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Robust Optimization Model for Truss Topology Design Problem Using Convex Programming CVX

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ABSTRACT

Topology optimization is one of the optimization applications in the field of infrastructure or truss structure design. Aiming to find the optimal connectivity bar by determining the best node leads to minimizing compliance. Robust optimization is used to conquer the uncertainty of external load parameters that are continuous and convex. The Robust Topology Optimization model uses semidefinite programming with an ellipsoidal uncertainty set. To solve the model, we use a modeling system called CVX, CVX uses the object-oriented features of MATLAB to turn it into an optimization modelling language: optimization variables can be declared and constraints and objectives specified using natural MATLAB syntax. The results of numerical simulations using CVX in the Robust Truss Topology Design (RTTD) model obtained an optimal robust solution, where the truss is resistant to load uncertainty for single-load or multi-load.

***Keywords*:** Truss Topology Design, semidefinite programming, Robust optimization, uncertainty, load, CVX

1. INTRODUCTION

Optimization is defined as a process of finding the best conditions that give the minimum or maximum value of a function that is bounded by certain conditions [1]. Optimization can be

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applied in various fields, one of which is infrastructure design by optimizing a truss design structure, this optimization can be grouped into three categories: optimization of size, shape, and topology [2]. Topology optimization, the most commonly used type, is a mathematical method for optimizing material layout in a particular design space. Aims to find the optimal connectivity bar by determining the best nodes, leading to minimizing weight or increasing the structure of stiffness, strength or dynamic response.

The first article about topology optimization was published in 1904 by the insightful Australian mechanical engineer Mitchell [3]. Mitchell's articel addressed the problem of least-volume topology of trusses with a single condition and a stress constraint. After approximately 70 years it was Rozvany [4] who extended Mitchell's theory from trusses to beam systems and introduced the first general theoritical background of topology optimizaton termed 'optimal layout theory' [5]. The scientific revolution in this field had begun and it has been mainly carried out the last 30 years with many interesting articles. There are three main approaches which deal with the topology optimizaton problem: element-based solution approaches, discrete approaches (evolutionary based algorhtms) and combined approaches [6]. The most known methods of topology optimization are: the solid isotropic material with penalization (SIMP) [7,8] and the evolutionary structural optimization (ESO) or the bi-directional evolutionary structural optimization (BESO). So do other method, gradient-based mathematical method [9], convex linearization [10], method of moving asymtotes [11], non-gradient algorhtms are the successive linear programming (SLP) and the successive quadratic programming (SQP) [12] were developed to support the theory of topology optimization [13].

Topology optimization is also known as the Truss Topology Design problem (TTD problem). Indication of the existence of uncertain parameters in this TTD problem, one of which is the uncertainty of the load, material or geometry. It is difficult if the model cannot be traced computationally, so a new modeling approach with Robust Optimization can be computationally calculated. Aims to find the optimal decision for the worst case of a given set of uncertainties.

Robust Optimization is an optimization method to solve optimization problems with data uncertainty and are only known in a set of uncertainties. The application of Robust optimization to topology optimization is known as Robust Topology Optimization (RTO), which was first studied by Ben-Tal and Nemirovski [14] with a semidefinite programming approach.

Table 1. State of the Art of Robust Truss Topology Design.

No	Authors	Title	Data Uncertainty	Year
1	Ben-Tal and Nemirovski [14]	Robust truss topology design via semidefinite programming	Load	1997
2	Laurent El Ghaoui and Giuseppe Calafiore [15]	Worst-Case Simulation of Uncertain Systems	Discrete-time System	1999
3	C. Roos and D. Chaerani [16]	Conic Optimization, with Applications to (Robust) Truss Topology Design	Load	2002

4	Yoshihiro Kanno and Izuru Takewaki [17]	Worst Case Plastic Limit Analysis of Trusses Under Uncertain loads via Mixed 0-1 Programming	Load	2007
5	Kazuo Yonekura and Yoshihiro Kanno [18]	Global optimization of robust truss topology via mixedinteger semidefinite programming	Load	2010
6	Daniel P. Mohr, Ina Stein, Thomas Matzies and Christina A. Knapek [19]	Robust Topology Optimization of Truss with regard to Volume	Volume	2012
7	C. Roos, Y. Bai and D. Chaerani [20]	Robust Resistance Network Topology Design by Conic Optimization	Electric Current	2012
8	Tristan Gally, Christoper M. Gehb, Philip Kolvenbach, Anja Kuttich, Marc E. Pfetsch and Stefan Ulbrich [21]	Robust Truss Topology Design with Beam Elements via Mixed Integer Nonlinear Semidefinite Programming	Load	2015
9	JianTao Liu and Hae Chang Gea [22]	Robust topology optimization under multiple independent unknown-but-bounded loads	Load	2017
10	Philip Kolvenbach, Stefan Ulbrich, Martin Krech and Peter Groche [23]	Robust Design of a Smart Structure under Manufacturing Uncertainty via Nonsmooth PDE-Constrained Optimization	Manufacturing	2018
11	Sayyed Ali Latifi Rostami and Ali Ghoddosian [24]	Robust Topology Optimization under Load and Geometry Uncertainties by Using New Sparse Grid Collocation Method	Geometry and Applied Load	2019
12	Jannis Greifenstein and Michael Stingl [25]	Topology optimization with worst-case handling of material uncertainties	Material	2020

RTOs are categorized into two, namely RTOs with load uncertainty [22] and RTOs with other uncertainties such as material [25] or geometry [24]. Robust optimization for the Truss Topology Design problem is still very well applied in various uncertainties that occur in a truss and uses the set of ellipsoidal uncertainty as an uncertain parameter characteristic [20, 21].

