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Effects of Internal Heat Generation on the Thermal Stability of a Porous Fin

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ABSTRACT

In this study, the effects of internal heat generation on thermal stability of porous fin is theoretical investigated using differential transform method. The parametric studies reveal increase in the internal heat generation leads to increase in the value or the range of the thermal stability of the fin. The internal heat generation can be used to control the thermal instability in the fin. Also, as the porosity parameter increase, the rate of heat transfer from the base of the fin and consequently improve the efficiency of the fin increase. However, a high value or an excessive internal heat generation results in an undesirable situation where some of the heat energy cannot escape to the sink and instead ends up flowing into the prime surface and the fin tends to store heat rather than dissipating it. This scenario defeats the prime purpose of the cooling fin. Therefore, the operational parameters must be carefully selected to ensure that the fin retains its primary purpose of removing heat from the primary surface.

***Keywords*:** Thermal analysis, Porous Fin, Thermal performance, Temperature-Dependent Internal Heat Generation, Differential Transformation Method

1. INTRODUCTION

In many thermal systems, heat is excessive generated that might lead to thermal damage of the systems. Over the years, different active and passive methods have been adopted to

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dissipate the excessive heat from the thermal systems. Although fins have been used as extended surfaces heat transfer augmentation in most devices, the presence of pores in the fins further increases the heat enhancement capacity of the extended surfaces. In fact, the use of porous fin with certain porosity may give same performance as convectional fin and save 100ϕ of the fin material [1]. Consequently, in some earlier works on extended surfaces, various studies have been put forward on the porous fins [1-9]. In recent times, different authors have adopted different techniques to study the heat transfer in porous fin [10-27]. However, a study on the effects of linear and non-linear temperature-dependent internal heat generation on the thermal performance of the porous fin have not been addressed. Therefore, in this work, thermal analysis of porous fin with linear and non-linear temperature-dependent internal heat generation using differential transform method is presented.

2. PROBLEM FORMULATION

Consider a straight porous fin of length L and thickness *t* exposed on both faces to a convective environment at temperature T_∞ as shown in Fig. 1. The dimension *x* pertains to the height coordinate which has its origin at the fin tip and has a positive orientation from fin tip to fin base. In order to analyze the problem, the following assumptions are made. Following the assumptions in our previous studies [27], the thermal energy balance could be expressed.

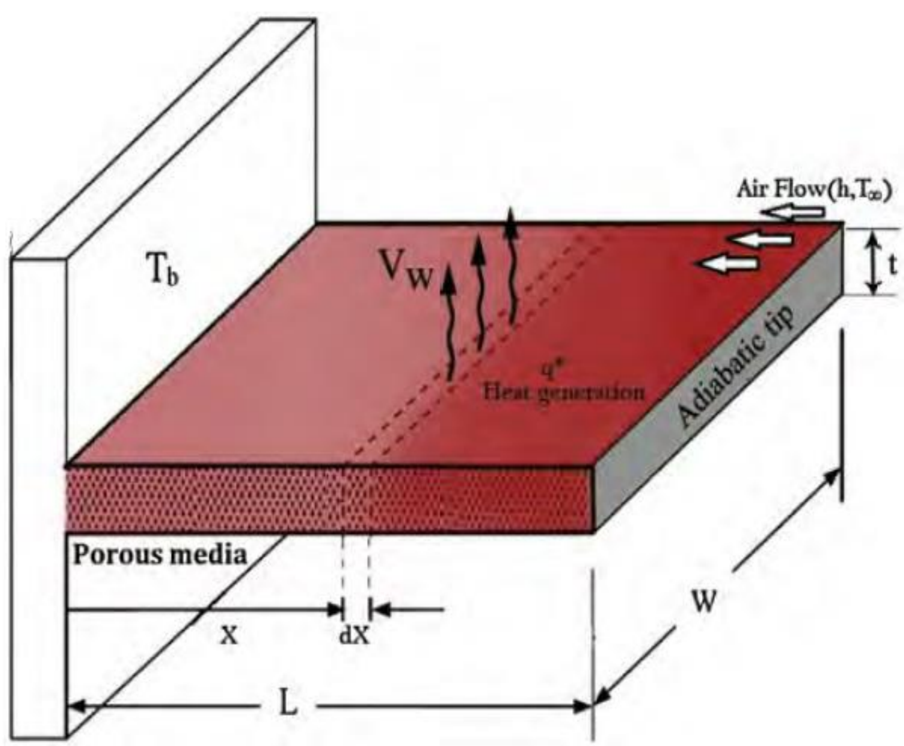


Fig. 1. Schematic of the longitudinal porous fin geometry with the internal heat generation [18]

$$\frac{d^2T}{dx^2}-\frac{h(T-T_\infty)}{k_{eff,a}t}-\frac{\rho c_p\, g\, K\beta^{'}(T-T_\infty)^2}{k_{eff,a}t\nu_f}+\frac{q(T)}{k_{eff,a}}=0 \tag{1}$$

The boundary conditions are

$$x=0,\quad T=T_b$$
$$x=L,\quad \frac{dT}{dx}=0 \tag{2}$$

For many engineering applications, the thermal conductivity and the coefficient of heat transfer are temperature-dependent. Therefore, the linear and non-linear temperature-dependent internal heat generations are given by

$$q_{\text{int}}(T)=q_a[1+\psi(T-T_\infty)] \tag{3}$$

$$q_{\text{int}}(T)=q_a[1+\psi_1(T-T_\infty)+\psi_2(T-T_\infty)^2] \tag{4}$$

Substituting eqs. (3) and (4) into eq. (1), we have

The governing equation for the linear temperature-dependent internal heat generation

$$\frac{d^2T}{dx^2}-\frac{h(T-T_\infty)}{k_{eff,a}t}-\frac{\rho c_p\, g\, K\beta^{'}(T-T_\infty)^2}{k_{eff,a}t\nu_f}+\frac{q_a}{k_{eff,a}}[1+(T-T_\infty)]=0 \tag{5a}$$

