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Separation axioms by virtue of soft semi*-open sets

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ABSTRACT

Soft set theory, proposed by Molodsov has been regarded as an effective mathematical tool to deal with uncertainties. Mohammed Shabir and Munazza Naz introduced soft topological spaces which are defined over an initial universe with the fixed set of parameters. Quite recently, the authors defined a new version of soft sets that using soft pre*-open sets and investigated some of their properties. In the present work, we introduce the concept of soft semi*- separation axioms for soft topological spaces using semi*-open and semi*-closed soft sets. We further investigate the relationship among them.

***Keywords*:** Soft semi*-open sets, soft semi*-closed sets, soft point, soft semi$^*T_0$ -space, soft semi$^*T_1$-space, soft semi$^*T_2$-space, soft semi$^*T_3$-space, soft semi$^*T_4$-space

AMS Subject Classification (2000): 54A05, 54A10, 54C05, 54C10

1. INTRODUCTION

Soft set theory was first introduced by Molodtsov (1999). A soft set is a collection of approximate descriptions of an object. Soft sets have many applications in various fields like Engineering, Social science, Medical science, etc. The notion of soft topological spaces was formulated by Shabir et.al. and Cagman et.al. (2011). Robert and Pious Missier (2012) defined

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semi*-open and semi*-closed sets using generalized closure operator. Soft separation axioms for soft topological spaces were studied by Shabir et. al. (2011). The notions of soft open sets soft closed sets, soft closure, soft interior and soft neighborhood of a point are also introduced and their basic properties are investigated by them. Generalized closed sets in topological spaces were introduced by Levine in 1970.

However it is extended to soft topological spaces by Kannan in the year 2012. Juthika Mahanta and P.K. Das (2014) studied soft semi-open sets and soft semi-closed sets and investigate their properties. Quite recently the authors introduced and studied semi*-open and semi*-closed soft sets. In the present paper we initiate the concept of separation axioms for soft topological spaces using semi*-open and semi*-closed soft sets. We further investigate the relationship among them.

2. PRELIMINARIES

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let $P(U)$ denote the power set of U, and let $A \subseteq E$. Here are some definitions required in the sequel. A subset A of a space (X, τ) is said to be generalized closed (briefly g-closed), if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. The intersection of all g-closed sets containing A is called the g-closure of A and denoted by $cl^*(A)$ and union of all g-open sets contained in A is called the g-interior of A and is denoted by $\text{int}^*(A)$. A subset S of a topological space (X, τ) is said to semi*-open if $S \subseteq cl^*(\text{int}(S))$. The complement of a semi*-open set is semi*-closed. It is well known that a subset S is semi*-closed if and only if $\text{int}^*(cl(S)) \subseteq S$.

Definition 2.1. A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$ where E is a set of parameters, $A \subseteq E$, $P(U)$ is the power set of U and $f_A : A \to P(U)$ such that $f_A(x) = \phi$ if $x \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary, some of them may be empty and some may have non-empty intersection.
Note that the set of all soft sets over U is denoted by $SS(U)_E$.

Example 2.2. Suppose U = set of real numbers on the $[a,b]$.
E = set of parameters. Each parameter is a word or a sentence.
E = {Compact, Closed, Connected, Open}.
In this case, to define a soft set means to point out closed set connected set and so on.
Let we consider below the same example in more detail.
$U = \{x : a \le x \le b\}$ and $E = \{e_1, e_2, e_3, e_4\}$ Where,
e_1 denotes parameter 'compact', e_2 - 'closed', e_3 - 'connected', e_4 - 'open'. Now
$f(e_1) = \{A \subseteq [a,b]$: Every open cover for A in $[a,b]$ has finite subcover$\}$, $f(e_2) = \{[\alpha, \beta] \subseteq [a,b] : \alpha, \beta \in R\}$, $f(e_3) = \{A \subseteq [a,b]$: Separation does not exists for A in

$[a,b]\}$, $f(e_4)=\{(\alpha,\beta)\subseteq[a,b]:\alpha,\beta\in R\}$.