In the last 10 years, by Publish or Perish Software [26] in Google Scholar database we found that there are 170 papers discuss about RTTD. Using a software for Bibliometric Vos Viewer [27], the research in RTTD topics still growing as we can see in Figure 1

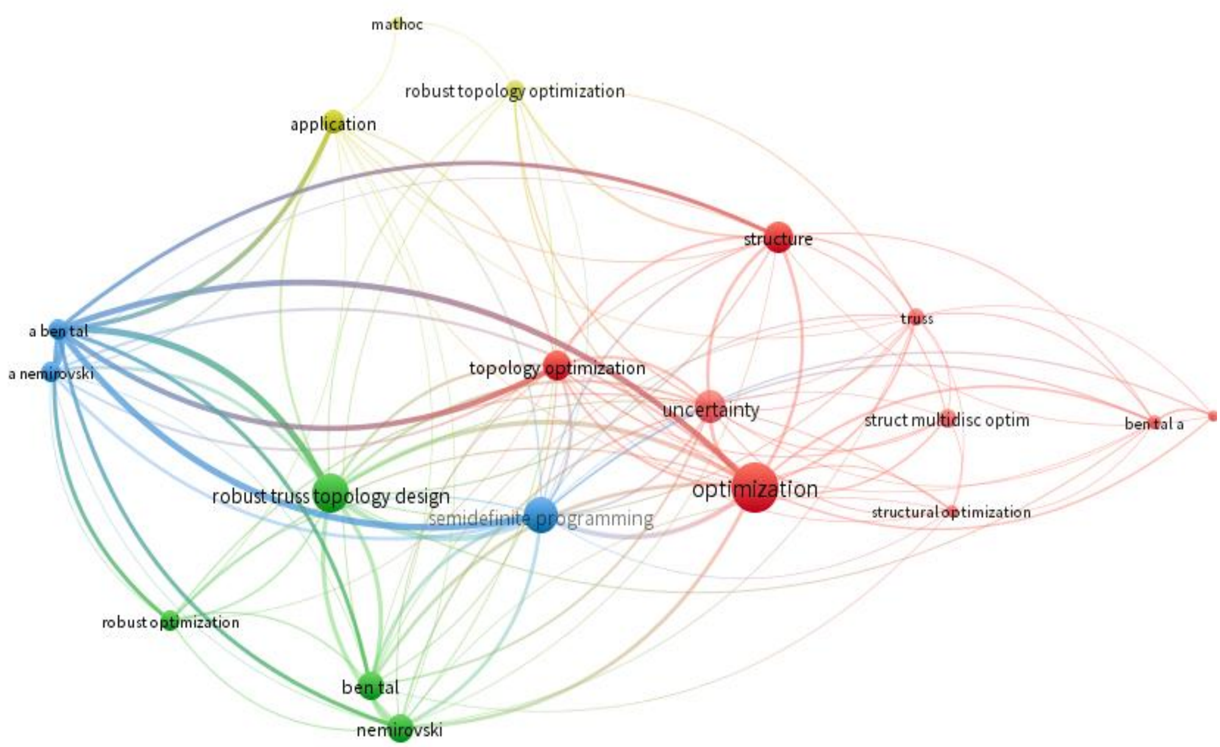

Figure 1. Diagram Bibliometric of Robust Truss Topology Design.

Table 2. Cluster of Diagram Bibliometric.

Cluster 1	Cluster 2	Cluster 3	Cluster 4
ben tal a	ben tal	a ben tal	application
nemirovski a robust truss topology design	nemirovski	a nemirovski	mathoc
optimization	robust optimization	semidefinite programming	robust topology optimization
struct multidisc optim	robust truss topology design		
structural optimization			
structure			

topologi optimization			
truss			
uncertainty			

The diagram shows that RTTD is closely related to optimization, is the application of optimization in the field of structural truss, conquering uncertainty parameters by robust optimization using semidefinite programming and research developments are still being carried out until now.

Whereas for Table 2 shows the cluster of keywords from the most frequently mentioned to the least. By using VosViewer we made a network based on abstracts and the titles of each paper from 170 papers that discussed RTTD.

Robust optimization results global optimal solutions, because basically the problem of topology optimization is a non-convex problem, so it must be approached by convex programming. Optimization of topology that is continuous and convex is one of them using semidefinite programming (SDP) [21, 28, 29].

