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Concepts Arising from Strong Efficient Domination Number. Part II

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ABSTRACT

Let G = (V,E) be a simple graph. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every v$\in V(G), |N_s[\boldsymbol{v}] \cap S| = 1.$ ($|N_w[v] \cap S| = 1$), where $N_s(v) = \{u \in V(G): uv \in E(G), degu \geq degv\}(N_w(v)\{u \in V(G), uv \in E(G), degv \geq degu\}$. The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and denoted by $\gamma_{se}(G)\big(\gamma_{we}(G)\big)$. The strong efficient non bondage number $b_{sen}(G)$ is the maximum cardinality of all sets of edge $X \subseteq E$ such that $\gamma_{se}(G - X) = \gamma_{se}(G)$. In this paper, the strong efficient non bondage number of some corona related graphs are studied.

***Keywords*:** Domination, strong efficient domination, strong efficient non bondage number

AMS Subject Classification (2010): 05C69

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1. INTRODUCTION

Throughout this paper is finite, undirected graphs without loops or multiple edges are considered. Let G = (V, E) be a simple graph. The concept of strong efficient domination in graphs was introduced by N. Meena et al. The concept of non bondage number was introduced by V.R. Kulli. The concept of strong efficient non bondage number was introduced by N. Meena. In this paper, strong efficient non bondage numbers of some corona related graphs and some corona related graphs and some special graphs are studied.

Definition1.1. The strong efficient non bondage number $b_{sen}(G)$ is the maximum cardinality of all sets of edge $X \subseteq E$such that $\gamma_{se}(G-X) = \gamma_{se}(G)$.

Definition1.2. Let G = (V, E) be a simple graph. A subset S of V (G) is called a strong (weak) efficient dominating set of G if for every v∈ $V(G), |N_w[v] \cap S| = 1$

where $N_s(v) = \{u \in V(G): uv \in E(G), degu \geq degv\}$ and $N_s[v] = N_s(v) \cup \{v\}, (N_w(v) = \{u \in V(G): uv \in E(G), degv \geq degu\}$and $N_w[V] = N_w(V) \cup \{v\})$.

Definition1.3. $[P_n: C_m{}^2]$ is a graph obtained from a path P_n by attaching vertex of 2 cycles of length m at each vertex of a path P_n.

Definition1.4. Let G = (V,E) be a graph with V = $S_1 \cup S_2 \cup ... \cup S_t \cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and T = V-∪ S_i. The degree splitting graph of G is denoted by DS(G) and is obtained from G by adding the vertices $W_1, W_2, ..., W_t$ and joining W_i to each vertex of S_i ,1≤ $i \leq t$.

Observation 1.5. 1.$\gamma_{se}(C_n)$ = n for all n∈ N.

2. γ_{se} (G) = 1 if and only if G has a full degree vertex.

3.$\gamma_{se}(K_n)$ = 1,n ≥ 1.

4.$\gamma_{se}(K_{1,n})$ = 1,n≥ 1.

5.$\gamma_{se}(w_n)$ = 1,n≥4.

Definition 1.6. The **Corona** $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

2. MAIN RESULTS

Theorem 2. 1. $b_{sen}(K_{1,n} \odot K_1) = n$, n≥ 1 .

Proof: Let G = $K_{1,n} \odot K_1$. Let V(G) = $\{u, v, u_i, v_i / 1 \leq i \leq n\}$ and E(G) = $\{uv, vv_i, v_i u_i / 1 \leq i \leq n\}$. S = $\{v, u_i / 1 \leq i \leq n\}$ is the unique strong efficient dominating set of G and $|S| = n + 1$. Therefore $\gamma_{se}(G) = n + 1$. Let X = $\{u_i v_i / 1 \leq i \leq n\}$ and $|X| = n$. Then G – X = $K_{1,n} \cup nK_1$. Therefore$\gamma_{se}(G - X) = n + 1 = \gamma_{se}(G)$. Hence $b_{sen}(G) \leq n$. Further it is verified that there is no set X with $|X| \leq n$ such that $\gamma_{se}(G - X) = \gamma_{se}(G)$. Therefore $b_{sen}(G) \geq n$. Hence $b_{sen}(G) = n$.