The soft set F_A is a parameterized family of subsets of the set U. Consider the mapping f in which $f(e_1)$ means set of all subsets of U which are compact whose functional value is the set $\{A\subseteq[a,b]:$Every open cover for A in $[a,b]$ has finite sub cover$\}$.

Here the soft set F_A is the collection of approximations given below:

{(compact, $\{A\subseteq[a,b]:$Every open cover for A in $[a,b]$ has finite sub cover$\}$), (closed, $\{[\alpha,\beta]\subseteq[a,b]:\alpha,\beta\in R\}$), (connected, $\{A\subseteq[a,b]:$Separation does not exists for A in $[a,b]\}$), (open, $\{(\alpha,\beta)\subseteq[a,b]:\alpha,\beta\in R\}$)}= F_A.

Definition 2.3. Let F_A and $F_B \in SS(U)_E$. Then

(i) F_A is a soft subset of F_B denoted by $F_A \tilde{\subseteq} F_B$ if $A\subseteq B$ and $f_A(x) \subseteq f_B(x)$ for all $x\in E$.

(ii) F_A and F_B are soft equal, denoted by $F_A = F_B$ if and only if $F_A \tilde{\subseteq} F_B$ and $F_B \tilde{\subseteq} F_A$.

(iii) Let $F_A\in SS(U)_E$. Power set of F_A is defined by $\tilde{P}(F_A)$=$\{F_{A_i} \tilde{\subseteq} F_A / i\in I\}$. Its cardinality is defined by $|\tilde{P}(F_A)|= 2^{\sum_{x\in E}|f_A(x)|}$ where $|f_A(x)|$ is cardinality of $f_A(x)$.

(iv) The soft union of soft sets F_A and G_B over a common universe U, denoted by $F_A \tilde{\bigcup} G_B$ is defined as the soft set H_C where $C=A\bigcup B$ and for all $c\in C$,

$$h(c)=\begin{cases} f(c), if\ \ c\in A\setminus B \\ g(c), if\ \ c\in B\setminus A \\ f(c)\bigcup g(c), if\ \ c\in A\bigcap B \end{cases}$$

(v) The soft intersection of two soft sets F_A and G_B over a common universe U, denoted by $F_A \tilde{\bigcap} G_B$ is defined as the soft set H_C, where $C=A\bigcap B$ and for all $e\in C$, $h(e)=f(e)$ or $g(e)$.

(vi) The soft complement $F_A^{\tilde{C}}$ of F_A is defined by the approximate function $f_{A^{\tilde{c}}}(x)= f_A^c(x)$ where $f_A^c(x)=U\setminus f_A(x)$ for all $x\in E$.

(vii) If $f_A(x)=\phi$ for all $x\in E$, then F_A is called an empty set denoted by ϕ_E

(viii) If $f_A(x)=U$ for all $x\in E$, then F_A is called a universal set (soft absolute set) denoted by U_E.

Definition 2.4. Let $\tilde{\tau}$ be a collection of soft sets over a universe U with a fixed set E of parameters, then $\tilde{\tau}\subseteq SS(U)_E$ is called a soft topology on U with a fixed set E if

(i) ϕ_E, U_E belong to $\tilde{\tau}$.

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

(iii) The intersection of any finite number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The pair $(U_E, \tilde{\tau})$ is called a soft topological space.

Definition 2.5. Let $(U_E, \tilde{\tau})$ be a soft topological space over U.

(i) A soft set F_B is called a soft generalized closed in U if $cl(F_B) \tilde{\subseteq} F_o$ whenever $F_B \tilde{\subseteq} F_0$ and F_0 is soft open in U.

(ii) A soft set F_B is called a soft generalized open in U if its soft complement $F_B^{\tilde{C}}$ is soft generalized closed in U. Equivalently $G_c \tilde{\subseteq} \text{int}(F_B)$ whenever $G_c \tilde{\subseteq} F_B$ and G_C is soft closed in U.

Definition 2.6. Let $(U_E, \tilde{\tau})$ be a soft topological space over U.

(i) The soft generalized closure of a soft set F_B is denoted by $cl^*(F_B)$ and it is defined as the soft intersection of all soft generalized closed sets contains F_B.