While the governing equation for the linear for the non-linear temperature-dependent internal heat generation

$$\frac{d^2T}{dx^2}-\frac{h(T-T_\infty)}{k_{eff,a}t}-\frac{\rho c_p\, g\, K\beta^{'}(T-T_\infty)^2}{k_{eff,a}t\nu_f}+\frac{q_a}{k_{eff,a}}[1+\psi_1(T-T_\infty)+\psi_2(T-T_\infty)^2]=0 \tag{5b}$$

On introducing the following dimensionless parameters in eq. (6) into eq. (5);

$$X=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_b-T_\infty},\quad Ra=Gr.\Pr=\left(\frac{\beta^{'}gT_bt^3}{\nu_f^2}\right)\left(\frac{\rho c_p\nu_f}{k_{eff,a}}\right),\quad Da=\frac{K}{t^2},\quad Q=\frac{q_aA_c}{h_bP(T_b-T_\infty)},\quad M^2=\frac{hL^2}{k_{eff,a}t}$$

$$S_h=\left(\frac{\beta^{'}g(T_b-T_\infty)t^3}{\nu_f^2}\right)\left(\frac{\rho c_p\nu_f K}{k_{eff,a}t^2}\right)\frac{(L/t)^2}{k_{eff,a}}=\frac{RaDa(L/t)^2}{k_{eff,a}},\quad \gamma=\psi(T_b-T_\infty),\quad \gamma_1=\psi_1(T_b-T_\infty),\quad \gamma_2=\psi_2(T_b-T_\infty) \tag{6}$$

We arrived at the dimensionless governing differential eq. (7) .

The dimensionless form of the governing equation for the linear temperature-dependent internal heat generation

$$\frac{d^2\theta}{dX^2} - M^2\theta - S_H\theta^2 + M^2Q\gamma\theta + M^2Q = 0 \tag{7a}$$

While the dimensionless form of the governing equation for the linear for the non-linear temperature-dependent internal heat generation

$$\frac{d^2\theta}{dX^2} - M^2\theta - S_H\theta^2 + M^2Q\gamma_1\theta + M^2Q\gamma_2\theta^2 + M^2Q = 0 \tag{7b}$$

The dimensionless boundary conditions are

$$X = 0, \quad \theta = 1$$
$$X = 1, \quad \frac{d\theta}{dX} = 0 \tag{8}$$

3. METHOD OF SOLUTION: DIFFERENTIAL TRANSFORM METHOD

The nonlinearities in Eqs. (7a) and (7b) call for the use of an approximate analytical method or a numerical method. In this study, we use differential transformation method. The definition and the operational properties of the method can be found in our previous study [28]. The differential transformation of the Eq. (7a) for the linear temperature-dependent internal heat generation is given as

$$(p+1)(p+2)\theta(p+2) - M^2\theta(p) - S_h\sum_{r=0}^{p}\theta(r)\theta(p-r) + M^2Q\gamma\theta(p) + M^2Q\delta(p) = 0 \tag{13}$$

where

$$\theta(p+2) = \frac{M^2\theta(p) + S_h\sum_{r=0}^{p}\theta(r)\theta(p-r) - M^2Q\gamma\theta(p) - M^2Q\delta(p)}{(p+1)(p+2)} \tag{14}$$

With the boundary conditions, we arrived at

$$\theta(0) = 1, \quad \theta(1) = a,$$

$$\theta(2) = \frac{-M^2Q}{2} + \frac{S_h}{2} + \frac{M^2}{2} - \frac{M^2Q\gamma}{2}$$

$$\theta(3)=\frac{aS_h}{3}+\frac{M^2a}{6}-\frac{aM^2Q\gamma}{6}$$

$$\theta(4)=\frac{-S_hM^2Q\gamma}{8}-\frac{S_hM^2Q}{12}+\frac{S_h^2}{12}+\frac{S_hM^2}{8}-\frac{M^4Q\gamma}{12}+\frac{a^2S_h^2}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma}{24}+\frac{M^4Q^2\gamma^2}{24}$$

$$\theta(5)=\frac{-S_haM^2Q}{20}-\frac{S_h^2a}{12}+\frac{S_haM^2}{12}-\frac{S_haM^2Q\gamma}{12}+\frac{aM^4}{120}-\frac{aM^4Q\gamma}{60}+\frac{M^4Q^2a\gamma^2}{120} \tag{15}$$

Therefore, from the definition

$$\theta(X)=1+aX+\left(\frac{S_h}{2}-\frac{M^2Q}{2}+\frac{M^2}{2}-\frac{M^2Q\gamma}{2}\right)X^2+\left(\frac{aS_h}{3}+\frac{M^2a}{6}-\frac{aM^2Q\gamma}{6}\right)X^2$$
$$+\left(\frac{S_hM^2}{8}-\frac{S_hM^2Q\gamma}{8}-\frac{S_hM^2Q}{12}+\frac{S_h^2}{12}-\frac{M^4Q\gamma}{12}+\frac{aS_h^2}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma}{24}+\frac{M^4Q^2\gamma^2}{24}\right)X^4$$
$$+\left(\frac{S_h^2aM^2}{12}-\frac{S_haM^2Q}{20}-\frac{S_h^2a}{12}-\frac{S_haM^2Q\gamma}{12}+\frac{aM^4}{120}-\frac{aM^4Q\gamma}{60}+\frac{M^4Q^2a\gamma^2}{120}\right)X^5+.... \tag{16}$$

For the porous fin with non-linear temperature-dependent internal heat generation as given in eq. (5b), we have the differential transformation of the Eq. (7b) is given as

$$\left(p+1\right)(p+2)\theta(p+2)-M^2\theta(p)-S_h\sum_{r=0}^{p}\theta\left(r\right)\theta\left(p-r\right)+M^2Q\gamma_1\theta(p)$$
$$+M^2Q\gamma_2\sum_{r=0}^{p}\theta\left(r\right)\theta\left(p-r\right)+M^2Q\delta\left(p\right)=0 \tag{17}$$

