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Extra Skolem Difference Mean Labeling of Some Graphs

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ABSTRACT

A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices x ϵ V with distinct elements $f(x)$ from the set $\{1,2 \dots p+q\}$ in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u) - f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u) - f(v)|$ is odd and the resulting labels of the edges are distinct and are from $\{1,2 \dots q\}$. A graph that admits skolem difference mean labeling is called a skolem difference mean graph. If one of the skolem difference mean labeling of G satisfies the condition that all the labels of the vertices are odd, then we call this skolem difference mean labeling an extra skolem difference mean labeling and call the graph G an extra skolem difference mean graph. In this paper, extra skolem difference mean labeling of some graphs are studied.

Keywords: Skolem difference mean labeling, extra skolem difference mean labeling, F-tree, Y-tree

1. INTRODUCTION

Graphs considered in this paper are finite, undirected and simple. Let $G = (V,E)$ be a graph with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges/both) then the labeling is called a vertex (edge/total) labeling. Rosa [16] introduced $\beta -$

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valuation of a graph and Golomb [6] called it as graceful labeling. There are several types of graph labeling and a detailed survey is found in [7]. Labeled graphs are becoming an increasing useful family of mathematical models for a broad range of applications like designing X-ray crystallography, formulating a communication network addressing system, determining an optimal circuit layouts, problems in additive number theory etc. A systematic presentation of diverse applications of graph labeling is given in [1,2,3,11,22].The concept of skolem difference mean labeling was introduced in [12] and various skolem difference mean labeling were studied in [4,5,8-10,13,14,17,19-21]. The concept of extra skolem difference mean labeling was introduced in [18] and further studied in [15]. The following definitions are necessary for the present study.

Definition 1.1: Bistar $B_{m,n}$ is the graph obtained from K_2 by joining *m* pendant edges to one end of K_2 and n pendant edges to the other end of K_2.

Definition 1.2: $B(m,n,k)$ is the graph obtained from a path of length *k* by attaching the star $K_{1,m}$ and $K_{1,n}$with its pendant vertices.

Definition 1.3: Coconut tree graph is obtained by identifying the central vertex of $K_{1,m}$ with a pendant vertex of the path P_n.

Definition 1.4: *F*-tree on *n+2* vertices, denoted by F_n , is obtained from a path P_n by attaching exactly two pendant vertices to the *n-1* and n^{th} vertex of P_n .

Definition 1.5: Y-tree on *n+1* vertices, denoted by Y_n , is obtained from a path P_n by attaching a pendant vertex to the n^{th} vertex of P_n .

Definition 1.6 [12]: A graph $G= (V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices x ϵ V with distinct elements $f(x)$ from the set $\{1,2 \ldots p+q\}$ in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are from $\{1,2 \ldots q\}$. A graph that admits skolem difference mean labeling is called a skolem difference mean graph.

Definition 1.7 [18]: Let *G = (V,E)* be a skolem difference mean graph with *p* vertices and *q* edges. If one of the skolem difference mean labeling of *G* satisfies the condition that all the labels of the vertices are odd, then we call this skolem difference mean labeling an extra skolem difference mean labeling and call the graph *G* an extra skolem difference mean graph.

2. RESULTS

Theorem 2.1: $K_{1,n}$ is extra skolem difference mean.

Proof: Let $\{v, v_i \,/\, 1 \le i \le n\}$ be the vertices and $\{vv_i \,/\, 1 \le i \le n\}$ be the edges of $K_{1,n}$.

Then $|V(K_{1,n})|$ = n+1 and $|E(K_{1,n})|$ = n.

Let $f: V(G) \rightarrow \{1,3,5, \dots ,2n+1\}$ be defined as follows.

$f(v) = 2n+1$

$f(v_i) = 2i-1; 1 \leq i \leq n$

Then the induced edge label f^* is calculated as follows.

$f^*(vv_1) = n$

$f^*(vv_{i+1}) = n-i; 1 \leq i < n$

Then the edge labels are 1,2,3,…, n which are distinct.

Therefore $K_{1,n}$ is extra skolem difference mean.

Example 2.2: An extra skolem difference mean labeling of $K_{1,6}$ is given in Fig. 1.