(ii) The soft generalized interior of a soft set F_B is denoted by $\text{int}^*(F_B)$ and it is defined as the soft union of all soft generalized open sets contained in F_B.

Definition 2.7. In a soft topological space, a soft set

(i) G_C is said to be semi*-open soft set if there exists an open soft set H_B such that $H_B \tilde{\subseteq} G_c \tilde{\subseteq} cl^*(H_B)$

(ii) L_A is said to be semi*-closed soft set if there exists a closed soft set K_D such that $\text{int}^*(K_D) \tilde{\subseteq} L_A \tilde{\subseteq} K_D$.

Definition 2.8. Let G_C be a soft set in a soft topological space $(U_E, \tilde{\tau})$.

(i) Then the soft semi*-closure of the soft set G_C , is denoted by $ss^*cl(G_C)$ and is defined as $ss^*cl(G_C) = \tilde{\bigcap}\{S_F / G_c \tilde{\subseteq} S_F \text{ } and \text{ } S_F \in S^*CSS(U)_E\}$ is a soft set.

(ii) Then soft semi*-interior of a soft set G_C , is denoted by $ss^*\text{int}(G_C)$, and is defined as $ss^*\text{int}(G_C) = \tilde{\bigcup}\{S_F / S_F \tilde{\subseteq} G_c \text{ } and \text{ } S_F \in S^*OSS(U)_E\}$ is a soft set.

Definition 2.9. Two soft sets G_C and H_B are said to be distinct if $G(e) \bigcap H(e) = \phi$ for all $e \in B \bigcap C$.

3. SOFT SEMI*T0-SPACE

Definition 3.1. A soft topological space $(U_E,\tilde{\tau})$ is said to be a soft semi*T0-space if for two disjoint soft points ℓ_G and ℓ_F, there exists a semi*-open set containing one but not the other.

Example 3.2. Consider the soft topological space $(U_E,\tilde{\tau})$ where $U=\{h_1,h_2\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\phi_E,(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\}),U_E\}$. In this soft topology the only semi*-open soft sets are $\phi_E,(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\}),U_E$. Clearly for all disjoint pairs of soft points in $(U_E,\tilde{\tau})$, we find a semi*-open soft set containing one but not the other.

Theorem 3.3. A soft subspace of a soft semi*T0-space is soft semi*T0.

Proof: Let F_A be a soft subspace of a soft semi*T0 space U_E. Let ℓ_G and ℓ_F be two distinct soft points of F_A. Then ℓ_G and ℓ_F are also soft points of U_E. As U_E is a soft semi*T0-space, there exists a semi*-open soft set H_C containing one of these soft points say ℓ_G, but not the other. This implies $H_C \tilde{\bigcap} V_B$ is a semi*-open soft set containing ℓ_G but not the other.

4. SOFT SEMI*T1-SPACE

Definition 4.1. A soft topological space $(U_E,\tilde{\tau})$ is said to be a soft semi*T1-space if for two distinct soft points ℓ_G and ℓ_F of U_E there exist soft semi*-open sets H_B and G_C such that $\ell_G \tilde{\in} H_B$ and $\ell_F \tilde{\notin} H_B$; $\ell_F \tilde{\in} G_C$ and $\ell_G \tilde{\notin} G_C$.

Example 4.2. In the example 3.2, $(e_1,\{h_1,h_2\})$ is the semi*-open soft set that contains all the soft points with respect to the parameter e_1 and $(e_2,\{h_1,h_2\})$ contains all the soft points with respect to the parameter e_2. Hence $(U_E,\tilde{\tau})$ is a soft semi*T1-space.

Theorem 4.3. Every soft point of a soft topological space $(U_E,\tilde{\tau})$ is a semi*-closed soft set if and only if $(U_E,\tilde{\tau})$ is a soft semi*T1-space.

Proof: Let ℓ_F and ℓ_G be two distinct soft points in U_E. Since ℓ_F and ℓ_G are semi*-closed soft sets their complements are semi*-open soft sets. Also $\ell_F \tilde{\in} (\ell_G)^c$, $\ell_G \tilde{\in} (\ell_F)^c$ and $\ell_F \tilde{\notin} \ell_G$, $\ell_G \tilde{\notin} \ell_F$. Hence $(U_E,\tilde{\tau})$ is a soft semi*T1-space.

