Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 146 (2020) 110-120 EISSN 2392-2192

Concepts Arising from Strong Efficient Domination Number. Part – III

M. Madhan Vignesh[1], **N. Meena**[2]

Department of Mathematics, The M.D.T. Hindu College, Tirunelveli, Tamil Nadu, India

[1,2]E-mail address: madhanvicky267@gmail.com , meenamdt@gmail.com

ABSTRACT

Let G = (V, E) be a simple graph. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every v ∈ V(G), $|N_s[v] \cap S|=1$. ($|N_w\ [v] \cap S|=1$), where $N_s\ (v)$ = {u ∈V(G) : uv ∈ E(G), deg u ≥ deg v}. ($N_w\ (v)$ = {u ∈V(G) : uv ∈ E(G), deg v ≥ deg u}). The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and is denoted by γ_{se}(G) (γ_{we}(G)). A graph G is strong efficient if there exists a strong efficient dominating set of G. The strong efficient co-bondage number $bcse$(G) is the maximum cardinality of all sets of edges X ⊆ E such that $\gamma_{se}(G + X) \leq \gamma_{se}$(G). In this paper, further results on strong efficient co-bondage number of some special graphs are determined.

***Keywords*:** domination, strong efficient domination, strong efficient co-bondage number

AMS Subject Classification (2010): 05C69

1. INTRODUCTION

Throughout this paper finite, undirected graphs without loops or multiple edges are considered. Let G = (V, E) be a simple graph. The concept of strong efficient domination in graphs was introduced by N. Meena. The concept of co-bondage number was introduced in

(Received 15 May 2020; Accepted 02 June 2020; Date of Publication 03 June 2020)

V.R. Kulli and B. Janakiram. The concept of strong efficient co-bondage number of graphs was introduced by N. Meena and M. Madhan Vignesh. In this paper, further results on strong efficient co-bondage numbers of some special graphs are studied.

2. PRELIMINARIES

Before proving the result some basic definitions, results and theorems are given.

Definition 2.1: Let G = (V, E) be a simple graph. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every v ∈ V(G) ($|N_w[v] \cap S|=1$) where $N_s(v)$ = {u ∈ V(G) : uv∈ E(G), deg u ≥ deg v} and $N_s[v] = N_s(v) \cup \{v\}$($N_w(v)$ = {u ∈ V(G) : uv∈ E(G), deg v ≥ deg u} and $N_w[v] = N_w(v) \cup \{v\}$).

Remark 2.2: The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and is denoted by γ_{se}(G) (γ_{we}(G)). A graph G is strong efficient if there exists a strong efficient dominating set of G.

Theorem 2.3: For any path P_m, $\gamma_{se}(P_m) = \begin{cases} n \text{ if } m = 3n, n \in N \\ n + 1 \text{ if } m = 3n + 1, n \in N \\ n + 2 \text{ if } m = 3n + 2, n \in N \end{cases}$

Theorem 2.4: $\gamma_{se}(C_{3n})$ = n for all n ∈ N.

Observation 2.5: γ_{se} (G) = 1 if and only if G has a full degree vertex.

Definition 2.6: A Gear graph G_n is obtained from the wheel graph W_n by adding a vertex between every pair of adjacent vertices in the cycle.

Definition 2.7: ${P_n}^2$ is a graph obtained from a path of length n – 1 by joining a vertex to another vertex which is away from a path of length 2.

Definition 2.8: Let G = (V, E) be a graph with V = $S_1 \cup S_2 \cup \ldots \cup S_t \cup$ T, where each S_i is a set of vertices having at least two vertices and having the same degree and T = V ∪ S_i. The degree splitting graph G is denoted by DS(G) and is obtained from G by adding the vertices $W_1, W_2, \ldots, W_t$ and joining W_i to each vertex of S_i, $1 \leq i \leq$ t.

Definition 2.9: Let G be a graph with fixed vertex v and let (P_m: G) be the graph obtained from m copies of G and the path P_m : $u_1, u_2, \ldots, u_m$ by joining u_i with the vertex v of the i^{th} copy of G by means of an edge for 1 ≤ i ≤ n.

Definition 2.10: The strong efficient co-bondage number bc_{se}(G) is the maximum cardinality of all sets of edges X ⊆ E such that $\gamma_{se}(G + X) \leq \gamma_{se}$(G).

3. MAIN RESULT

Theorem 3.1: Let G_n be a gear graph. Then $bc_{se}(G_n)$ = 1, n ≥ 3.