To solve the RTTD problem with semidefinite programming that is computationally tractable, specified and solved using the modelling system for convex optimization CVX. which enables them to be analyzed and solved efficiently. CVX provides a special SDP mode which allows this LMI convention to be employed inside CVX models using Matlab's standard equality operators >=, <=, etc. [30]. In order to use it, one must simply begin a model with the statement cvx_begin sdp or cvx_begin SDP instead of simply cvx_begin. When SDP mode is engaged, CVX interprets certain inequality constraints in a different manner.

The use of CVX on RTTD issues is still a few article that discusses. Previously Mathias Stolpe [26] used CVX for TTD with fail-safe relaxations and Kucherenko [31] used CVX for topology of truss-like elastic structures. In this paper discusses the formulation of the Robust Truss Topology Design model with the uncertainty of the load parameters in the set of ellipsoidal uncertainties. Topology optimization programming uses the semidefinite programming (SDP) approach so that the convex problem is to formulate a computationally tractable Robust optimization model. Calculation of the solution of numerical simulations is performed using MATLAB software with CVX modeling system.

2. MATERIALS AND METHODS

Formulating and solving the Robust Optimization model for the Truss Topology Design problem needs to be known formerly about the deterministic model of TTD, Robust Optimization and cvx.

2. 1. Truss Topology Design problem

Referring to [32] a topology or a truss formed by a set of nodes $V = \{v_1, v_2, \ldots, v_n\} \subseteq \Re^d$ where fixed nodes static while free nodes $V_f \subseteq V$ are movable and a set of bars $\varepsilon \subseteq V \times V$ that have length L_k, volume t_k and Young's modulus E_k in each bar. The Truss Topology Design problem aims to find the optimal connectivity bar which determines the location of the best node by minimizing the weight or increasing the structural stiffness due to the external load applied.

The external load received by the truss effects in displacement of points u and distortions of the bar, so there is an internal force $f_k = -\frac{E_k}{L_k^2} t_k A^T u$ which A the bar projection matrix $N \times m$, so that the stability of the truss satisfying the equilibrium conditions $Af = -F$ is evaluated using compliance $\frac{1}{2} F^T u$ which is energy or work due to bar distortions is a sum of the forces times displacements. The larger the compliance, the more work of displacement that the truss undergoes under the external loads, so we would like the compliance to be a small number or minimized with the volume bar t_k being the design parameter objectived. Upper and lower bounds on the volumes of certain bars $Mt \le d, t \ge 0$ and overall volume $\sum_{k=1}^{m} t_k \le V$.

$$
\begin{aligned}
&\min_{t,u} \frac{1}{2} F^T u \\
&s.t. K(t)u = F \\
&Mt \le d \\
&t \ge 0 \\
&u \in \Re^N, t \in \Re^m
\end{aligned}
\qquad (1)
$$

where $K(t) = \sum_{k=1}^{m} t_k \frac{E_k}{L_k^2} (a_k)(a_k)^T$ with a_k is the k-column in matrix A.

Problem (1) is a non-convex problem, so that the optimal global solution must be approached by convex programming. In this paper, semidefinite programming will be used that span convex optimization, discrete optimization and control theory [32].

$$
\begin{aligned}
&\min_{t,\theta} \theta \\
&s.t. \begin{pmatrix} \theta & F \\ F & K(t) \end{pmatrix} \succeq 0 \\
&Mt \le d \\
&t \ge 0 \\
&\theta \in \Re, t \in \Re^m
\end{aligned}
\qquad (2)
$$

2. 2. Robust Optimization

In this section discussion about the theory of Robust Optimization (RO) as proposed by Ben-Tal and Nemirovski [33] and discussed in Den Hertog et al. [34], Gorissen et al. [35], also discussed in Chaerani [36]. Robust Optimization is an optimization method to solve optimization problems with data uncertainty and are only known in a set of uncertainties.

The linear programming problem with uncertainty in Robust Optimization can be formulated as follows:

$$\begin{aligned} &\min_x c^T x \\ &s.t. Ax \le b \\ &(c, A, b) \in U \end{aligned} \tag{3}$$

which data $c \in \Re^n, A \in \Re^{m\times n}, b \in \Re^m$ are uncertain, but are known to reside in an uncertain set U.

Robust Optimization is based on assumptions "decision making environment" i.e., first, decision is offline it means the entire decision vector x is "here-and-now-decision"—the desicion variable must be fixed prior before knowing the actual parameter values. Second, only some the variables $(x_1,..., x_n)$ may be determined after the uncertain parameters become known ("wait and see" decision). Third, information about data (c, A, b) is crude and is in a compact uncertainty set U. Fourth, $Ax \le b$ are hard constraints, i.e., they must all be satisfied the uncertain parameters in U.