Conversely let $(U_E,\tilde{\tau})$ be a soft semi*T1-space. Let ℓ_F be a soft point of U_E. Since $(U_E,\tilde{\tau})$ is a soft semi*T1-space, for each soft point $\ell_G \neq \ell_F$ we can find a soft semi*-open set K_D of ℓ_G such that $\ell_F \tilde{\notin} K_D$. The union of all these soft semi*-open sets is a soft semi*-open set and it is the complement of ℓ_F in U_E. Hence ℓ_F is a soft semi*-closed.

Theorem 4.4. A soft subspace of a soft semi*T_1-space is soft semi*T_1.

Proof: Let K_D be a soft subspace of a soft semi*T_1-space U_E. Let ℓ_F and ℓ_G be two distinct soft points of K_D. Also they are distinct soft points of U_E. As U_E is a soft semi*T_1-space, there exist semi*-open soft sets V_B and H_C such that $\ell_F \tilde{\in} V_B$ and $\ell_G \tilde{\notin} V_B$, $\ell_G \tilde{\in} H_C$ and $\ell_F \tilde{\notin} H_C$. This implies $V_B \tilde{\cap} K_D$ and $H_C \tilde{\cap} K_D$ are semi*-open soft sets of K_D such that $\ell_F \tilde{\in} V_B \tilde{\cap} K_D$ and $\ell_G \tilde{\notin} V_B \tilde{\cap} K_D$, $\ell_G \tilde{\in} H_C \tilde{\cap} K_D$ and $\ell_F \tilde{\notin} H_C \tilde{\cap} K_D$.

5. SOFT SEMI*T_2-SPACE

Definition 5.1. A soft topological space $(U_E, \tilde{\tau})$ is said to be a soft semi*T_2-space if and only if for distinct soft points ℓ_F and ℓ_G of U_E there exist disjoint soft semi*-open soft sets H_B and G_C such that $\ell_F \tilde{\in} H_B$ and $\ell_G \tilde{\in} G_C$.

Example 5.2. In the example 3.2, $(e_1, \{h_1, h_2\})$ is a semi*-open soft set contains all soft points with respect to the parameter e_1 and $(e_2, \{h_1, h_2\})$ is a semi*-open soft set contains all soft points with respect to the parameter e_2 and also they are disjoint semi*-open soft set. Hence $(U_E, \tilde{\tau})$ is a soft semi*T_2-space.

Theorem 5.3. A soft subspace of a soft semi*T_2-space is soft semi*-T_2.

Proof: Let $(U_E, \tilde{\tau})$ be a soft semi*T_2-space and V_B be a soft subspace of U_E. Let ℓ_F and ℓ_G be two distinct soft points of V_B. Then they are soft points of U_E also. As U_E is a soft semi*T_2-space, there exists two distinct soft semi*-open sets K_D and G_C such that $\ell_F \tilde{\in} K_D$ and $\ell_G \tilde{\in} G_C$. Then $K_D \tilde{\cap} V_B$ and $G_C \tilde{\cap} V_B$ are disjoint semi*-open soft sets of V_B containing ℓ_F and ℓ_G respectively. Hence V_B is a soft semi*T_2-space.

Theorem 5.4. A soft topological space $(U_E, \tilde{\tau})$ is soft semi*T_2 if and only if for distinct soft points ℓ_G and ℓ_K of U_E there exists a semi*-open soft set S_D containing ℓ_G but not ℓ_K such that $\ell_K \tilde{\notin} ss^*cl(S_D)$.

