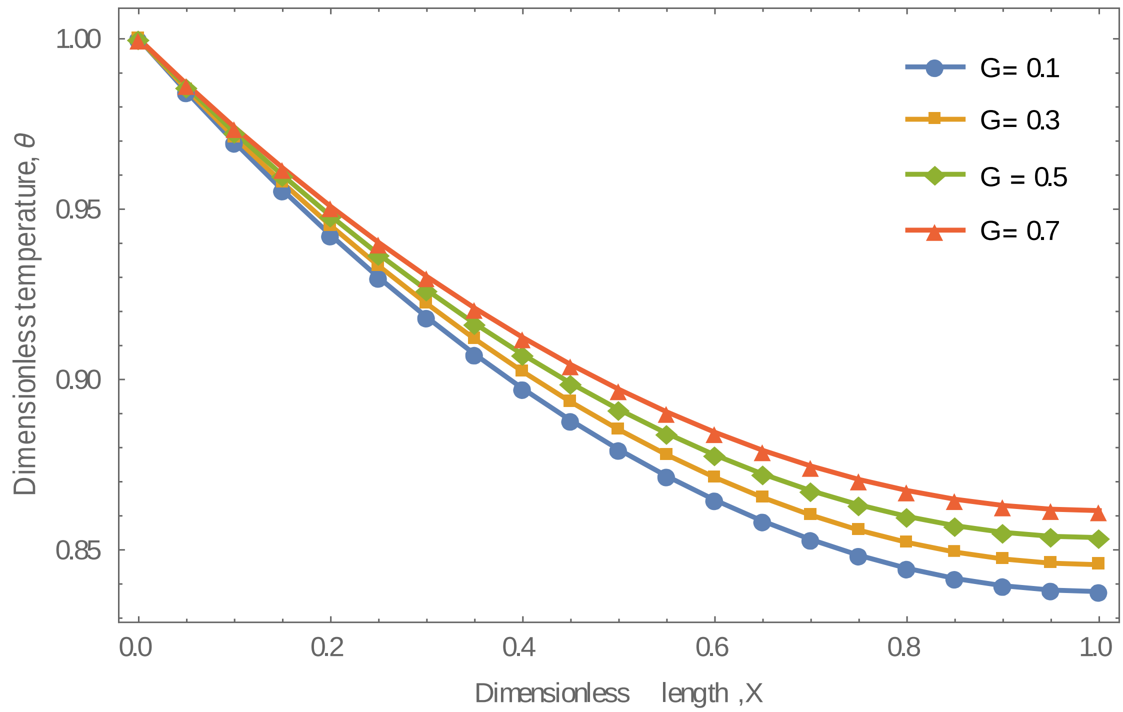

Fig. 7. Dimensionless temperature distribution in the fin parameters for varying generation parameter when $\beta = 0.2; M = 0.3; S_p = 0.5; \mathrm{Pe} = 0.5; H_t = 0.6$