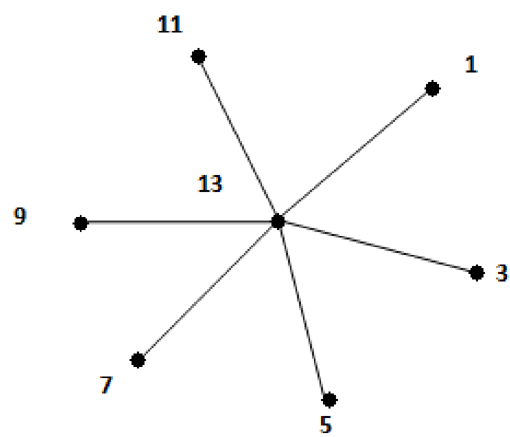

Fig. 1.

Theorem 2.3: B(m,n) is extra skolem difference mean.

Proof: Let $\{u, u_i, v, v_j: 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertices and let $\{uu_i, uv, vv_j: 1 \leq i \leq m, 1 \leq j \leq n\}$ be the edges of B(m,n).

Then $|V(\mathrm{B(m,n)})| = m+n+2$ and $|E(\mathrm{B(m,n)})| = m+n+1$

Let $f: V(G) \rightarrow \{1,2,3, \dots ,2m+2n+3\}$ be defined as follows.

$f(u) = 1$

$f(u_{i+1}) = 2m+2n+3-2i \,, 0 \leq i < m$

$f(v) = f(u_m) - 2$

$f(v_j) = f(v) - 2j \,, 1 \leq j \leq n$

Then the induced edge label f^* is calculated as follows.

$f^*(uu_{i+1}) = m+n+1-i \,, 0 \leq i < m$

$f^*(uv) = n+1$

$f^*(vv_j) = j\,, 1 \leq j \leq n.$

Thus B(m,n) is extra skolem difference mean.

Example 2.4: Extra skolem difference mean labeling of B (5,3) is shown in Fig. 2.

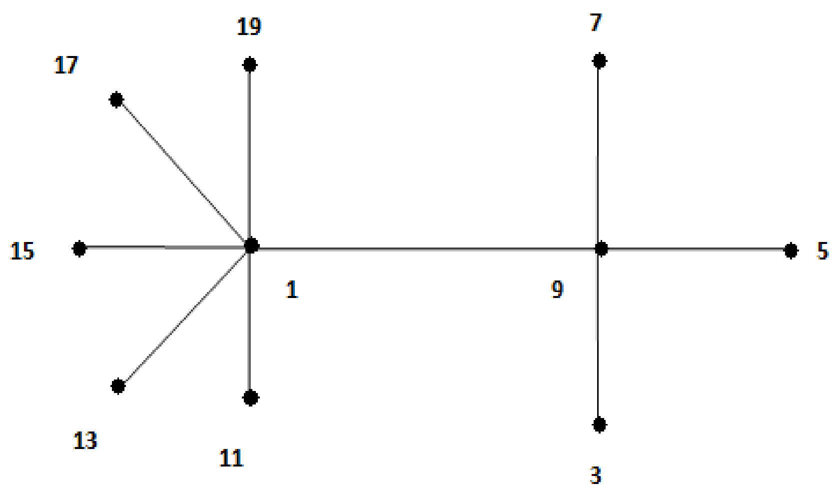

Fig. 2.

Theorem 2.5: $B(m,n,k)$ is extra skolem difference mean.

Proof: Let $\{u_i, v_j, w_t / \ 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq t \leq k+1\}$ be the vertices and let $\{w_1 u_i / 1 \leq i \leq m\} \cup \{w_{k+1} v_j / 1 \leq j \leq n\} \cup \{w_t w_{t+1} / 1 \leq t \leq k\}$ be the edges of $B(m,n,k)$.

Then $|V(B(m,n,k))| = m+n+k+1$ and $|E(B(m,n,k))| = m+n+k$

Let $f: V(G) \to \{1,3,5, \ldots, 2m+2n+2k+1\}$ be defined as follows.

Case i: k is odd.