Proof: Let G = G_n , n ≥ 3. Let V(G) = $\{v, v_i, u_i \,/\, 1 \leq i \leq$ n} be the vertices of the gear graph G. Then the vertex v is adjacent with v_i ; $1 \leq i \leq$ n. deg(v) = n = Δ(G), deg(v_i) = 3 ; $1 \leq i \leq$ n, deg(u_i) = 2 ; $1 \leq i \leq$ n. v strongly efficiently dominates all the vertices v_i ; $1 \leq i \leq$ n. The vertices u_i *and* v_i are mutually non-adjacent with each other. Therefore $\{v, u_i \,/\, 1 \leq i \leq$ n} is the unique strong efficient dominating set of G. Thus γ_{se}(G) = n + 1. Let e = u_i ; for some i, $1 \leq i \leq$ n . Let H = G + e. Then S_i = $\{v, u_j \,/\, 1 \leq j \leq$ n, $i \neq j$ }, $1 \leq i \leq$ n is the strong efficient dominating set of G and $|S_i|$ = n. Therefore γ_{se}(H) = n < γ_{se}(G). Hence $bc_{se}(G)$ = 1.

Example 3.2: Consider the following graphs G = G_4 and G + e

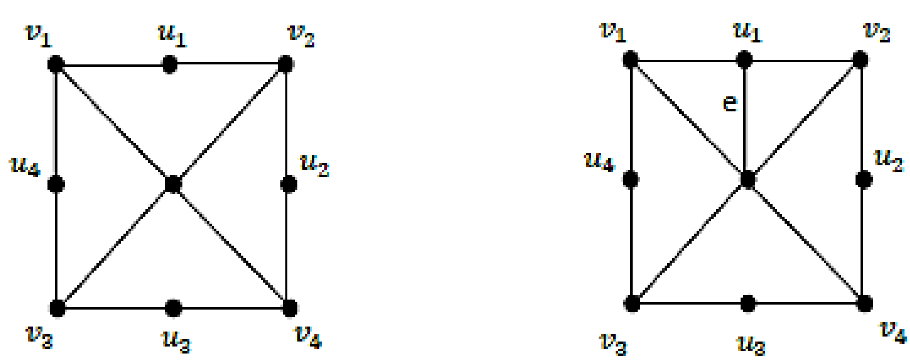


Figure 3.1

Let G = G_4. Let V(G) = $\{v, v_i, u_i \,/\, 1 \leq i \leq 3\}$ be the vertices of the gear graph G. $\{v, u_i \,/\, 1 \leq i \leq 4\}$ is the unique strong efficient dominating set of G. Thus γ_{se}(G) = 5. Let e = vu_1.

Let H = G + e. Then S = { v, u_2, u_3, u_4} is the unique strong efficient dominating set of G and |S| = 4. Therefore γ_{se}(H) = 4<γ_{se}(G). Hence $bc_{se}(G)$ = 1.

Theorem 3.3: Let G = $P_m{}^2$, m ≥ 6.

$$\text{Then } bc_{se}(G) = \begin{cases} 1 \; if \; m = 5n+1, \; m = 5n+3 \\ 2 \; if \; m = 5n+2 \\ 4 \; if \; m = 5n+4 \\ 5 \; if \; m = 5n+5 \end{cases} \text{, for all n} \geq 1$$

Proof: Let G = $P_m{}^2$, m ≥ 6. Let V(G) = {$v_1, v_2, v_3, \ldots, v_m$} and E(G) = {$v_i v_{i+1}$, $v_j v_{j+2}$: 1 ≤ i ≤ m – 1 and 1 ≤ j ≤ m – 2 }.

Case - (i): Let m = 5n , n ≥ 2. Then {v_3,v_8, v_{13}, …,v_{5n-2}} is the unique strong efficient dominating set of G. Therefore γ_{se}(G) ≤ n. Every maximum degree vertex can strongly efficiently dominates five vertices. At least n vertices are needed to strongly efficiently dominate 5n vertices. Therefore γ_{se}(G) ≥ n. Hence γ_{se}(G) = n . Let e_1= $v_1 v_8$, e_2 = $v_2 v_8$, e_3 = $v_3 v_8$, e_4 = $v_4 v_8$, e_5 = $v_5 v_8$. Let H = G + {e_1, e_2, e_3, e_4, e_5}. Then {v_8, v_{13}, …,v_{5n-2}} is the unique strong efficient dominating set of H. Therefore γ_{se}(H) = n – 1 < γ_{se}(G). Hence bc_{se}(G) ≤ 5. But five edges are needed to G. Therefore bc_{se}(G)≥ 5. Hence bc_{se}(G)= 5.