Example 2.2: Consider the flowing graph $K_{1,3} \odot K_1 - \{u_1 v_1, u_2 v_2, u_3 v_3\}$

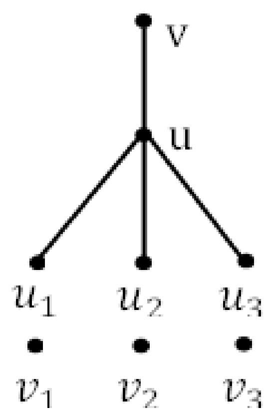

Figure 2.1.

{u,v_1, v_2, v_3} is the unique strong efficient dominating set of G. Here X = $\{u_1 v_1,\ u_2 v_2, u_3 v_3\}$. Then γ_{se}(G – X) = 4 = $\gamma_{se}(G)$. Hence $b_{sen}(K_{1,3} \odot K_1) = 3$.

Theorem 2.3. $b_{sen}(W_n \odot K_1) = 2n, n \geq 3$.

Proof: Let G = $W_n \odot K_1$. Let V(G) = $\{\ v, u, u_i, v_i / 1 \leq i \leq n\}$ and $E(G) = \{uv, vv_i, v_i u_i, v_j v_{j+1}, v_n v_1 / 1 \leq i \leq n, 1 \leq j \leq n - 1\}$. S = $\{u_1 v_1, u_2 v_2, \dots, u_n v_n, v_1 v_2, v_2 v_3 \dots, v_n v_1\}$ is the unique strong efficient dominating set of G and $|S| = 2n$. Therefore $\gamma_{se}(G)\ = n + 1$. Let H be a new graph obtained by deleting the edges of S from $W_n \odot K_1$. Then H = $K_{1,n+1} \cup nK_1$. Hence $\gamma_{se}(H) = 1 + n = \gamma_{se}(G)$. Therefore $b_{sen}(G) \leq 2n$. Further it is verified that there is no set S with $|S| \leq n+1$ such that $\gamma_{se}(H) = 1+n = \gamma_{se}(G)$. Therefore $b_{sen}(G) \geq 2n$. Hence $b_{sen}(G) = 2n$.

Example 2.4. Consider the following graph G = $W_5 \odot K_1$ - {u_1v_1, u_2v_2, u_3v_3, u_4v_4, u_5v_5, u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1}.

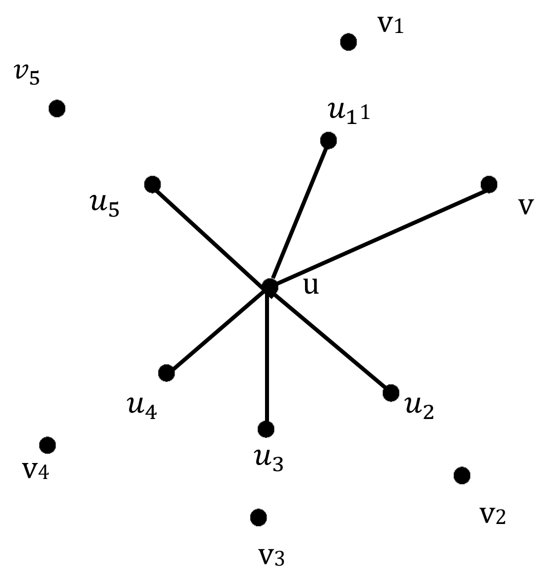

Figure 2.2

{u,v_1, v_2, v_3, v_4, v_5} is the unique strong efficient dominating set of G. $\gamma_{se}(G) = 6$

Here X = $\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, v_1u_1, v_2u_2, v_3u_3, v_4u_4, v_5u_5\}$. Then γ_{se}(G – X) = 6.

Hence $b_{sen}(W_5 \odot K_1) = 10$.

Theorem 2.5. $b_{sen}(D_{r,s} \odot K_1) = r+s+1, r \leq s$.