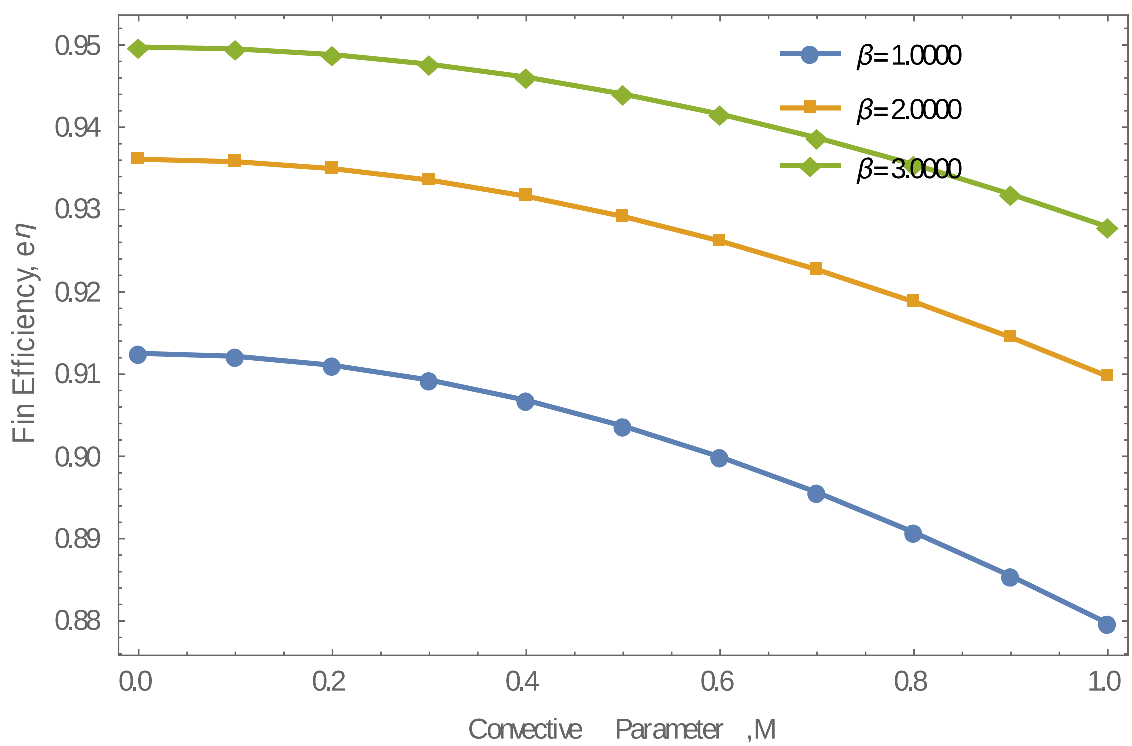

Fig. 8. Efficiency versus convective for $H_t = 0.6; S_p = 0.7; G = 0.4; \mathrm{Pe} = 0.5$

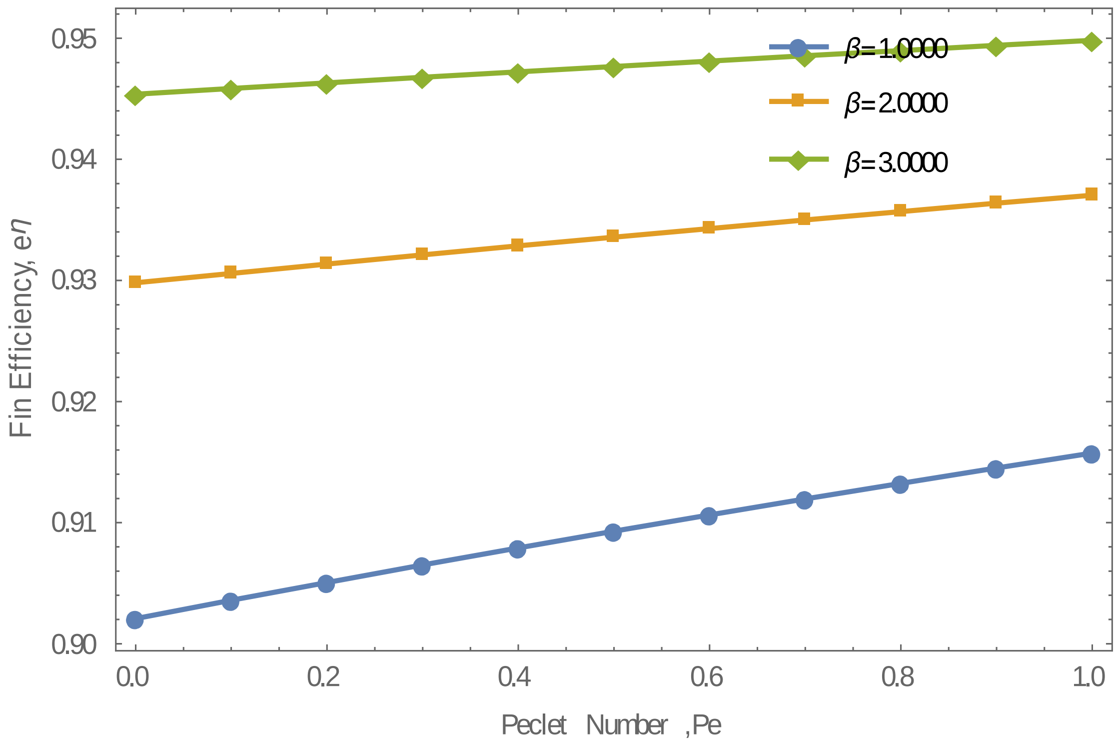

Fig. 9. Efficiency versus Peclet number for varying thermal conductivity parameter when $H_t = 0.6; M = 0.3; S_p = 0.7; G = 0.4$