Case - (ii): Let m = 5n + 1, n ≥ 1. Then {v_3,v_8, v_{13}, …,v_{5n-2}, v_{5n+1}} and {v_1,v_4, v_9, …,v_{5n-1}} are two strong efficient dominating set of G. Therefore γ_{se}(G) ≤ n + 1. Every maximum degree vertex can strongly efficiently dominates five vertices. At least n + 1 vertices are needed to strongly efficiently dominate 5n + 1vertices. Therefore γ_{se}(G) ≥ n + 1. Hence γ_{se}(G) = n + 1 . Let e = $v_{5n-2} v_{5n+1}$. Let H = G + e. Then {v_3, v_8, v_{13}, …,v_{5n-2}} is the unique strong efficient dominating set of H. Therefore γ_{se}(H) = n < γ_{se}(G). Hence bc_{se}(G)= 1.

Case - (iii): Let m = 5n + 2, n ≥ 1. Then {v_3,v_8, v_{13}, …,v_{5n-2}, v_{5n+1}} and {v_2,v_5, v_{10}, …,v_{5n}} are two strong efficient dominating set of G. Therefore γ_{se}(G) ≤ n + 1.As in Case-(i), at least n + 1 vertices are needed to strongly efficiently dominate 5n + 2 vertices. Therefore γ_{se}(G) ≥ n + 1. Hence γ_{se}(G) = n + 1 . Let e_1= $v_{5n-2} v_{5n+1}$, e_2 = $v_{5n-2} v_{5n+2}$. Let H = G + {e_1, e_2}. Then {v_3, v_8, v_{13}, …,v_{5n-2}} is the unique strong efficient dominating set of H. Therefore γ_{se}(H) = n < γ_{se}(G). Hence bc_{se}(G) ≤ 2. Suppose one edge is added in such a way that the existence of strong efficient dominating set is guaranteed. Let H = G + e. Let S be any strong efficient dominating set of H. For any two vertices v_i, v_j in S, d(v_i, v_j) ≥ 5. The maximum degree vertex strongly efficiently dominates exactly 5 vertices of H. To strongly efficiently dominate the remaining (5n – 5) + 1 vertices, n vertices are needed. γ_{se}(H) ≥ n + 1, a contradiction. Therefore bc_{se}(G)≥ 2. Hence bc_{se}(G)= 2.

Case - (iv): Let m = 5n + 3, n ≥ 1. Then $\{v_1, v_4, v_9, \ldots, v_{5n-1}, v_{5n+2}\}$ and $\{v_1, v_5, v_{10}, \ldots, v_{5n-1}, v_{5n+3}\}$ are two strong efficient dominating set of G. Therefore γ_{se}(G) ≤ n + 2. As in Case-(i), at least n + 2 vertices are needed to strongly efficiently dominate 5n + 3 vertices. Therefore γ_{se}(G) ≥ n + 2. Hence γ_{se}(G) = n + 2 .Let e = $v_1 v_4$. Let H = G + e. Then $\{v_4, v_9, \ldots, v_{5n-1}, v_{5n+2}\}$ is the unique strong efficient dominating set of H. Therefore γ_{se}(H) = n + 1 < γ_{se}(G). Hence bc_{se}(G)= 5.

Case - (v): Let m = 5n + 4 , n ≥ 1. Then $\{v_3, v_8, v_{13}, \ldots, v_{5n-2}, v_{5n+2}\}$ is the unique strong efficient dominating set of G. Therefore γ_{se}(G) ≤ n + 1. As in Case-(i), at least n + 1 vertices are needed to strongly efficiently dominate 5n + 4 vertices. Therefore γ_{se}(G) ≥ n +1. Hence γ_{se}(G) = n + 1. Let $e_1 = v_{5n-2}v_{5n-1}$, $e_2 = v_{5n-2}v_{5n}$, $e_3 = v_{5n-2}v_{5n+1}$, $e_4 = v_{5n-2}v_{5n+2}$. Let H = G + $\{e_1, e_2, e_3, e_4\}$. Then $\{v_3, v_8, v_{13}, \ldots, v_{5n-2}\}$ is the unique strong efficient dominating set of H. Therefore γ_{se}(H) = n < γ_{se}(G). Hence bc_{se}(G) ≤ 4. But four edges are needed to G. Therefore bc_{se}(G) ≥ 4. Hence bc_{se}(G)= 4.