From which

$$\theta(p+2)=\frac{M^2\theta(p)+S_h\sum_{r=0}^{p}\theta\left(r\right)\theta\left(p-r\right)-M^2Q\gamma_2\sum_{r=0}^{p}\theta\left(r\right)\theta\left(p-r\right)-M^2Q\gamma\theta(p)-M^2Q\delta\left(p\right)}{\left(p+1\right)(p+2)} \tag{18}$$

With the boundary conditions, we arrived at

$$\theta(0)=1,\quad \theta(1)=a$$

$$\theta(2)=\frac{-M^2Q}{2}+\frac{S_h}{2}-\frac{M^2Q\gamma_2}{2}+\frac{M^2}{2}-\frac{M^2Q\gamma}{2}$$

$$\theta(3)=\frac{aS_h}{3}-\frac{aM^2Q\gamma_2}{3}+\frac{M^2a}{6}-\frac{aM^2Q\gamma}{6}$$

$$\theta(4)=\frac{-S_hM^2Q\gamma}{8}-\frac{S_hM^2Q}{12}+\frac{S_h^2}{12}+\frac{S_hM^2}{8}+\frac{M^4Q^2\gamma_1\gamma_2}{8}+\frac{M^4Q^2\gamma_2}{12}+\frac{M^4Q^2\gamma_2^2}{12}-\frac{M^4Q\gamma_2}{8}-\frac{M^4Q\gamma}{12}+\frac{aM^4Q^2\gamma_2^2}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma}{24}+\frac{M^4Q^2\gamma^2}{24}$$

$$\theta(5)=\frac{S_h^2aM^2}{12}-\frac{S_haM^2Q}{20}-\frac{S_h^2a}{12}-\frac{S_haM^2Q\gamma}{12}+\frac{aM^4Q^2\gamma_2}{20}-\frac{M^4Q^2\gamma_2^2a}{12}+\frac{M^4Q^2\gamma_2^2aM^2}{12}+\frac{aM^4Q^2\gamma_1\gamma_2}{12}+\frac{aM^4}{120}-\frac{aM^4Q\gamma}{60}+\frac{M^4Q^2a\gamma^2}{120} \quad (19)$$

From the definition, we have,

$$\theta(X)=1+aX+\left(\frac{S_h}{2}-\frac{M^2Q}{2}-\frac{M^2Q\gamma_2}{2}+\frac{M^2}{2}-\frac{M^2Q\gamma}{2}\right)X^2+\left(\frac{aS_h}{3}-\frac{aM^2Q\gamma_2}{3}+\frac{M^2a}{6}-\frac{aM^2Q\gamma}{6}\right)X^3$$
$$+\left(\begin{array}{l}\frac{-S_hM^2Q\gamma}{8}-\frac{S_hM^2Q}{12}+\frac{S_h^2}{12}+\frac{S_hM^2}{8}+\frac{M^4Q^2\gamma_1\gamma_2}{8}+\frac{M^4Q^2\gamma_2}{12}+\frac{M^4Q^2\gamma_2^2}{12}\\ -\frac{M^4Q\gamma_2}{8}-\frac{M^4Q\gamma}{12}+\frac{aM^4Q^2\gamma_2^2}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma}{24}+\frac{M^4Q^2\gamma^2}{24}\end{array}\right)X^4$$
$$+\left(\begin{array}{l}\frac{S_h^2aM^2}{12}-\frac{S_haM^2Q}{20}-\frac{S_h^2a}{12}-\frac{S_haM^2Q\gamma}{12}+\frac{aM^4Q^2\gamma_2}{20}-\frac{M^4Q^2\gamma_2^2a}{12}\\ +\frac{M^4Q^2\gamma_2^2aM^2}{12}+\frac{aM^4Q^2\gamma_1\gamma_2}{12}+\frac{aM^4}{120}-\frac{aM^4Q\gamma}{60}+\frac{M^4Q^2a\gamma^2}{120}\end{array}\right)X^5+.... \quad (20)$$

For solid fin with linear temperature-dependent internal heat generation, we have the governing equation as

$$\frac{d^2\theta}{dX^2}-M^2\theta+M^2Q\gamma\theta+M^2Q=0 \quad (21)$$

and the recursive relation of the governing equation is given as

$$(p+1)(p+2)\theta(p+2)-M^2\theta(p)+M^2Q\gamma\theta(p)+M^2Q\delta(p)=0 \quad (22)$$

Then

$$\theta(p+2)=\frac{M^2\theta(p)-M^2Q\gamma\theta(p)-M^2Q\delta(p)}{(p+1)(p+2)}$$

With the boundary conditions, we arrived at

$$\begin{aligned}
&\theta(0)=1,\quad \theta(1)=a,\\
&\theta(2)=\frac{-M^2Q}{2}+\frac{M^2}{2}-\frac{M^2Q\gamma}{2}\\
&\theta(3)=\frac{M^2a}{6}-\frac{aM^2Q\gamma}{6}\\
&\theta(4)=-\frac{M^4Q\gamma}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma}{24}+\frac{M^4Q^2\gamma^2}{24}\\
&\theta(5)=\frac{aM^4}{120}-\frac{aM^4Q\gamma}{60}+\frac{M^4Q^2a\gamma^2}{120}
\end{aligned} \tag{23}$$