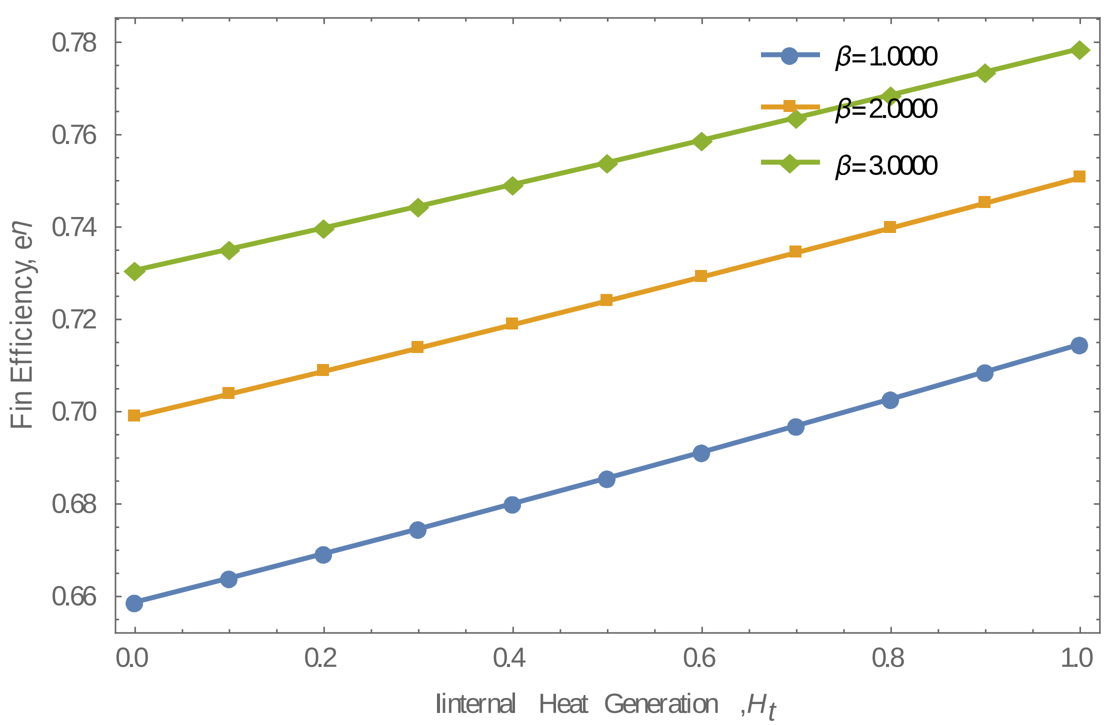

Fig. 10. Efficiency versus internal heat generation for varying Thermal conductivity parameter when $M = 2; S_p = 5; G = 0.4; \text{Pe} = 2$