Proof: Let G = $D_{r,s} \odot K_1$. Let V(G) = $\{u, v, u_0, v_0, u_i, v_j, u_{ik}, v_{jt} / 1 \leq i \leq r, 1 \leq j \leq s, 1 \leq k \leq r, 1 \leq t \leq s\}$ and E(G) = $\{uv, uu_0, vv_0, uu_i, u_iu_{ik}, v_jv_{jt}, vv_j / 1 \leq i \leq r, 1 \leq j \leq s, 1 \leq k \leq r, 1 \leq t \leq s\}$. deg u = r + 2, deg v = s + 2, deg$u_i = degv_j = 2, degu_{ik} =$ deg v_{jt}= 1, deg$u_0 = degv_0 = 1$. S = {v,u_0,u_1,u_2,…,u_r,v_{11},v_{21},….,v_{s1}} is the unique strong efficient dominating set of G. Therefore $\gamma_{se}(G) = 1+s+r+1 = r+s+2$.

Let X = $\{uu_i, uu_0, v_jv_{jt} / 1 \leq ir, 1 \leq j \leq s, 1 \leq k \leq r, 1 \leq t \leq s\}$. Remove the edges of X from G. Then G – X = $K_{1,s+2} \cup rK_2 \cup (s+1)K_1$. Therefore $\gamma_{se}(G-X) = 1+r+s+1 =$

$r + s + 2 = \gamma_{se}(G)$. Hence $b_{sen}(G) \leq r + s + 1$. Further it is verified that there is no set X with $|X| \leq r + s + 1$ such that $\gamma_{se}(G - X) = \gamma_{se}(G)$. Therefore $b_{sen}(G) \geq r + s + 1$. Hence $b_{sen}(G) = r + s + 1$.

Example 2.6. Consider the graph G = $D_{3,5} \odot K_1$ - {uu_0, uu_1, uu_2, uu_3, v_1v_{11}, v_2v_{21}, v_3v_{31}, v_4v_{41}, v_5v_{51}}

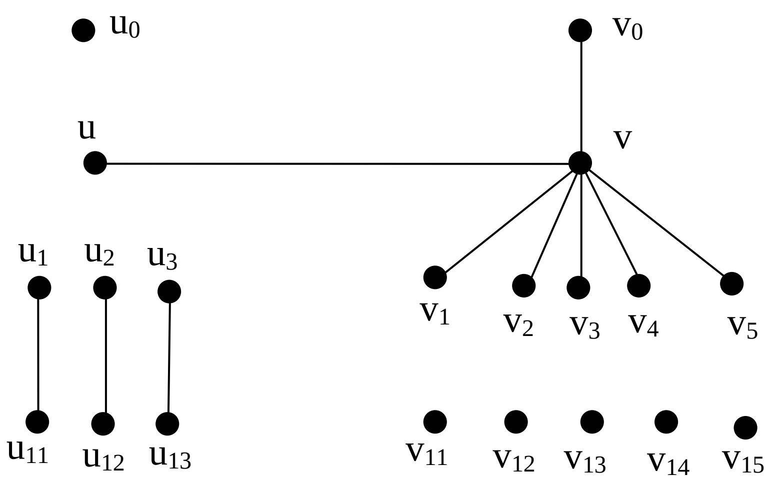

Figure 2.3.

Here {v,u_0,u_1,u_2,u_3,v_{11},v_{21},v_{31},v_{41},v_{51} } is a strong efficient dominating set of G. Therefore γ_{se}(G) = 10. Hence b_{sen}(G) = 9.

Theorem 2.7. $b_{sen}(C_{3n} \odot K_1) = 3n$.

Proof: Let G =($C_{3n} \odot K_1$). Let V(G) = $\{u_i v_i / 1 \leq i \leq n\}$ and $E(G) = \{v_i v_{i+1}, u_j v_j v_{3n} v_1, / 1 \leq i \leq n - 1, 1 \leq j \leq n\}$. $S = \{v_2v_3, v_5v_6, \dots, v_{3n-1}v_{3n}, u_2v_2, \dots, v_{3n-1}u_{3n-1}, v_3u_3, \dots, v_{3n}u_{3n}\}$ is the unique strong efficient dominating set of G and $|S| = 3n$. Therefore $\gamma_{se}(G) = 3n$. Let H be a new graph obtained by deleting the edges of S from $C_{3n} \odot K_1$. Now H = n$K_{1,3} \cup 2nK_1$. Hence $\gamma_{se}(H) = n + 2n = 3n = \gamma_{se}(G)$. Therefore $b_{sen}(G) \leq 3n$. Further it is verified that there is no set S with $|S| \leq 3n$ such that $\gamma_{se}(H) = \gamma_{se}(G)$. Therefore $b_{sen}(G) \geq 3n$. Hence $b_{sen}(G) = 3n$.