Proof: Let ℓ_G and ℓ_K be distinct soft points in a soft semi*T_2-space $(U_E, \tilde{\tau})$. Then there exist disjoint semi*-open soft sets H_A and W_D such that $\ell_G \tilde{\in} H_A$ and $\ell_K \tilde{\in} W_D$. As H_A and W_D are disjoint H_A containing ℓ_G but not ℓ_K. We have to prove $\ell_K \tilde{\notin} ss^*cl(H_A)$. Suppose $\ell_K \tilde{\in} ss^*cl(H_A)$. Then by theorem 4.9 every semi*-open soft set containing ℓ_K must intersect H_A. This is a contradiction. Hence $\ell_K \tilde{\notin} ss^*cl(H_A)$. Conversely let ℓ_G and ℓ_K be a pair of distinct soft points of U_E. Then by our assumption there exists a semi*-open soft set S_D

containing ℓ_G but not ℓ_K such that $\ell_K \tilde{\not\in} ss^*cl(S_D)$. This implies $\ell_K \tilde{\in} (ss^*cl(S_D))^c$. As $ss^*cl(S_D)$ is soft semi*-closed, $(ss^*cl(S_D))^c$ is soft semi*-open. Hence S_D and $(ss^*cl(S_D))^c$ are disjoint semi*-open soft sets containing ℓ_G and ℓ_K respectively. Therefore (U_E, τ) is soft semi*T$_2$-space.

6. SOFT SEMI*T$_3$-SPACE

Definition 6.1. A soft topological space $(U_E, \tilde{\tau})$ is said to be a soft semi*-regular space if for every soft point ℓ_K and semi*-closed soft set L_A not containing ℓ_K there exist disjoint semi*-open soft sets H_{B_1} and H_{B_2} such that $\ell_K \tilde{\in} H_{B_1}$ and $L_A \tilde{\subseteq} H_{B_2}$ where $B_1, B_2 \subseteq E$.
A soft semi*-regular space together with soft semi*T$_1$-space is called soft semi*T$_3$-space

Example 6.2. The soft topological space given in example 3.2 is a soft semi*-regular space and from the example 4.2 it is also semi*T$_1$-space and hence is a soft semi*T$_3$-space.

Lemma 6.3. Every soft subspace of a soft semi*-regular space is semi*-regular.

Proof: Let $(U_E, \tilde{\tau})$ be a soft topological space and V_B be a soft subspace of U_E. Let ℓ_F be a soft point in V_B and K_D be soft semi*-closed set in V_B such that $\ell_F \tilde{\not\in} K_D$. As V_B is a subspace of U_E $K_D = H_C \tilde{\bigcap} V_B$ where H_C is soft semi*-closed in U_E. Since $\ell_F \tilde{\not\in} K_D$ and $\ell_F \tilde{\in} V_B$, $\ell_F \tilde{\not\in} H_C$. Then there exist soft semi*-open soft sets G_{A_1} and G_{A_2} of U_E such that $\ell_F \tilde{\in} G_{A_1}$ and $H_C \tilde{\subseteq} G_{A_2}$ .Then $\ell_F \tilde{\in} G_{A_1} \tilde{\bigcap} V_B$ where $G_{A_1} \tilde{\bigcap} V_B$ is a soft semi*-open set in V_B. Also $H_C \tilde{\bigcap} V_B \tilde{\subseteq} G_{A_2} \tilde{\bigcap} V_B$. That is $K_D \tilde{\subseteq} G_{A_1} \tilde{\bigcap} V_B$ where $G_{A_1} \tilde{\bigcap} V_B$ is a soft semi*-open set in V_B.
Hence soft subspace of a soft semi*-regular space is semi*-regular.

Theorem 6.4. Soft subspace of a soft semi*T$_3$-space is soft semi*T$_3$.

Proof: Let $(U_E, \tilde{\tau})$ be a soft topological space and V_B be a soft subspace of U_E. We know that by theorem 8.9 soft subspace of a soft semi*T$_1$-space is soft semi*T$_1$ and by lemma 8.16 soft subspace of a soft semi*-regular space is semi*-regular. Hence V_B is a soft semi*T$_3$-space.

7. SOFT SEMI*T$_4$-SPACE

Definition 7.1. A soft topological space $(U_E, \tilde{\tau})$ is said to be soft semi*-normal if for every pair of disjoint semi*-closed soft sets L_A and K_D there exist two disjoint semi*-open soft sets H_{A_1} and H_{A_2} such that $L_A \tilde{\subseteq} H_{A_1}$ and $K_D \tilde{\subseteq} H_{A_2}$.
A soft semi*-normal T$_1$-space is called soft semi*T$_4$-space.