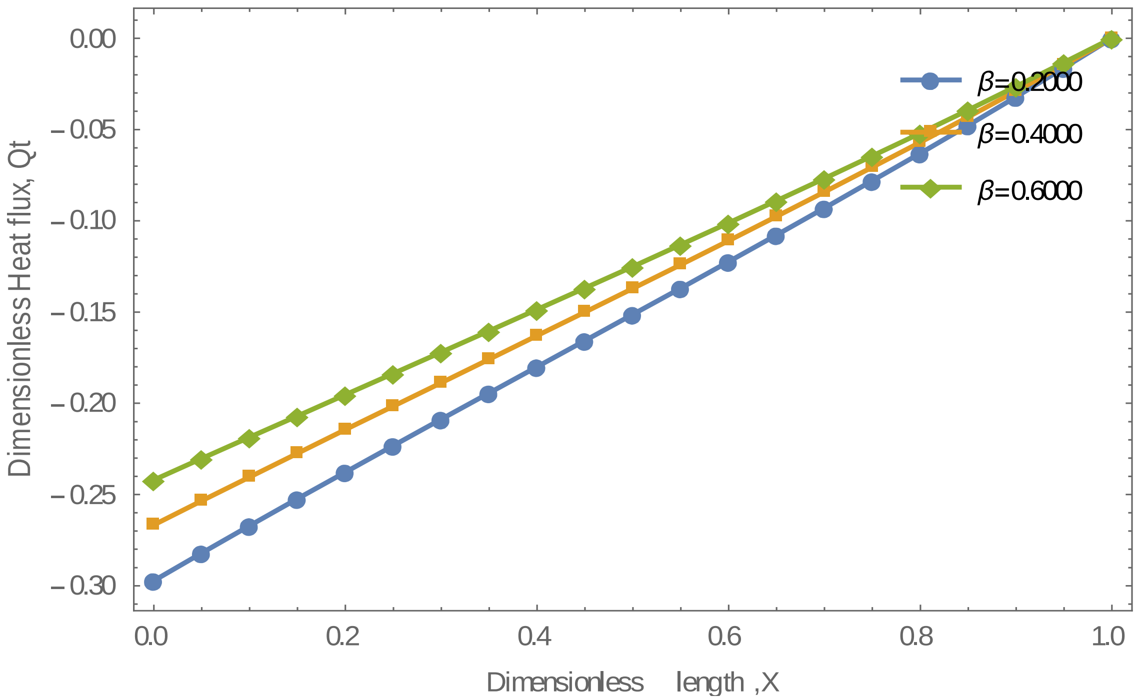

Fig. 11. Heat flux across fin length for varying thermal conductivity parameter $H_t = 0.6; M = 0.3; S_p = 0.5; G = 0.4; \mathrm{Pe} = 0.5;$