Therefore,

$$\begin{aligned}
\theta(X)=1+aX&+\left(\frac{M^2}{2}-\frac{M^2Q}{2}-\frac{M^2Q\gamma}{2}\right)X^2+\left(\frac{M^2a}{6}-\frac{aM^2Q\gamma}{6}\right)X^2\\
&+\left(\frac{M^4}{24}-\frac{M^4Q\gamma}{12}-\frac{M^4Q}{24}+\frac{M^4Q^2\gamma}{24}+\frac{M^4Q^2\gamma^2}{24}\right)X^4\\
&+\left(\frac{aM^4}{120}-\frac{aM^4Q\gamma}{60}+\frac{M^4Q^2a\gamma^2}{120}\right)X^5+....
\end{aligned} \tag{24}$$

For solid fin with non-linear temperature-dependent internal heat generation, we have the governing equation as

$$\frac{d^2\theta}{dX^2}-M^2\theta+M^2Q\gamma_1\theta+M^2Q\gamma_2\theta^2+M^2Q=0$$
(25)

and the recursive relation of the governing equation is given as

$$(p+1)(p+2)\theta(p+2)-M^2\theta(p)+M^2Q\gamma_2\sum_{r=0}^{p}\theta(r)\theta(p-r)+M^2Q\gamma\theta(p)+M^2Q\delta(p)=0 \tag{26}$$

From which we arrived at

$$\theta(p+2)=\frac{M^2\theta(p)-M^2Q\gamma_2\sum_{r=0}^{p}\theta(r)\theta(p-r)-M^2Q\gamma\theta(p)-M^2Q\delta(p)}{(p+1)(p+2)} \tag{27}$$

With the boundary conditions, we arrived at

$$\theta(0)=1,\quad \theta(1)=a,\quad \theta(2)=\frac{-M^2Q}{2}-\frac{M^2Q\gamma_2}{2}+\frac{M^2}{2}-\frac{M^2Q\gamma_1}{2}$$

$$\theta(3)=-\frac{aM^2Q\gamma_2}{3}+\frac{M^2a}{6}-\frac{aM^2Q\gamma_1}{6},$$

$$\theta(4)=\frac{M^3Q^2\gamma_1\gamma_2}{8}+\frac{\gamma_2M^4Q^2}{12}+\frac{M^4Q^2\gamma_2^2}{12}+\frac{M^4Q\gamma_2}{8}-\frac{M^4Q\gamma_1}{12}+\frac{aM^4Q^2\gamma_2^2}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma_1}{24}+\frac{M^4Q^2\gamma^2}{24}$$

$$\theta(5)=\frac{aM^4Q^2\gamma_2}{20}-\frac{M^4Q^2\gamma_2^2a}{12}+\frac{M^4Q^2\gamma_2^2M^2}{12}+\frac{M^4Q^2\gamma_1\gamma_2}{12}+\frac{aM^4}{120}-\frac{aM^4Q\gamma_1}{60}+\frac{M^4Q^2a\gamma_1^2}{120} \tag{28}$$

Therefore,

$$\begin{aligned}\theta(X)=1+aX&+\left(\frac{-M^2Q}{2}-\frac{M^2Q\gamma_2}{2}+\frac{M^2}{2}-\frac{M^2Q\gamma_1}{2}\right)X^2+\left(-\frac{aM^2Q\gamma_2}{3}+\frac{M^2a}{6}-\frac{aM^2Q\gamma_1}{6}\right)X^3\\&+\left(\begin{aligned}&\frac{M^3Q^2\gamma_1\gamma_2}{8}+\frac{\gamma_2M^4Q^2}{12}+\frac{M^4Q^2\gamma_2^2}{12}+\frac{M^4Q\gamma_2}{8}-\frac{M^4Q\gamma_1}{12}\\&+\frac{aM^4Q^2\gamma_2^2}{12}-\frac{M^4Q}{24}+\frac{M^4}{24}+\frac{M^4Q^2\gamma_1}{24}+\frac{M^4Q^2\gamma^2}{24}\end{aligned}\right)X^4\\&+\left(\begin{aligned}&\frac{aM^4Q^2\gamma_2}{20}-\frac{M^4Q^2\gamma_2^2a}{12}+\frac{M^4Q^2\gamma_2^2M^2}{12}+\frac{M^4Q^2\gamma_1\gamma_2}{12}\\&+\frac{aM^4}{120}-\frac{aM^4Q\gamma_1}{60}+\frac{M^4Q^2a\gamma_1^2}{120}\end{aligned}\right)X^5+....\end{aligned} \tag{29}$$

The unknown value of "a" is found using the tip boundary condition.

4. EXACT ANALYTICAL SOLUTIONS

If the nonlinear terms are removed, the dimensionless form of the governing equation reduces to

$$\frac{\partial^2\theta}{\partial X^2} - M^2(1-\gamma Q)\theta = -M^2 Q \tag{30}$$

and the exact analytical solution is given by

$$\theta(X) = \left\{ \left\{ \frac{\left[\frac{Q}{(1-\gamma Q)} - 1\right]\left[e^{\left(\frac{\left[2M\sqrt{(1-\gamma Q)}\right]}{2}\right)X} + e^{-\left(\frac{\left[2M\sqrt{(1-\gamma Q)}\right]}{2}\right)X} \right]}{e^{\left(\frac{\left[2M\sqrt{(1-\gamma Q)}\right]}{2}\right)} - e^{\left(\frac{-\left[2M\sqrt{(1-\gamma Q)}\right]}{2}\right)}} \right\} \right\} + \frac{Q}{(1-\gamma Q)}$$