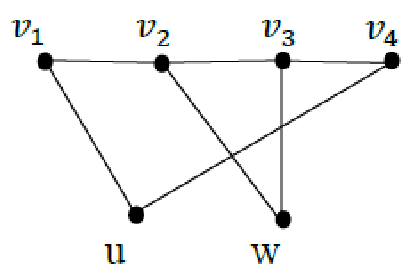
Theorem 3.4: Let G = DS(P_n), n ≥ 4. Then $bc_{se}(G) = \begin{cases} 2 \ if \ n = 4 \\ 3 \ \ if \ n \geq 5 \end{cases}$.

Proof: Let G = DS(P_n) , n ≥ 4. Let V(G) = {u, w, v_i / 1 ≤ i ≤ n} and E(G) = {uv_1, uv_n, wv_j, $v_i v_{i+1}$ / 2 ≤ j ≤ n – 1 and 1 ≤ i ≤ n – 1 }.

Case – 1: Take n = 4. Let G = DS(P_4). Since there is no full degree vertex in G, then γ_{se}(G) ≥ 2. Therefore, $\{v_2, v_4\}$ and $\{v_1, v_3\}$ are the two strong efficient dominating set of G. Hence γ_{se}(G) = 2. Let e_1 = uv_2 and $e_2 = v_2 v_4$. Let H = G + $\{e_1, e_2\}$. Then $\{v_2\}$ is the unique strong efficient dominating set of H. Therefore, γ_{se}(H) = 1 < γ_{se}(G). Hence bc_{se}(G) ≤ 2. Here deg(v_2) = deg(v_4) = Δ(G) = 3. To make γ_{se}(G) = 1, deg(v_2) or deg(v_4) must be 5. Two more edges are added either with v_2 or v_4. Therefore, bc_{se}(G) ≥ 2. Hence bc_{se}(G) = 2.

Case – 2: Let n ≥ 5. Then {u, w} is the unique strong efficient dominating set of G. Therefore, γ_{se}(G) = 2. Let H = G + {uw, wv_1, wv_n}. Then {w} is the unique strong efficient dominating set of H. Therefore, γ_{se}(H) = 1 < γ_{se}(G). Hence bc_{se}(G) ≤ 3. Here, deg w = n – 2 = Δ(G). Therefore, w is the unique maximum degree vertex of G. To get γ_{se}(G) = 1, there must be a full degree vertex. Three edges are needed to make 'w' as a full degree vertex. Therefore bc_{se}(G) ≥ 3. Hence bc_{se}(G) = 3.

Illustration 3.5: Consider the following graphs G = DS(P_4) and G + {e_1, e_2}.



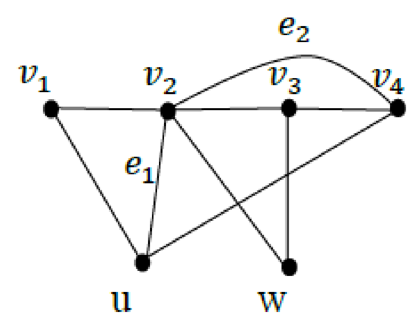
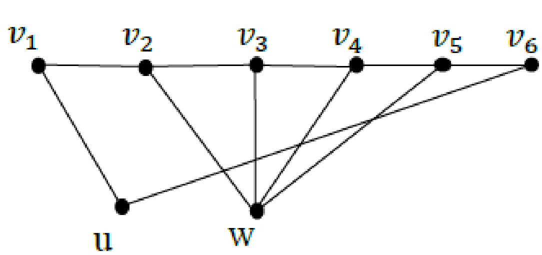


Figure 3.2

Let G = DS(P_4). {v_2, v_4} and {v_1, v_3} are the two strong efficient dominating sets of G. Hence γ_{se}(G) = 2. Let e_1 = uv_2 and e_2= $v_2 v_4$. Let H = G + {e_1, e_2}. Then {v_2} is the unique strong efficient dominating set of H. Therefore, γ_{se}(H) = 1 < γ_{se}(G). Hence bc_{se}(G) = 2.

Example 3.6: Consider the following graphs G = DS(P_6) and G + {e_1, e_2}.



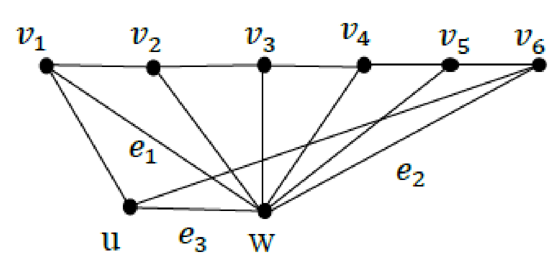


Figure 3.3

Let G = DS(P_6). Then {u, w} and {v_1, v_3} are the two strong efficient dominating sets of G. Hence γ_{se}(G) = 2. Let e_1 = uv_2 and e_2= $v_2 v_4$. Let H = G + {e_1, e_2}. Then {v_2} is the unique strong efficient dominating set of H. Therefore, γ_{se}(H) = 1 < γ_{se}(G). Hence bc_{se}(G) = 2.