If there are uncertainties in the objective function, then the problem can be formulated by replacing the objective function with a single variable function such that uncertainty arises in a constraint function.

$$\begin{aligned} &\min_{x,t} t \\ &s.t. c^T x - t \le 0 \\ &a_i^T x - b_i \le 0, i = 1,\dots,m \\ &\forall (c, A, b) \in U \end{aligned} \tag{4}$$

Refers to [34] in dealing with Robust Linear Optimization (RLO) assume, firstly, the objective $c^T x$ is certainly valuable, see (4). Secondly, the right-hand-side b is certain, if b is uncertain, an extra variable x_{n+1} can be introduced,

$$\begin{aligned} &\min_x c^T x \\ &s.t. a_i^T x - b_i x_{n+1} \le 0, i = 1,\dots,m \\ &\forall (A, b) \in U \end{aligned} \tag{5}$$

this assumption makes the notation for uncertainty region and the resulting robust counterpart easier. Third, the robustness of U can be formulated as constraint-wise problem and the last, the uncertainty set U is closed and convex. Refers to Den Hertog et al. [34], consider the following canonical robust semi-infinite constraint

$$a^T x - b \le 0, \forall (A, b) \in U \tag{6}$$

here a is a vector in $\Re^n$ and b which are the general representatives of a_i and b_i. Similarly, U can represent U_i. Describe the uncertainty parameters a, b and the uncertainty set U in the form of a primitive factor $\zeta \in \Re^L$. Namely,

$$a = \bar{a} + Q\zeta, b = \bar{b} + q^T\zeta \tag{7}$$

where $\bar{a} \in \Re^n, Q \in \Re^{n\times L}, \bar{b} \in \Re, q \in \Re^L$ and

$$U = \left\{ \begin{pmatrix} a = \bar{a} + Q\zeta \\ b = \bar{b} + q^T\zeta \end{pmatrix} : \zeta \in Z \right\} \tag{8}$$

where $Z \subset \Re^L$ is the uncertainty set for the primitive factors. The vector $\bar{a}$ and the scalar $\bar{b}$ called nominal. Representation (8) is an alternative formulation of (6). By replacing a and b in the ζ expression, it is obtained:

$$\left(\bar{a}^T x - \bar{b}\right) + \left(Q^T x - q\right)^T \zeta \le 0, \forall \zeta \in Z \tag{9}$$

Theorem 1 (Ben-Tal and Nemirovski [37]): Assume that uncertainty set U in (5) is given as the affine image of a bounded set $Z = \{\zeta\} \subset \Re^L$ and Z is given
either

(i) by a system of linear inequality constraints

$$P\zeta \le p$$

or

(ii) by a system of Conic Quadratic inequalities

$$\|P_i\zeta - p_i\|_2 \le q_i^T\zeta - r_i, i = 1,\ldots,m$$

or

(iii) by a system of Linear Matrix Inequalities

$$P_0 + \sum_{i=1}^{\dim \zeta} \zeta_i P_i \ge 0$$

In the cases (ii), (iii) assume also that the system of contraints defining U is strictly feasible. Then the robust counterpart (4) of the uncertain LP (5) is equivalent to

- a Linear Programming problem in case (i),
- a Conic Quadratic problem in case (ii),
- a Semidefinite program in case (iii).

Table 3 refers to Gorissen et al. [35] presents the tractable robust counterparts of an uncertain linear optmization problems for different classes of uncertainty sets.

Table 3. Tractable reformulations for the uncertain constraints $\left[(\bar{a}+Q\zeta)^T x \le 0, \forall \zeta \in Z\right]$ for different types of uncertainty sets

Uncertainty Set	Z	Robust Counterpart	Tractability
Box	$\lVert\zeta\rVert_\infty \le 1$	$a^T x + \lVert P^T x\rVert_1 \le b$	LP
Ellipsoidal	$\lVert\zeta\rVert_2 \le 1$	$a^T x + \lVert P^T x\rVert_2 \le b$	CQP
Polyhedral	$D\zeta + q \ge 0$	$\begin{cases} a^T x + q^T w \le b \\ D^T w = -P^T x \\ w \ge 0 \end{cases}$	LP

2. 3. Formulation of Robust Truss Topology Design (RTTD) Model in the Set of Ellipsoidal Uncertainties

Refers to C. Roos et al. [20], RTTD model formulation is done to carry the uncertainty of external loads received by the truss so that $F \in U$. Note the constraints in (2) which contain uncertain parameters.

$$\begin{pmatrix} \theta & F \\ F & K(t) \end{pmatrix} \succeq 0 \tag{10}$$

describe the uncertainty parameters F the uncertainty set U in terms of a primitive factor $\zeta \in Z, Q \in \Re^{n \times L}$ in the form of affine image $F = \bar{F} + Q\zeta$ so as $U = \left(\left\{F = \bar{F} + Q\zeta\right\} : \zeta \in Z\right)$ where $Z \subset \Re^L$ is the uncertainty set for the primitive factors. The the scalar $\bar{F}$ will thereafter be called nominal.

$$\begin{pmatrix} \theta & (\bar{F} + Q\zeta)^T \\ \bar{F} + Q\zeta & K(t) \end{pmatrix} \succeq 0 \tag{11}$$