Example 2.8. Consider the following graph G = $C_6 \odot K_1 - \{u_2u_3, u_6u_5, v_6u_6, v_5u_5, v_2u_2, u_3v_3\}$.

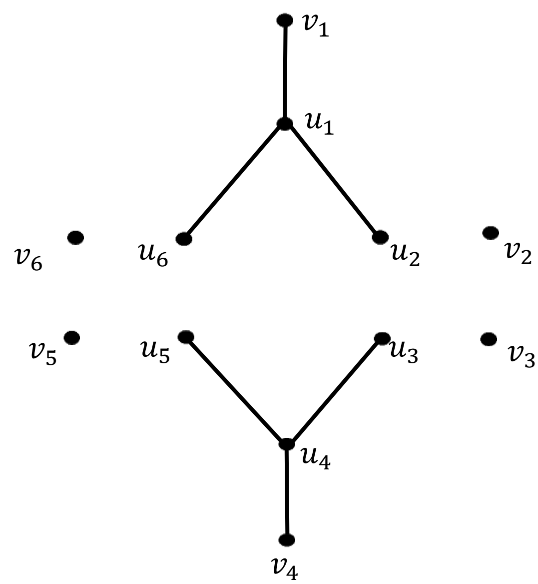

Figure 2.4

Here $\{u_1, u_4, v_2, v_3, v_6, v_5\}$ is a unique strong efficient dominating set of G. Therefore $\gamma_{se}(G) = 6$. Let X = $\{u_2u_3, u_6u_5, v_6u_6, v_5u_5, v_2u_2, u_3v_3\}$. Then $\gamma_{se}(G - X) = 6$. Hence $b_{sen}(C_6 \odot K_1) = 6$.

Theorem 2.9. Let G = DS(P_n). Then b_{sen}(G) = n-1, n ≥ 5.

Proof: Let V(P_n) = $\{v_i / 1 \leq i \leq n\}$. deg$v_1 = deg v_n = 1$, deg$v_i$ = n, $2 \leq i \leq n - 1$. Let S_1 = $\{v_1, v_n\}$ and $S_2 = \{v_2, v_3, \ldots, v_{n-1}\}$. Then $S_1 \cup S_2 = V(P_n)$. Let w_1 and w_2 be two new vertices. Let G = DS(P_n). Then V(G) = $V(P_n) \cup \{w_1, w_2\}$. Let T =$\{w_1 v_i, w_2 v_j / i = 1, n, 2 \leq j \leq n - 1\}$. Then E(G) = $E(P_n) \cup T$. $\{w_1, w_2\}$ is the unique dominating set of G. Therefore $\gamma_{se}(G) = 2$. Let X = E(P_n). Then G – X = $K_{1,n-2} \cup K_{1,2}$. Therefore $\gamma_{se}(G - X) = \gamma_{se}(G) = 2$. Hence $b_{sen}(G) \geq n - 1$. If the edges of T are removed, then w_1 and w_2 will be isolated. That is, n edges are removed and $\gamma_{se}(P_n) + 2 > \gamma_{se}(G)$. Therefore $b_{sen}(G) \leq n - 1$. Hence $b_{sen}(G) = n - 1$.

Example 2.10. Consider the following graph G = D S(P_5) $- \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$

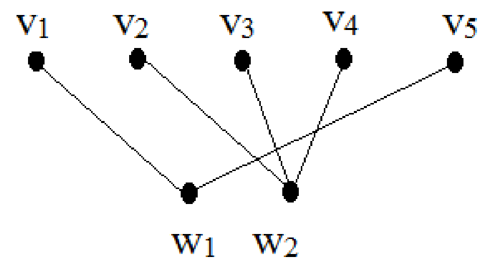

Figure 2.5.