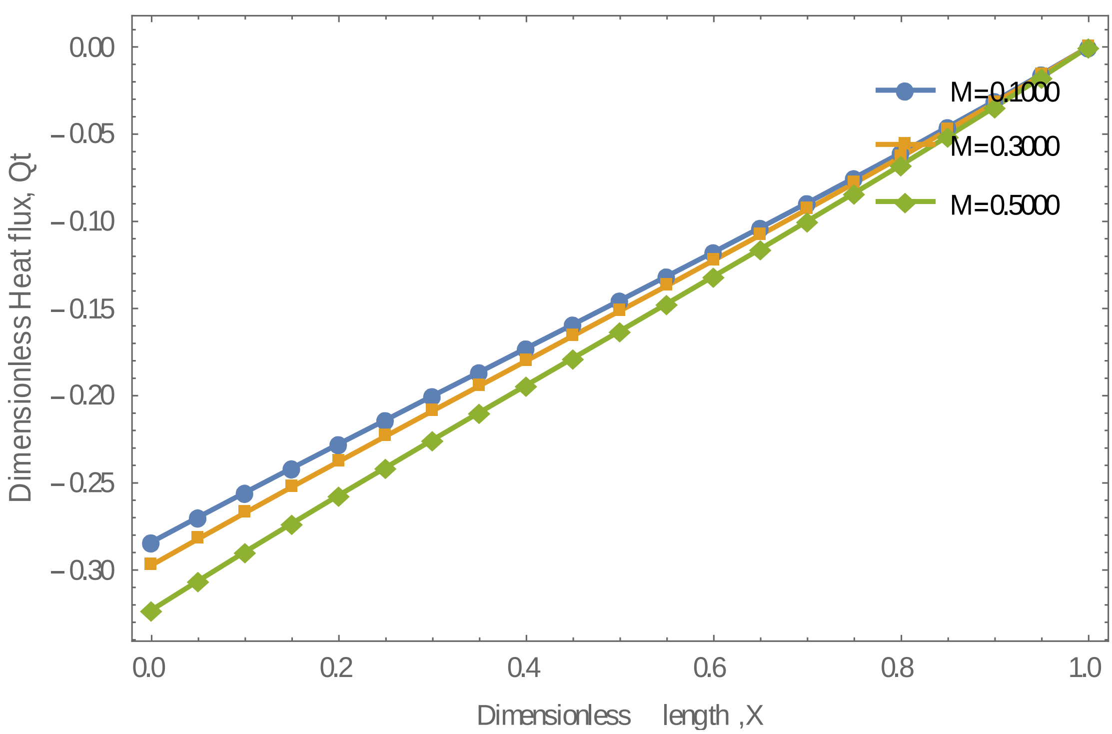

Fig. 12. Heat flux across fin length for varying convective parameter $\beta = 0.2; H_t = 0.6; 0.5; G = 0.4; \mathrm{Pe} = 0.5$

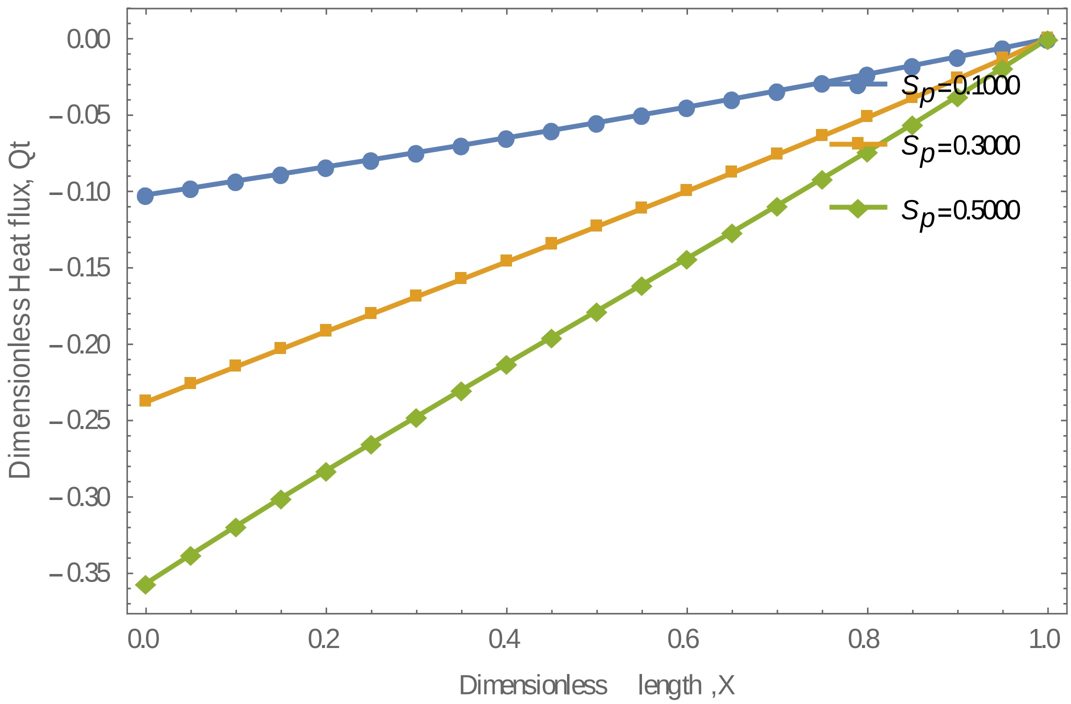

Fig. 13. Heat flux across fin length for varying porosity $\beta = 0.2; H_t = 0.6; 0.5; G = 0.4; \mathrm{Pe} = 0.5$