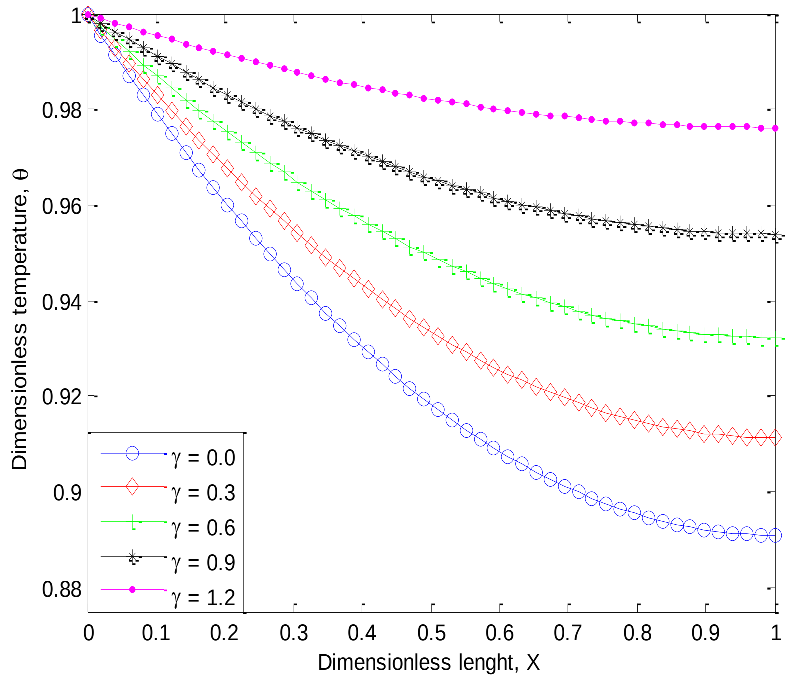
5. RESULTS AND DISCUSSION

Using the developed models, the graphical representations of the results are presented in this section. The results show that the dimensionless temperature distribution falls monotonically along fin length.

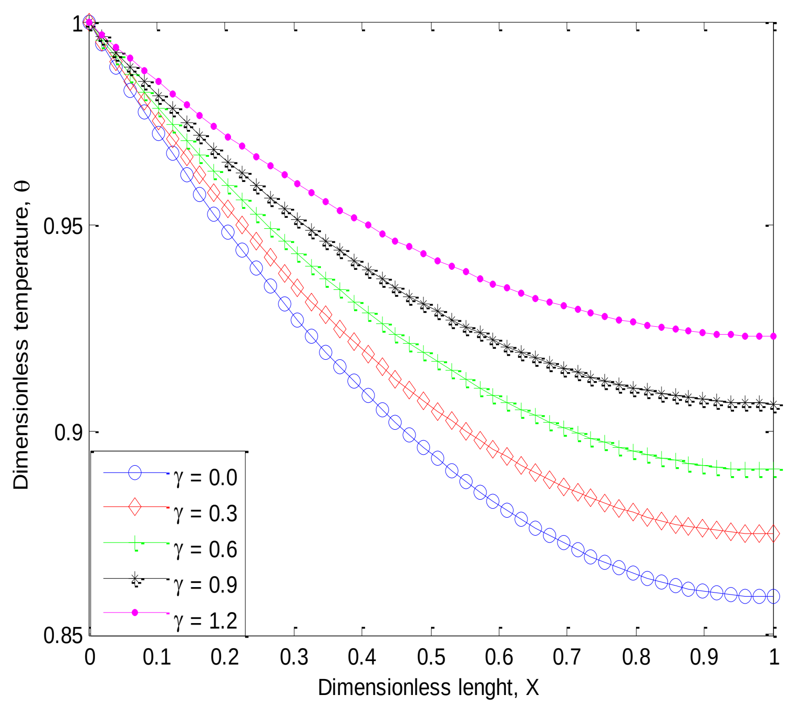
5. 1. Effects of Internal heat generation on temperature distribution in the porous fin

The effects of the internal heat generation on the thermal response of the porous fin is shown in Fig. 2a and b and Fig. 3a and b. From the figures, it could be deducted that the internal heat generation terms have direct and significant effects on the rate of heat transfer from the fin. The temperature gradient at the fin base is such that the fin is extracting heat from the prime surface and dissipating this energy together with the internal generated energy in the fin to the environment. Increase in the internal heat generation leads to increase in the value or the range of the thermal stability of the fin.

However, a high value or an excessive internal heat generation results in an undesirable situation where some of this energy cannot escape to the sink and instead ends up flowing into the prime surface and the fin tends to gain heat rather than losing it. This scenario defeats the purpose of the fin. Thus, the operational parameters must be carefully chosen to ensure that the fin retains its primary purpose of removing heat from the primary surface.

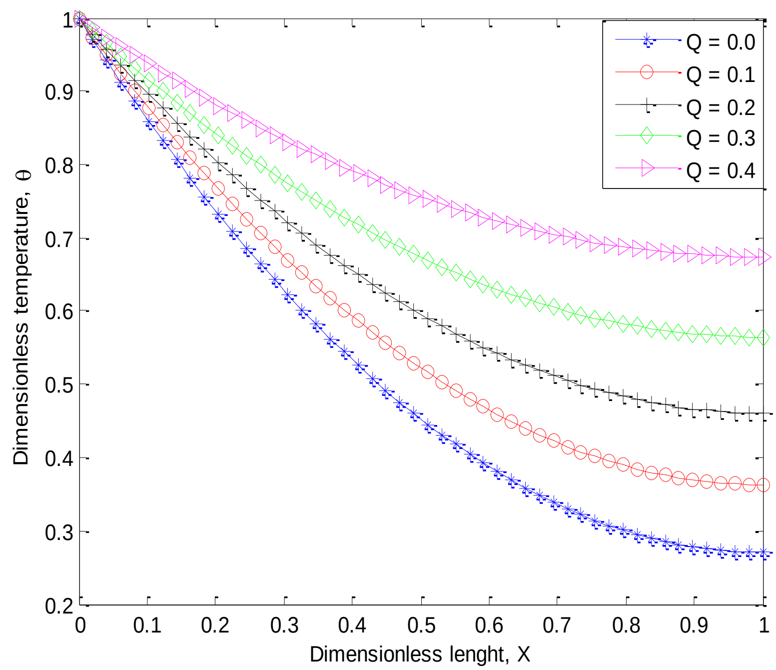


(a)