$f(u_i) = 2i-1;\ 1 \leq i \leq m$

$f(w_{2t+1}) = 2m+2n+2k-1-2t;\quad 0 \leq t < \dfrac{k+1}{2}$

$f(w_{2t}) = 2m-1+2t;\quad 1 \leq t < \dfrac{k+1}{2}$

$f(v_j) = f(w_k) + 2j;\ \ 1 \leq j \leq n$

Case ii: k is even.

$f(u_i) = 2i-1;\ 1 \leq i \leq m$

$f(w_{2t+1}) = 2m+2n+2k-1-2t;\quad 0 \leq t < \dfrac{k}{2}$

$f(w_{2t}) = 2m - 1 + 2t; \quad 1 \leq t < \frac{k}{2}$

$f(v_j) = f(w_k) - 2j; \;\; 1 \leq j \leq n$

Then the induced edge label f^* is calculated as follows:

$f^*(\mathrm{w_1 u_i}) = \mathrm{m + n + k - i; 1 \leq i} \leq m$

$f^*(\mathrm{w_t w_{t+1}}) = n + k - \mathrm{t; 1 \leq t} < k$

$f^*(\mathrm{w_{k+1} v_j}) = n + 1 - \mathrm{j; 1 \leq j \leq n}$

Therefore $B(m, n, k)$ is extra skolem difference mean.

Example 2.6: Extra skolem difference mean labeling of $B(2,4,4)$ and $B(3,4,7)$ are given in Fig. 3 and Fig. 4 respectively.

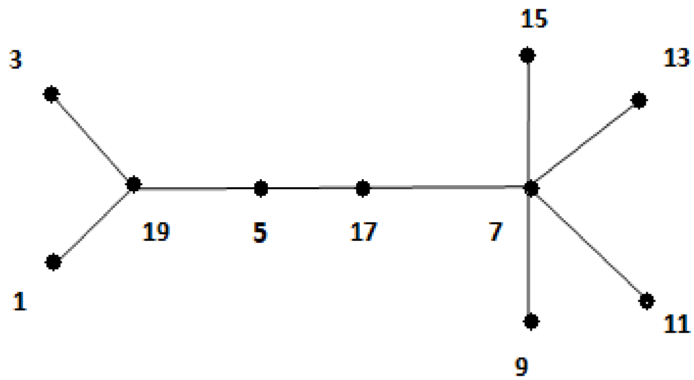

Fig. 3.

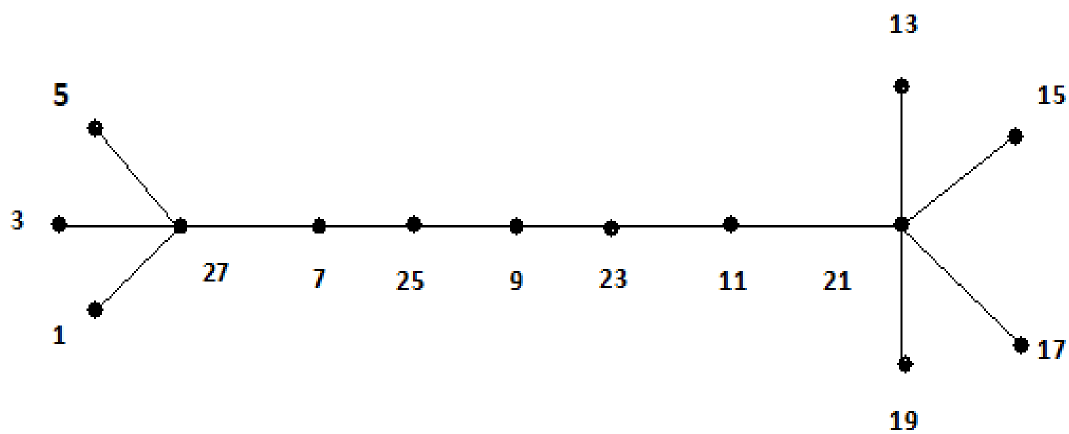

Fig. 4.

Theorem 2.7: Coconut tree *CT(m,n)* is extra skolem difference mean.