Equivalently

$$\begin{aligned} &\begin{pmatrix} \alpha & \beta \end{pmatrix} \begin{pmatrix} \theta & (\bar{F} + Q\zeta)^T \\ \bar{F} + Q\zeta & K(t) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \ge 0 \\ &\Rightarrow \alpha^2\theta + 2\alpha(\bar{F} + Q\zeta)^T \beta + \beta^T K(t)\beta \ge 0, \forall \alpha \in \Re, \forall \beta \in \Re^m \\ &\Rightarrow \alpha^2\theta + 2\alpha(\bar{F})^T \beta + \beta^T K(t)\beta + 2\alpha\zeta^T Q^T \beta \ge 0, \forall \alpha \in \Re, \forall \beta \in \Re^m \end{aligned} \tag{12}$$

Assume that uncertain parameters are in the set of ellipsoidal uncertainty. The set of ellipsoidal uncertainty is defined as follows:

$$Z = \{\zeta : \|\zeta\|_2 \le 1\} \tag{13}$$

Robust Counterpart formulation for constraints with uncertain parameters is in the set of ellipsoidal uncertainty

$$\begin{aligned} &(\bar{a} + P\zeta)^T x \le b, \forall \zeta : \|\zeta\|_2 \le 1 \\ &\equiv \bar{a}x + \min_{\zeta:\|\zeta\|_2 \le 1} (P^{tT} x)^T \zeta \le b \\ &= \bar{a}x - \|P^T x\|_2 \le b \end{aligned} \tag{14}$$

where $\bar{a} \in \Re^n$ vector nominal, $P \in \Re^{n\times L}$ confounding matrix and $\zeta \in \Re^L$ vector primitive uncertainties.

Assume that uncertainty are in the set of ellipsoidal uncertainty,

$$U = (\{F = \bar{F} + Q\zeta\} : \zeta : \zeta^T \zeta \le 1) \tag{15}$$

Since the set F is infinite, we meet a difficulty not present in the case of finite F, namely that objective now is to minimize the supremum of infinitely many semidefinite representable (SDR) functions [20], so that (12) equivalent to

$$\begin{aligned} &\alpha^2\theta + 2\alpha(\bar{F})^T \beta + \beta^T K(t)\beta + 2 \min_{\zeta:\zeta^T\zeta \le 1} \alpha\zeta^T Q^T \beta \ge 0, \forall \alpha \in \Re, \forall \beta \in \Re^m \\ &\Rightarrow \alpha^2\theta + 2\alpha(\bar{F})^T \beta + \beta^T K(t)\beta - 2\|\alpha Q^T \beta\|_2 \ge 0, \forall \alpha \in \Re, \forall \beta \in \Re^m \\ &\Rightarrow \alpha^2\theta + 2\alpha(\bar{F})^T \beta + \beta^T K(t)\beta - 2\|\alpha\|_2 \|Q^T \beta\|_2 \ge 0, \forall \alpha \in \Re, \forall \beta \in \Re^m \end{aligned} \tag{16}$$

We now give a second order representation for the final negative term in the above inequality, which, combined with the Schur Complement representation of an SOCP,

Lemma 1. Schur Complement

Let $A = \begin{pmatrix} B & C^T \\ C & D \end{pmatrix}$ be a symmetric matrix with $k \times k$ block B and $l \times l$ block D. Assume that B is possitive (semi-)definite. Then $D - CB^{-1}C^T$ is called the Schur complement of B in . One has

$$A \succeq 0 \Leftrightarrow D - CB^{-1}C^T \succeq 0$$

gives us a linear matrix inequality that is equivalent to the robust inequality. Indeed, note that

$$\sup\left\{\eta^T Q^T \beta \mid \|\eta\|_2^2 \le \|\alpha\|_2^2\right\} = \|\alpha\|_2 \left\|Q^T \beta\right\|_2 \tag{17}$$

So we have that (16) is equivalent to

$$\alpha^2\theta + 2\alpha\left(\overline{F}\right)^T \beta + \beta^T K(t)\beta + 2\eta^T Q^T \beta \ge 0, \forall \alpha \in \Re, \forall \beta \in \Re^m, \forall\left(\eta : \eta^T\eta \le \alpha^2\right) \tag{18}$$

for all α, β, η:

$$\eta^T \eta \le \alpha^2 \Rightarrow \alpha^2\theta + 2\alpha\left(\overline{F}\right)^T \beta + \beta^T K(t)\beta + 2\eta^T Q^T \beta \ge 0 \tag{19}$$

Lemma 2. S-Lemma

Let A, B be symmetric $n \times n$ matrices, and assume that $x^T Ax > 0$ holds for some x. Then

$$x^T Ax \ge 0 \Rightarrow x^T Bx \ge 0$$

holds for all x if and only if there exists a $\lambda \ge 0$ such that $B \succeq \lambda A$.
Based on *S*-Lemma, the inequality (19) if and only if there is $\lambda \ge 0$

$$\begin{gathered} \alpha^2\theta + 2\alpha\left(\overline{F}\right)^T \beta + \beta^T K(t)\beta + 2\eta^T Q^T \beta \ge \lambda(\alpha^2 - \eta^T\eta) \\ \Rightarrow \alpha^2(\theta - \lambda) + 2\alpha\left(\overline{F}\right)^T \beta + \beta^T K(t)\beta + 2\eta^T Q^T \beta + \lambda\eta^T\eta \ge 0 \end{gathered} \tag{20}$$

that is, we must have that

$$\begin{pmatrix}\alpha & \beta & \eta\end{pmatrix} \begin{pmatrix} \theta - \lambda & \left(\overline{F}\right)^T & 0 \\ \overline{F} & K(t) & Q \\ 0 & Q^T & \lambda I \end{pmatrix} \begin{pmatrix}\alpha \\ \beta \\ \eta\end{pmatrix} \ge 0 \tag{21}$$