Example 7.2. The soft topological space $(U_E,\tilde{\tau})$ given in the example 3.2 is soft semi*-normal also by the example 4.2, $(U_E,\tilde{\tau})$ is soft semi*T$_1$-space. Hence $(U_E,\tilde{\tau})$ is a soft semi*T$_4$-space.

Theorem 7.3. A soft topological space $(U_E,\tilde{\tau})$ is soft semi*-normal if and only if for any semi*-closed soft set L_A and semi*-open soft set G_C containing L_A there exists a semi*-open soft set H_B such that $L_A \tilde{\subseteq} H_B$ and $ss^*cl(H_B) \tilde{\subseteq} G_C$.

Proof: Let $(U_E,\tilde{\tau})$ be a semi*-normal space and L_A be a semi*-closed soft set and G_C be a semi*-open soft set containing L_A. This implies L_A and $(G_C)^c$ are disjoint semi*-closed soft sets. As $(U_E,\tilde{\tau})$ is semi*-normal, there exist disjoint semi*-open soft sets H_{A_1} and H_{A_2} such that $L_A \tilde{\subseteq} H_{A_1}$ and $(G_C)^c \tilde{\subseteq} H_{A_2}$.Now $H_{A_1} \tilde{\subseteq} (H_{A_2})^c$. This implies $ss^*cl(H_{A_1}) \tilde{\subseteq} ss^*cl(H_{A_2})^c = (H_{A_2})^c$. Also $(H_{A_2})^c \tilde{\subseteq} G_C$. Therefore $ss^*cl(H_{A_1}) \tilde{\subseteq} G_C$.
Conversely let S_A and K_D be a pair of disjoint semi*-closed soft sets. Then $S_A \tilde{\subseteq} (K_D)^c$. Then by our hypothesis there exists an semi*-open soft set H_B such that $S_A \tilde{\subseteq} H_B$ and $ss^*cl(H_B) \tilde{\subseteq} (K_D)^c$. This implies $(K_D) \tilde{\subseteq} (ss^*cl(H_B))^c$. Hence H_B and $(ss^*cl(H_B))^c$ are soft semi*-open sets containing S_A and K_D respectively.

Theorem 7.4. Let $f: SS(U)_E \to SS(V)_{E'}$ be a soft surjective function which is both soft semi*-irresolute and soft semi*-open where $(U_E,\tilde{\tau})$ and $(V_{E'},\delta)$ are soft topological space. If U_E is soft semi*-normal then so is $V_{E'}$.

Proof: Let L_A and K_D be a disjoint pair of semi*-closed soft sets of $V_{E'}$. As f is soft semi*-irresolute $f^{-1}(L_A)$ and $f^{-1}(K_D)$ are disjoint semi*-closed soft sets of U_E. This implies there exist disjoint semi*-open soft sets H_{A_1} and H_{A_2} such that $f^{-1}(L_A) \tilde{\subseteq} H_{A_1}$ and $f^{-1}(K_D) \tilde{\subseteq} H_{A_2}$ since U_E is normal. Then $L_A \tilde{\subseteq} f(H_{A_1})$ and $K_D \tilde{\subseteq} f(H_{A_2})$. This implies $f(H_{A_1})$ and $f(H_{A_2})$ are disjoint semi*-open soft sets of $V_{E'}$ containing L_A and K_D respectively.

Theorem 7.5. A semi*-closed soft subspace of a soft semi*-normal space is semi*-normal.

Proof: Let V_B be a soft semi*-closed subspace of a soft semi*-normal space U_E. Let L_{A_1} and L_{A_2} be a disjoint pair of soft semi*-closed sets. Since V_B is a soft subspace of U_E there exist soft semi*-closed sets K_{D_1} and K_{D_2} of U_E such that $L_{A_1} = K_{D_1} \tilde{\cap} V_B$ and $L_{A_2} = K_{D_2} \tilde{\cap} V_B$. As V_B, K_{D_1} and K_{D_2} are soft semi*-closed sets of U_E, $K_{D_1} \tilde{\cap} V_B$ and $K_{D_2} \tilde{\cap} V_B$ are soft semi*-closed sets U_E. As U_E is normal there exist disjoint semi*-open soft sets G_{C_1} and G_{C_2} such that $K_{D_1} \tilde{\cap} V_B \tilde{\subseteq} G_{C_1}$ and $K_{D_2} \tilde{\cap} V_B \tilde{\subseteq} G_{C_2}$. Then $L_{A_1} \tilde{\subseteq} G_{C_1}$ and $L_{A_2} \tilde{\subseteq} G_{C_2}$. Hence $G_{C_1} \tilde{\cap} V_B$ and