Proof: Let $\{u_i, v_j \, / \;\; 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertices and let $\left\{\frac{u_i v_1, v_i v_{i+1}}{1} \leq i \leq m, 1 \leq j \leq n - 1\right\}$ be the edges of *CT(m,n)* .

Then $|V(CT(m,n))| = m+n$ and $|E(CT(m,n))| = m+n-1$.

Let $f: V(G) \rightarrow \{1,3,5,\dots,2m+2n-1\}$ be defined as follows.

Case i: n is odd.

$f(u_{i+1}) = 2m+2n-1-2i; 0 \leq i < m$

$f(v_{2j+1}) = 2j+1; \quad 0 \leq j < \frac{n+1}{2}$

$f(v_{2j}) = 2n+1-2j; 1 \leq j < \frac{n+1}{2}$

Case ii: n is even.

$f(u_{i+1}) = 2m+2n-1-2i; 0 \leq i < m$

$f(v_{2j+1}) = 2j+1; \quad 0 \leq j < \frac{n}{2}$

$f(v_{2j}) = 2n+1-2j; 1 \leq j \leq \frac{n}{2}$

Then the induced edge label f^* is calculated as follows.

$f^*(\mathrm{u_i v_1}) = \mathrm{m+n-i}; \quad 1 \leq \mathrm{i} \leq m$

$f^*(\mathrm{v_i v_{i+1}}) = n - \mathrm{i}; 1 \leq \mathrm{i} < n$

Therefore coconut tree $CT(m,n)$ is extra skolem difference mean.

Example 2.8: Extra skolem difference mean labeling of $CT(4,5)$ and $CT(7,4)$ are given in Fig. 5 and Fig. 6 respectively.

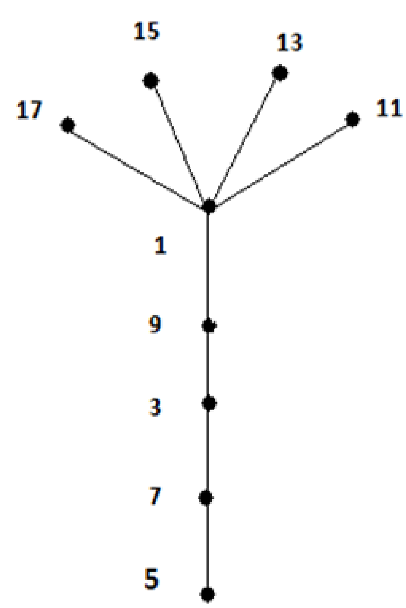

Fig. 5.

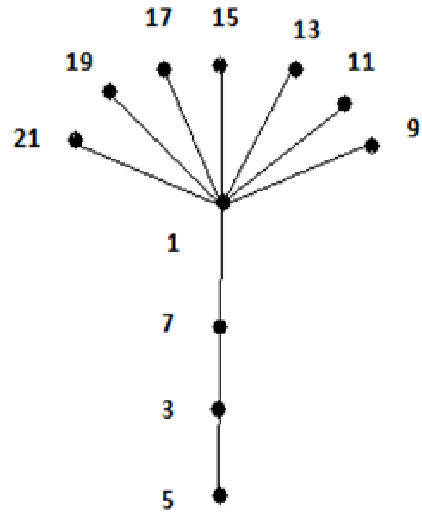

Fig. 6.

Theorem 2.9: Let G be a graph obtained by identifying a pendant vertex of P_m with a leaf of $K_{1,n}$. Then G is extraskolem difference mean for all values of m and n.

Proof: Let $\{u, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ be the vertices and let $\{uu_i, u_n v_2, v_j v_{j+1} : 2 \leq i \leq n, 1 \leq j \leq m-1\}$ be the edges of G. Then $|V(G)| = m + n$ and $|E(G)| = m + n - 1$. Let $f: V(G) \to \{1,3,5, \dots, 2m + 2n - 1\}$ be defines as follows