So that the Robust Counterpart model with the Ellipsoidal Uncertainty set for the Truss Topology Design problem is as follows,

$$\begin{aligned} &\min_{t,\theta} \theta \\ &s.t. \begin{pmatrix} \theta - \lambda & \left(\overline{F}\right)^T & 0 \\ \overline{F} & K(t) & Q \\ 0 & Q^T & \lambda I \end{pmatrix} \succeq 0 \\ &Mt \le d \\ &t \ge 0 \\ &\theta \in \Re, t \in \Re^m \end{aligned} \tag{22}$$

3. RESULT AND DISCUSSION

Calculation of the solution of numerical simulations is performed using Matlab software with cvx modeling system.

3. 1. Result and Discussion: The CVX Matlab Codes for Truss Topology Design Problem

The structural analysis and the optimization problem modelling are entirely implemented in Matlab version 2019a (The Mathworks Inc 2019). The optimization problem are specified and solved using the modelling system for convex optimization, CVX. The problem of convex optimization numerically solved using SeDuMi. Default parameters precision flag is set to high in CVX (cvx_precision high) is scaled in the computations such that the convex optimization problems can be numerically solved to high accuracy, and this information is passed to the solver. Refers to [30] CVX is a modelling system for disciplined convex programming. Disciplined convex programs, or DCPs, is a methodology for constructing convex optimization problems proposed by Michael Grant, Stephen Boyd, and Yinyu Ye, that are described using a limited set of construction rules, which enables them to be analyzed and solved efficiently. CVX can solve standard problems such as linear programs (LPs), quadratic programs (QPs), second-order cone programs (SOCPs), and semidefinite programs (SDPs); but compared to directly using a solver for one or these types of problems, CVX can greatly simplify the task of specifying the problem. This version of CVX supports two core solvers, SeDuMi and SDPT3, which is the default, are open-source interior-point solvers written in Matlab.

We discuss how we generate a code with CVX for TTD problem. We present two cases, i.e., the original problem without uncertainty and the case for robust relaxation problem.

3. 1. 1. The Code for The TTD Nominal Problem

The following is the CVX Matlab Codes to calculate the TTD problem without data uncertainty,

```
cvx_begin
    variables theta a(m);
    k = length(F);
    Z = zeros(k, k);
    for i = 1:m
        Z = Z + a(i)*E(i)*(L(i)^(-2))*(A(:, i)*A(:, i)');
    end
    Q = [theta, F'; F, Z];
    minimize( theta )
    subject to
        Q == semidefinite(k+1);
        L'*a <= V;
        a >= 0;
        M*a <= d;
cvx_precision high
cvx_solver sedumi
cvx_end
```

3. 1. 2. The Code for Robust Relaxations

For the case of uncertainty involved, the CVX codes the following,

```
cvx_begin
        variables theta a(m) lambda;
        lambda>=0;
        k = length(F);
        Z = zeros(k, k);
        Q = sprand(F);
        for i = 1:m
                Z = Z + a(i)*E(i)*(L(i)^(-2))*(A(:, i)*A(:, i)');
        end
        C = [theta-lambda F' 0; ...
                        F Z Q; ...
                        0 Q' lambda*1];
        minimize( theta )
        subject to
                C == semidefinite(k+2);
                        L'*a <= V;
                        a >= 0;
                        M*a <= d;
cvx_precision high
cvx_solver sedumi
cvx_end
```

3. 2. Numerical Simulation for First Case

As an example for numerical simulation, the data is used in this paper is taken from Mathias Stolpe [28], the third set of examples is based on a design domain with dimensions 1 x 1, the ground structure consists of 9 x 9 equidistant nodes. There are 81 nodes with fixed points on the left side of the truss (wall), 272 bars and the load at the bottom right of the truss is 1N vertical to the bottom. The volume limit is 10% of the total volume on the truss and Young's modulus is 100. The 9 x 9 truss is as follows,