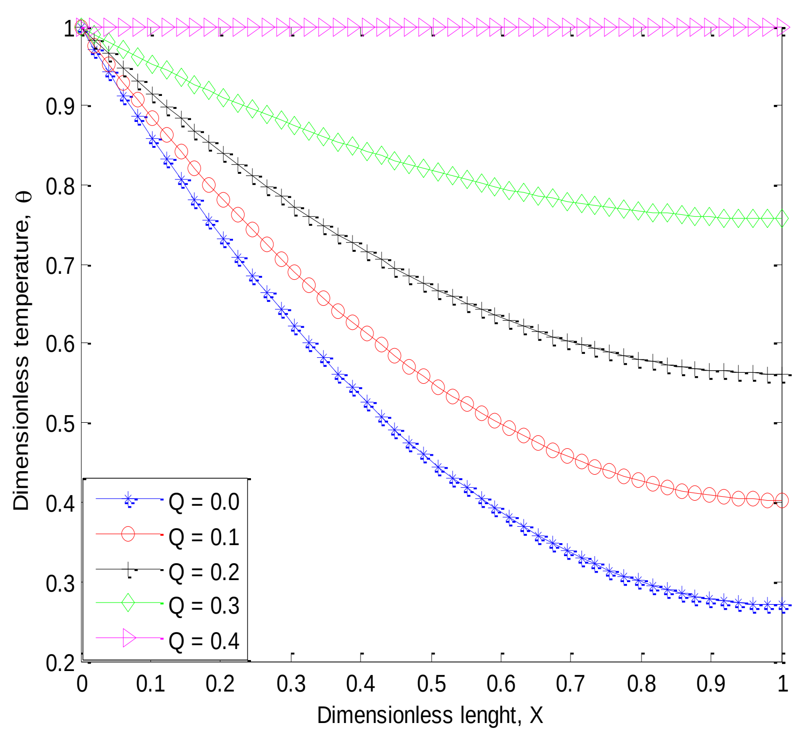


(b)

Fig. 2. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when **(a)** M = 0.5, Q = 0.4, S_h = 0.5, **(b)** M = 1.0, Q = 0.4, S_h = 0.5



(a)



(b)

Fig. 3. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when **(a)** S = 0, M = 2, G = 0.5 **(b)** S = 0, M = 2, G = 1.5,

The influence on internal heat generation on the dimensionless temperature distribution in the fin. It is established in our previous work [23] that as porous parameter, S_h increases to a certain value, the dimensionless temperature distribution at the fin tip results in negative value (which shows thermal instability) at the tip of the fin, contradicting the assumption made in the analysis. From the analysis, the limiting value of S_h for for thermal instability for constant thermal properties without internal heat generation is approximately 4√34. However, value of porosity parameter for the thermal stability increases with increase in internal heat generation parameter.

5. 2. Effects of Internal heat generation on thermal stability and the limitation of the exact analytical solution in predicting the thermal instability in the fin