$G_{C_2} \tilde{\bigcap} V_B$ are disjoint pair of soft semi*-open sets of V_B containing L_{A_1} and L_{A_2} respectively. Therefore V_B is soft semi*-normal.

Theorem 7.6. Semi*-$T_4 \Rightarrow$ Semi*-$T_3 \Rightarrow$ Semi*-$T_2 \Rightarrow$ Semi*-$T_1 \Rightarrow$ Semi*-T_0.

Proof: Let $(U_E, \tilde{\tau})$ be a soft semi* T_4-space. Let ℓ_F and V_B be a pair of soft point and soft semi*-closed set of U_E such that $\ell_F \tilde{\not\in} V_B$. Then $ss^*cl(\ell_F)$ is the soft semi*-closed set containing ℓ_F. As soft semi* T_4-space is soft semi*T_1-space by theorem 6.7 every soft point in U_E is soft semi*-closed. Hence $ss^*cl(\ell_F)$ and V_B are disjoint soft semi*-closed sets of U_E. Then by the definition of soft semi*-normal there exist two disjoint soft semi*-open sets G_{A_1} and G_{A_2} such that $\ell_F \tilde{\in} G_{A_1}$ and $V_B \tilde{\subseteq} G_{A_2}$. Therefore $(U_E, \tilde{\tau})$ is a soft semi*T_3-space.

Now let us take $(U_E, \tilde{\tau})$ be a soft semi* T_3-space. Let ℓ_F and ℓ_G be two distinct soft points of U_E. Then by the definition of soft semi* T_1-space ℓ_F is a soft semi*-closed set. Hence we have a soft point and a soft semi*-closed set such that $\ell_G \tilde{\not\in} ss^*cl(\ell_F)$. Hence there exist disjoint soft semi*-open sets H_{A_1} and H_{A_2} such that $\ell_F \tilde{\in} H_{A_1}$ and $\ell_G \tilde{\in} H_{A_2}$ and hence $(U_E, \tilde{\tau})$ is a soft semi*T_2-space.

Now let us take $(U_E, \tilde{\tau})$ be a soft semi* T_2-space. Let ℓ_F and ℓ_G be two distinct soft points of U_E. Then by the definition of soft semi* T_2-space there exist disjoint soft semi*-open sets H_{A_1} and H_{A_2} such that $\ell_F \tilde{\in} H_{A_1}$ and $\ell_G \tilde{\in} H_{A_2}$. As H_{A_1} and H_{A_2} are disjoint $\ell_F \tilde{\not\in} H_{A_2}$ and $\ell_G \tilde{\not\in} H_{A_1}$. Hence $(U_E, \tilde{\tau})$ is a soft semi*T_1-space.

Now let us take $(U_E, \tilde{\tau})$ be a soft semi* T_1-space. Let ℓ_F and ℓ_G be two distinct soft points of U_E. Then by the definition of soft semi* T_1-space there exist soft semi*-open sets H_{A_1} and H_{A_2} containing ℓ_F and ℓ_G. That is H_{A_2} is a soft semi*-open set containing ℓ_G and $\ell_F \tilde{\not\in} H_{A_2}$. Therefore $(U_E, \tilde{\tau})$ is a soft semi*T_0-space.

8. CONCLUSION

In this article, we introduce four separation axioms and explore some of their fundamental properties. In general topology, Normal$\Rightarrow$Regular$\Rightarrow$Hausdorff (T_2) $\Rightarrow T_1 \Rightarrow T_{1/2} \Rightarrow T_0$. We have established this relation in soft topology via semi-open sets. Further we planned to introduce semi*-$T_{1/2}$ spaces and interrogate its properties.

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