Theorem 3.7: Let G = $C_n \odot K_1$, n $\geq$ 4. Then bc_{se}(DS(G)) = n + 1.

Proof: Let G = $C_n \odot K_1$, n $\geq$ 4. Let V(DS(G)) = {u, v, u_i, v_i / 1 $\leq i \leq$ n} and E(DS(G)) = {uu_i, vv_i, $v_n v_1$, $v_i v_{i+1}$,$v_j u_j$ / 1 $\leq i \leq$n – 1 and 1 $\leq j \leq$ n }. Here {u, v} is the unique strong efficient dominating set of DS(G). Therefore γ_{se}(DS(G)) = 2. Let X = {vu_i, uv / 1 $\leq i \leq$ n}. Let H = DS(G) + X. Then v is the unique full degree vertex in H. Therefore γ_{se}(H) = 1 and |X| = n + 1. Hence bc_{se}(DS(G)) $\leq$ n + 1. Now reduce γ_{se}(DS(G)) to 1, there must be full degree vertex. Here u and v are the maximum degree vertex and deg u = deg v = n. Therefore n + 1 edges must be added to DS(G) to make either u or v as a full degree vertex. Therefore bc_{se}(DS(G)) $\geq$ n + 1. Hence bc_{se}(DS(G)) = n + 1.

Illustration 3.8: Consider the following graphs G = DS($C_5 \odot K_1$) and G + {e_1, e_2, e_3, e_4, e_5, e_6}. Let G = DS($C_5 \odot K_1$). Then {u, v} is the unique strong efficient dominating set of G. Therefore γ_{se}(G) = 2. Let e_1 = uv_1, e_2 = uv_2, e_3 = uv_3, e_4 = uv_4, e_5 = uv_5, e_6 = uv. Let H = G + {e_1, e_2, e_3, e_4, e_5, e_6}. Then v is the full degree vertex of H. Therefore γ_{se}(H) = 1 <γ_{se}(G). Hence bc_{se}(G) = 6.