Here $\{w_1, w_2\}$ is the unique strong efficient dominating set of G. $\gamma_{se}(G) = 2$. Let X = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$. Then $\gamma_{se}(G - X) = 2$. Hence $b_{sen}(\mathrm{DS}(P_5))$ = 4.

Remark 2.11. $b_{sen}(DS(P_2)) = 1$**.**

Proof. DS(P_2) = C_3. Therefore $\gamma_{se}(C_3) = 1$. Let e be any edge of C_3. Then C_3 −e = $K_{1,2}$. Therefore $\gamma_{se}(C_3 - e) = 1 = \gamma_{se}(C_3)$. Hence $b_{sen}(DS(P_2)) = 1$.

Remark 2.12. $DS(P_3) = C_4$ which has no strong efficient dominating set.

Remark 2.13. $DS(P_4)$ – $\{v_1v_2, v_2v_3, v_3v_4\}$ is given in the following figure.

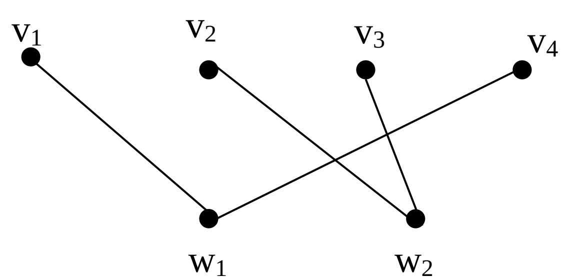

Figure 2.6

$\{v_3, v_1\}$ and $\{v_2, v_4\}$ are strong efficient dominating sets of G, where G = D $S(P_4)$. Therefore $\gamma_{se}(G) = 2$. Let X = $\{v_1v_2, v_2v_3, v_3v_4\}$. Then G-X = $2K_{1,2}$ = 2. Therefore $\gamma_{se}(G - X) = 2$. Hence $b_{sen}(G) = 3$.

Theorem 2.14. Let G = D S(K_n). Then $b_{sen}(G) = \frac{n(n-1)}{2}$, n ≥ 2.

Proof: Let G = D S(K_n) $= K_{n+1.}$V(K_n) $= \{v_i/1 \leq i \leq n\}$. Let w be a new vertex. V(G) = {w, v_i / 1 ≤ i ≤ n} and E(G) = E(K_n) ∪ {wv_i / 1 ≤ i ≤ n}. {w} is the unique strong efficient dominating set of G. Therefore $\gamma_{se}(G) = 1$. Let X = E(K_n). Then $G - X = K_{1,n.}$. Therefore $\gamma_{se}(G - X) =$ 1 = $\gamma_{se}(G)$ and $|X| = \frac{n(n-1)}{2}$. Since we have $\gamma_{se}(G - X) = 1$, there must be at least one full degree vertex. The graph must be $K_{1,n.}$ Hence $b_{sen}(G) = \frac{n(n-1)}{2}$.

Example 2.15. Consider the graph G =DS(K_4) – $\{u_1u_2, u_2u_3, u_3u_4, u_4u_1, u_1u_3, u_2u_4\}$.

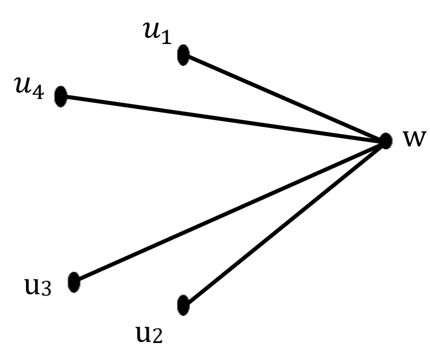

Figure 2.7

{w} is the unique strong efficient dominating set of G. Therefore $\gamma_{se}(G) = 1$. Hence b_{sen}(DS(K_4)) = 6.

Theorem 2.16. Let G = DS(C_n). Then b_{sen}(G) = n, n≥ 3.