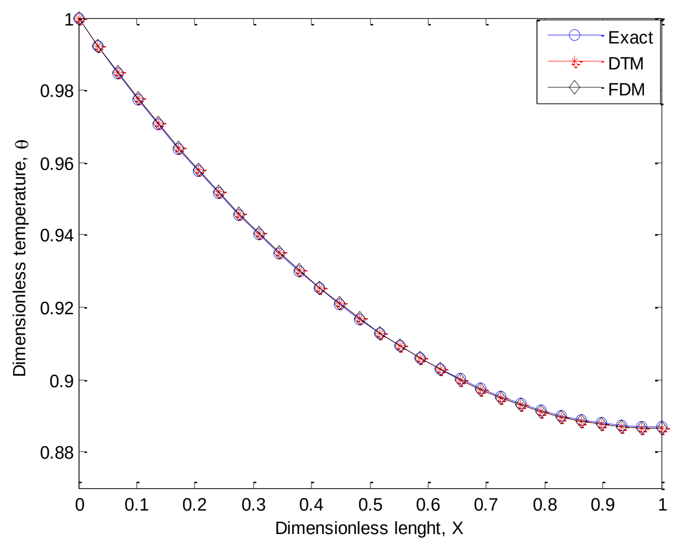


Fig. 4. Heat flux across fin length for varying porosity $M = 0.5$

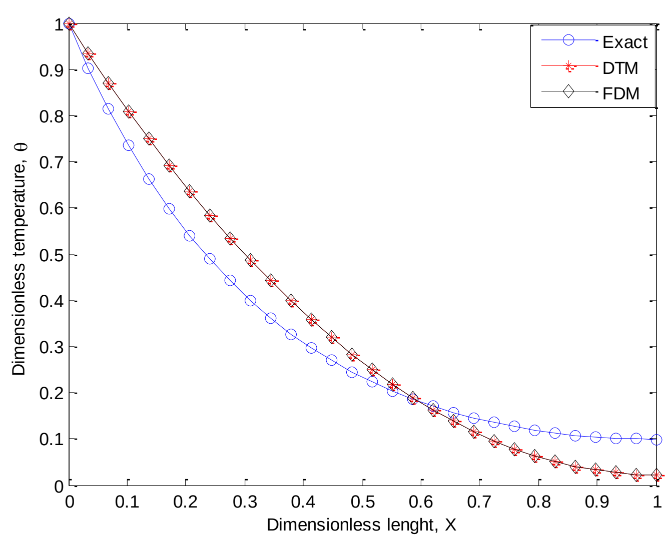


Fig. 5. Heat flux across fin length for varying porosity $M = 1.0$

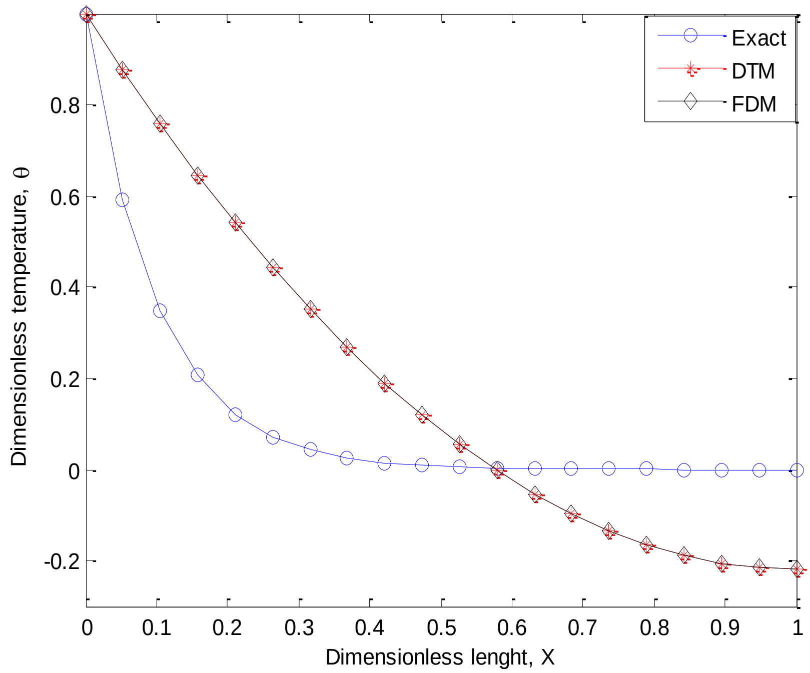


Fig. 6. Heat flux across fin length for varying porosity $M = 10$

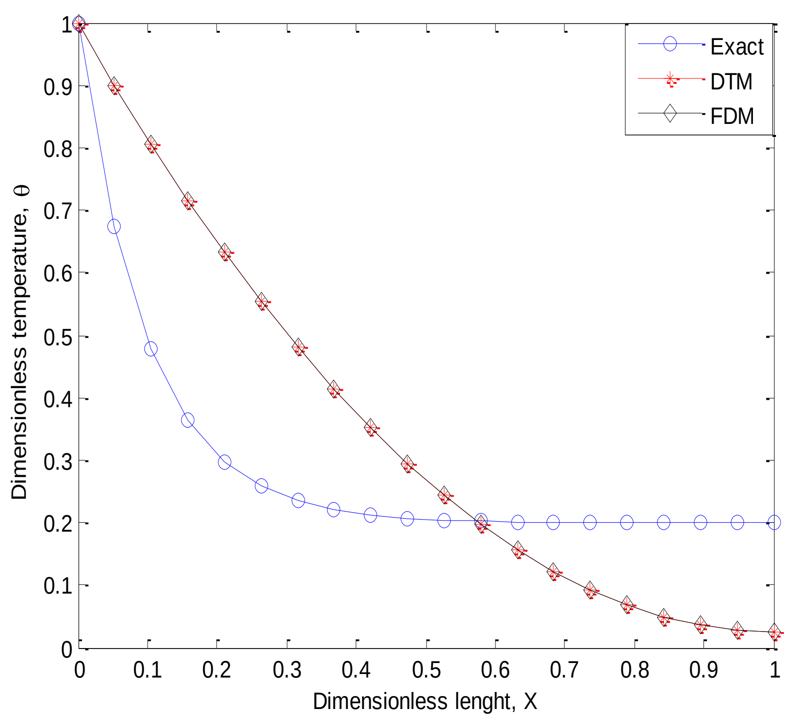


Fig. 7. Heat flux across fin length for varying porosity $M = 10;\ G = 0.2$

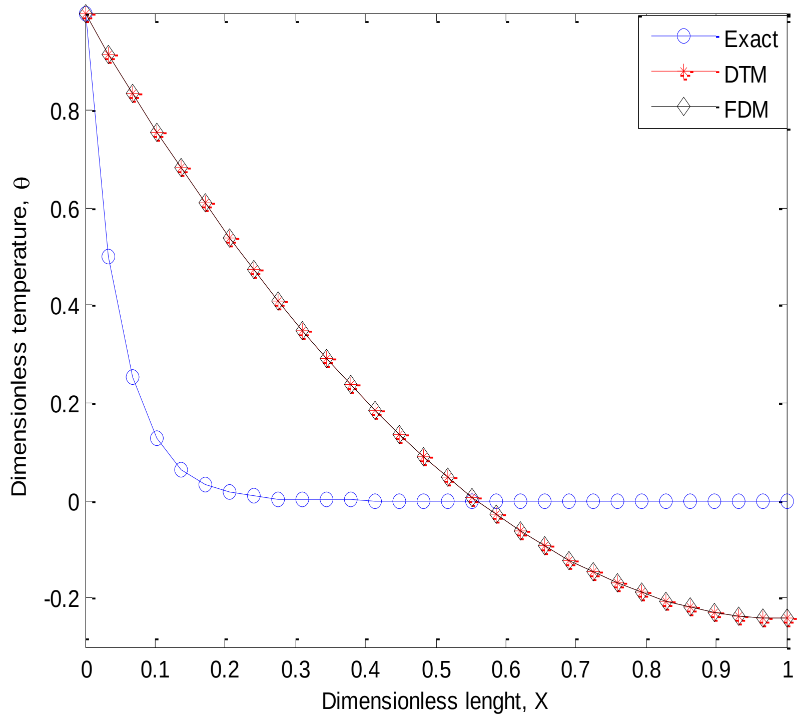


Fig. 8. Heat flux across fin length for varying porosity $M = 20$

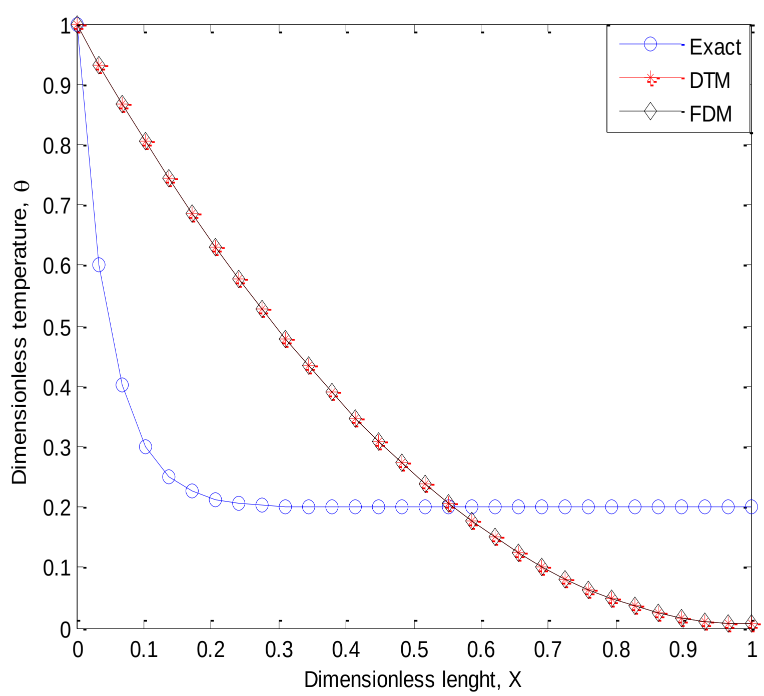


Fig. 9. Heat flux across fin length for varying porosity $M = 20;\ Q = 0.2$

From Fig. 4, it is depicted that the results of differential transform and finite difference methods are very good agreement with the results of the exact analytical method. However, this only happen in the range of thermal stability of the fin for the linear problem. As the value of the thermo-geometric parameter, M increases, the differences between the results become widened. Also, it should be added that at the specific value of the thermo-geometric parameter, M the tip temperature will eventually drop to the ambient fluid temperature. However, when the thermo-geometric parameter exceeds this value, the dimensionless temperature distribution results in negative value (which shows thermal instability) at the tip of the fin, contradicting the assumption at the boundary condition as shown in Fig. 6 and 8. From the analysis, the limiting value of M for thermal for thermal instability to the initially in the fin of constant thermal properties without internal heat generation is $\sqrt{2}$. However, when the temperature-dependent properties and internal heat generation in the fin are considered, the value of M for the thermal stability range increases as shown in Fig 7 and 9. This shows that although, the internal heat generation increases the fin temperature, it can be used to control the thermal instability in the fin. However, this must be done with the caution and proper selection of the operating parameters because when the internal heat parameter increases to some certain values, some negative effects are recorded where the fin stores heat rather than dissipating it. The internal heat generation can produce negative effect which greatly defeats the ultimate purposes of the fin for effective heat dissipation and heat transfer enhancement. Therefore, it is highly recommended that the operational parameters of the extended surface must be properly selected for required purpose.

Also, it should be pointed out that with the agreements of the results of the differential transform and finite difference methods when the fin parameter exceed the specific value, the methods are able to predict the thermal in the extended surface but the exact analytical method only show that its solution is not physically valid for a steady state mode to exist when the thermo-geometric parameter exceeds a specific value. As a consequence, it can no longer satisfy the adiabatic condition prescribed at the fin tip and hence no solution is obtained with the exact analytical method. With the development of an exact analytical solution, Yeh and Liaw [18] had earlier stated that no solution is found when the fin parameter exceeds a specific value, and which is believed to be related to thermal instability.