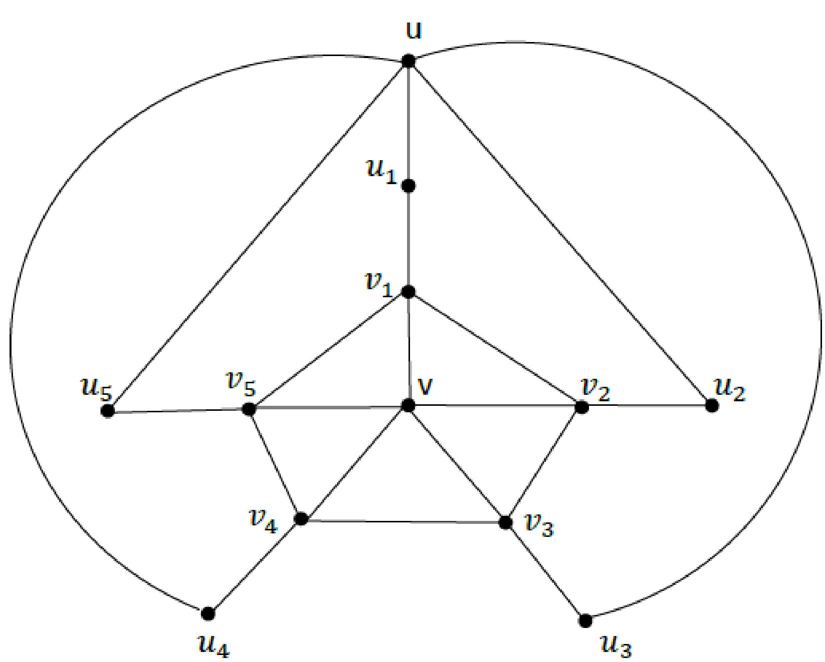


Figure 3.4

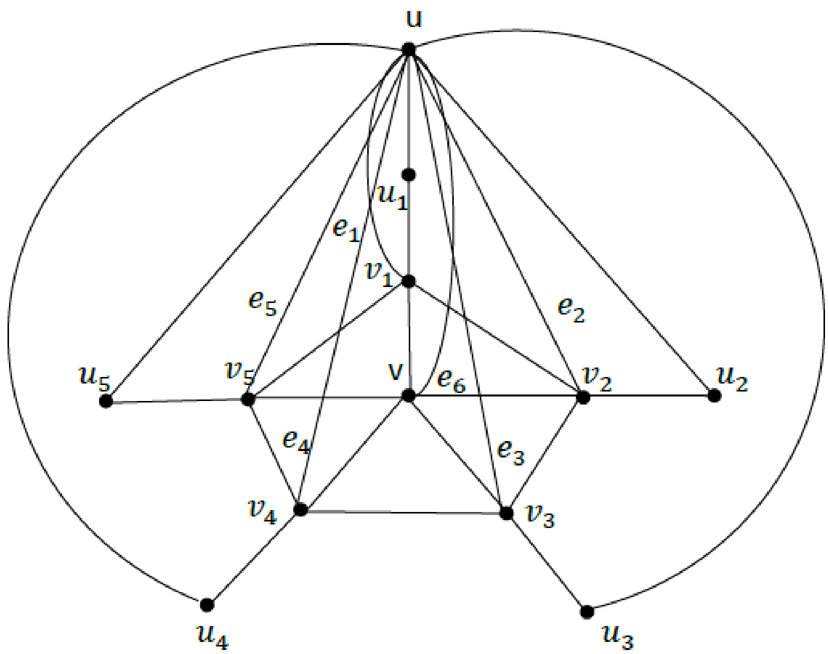


Figure 3.5

Theorem 3.9: Let G = $P_n : S_m$, $n \geq 2$ and $m \geq 2$. Then bc_{se}(G) = m + 2.

Proof: Let G = $P_n : S_m$, $n \geq 2$ and $m \geq 2$. Let V(P_n) = $\{v_1, v_2, v_3, \ldots, v_n\}$. Let V(G) = V($P_n$) ∪ $\{u_i, u_{ij} \,/\, 1 \leq i \leq n$ and $1 \leq j \leq m\}$ and E(G) = $\{v_i v_{i+1}, v_k v_k, v_k v_{kj} \,/\, 1 \leq i \leq n-1, 1 \leq j \leq m$ and $1 \leq k \leq n\}$. Then $\{u_1, u_2, u_3, \ldots, u_n\}$ is the unique strong efficient dominating set of G. Therefore γ_{se}(G) = n. Let $X_i = \{u_i u_{kj}, u_i u_k, u_i v_k \,/\, 1 \leq j \leq m\}$ and Let H = G + X_i. Then $|X_i|$ = m + 2. Therefore $\{u_i \,/\, 1 \leq i \leq n, i \neq k\}$ is the unique strong efficient dominating set of H. Hence γ_{se}(H) = $n - 1$ < γ_{se}(G). Therefore bc_{se}(G) ≤ m + 2.$u_1, u_2, u_3, \ldots, u_n$ are maximum degree vertices. Each $u_i$strongly efficiently dominates v_i, u_{ij}, $1 \leq j \leq m$. To reduce the strong efficient domination number to $n - 1$ at least m + 2 edges should be added. Therefore bc_{se}(G) ≥ m + 2. Hence bc_{se}(G) = m + 2.

Illustration 3.10: Consider the following graphs G = $P_4 : S_4$ and G + $\{e_1, e_2, e_3, e_4, e_5, e_6\}$. Let G = $P_4 : S_4$. Then $\{u_1, u_2, u_3, u_4\}$ is the unique strong efficient dominating set of G. Therefore γ_{se}(G) = 4. Let $e_1 = v_1 u_2$, $e_2 = u_1 u_2$, $e_3 = u_{11} u_2$, $e_4 = u_{12} u_2$, $e_5 = u_{13} u_2$, $e_6 = u_{14} u_2$. Let H = G + $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ Then $\{u_2, u_3, u_4\}$ is the unique strong efficient dominating set of H. Therefore γ_{se}(H) = 3 < γ_{se}(G). Hence bc_{se}(G) = 6.