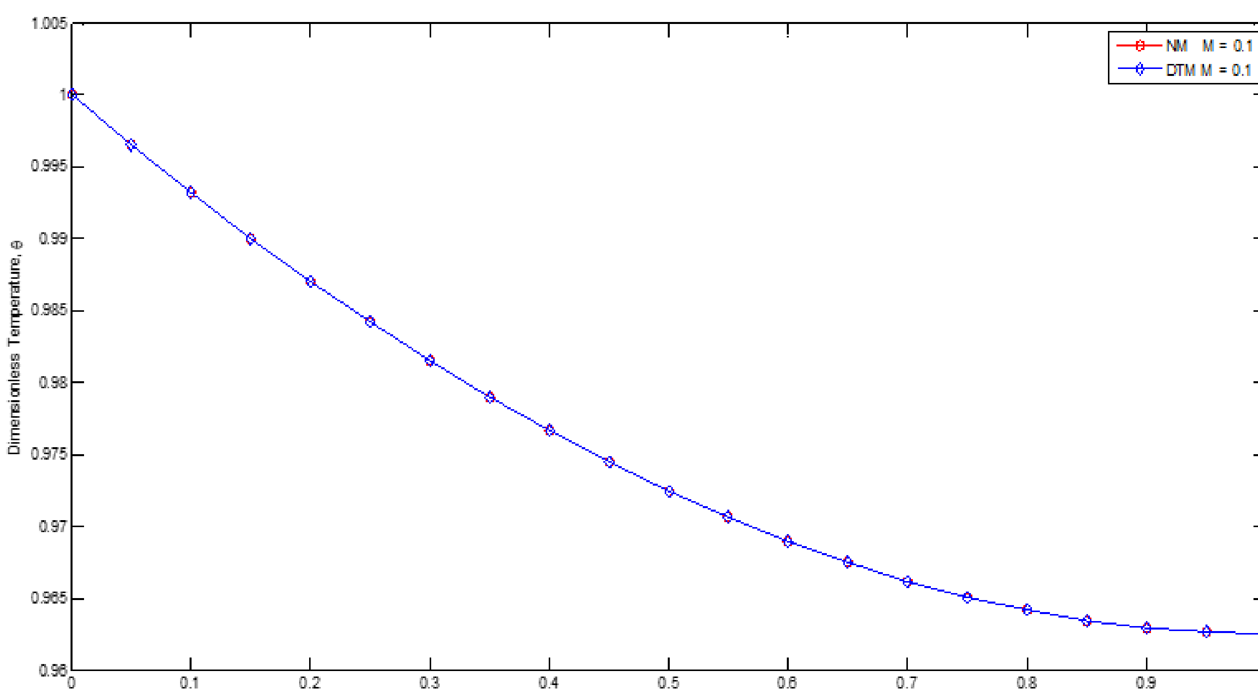

Fig. 14. Dimensionless temperature distribution in the fin parameters for varying convective parameter when $\beta = 0.2; S_p = 0.1; \mathrm{Pe} = 0.5; G = 0.1; H_t = 0.1$

Fig. 2 shows the effects of porous parameter or porosity on the temperature distribution in the porous fin are shown. From the figures, as the porosity parameter increases, the temperature decreases rapidly and the rate of heat transfer (the convective-radiative heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates) as depicted in the figures. The rapid decrease in fin temperature due to increase in the porosity parameter is because as porosity parameter increases, the permeability of the porous fin increases and therefore the ability of the working fluid to penetrate through the fin pores increases, the effect of buoyancy force increases and thus the fin convects more heat, the rate of heat transfer from the fin is enhanced and the thermal performance of the fin is increased. High value of porosity parameter not only decreases the effective thermal conductivity but also reduces ideal heat transfer.

Fig. 3 shows temperature distribution along the length of a moving porous fin while varying the thermogeometric parameter M, from the figure below, it can be seen that the magnitude of the temperature is increased by decreasing the thermogeometric parameter M. It is evident that as the convection-conduction parameter is increased it contributes to more heat loss from the fin and hence cooling of the fin occurs which shows a decrease in the temperature profile. Fig. 4 indicates that the fin temperature increases with an increase in the Peclet number. This is expected because with an increase in Peclet number, the material moves faster and the time for which the material is exposed to the environment gets shorter as well as the losing heat form fin surface gets stronger, thus the fin temperature increases. The influence of the thermal conductivity parameter β on the temperature distribution along the fin is presented in Fig. 5.

It is clear from the figure that as the thermal conductivity parameter is increased the temperature distribution along the fin increases. Physically speaking, the effect of an increase in the thermal conductivity parameter enhances the heat conduction process and results in an increase in the local temperature of the fin. It is further observed that the fin-tip temperature increases with an increase in the thermal conductivity parameter. In order to get the best efficiency from the fin, the dimensionless thermal conductivity parameter β should be kept as low as possible.