Proof: Let G = DS(C_n). Let V(C_n) ={v_i/1≤ I ≤ n}. deg v_i=2,1 ≤ i≤ n. Let S = { v_1, v_2 v_3, ..., v_n}. Then S = V(C_n) . Let w be a new vertex . Let G = D S(C_n). Then V(G) = V(C_n) ∪{w} and E(G) = E(C_n) ∪ {w v_i/1 ≤ i ≤ n}. w is the unique full degree vertex of G. Then {w} is the unique strong efficient dominating set of G. Therefore $\gamma_{se}(G)$= 1. Let X = E(C_n). Then G – X = $K_{1,n}$. Therefore $\gamma_{se}(G - X)$ = 1 = $\gamma_{se}(G)$ and $|X|$ = n. Hence $b_{sen}(G) \geq$ n. since there is no full degree vertex other than w, n+1 edges removal results in a graph which $\gamma_{se}(G)$ increases. Therefore$b_{sen}(G) \geq$ n. Hence $b_{sen}(G)$ = n.

Example 2.17. Consider the following graph G = DS(C_5) - X, where X = {u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1}.

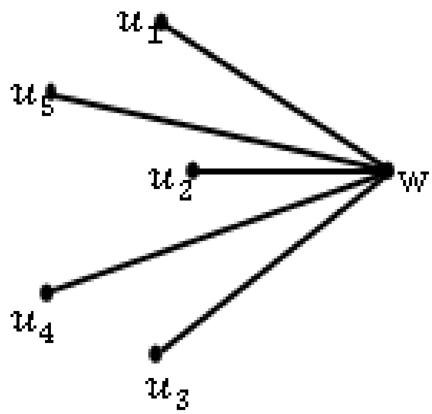

Figure 2.8.

{w} is the unique strong efficient dominating set of G. $\gamma_{se}(G) = 1$.

Hence b_{sen}(DS(C_5)) = 5.

Theorem 2.18. Let G be a connected graph. Then b_{sen}(G+K$_1$) = $|E(G)|$.

Proof. Let H =$G + K_1$. let $V(G) = \{v_i/1 \leq i \leq n\}$ and V(K$_1$) = {u}. $V(H) = \{v_i, u/1 \leq i \leq$ n}. E(H) = $E(G)$ ∪{u$v_i/1 \leq i \leq n$}. u is a full degree vertex of H. Therefore $\gamma_{se}(H) = 1$.

Case (1): Suppose u is the unique full degree vertex of H. Let X be set of edges of H. since we must have $\gamma_{se}(H - X)$ = 1, let X = E(G). Then $H - X = K_{1,n}$. Hence $b_{sen}(H) \geq |E(G)|$.It is verified that there is no set with $\gamma_{se}(H - X)$ = 1 = $\gamma_{se}(H)$. Therefore $b_{sen}(G) \leq |E(G)|$. Hence $b_{sen}(G)$ = $|E(G)|$.

Case (2): Suppose v_i other than u is a full degree vertex of H. Then $|E(H)| = |E(G)| + n$. Let S = $\{v_i v_j, uv_i/1 \leq j \leq n, i \neq j\}$ and $|S| = n$. As in Case(1), we must have γ_{se}(G–X) = 1, where X = E(H) – S. Hence $b_{sen}(H) = |E(G)|$.

Example 2.19: Consider the following graphs G and G – X where X = {v_1v_2, v_2v_3, v_2v_4, v_4v_5, v_4v_6}

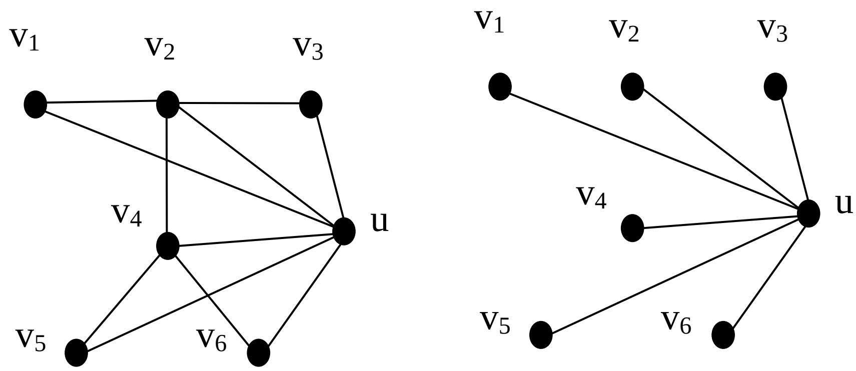

Figure 2.9

Then {u} is the unique strong efficient dominating set G. Therefore γ_{se} (G) = 1. |x| = 5.