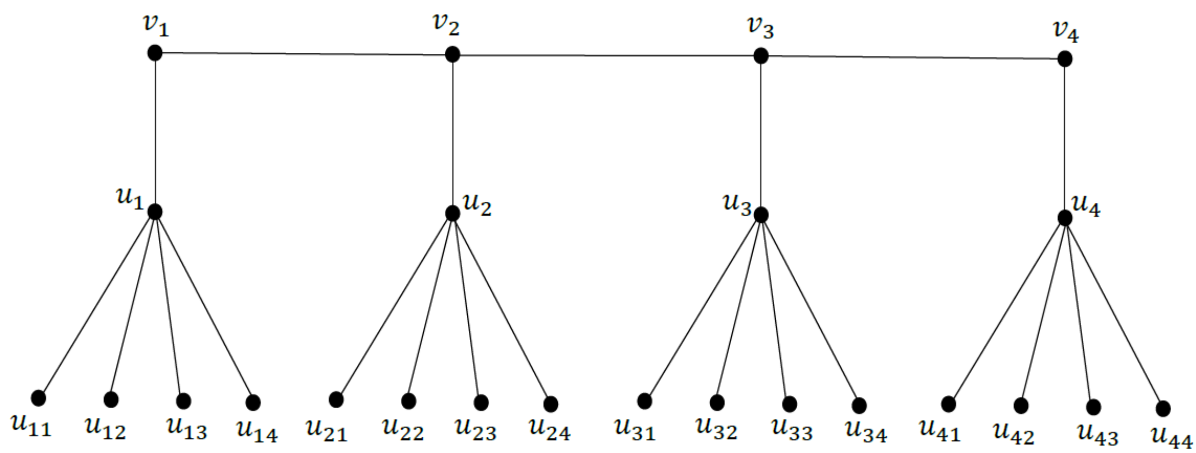


Figure 3.6

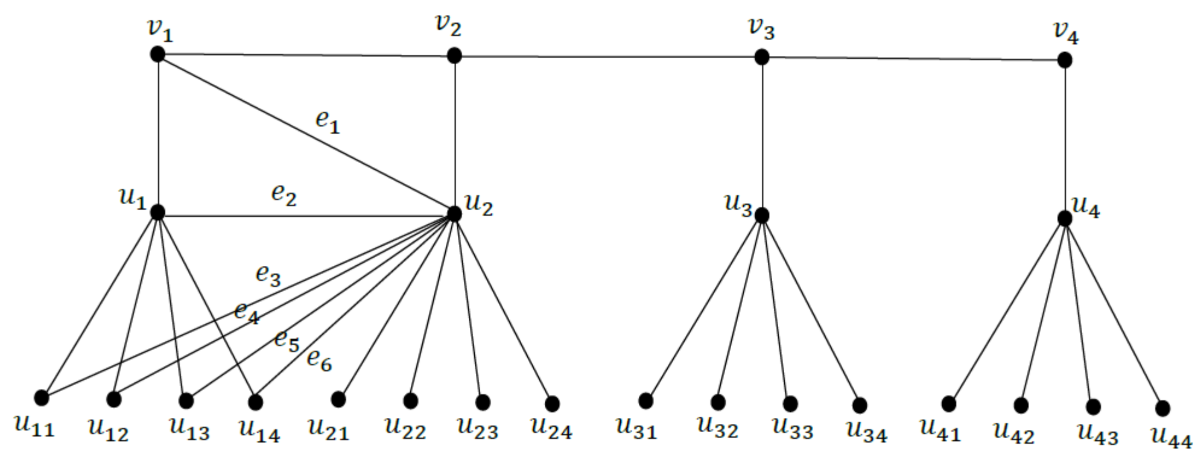


Figure 3.7

4. CONCLUSION

Thus in this paper the authors studied further results on strong efficient co-bondage number of some special graphs. Similar studies can be made on strong efficient co-bondage number for path and its derived graphs such as line graph, subdivision graph, paraline graph, total graph etc. Also sum of this strong efficient co-bondage number and chromatic number of graphs can be found. The relation between strong efficient co-bondage number of some and their middle graphs can be determined.

References

[1] M. Anitha and S. Balamurugan, Strong Efficient Open Domination in Graphs. *Int. J. Math. And Appl.* *6*(1–B) (2018), 337–342

[2] S. Balamurugan, M. Anitha and N. Anbazhagan, Some Results On Strong Efficient Open Domination. *International Journal of Pure and Applied Mathematics* Volume 119 No. 15, 2018, 641-648

[3] D.W. Bange, A.E. Barkauskas and P.J. Slater, Efficient dominating sets in graphs. *Application of Discrete Mathematics*, 189 – 199, SIAM, Philadephia, 1988.

[4] D.W. Bange, A.E. Barkauskas, L.H.Host and P.J. Slater, Generalized domination and efficient domination in graphs. *Discrete Mathematics*, 159, 1-11, 1996.

[5] Frank Harary, Teresa W. Haynes, Peter J. Slater, Efficient and Excess Domination in Graphs. *JCMCC* 26, 83-95, 1998.