Case 1: m is odd

$f(u) = 2m + 2n - 1$

$f(u_{i+1}) = 2i + 1\,; 0 \leq i < n$

$f(v_{2j+1}) = f(u_n) + 2j\,; 1 \leq j < \dfrac{m+1}{2}$

$f(v_{2j}) = f(u) - 2j\,; 1 \leq j < \dfrac{m+1}{2}$

Case 2: m is even

$f(u) = 2m + 2n - 1$

$f(u_{i+1}) = 2i + 1\,; 0 \leq i < n$

$f(v_{2j+1}) = f(u_n) + 2j\,; 1 \leq j < \dfrac{m}{2}$

$f(v_{2j}) = f(u) - 2j\,; 1 \leq j \leq \dfrac{m}{2}$

Then the induced edge label f^* is calculated as follows:

$f^*(uu_{i+1}) = m + n - 1 - i\,, 0 \leq i < n$

$f^*(u_n v_2) = m - 1$

$f^*\left(v_j v_{j+1}\right) = m - j\ ;\ 2 \le i < m - 1$

Thus G is an extra skolem difference mean.

Example 2.10: An extra skolem difference mean labeling graph obtained by identifying a pendant vertex of P_5 with a leaf of $K_{1,6}$ is given in Fig. 7.

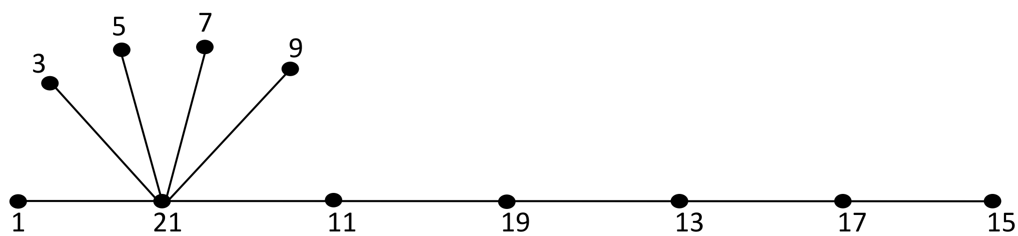

Fig. 7.

Theorem 2.11: A F-tree FP_n , $n \ge 3$ is extra skolem difference mean.

Proof: Let $\{u, v, v_i / 1 \le i \le n\}$ be the vertices and $\{v_i v_{i+1}, 1 \le i \le n-1\} \cup \{uv_{n-1}, vv_n\}$ be the edges of FP_n.

Then $|V(FP_n)|$ = n+2 and $|E(FP_n)|$ = n+1

Let $f: V(G) \to \{1,3,5, \dots, 2n+3\}$ be defined as follows.

Case 1: n is odd

$f(v) = 1$

$f(v_n) = 2n + 3$

$f(u) = f(v_n) - 2$

$f(v_{2i+1}) = n + 2 + 2i; 0 \le i < \dfrac{n-1}{2}$

$f(v_{2i}) = f(v_1) - 2i;\ \ 1 \le i < n - 2$

Case 2: n is even

$f(v) = 1$

$f(v_n) = 2n + 3$

$f(u) = f(v_n) - 2$

$f(v_{2i+1}) = n + 1 - 2i; 0 \le i < \dfrac{n}{2}$

$f(v_{2i}) = f(v_1) + 2i;\ \ 1 \le i \le \dfrac{n-2}{2}$

Then the induced edge label f^* is calculated as follows.

$f^*(v_i v_{i+1}) = i;\ \ 1 \le i \le n-2$

$f^*(uv_{n-1}) = n-1;$

$f^*(v_{n-1}v_n) = n$

$f^*(v_n v) = n+1$

Thus the F-tree FP$_n$, $n \ge 3$ is extra skolem difference mean.

Example 2.12: Extra skolem difference mean labeling of FP$_5$ and FP$_6$, $n \ge 3$ are shown in Fig.8 and Fig. 9 respectively.

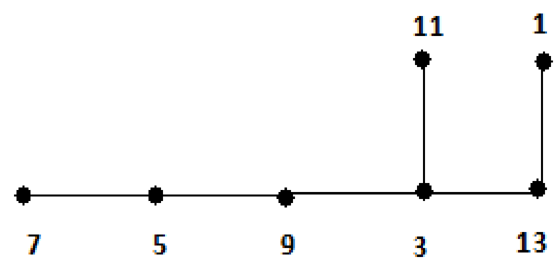

Fig. 8.