Let G – X = $k_{1,6}$. Then γ_{se}(G – X) = 1 = γ_{se} (G). Hence b_{sen}(H) = 5.

Theorem 2.20. $b_{sen}([P_2: C_{3n}{}^2])$ = 4n+2.

Proof: Let G = $[P_2: C_{3n}{}^2]$. Let V(G) = $\{u,v,u_{j1}, u_{1t}, v_{j1,}v_{1t}/1 \leq j \leq 3n-1, 2\leq t\leq 3n\}$ and E(G) $=\{uv,uu_{11},, u_{j1}u_{j+1\,1,}uu_{3n-1\,1,}uu_{12}, u_{1t}u_{1t+1}, uu_{13n}, vv_{11,}v_{11}v_{j+1\,1}, vv_{3n-1\,1}, vv_{1t+1}, vv_{13n}\}$. S = $\{v, v_{31}, v_{14}, u_{21}, \dots, u_{13n}\}$, $\{u, u_{31}, u_{14}, v_{21}, v_{41,\dots,}v_{13n}\}$ are some strong efficient dominating sets of G. Therefore γ_{se}(G) = 4n-1. Let H be a new graph obtained by deleting the edges of S from G. Then H = $K_{1,5} \cup (4n-4)K_{1,2} \cup 2K_1$. Therefore $\gamma_{se}(H)$ = 4n–1 = γ_{se} (G). Hence $b_{sen}(G) \leq 4n+2$. Further it is verified that there is no set with $|S| \leq 4n-1$ such that $\gamma_{se}(G-S) = \gamma_{se}(G)$. Therefore $b_{sen}(G) \geq 4n+2$. Hence$b_{sen}(G) = 4n+2$.

Example 2.21. Consider the following graph G – X = $[P_2.C_6{}^2]$ – $\{u_{11}u_{12}, u_{14}u_{15}, u_{21}u_{22}, u_{24}u_{25}, vv_{11}, v_{12}v_{13}, vv_{15}, vv_{21}, v_{23}v_{24}, vv_{25}\}$

Here $\{u, u_{13}, u_{23}, v_{11}, v_{14}, v_{22}, v_{24}\}$, $\{v, v_{13}, v_{23}, u_{11}, u_{14}, u_{22}, u_{24}\}$ are some strong efficient dominating sets of G.

Therefore γ_{se}(G) = γ_{se}(G – X) = 7. Hence b_{sen}(G) = 10.

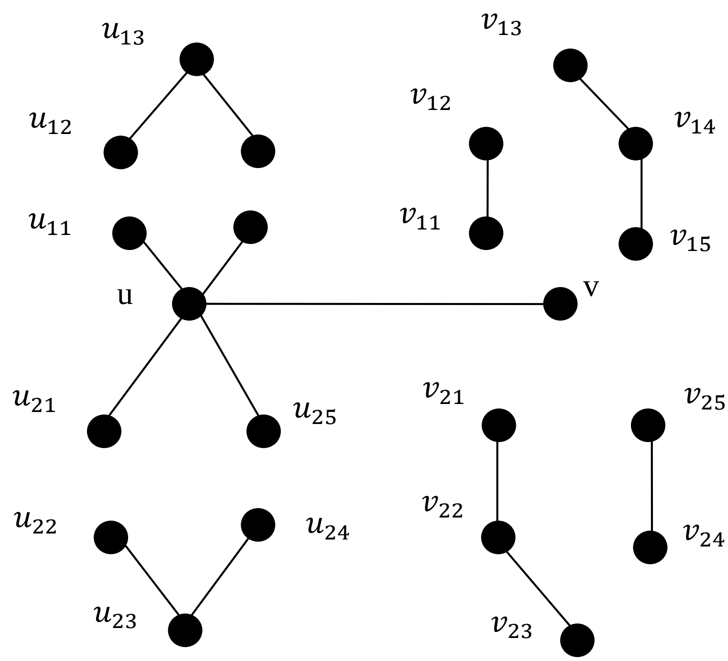

Figure 2.10.