[6] H. Gavlas and K. Schultz, Efficient Open Domination, *Electron. Notes Discrete Math.* 11 (2002) 681-691

[7] J.H. Hattingh, M.A. Henning. On strong domination in graphs. *J. Combin. Math. Combin. Compute.* 26, 73-82, 1998.

[8] R. M. Karthikeyan, R. Jahir Hussain, The Co-Bondage (Bondage) number of fuzzy graphs and its properties. *Malaya Journal of Matematik*, 5(3) (2017) 483-493

[9] V.R. Kulli, Nonbondage number of graphs and diagraphs. *International Journal of Advanced Research in Computer Sci and Technology*, 3(1), 55-65 (2015).

[10] V.R. Kulli and B. Janakiram, The Cobondage Number of a graph. *Discussiones Mathematicae, Graph Theory* 16 (1996) 111-117

[11] V.R. Kulli and B. Janakiram. The maximal domination number of a graph. *Graph Theory Notes of New York, New York Academy of Sciences*, 33, 11-13 (1997).

[12] V. R. Kulli and B. Janakiram, The total global domination number of a graph. *Indian J. Pure Appl. Math.* 27, 537-542 (1996).

[13] V.R. Kulli and D.K. Patwari, Total Efficient domination in graphs. *IRJPA*, 6(1) (2016), 227-232

[14] V.R. Kulli and N. D. Soner, Efficient bondage number of a graph. *Nat. Acad. Sci. Lett.* 19 (9 and 10), 197-202 (1996).

[15] M. Madhan Vignesh, N. Meena, Strong Efficient Co-Bondage Number of some Graphs, *World Scientific News*, 145 (2020) 234-244

[16] N. Meena, A. Subramanian, V. Swaminathan, Strong Efficient Domination and Strong Independent Saturation Number of Graphs. *International Journal of Mathematics and Soft Computing*, Vol. 3, no. 2, 41-48, 2013

[17] N. Meena, A. Subramanian and V. Swaminathan, Strong Efficient Domination in Graphs. *International Journal of Innovative Science, Engineering & Technology*, Vol. I Issue 4, 172-177, June 2014.

[18] N. Meena, Strong Efficient Edge Domination Number of Some Corona Related Graphs. *Journal Emerging Technologies and Innovative Research*, Vol. 6 Issue 2, 778-787, February 2019.

[19] N. Meena, Strong Efficient Edge Domination Number of Some Cycle Related Graphs. *International Journal of Mathematical Trends and Technology*, Vol. 59, Issue 3, 201-207, July 2018.

[20] N. Meena, Strong Efficient Domination Number of Inflated graphs of some standard graphs. *International Journal of Scientific and Innovative Mathematical Research*, Vol. 2, Issue 5, 435-440, May 2014.

[21] K. Murugan, Square Graceful Labeling of Some Graphs. *International Journal of Innovative Research in Science, Engineering and Technology*, Vol. 4, Issue 2, p. 511-520, February 2015.

[22] K. Murugan and N. Meena, Some Nordhaus-Gaddum Type Relations on strong efficient dominating sets. *Journal of New Results in Science,* 11, 04-16, 2016.

[23] N. Pratap Babu Rao, N. Sweta, Strong Efficient Domination in Jump Graphs. *International Journal of Innovative Research in Science, Engineering and Technology*, Vol. 7, Issue 4, April 2018, 4011-4014

[24] D. Rautenbach, Bounds on strong domination number. *Discrete Math* 215 (2000) 201-212

[25] E. Sampathkumar and L. Pushpalatha, Strong weak domination and domination balance in a graph. *Discrete Math.* 161: 235-242, 1996

[26] A. Senthil Thilak, Sujatha V Shet and S.S. Kamath, Changing and unchanging efficient domination in graphs with respect to edge addition. *Mathematics in Engineering, Science and Aerospace* Vol. 11, No 1, 201-213, 2020.

[27] Teschner, U, New results about the bondage number of a graph. *Discret. Math.* 171, 249-259, 1997.

[28] P. Thangaraju, Strong and weak co-bondage numbers of a graph. *Journal of Information and Optimization Sciences,* Vol. 39, Issue 4 (2018) 879-887

[29] J.M. Xu, On bondage numbers of graphs - a survey with some comments. *International Journal of Combinatorics,* vol. 2013, Article ID 595210, 34 pages, 2013.

[30] Ulrich Teschner, New results about the bondage number of a graph. *Discrete Mathematics,* Volume 171, 249-259, 1994.