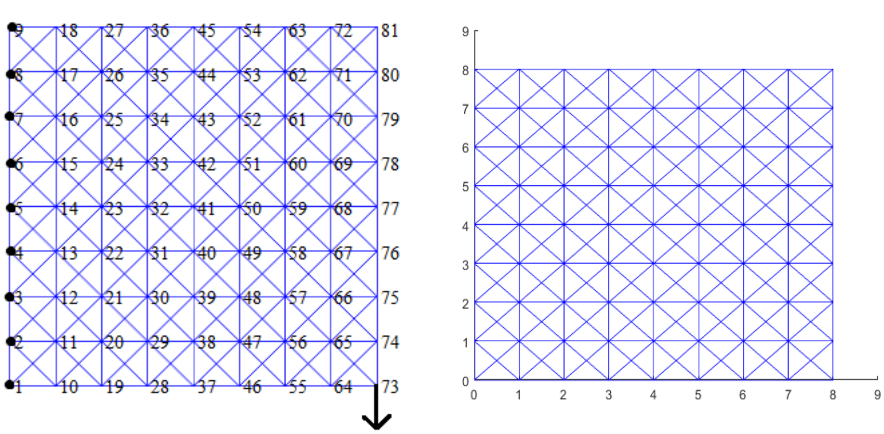

Figure 2. First Case

Obtained optimal compliance and volume from the results of numerical calculations RTTD with ellipsoidal uncertainty set for the first case

$$\begin{aligned}
&\min_{t,\theta} \theta \\
&s.t. \begin{pmatrix} \theta-\lambda & (\overline{F})^T & 0 \\ \overline{F} & K(t) & Q \\ 0 & Q^T & \lambda I \end{pmatrix} \succeq 0 \\
&Mt \le d \\
&t \ge 0 \\
&\theta \in \Re, t \in \Re^m
\end{aligned} \qquad \text{(RTTD)}$$

in Table 4,

Table 4. Results of Numerical Calculations RTTD for First Case.

N_x	N_y	Bars	Q	t^*		θ^*	
				Nominal	Robust	Nominal	Robust
9	9	272	Matrix Q sized 144x1 (128,1) 0.9528	$t^* = \begin{pmatrix} 0.0000 \\ 2.0000 \\ 0.0000 \\ \vdots \\ 0.6607 \\ 0.0000 \\ 0.6607 \\ 2.0000 \\ 0.7857 \\ \vdots \\ 0.0000 \\ 0.0000 \end{pmatrix}$	$t^* = \begin{pmatrix} 0.0000 \\ 2.0000 \\ 0.0000 \\ \vdots \\ 0.6607 \\ 0.0000 \\ 0.6607 \\ 2.0000 \\ 0.7857 \\ \vdots \\ 0.0000 \\ 0.0000 \end{pmatrix}$	0.1382	0.5270

and obtained the optimal design for the first case as follows,

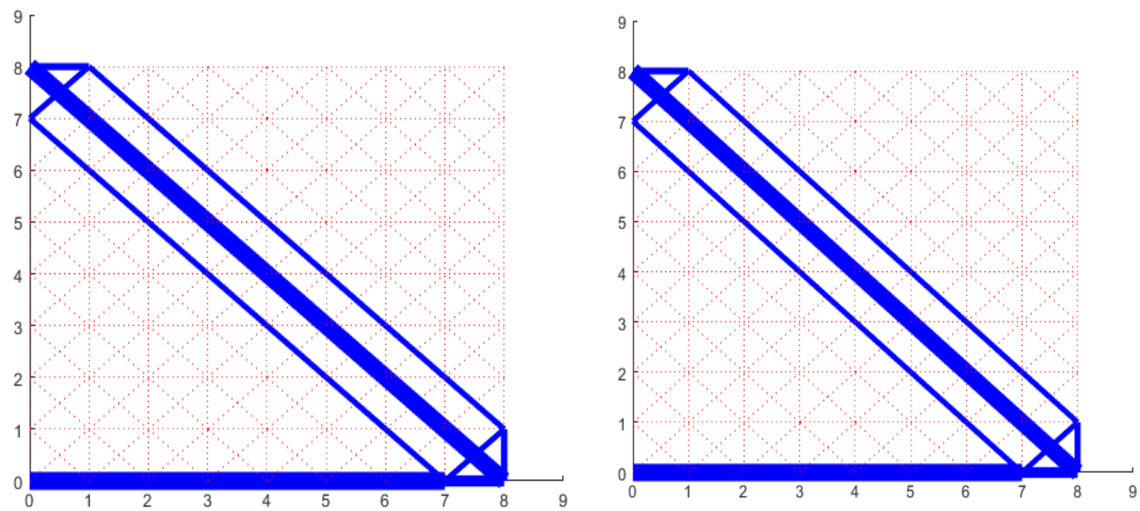

Figure 3. Results of Optimal Design for The First Case, Nominal (left) and Robust (right).

3. 3. Numerical Simulation for Second Case

As an example for numerical simulation, the data is used in this paper is taken from Hao Yi Ong and Conrad Stansbury [38], applied to the problem of designing a 2-D bridge. There are 32 nodes with two fixed nodes in the left and right bottom corner of the truss, 94 bars and a load at each node under the truss is 1N vertical to the bottom. The volume limit is 10% of the total volume on the truss and Young's modulus is 100. The 8 x 4 truss is as follows,