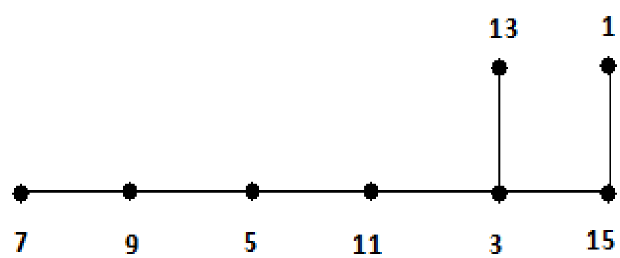

Fig. 9.

Theorem 2.13: A Y-tree is extra skolem difference mean.

Proof: Let $\{v, v_i/1 \le i \le n\}$ be the vertices and $\left\{v_i v_{i+1}, \frac{v_{n-1}v}{1} \le i \le n-1\right\}$ be the edges of Y-tree .

Then $|V|$ = n+1 and $|E|$ = n

Let $f: V(G) \to \{1,3,5, \dots ,2n+1\}$ be defined as follows.

Case 1: n is odd

$$f(v_{2i+1}) = 2i+1\,;0 \le i < \frac{n-1}{2}$$

$$f(v_{2i+2}) = 2n + 1 - 2i;\ 0 \le i < \frac{n-1}{2}$$

$$f(v_n) = f(v_{n-1}) - 2$$

$$f(v) = f(v_n) - 2$$

Case 2: n is even

$$f(v_{2i+1}) = 2i + 1\ ;0 \le i < \frac{n}{2}$$

$$f(v_{2i+2}) = 2n + 1 - 2i;\ 0 \le i < \frac{n-2}{2}$$

$$f(v_n) = f(v_{n-1}) + 2$$

$$f(v) = f(v_n) + 2$$

Then the induced edge label f^* is calculated as follows.

$$f^*(v_i v_{i+1}) = n + 1 - i;\ \ 1 \le i < n - 1$$

$$f^*(v_{n-1} v) = 2$$

$$f^*(v_{n-1} v_n) = 1$$

Thus Y-tree is an extra skolem difference mean.

Example 2.14: Extra skolem difference mean labeling of Y_7 and Y_{10} are shown in Fig. 10 and Fig. 11 respectively.

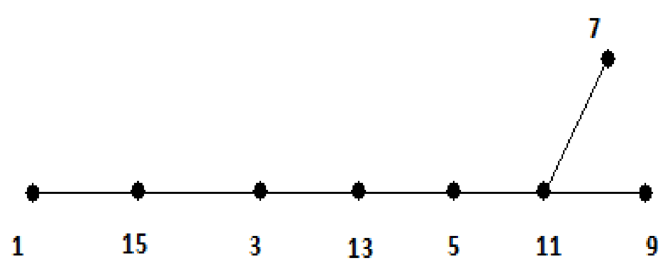

Fig. 10.

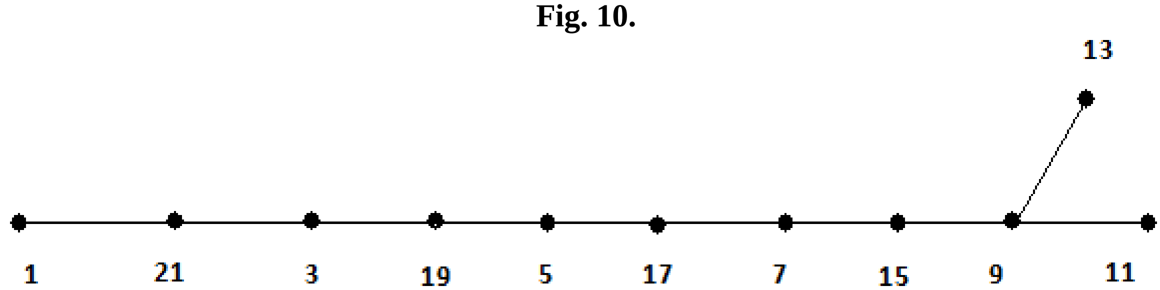

Fig. 11.

3. CONCLUSION

In this paper, the authors studied the extra skolem difference mean labeling of some graphs. Similar study can be extended for other graphs.

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