Theorem 2.22. $b_{sen}(C_{3n} \odot K_2)$ = 6

Proof: Let G = $C_{3n} \odot K_2$. Let V(C_{3n}) = $\{v_i/1 \le i \le 3n\}$ and E(C_{3n}) = $\{v_i v_{I+1}/1 \le i \le 3n-1\}$. Let V(G) = V($C_{3n}$)∪$\{u_i, w_i/1 \le i \le 3n\}$ and E(G) = E(C_{3n}) ∪$\{v_i u_i, v_i w_i$ /1≤ i ≤3n}. S = $\{v_1,v_4,\ldots,v_{3n-2},u_2, u_5,\ldots,u_{3n-1},u_3,u_6,\ldots u_{3m}\}$,$\{v_3,v_6,\ldots,v_{3n},w_2,w_5,\ldots,w_{3n-1},w_1,w_4,\ldots,$ $w_{3n-2}\}$ are some strong efficient dominating set of G and $|S|$ = n + n + n = 3n. Let X = E(C_{3n})∪$\{u_i w_i/1 \le i \le 3n\}$. Then G -X = 3n$k_{1,2}$. Therefore$\gamma_{se}(G - X)$ = 3n = $\gamma_{se}(G)$. Hence b_{sen}(G) ≥ 6n. To have $\gamma_{se}(G)$ = 3n, 3n edges from $C_{3n}$and one edge from each cycle C_3 should be removed. Therefore$b_{sen}(G)$ ≥6n. Hence $b_{sen}(G)$ = 6n.

Example 2.23. Consider the following graph G = $C_6 \odot K_2$ – { v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1, u_1w_1, u_2w_2, u_3w_3, u_4w_4, u_5w_5, u_6w_6}

Here $\{v_1,v_2,v_3,v_4,v_5,v_6\}$ is the unique strong efficient dominating set of G. Therefore γ_{se}(G) = 6. Let X = $\{v_1v_2, v_2v_3,\ldots, v_6v_1, u_1w_1, u_2w_2,\ldots, u_6w_6\}$. Then b_{sen}(G) = 12.

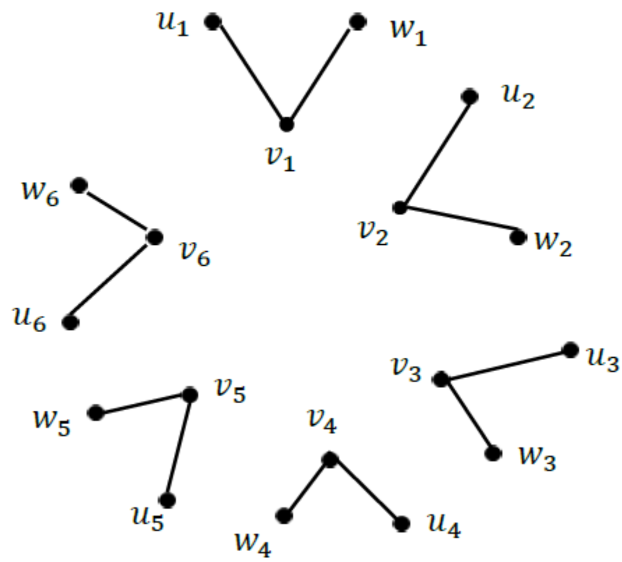

Figure 2.11.

3. CONCLUSION

In this paper the authors studied the strong efficient non bondage number of some special graphs. Further studies can be made on this type for various derived graphs. Also the strong efficient non bondage number of joint sum of cycles, product graphs, inflated graphs of some graphs. The graphs with specified strong efficient non bondage number can be characterized.

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