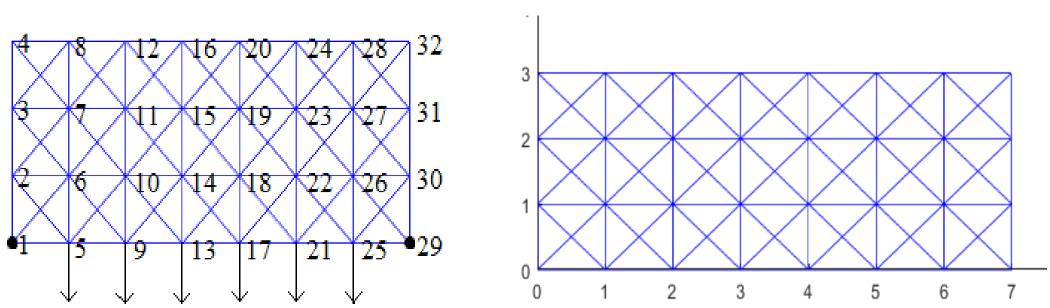

Figure 4. Second Case

Obtained optimal compliance and volume from the results of numerical calculations RTTD with ellipsoidal uncertainty set for the second case

$$\begin{aligned}
&\min_{t,\theta} \theta \\
&s.t. \begin{pmatrix} \theta-\lambda & (\bar{F})^T & 0 \\ \bar{F} & K(t) & Q \\ 0 & Q^T & \lambda I \end{pmatrix} \succeq 0 \\
&Mt \le d \\
&t \ge 0 \\
&\theta \in \Re, t \in \Re^m
\end{aligned} \qquad \text{(RTTD)}$$

in Table 5,

Table 5. Results of Numerical Calculations RTTD for Second Case.

N_x	N_y	Bars	Q	t^*		θ^*	
				Nominal	Robust	Nominal	Robust
8	4	94	Matrix Q sized 60x1 (10,1) 0.9528 (18,1) 0.7041 (26,1) 0.9539 (34,1) 0.5982 (42,1) 0.8407 (50,1) 0.4428	$t^* = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 2.0000 \\ \vdots \\ 0.3700 \\ 0.1184 \\ 0.2996 \\ 0.1329 \\ 0.6689 \\ \vdots \\ 0.0000 \\ 0.0000 \end{pmatrix}$	$t^* = \begin{pmatrix} 0.0884 \\ 0.0000 \\ 2.0000 \\ \vdots \\ 0.2983 \\ 0.0997 \\ 0.3123 \\ 0.0997 \\ 0.6341 \\ \vdots \\ 0.0000 \\ 0.0000 \end{pmatrix}$	1.0119	3.1668

and obtained the optimal design for the second case as follows,

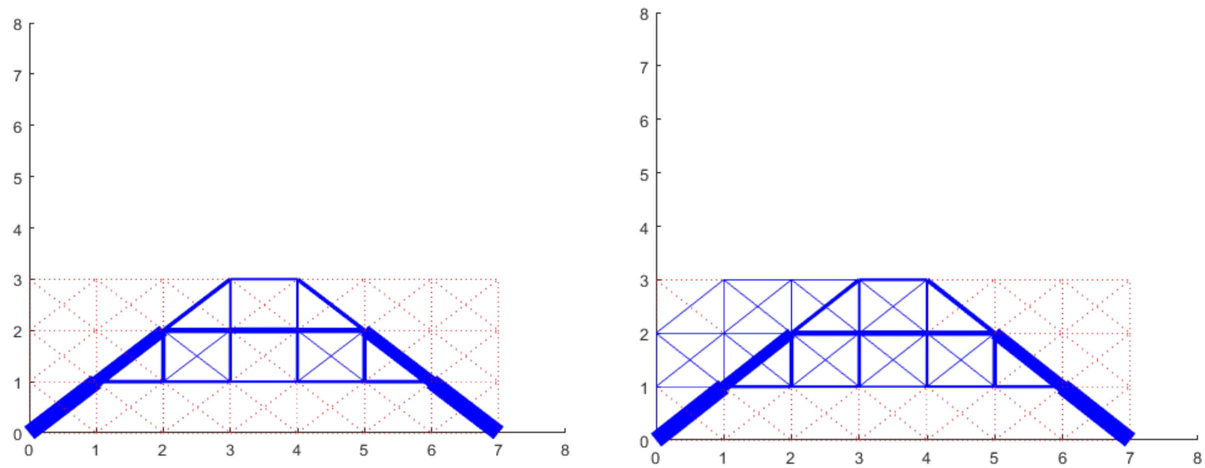

Figure 5. Results of Optimal Design for The First Case, Nominal (left) and Robust (right).

4. CONCLUSIONS

Truss Topology Design problem is a problem that aims to find the optimal connectivity bar by determining the best node so that a feasible truss is obtained. Optimizing a truss with a certain load can be done with a convex programming approach even though the origin of the TTD problem is non-convex. Semidefinite programming is used both for trusses that accept single and multiple loads. To conquer the indication of parameter uncertainty, the Robust Truss Topology Design (RTTD) optimization model for single-load and multi-load cases with uncertain load parameters in the Ellipsoidal Uncertainty Set, provides robust results for uncertainties in load and computationally tractable which the truss design is resistant to load uncertainty and obtained optimal compliance. Numerical experimental using modelling system CVX which enables them to be analyzed and solved efficiently.

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