Figure 6 shows the effect of internal heat generation on the temperature distribution in the fin. with a decrease in the internal heat generation number, the losing heat fin form surface gets stronger, thus the fin temperature decreases. Figure 7 shows the effect of generation parameter on the transient temperature distribution in the fin. As the generation parameter increases the fin temperature increases.

Fig. 8 shows a comparison of the fin efficiency for porous fins against varying thermo geometric parameter M. The fin efficiency decreases monotonically (for different thermal conductivity) with increasing thermo geometric parameter. From the figures, it is shown that as the thermo geometric parameter increases, the efficiency of the fin decreases. From Fig.9, Fin efficiency increases as the porosity increases. As the porosity value approaches a unity value, the fin efficiency becomes high as the effective thermal conductivity is reduced to a very small quantity. Fig 10 shows that the fin efficiency decreases with increase in internal heat generation. Fin efficiency is improved by increasing the Peclet number. An increase in Peclet number indicates the speed of the fin material is high, if a material moves in high speed, the heat transfer rate is augmented, and hence efficiency is enhanced.

Figs 11-13 shows a plot of the temperature gradient across the length of the fin for varying thermal conductivity parameter, convective parameter and porosity. As expected, the temperature gradient decreases as the thermal conductivity parameter increases, but increases as the porosity and convective parameter increases. But generally, heat flux increases across the length of the fin.

Figure 14 shows the verification of the present analytical method. Figure shows that both analytical method and numerical method are in good agreement with each other.

7. CONCLUSIONS

In this paper, the differential transformation method has been successfully employed in the thermal analysis of a moving convective porous fin with temperature dependent thermal conductivity and internal heat generation. It is always difficult to construct exact solutions for highly nonlinear equations but the differential transformation method technique has shown to be more effective and produces more accurate and suitable results.

This method provides solution in the form of infinite power series and the method possess high accuracy. We have studied the effects of the parameters appearing in the model on the temperature distribution in a longitudinal rectangular fin. Some interesting results are obtained. From the analysis, it is found that increase in porosity, convective, increase the rate of heat transfer from the fin and consequently improve the efficiency of the fin while increase in thermal conductivity and internal heat generation decreases the rate of heat transfer from the fin.

Nomenclature

A Cross sectional area of the fins, (m^2).
h Heat transfer coefficient,($Wm^{-2}k^{-1}$).
C_p Specific heat of the fluid passing through porous fin (J/kg-K).
Da Darcy number.
g Gravity constant(m/s^2).
h Heat transfer coefficient over the fin surface (W/m^2K).
H Dimensionless heat transfer coefficient at the base of the fin, ($Wm^{-2}k^{-1}$).
k Thermal conductivity of the fin material, ($Wm^{-1}k^{-1}$).
k_{eff} Effective thermal conductivity of the porous fin.
k_0 Permeability of the porous fin (m^2).
b Length of the fin, (m).
M Dimensionless Convective parameter.
$\dot{m}$ Mass flow rate of fluid passing through porous fin (kg/s).
p Perimeter of the fin (m).
Pe Peclet number.
Q Dimensionless heat transfer rate per unit area.
q internal heat generation
q_T Heat flux.
Sp Porosity parameter.
t^* Thickness of the fin.
T Fin temperature (K).
T_s Ambient temperature, (K).
T_b Temperature at the base of the fin, (K).

U Velocity of fin (m/s).
v_w velocity of fluid passing through the fin at any point (m/s).
w Width of the fin (m).
x Axial length measured from fin tip (m).
X Dimensionless axial length of the fin.
Ht Dimensionless internal heat generation parameter
G Heat Generation number

Greek Symbols

β Thermal conductivity parameter or non-linear parameter.
θ Dimensionless temperature.
η Efficiency of the fin.
υ Kinematic viscosity(m^2/s).
ρ Density of the fluid(kg/m^3).
λ Measure of thermal conductivity variation with temperature.

Subscripts

s Solid properties.
f Fluid properties.
eff Effective porous properties.

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