Therefore, it could be stated that the numerical and the approximate analytical methods are used for deeper understanding to predict the anomalies which are not possible in the exact analytical method. However, the numerical approach is not elegant as approximate analytical solutions. Moreover, it can be stated that the numerical solution will not provide any insight into generalizations.

6. CONCLUSION

In this work, thermal performance analysis in a porous fin temperature-dependent internal heat generation has been analyzed using differential transformation method. The developed symbolic heat transfer models were used to investigate the effects of various parameters on the thermal performance of the porous fin. From the study, it was shown that increase in the internal heat generation leads to increase in the value or the range of the thermal stability of the fin. Therefore, it can be used to control thermal instability in the fin. Furthermore, it was established that a high value or an excessive internal heat generation results in an undesirable situation where some of the heat energy cannot escape to the sink and instead ends up flowing into the prime surface and the fin tends to store heat rather than dissipating it. This scenario defeats the prime purpose of the cooling fin. Therefore, it is highly recommended that the operational parameters must be carefully selected to ensure that the fin retains its primary purpose.

Nomenclature

h heat transfer coefficient, $Wm^{-2}k^{-1}$
h_b heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
c_p specific heat of the fluid passing through porous fin(J/kg-K)
Da Darcy number
g gravity constant(m/s^2)
H dimensionless heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
k thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
k_b thermal conductivity of the fin material at the base of the fin, $Wm^{-1}k^{-1}$
k_{eff} effective thermal conductivity ratio
K permeability of the porous fin (m^2)
L Length of the fin, m
M dimensionless thermo-geometric parameter
m mass flow rate of fluid passing through porous fin (kg/s)
q internal heat generation in W/m^3
Q dimensionless heat transfer rate per unit area
Q_b dimensionless heat transfer rate the base in porous fin
Q_s dimensionless heat transfer rate the base in solid fin
S_h Porosity parameter
t thickness of the fin
T_b base temperature (K)
T fin temperature (K)
T_a ambient temperature, K
v average velocity of fluid passing through porous fin(m/s)
x axial length measured from fin tip (m)
X dimensionless length of the fin
w width of the fin

Greek Symbols

δ thickness of the fin, m
δ_b fin thickness at its base.
γ dimensionless internal heat generation parameter
θ dimensionless temperature
θ_b dimensionless temperature at the base of the fin
υ kinematic viscosity(m^2/s)
ρ density of the fluid(kg/m^3)

Subscripts

solid properties, fluid properties, eff effective porous properties

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