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# Extended Lognormal Distribution: Properties and Applications

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**ABSTRACT**

This paper is devoted to study a form of lognormal distributions family and introduce some of its basic properties. It presents new derivate models that find many applications will be useful for practitioners in various fields. The abstract explores some of the basic characteristics of the family of abnormal distributions and provides some practical methods for analyzing some different applied fields related to theoretical and applied statistical sciences. The proposed model allows for the improvement of the relevance of near-real data and opens broad horizons for the study of phenomena that can be addressed through the results obtained. This study is devoted to a review of fitting data, discuss distribution laws for models and describe the approaches used for parameterization and classification of models. Finally, a set of concluding observations has been developed to track the mix distributions and their adaptation mechanisms.

***Keywords*:** Lognormal Distribution, Mixture, Generating and Quantile Functions, Model, Hazard Rate Function, Characteristics and Survival Function

## 1. HISTORY OF THE LOGNORMAL DISTRIBUTION

The exploratory study of the subject indicates that the first to publish information on the logarithmic distribution is the publication of Scottish mathematician John Napier in the study

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of a table of logarithmic values in 1914 [13]. The logarithmic distribution is characterized by many tests and statistical models suitable for data that follow a particular distribution can be modified using the logarithmic function. It was first used to represent environmental data that have near-natural representation in the positive period, and because extended normal distribution and extended normal inverse Gaussian distribution have some similarities in the representation of research phenomena [6].

More recently, the search for an effective method of distribution represents the volume of data that follow the logarithmic distribution of the subjects that are of great interest to researchers, and by tracking the results obtained during the last quarter of a century in various regions of the world, which allowed many researchers to propose distribution As a possible alternative to representing energy law allocations in many studies [19].

Researchers Galton (1879) and McAlister (1879) began to use abnormal distribution in their joint article as an estimate of the site. He then discussed Kapteyn (1903) the most important properties of distribution and reviewed in his article the most important changes and relations between this distribution and the normal distribution. Both encouraged Kapteyn and Van Uven (1916) to devise a method for estimating the distribution parameters using a graph. In view of the results achieved, other statisticians, such as Pearson and others, continued to study the distribution in general B and to find a way to convert the target data for normal distribution. Due to some special cases, abnormal distribution has been used with extreme caution, especially for evaluation purposes in financing and the representation of financial phenomena [7, 19].

Dickson, GT (1932) managed in an unpublished study cited by some researchers such as Wilkinson's (1934), Marlow (1967), and Fenton, L.F. (1960) [19] proceeds from the basic idea and explains the abnormal logarithmic approximation of the sum of the logarithmic data based on an instantaneous time-match, later called the Vinton-Wilkinson approximation cited by Mitchell (1968) [20].

The hard work and classical manifestation of this subject were clearly written by Aitchison and Brown (1957). Researchers Johnson and Coetz were able to present an interesting and wide-ranging presentation in a field study in which the natural logarithmic distribution of a large sample was used and was not applied in small data until the late 1940s [17].

The Lognormal distribution is commonly used to model the life of units in which failure patterns are obviously stressful, but recent studies have shown that distribution can be generalized to other uses by converting data using the logarithm function to represent natural data. Due to the similarities with the normal distribution by changing the nature of the natural random variable to the lognormal random variable [14].

Some researchers in the 1970s noted attention and caution when using the Lognormal distribution; interpreting the results of representing certain funding issues due to differences and jumps in guessing periods, due to the fluctuation caused by the conversion function of natural data, several statisticians such as Pearson, who had general lack of confidence in the mode of transformation [1].

The upper tail of the lognormal distribution, which is very similar to the Pareto distribution, allowed to draw an accurate integral expression of the characteristic function of this distribution, which is indicated by researcher Leipnik, R.B. (1991) [16].

Some studies have indicated that sometimes the Pareto distribution is a uniform slope, which means that the order of output and size between the main data, especially when there are constant values, makes the conversion give repetitive values for modulation and some of these alerts are mentioned in the sources [8-11].

Usually when modelling the life of production units where the patterns of failure are of great stress nature, the logarithmic distribution is mentioned by name and is the first preferred to represent the phenomenon data because it has some similarities with the normal distribution, and for this reason, there are many mathematical similarities between the two distributions [14].

Due to the many mathematical similarities between the two distributions, the data are practically represented by the distribution of the random variable naturally, and then the data is modified to fit the logarithm distribution through the function of the random variable naturally. For this reason, mathematical thinking has evolved in constructing probability measures and biasing of the capabilities of the parameters in which the random variable is defined, much like these two distributions [18].

The study of the logarithmic distributions in more detail shows that there is valuable information from the results, especially when the sample obtained is well suited. The results showed the distinct distinction of this distribution in biological systems, income distribution, and tracking natural phenomena such as river flooding and climate degradation.

The central boundary theory binds these properties to each distribution and relates them to the normal distribution. Since the distribution of event rates usually tends to be abnormal and errors are just a random subset or sample of events, the distribution of failure rates and emergency phenomena also tends to be abnormal, all of which support the idea of expanding the field of representation by adding parameters And the scope for abnormal distribution [17].

In this article, a new probability distribution called an extended logarithmic distribution is introduced that aims to stabilize the defect in the old formula. The tail end of the constituent data of the target data, especially when the sample data is limited, and can also be used to calculate the probability of the instantaneous situation in a simple and easy to use manner.

Our aim in this study is to lay the groundwork for analyzing and interpreting data assuming some knowledge of mathematics and analyzes sufficient to portray the ability to understand and apply basic statistical methods.

## 2. DEFINITION OF LOGNORMAL DISTRIBUTION

The logarithmic distribution is usually used to assign continuous random sample data when it is clearly believed that the distribution is right-handed, and there are many examples including income and age variables, and flooding of rivers. Logarithmic distribution data can be directly derived from the normal distribution by looking at the abnormal and normal distribution $X = \exp(Y) = g(Y)$ by transformation $Y = \log(X) = g^{-1}(X)$ and the derivative of $g^{-1}(X)$ the Jacobean with respect to $X$ is $(X^{-1})$.

Thus,

$$f_X(X) = f_Y(\ln(X)) * (X^{-1})$$

This is equal to the lognormal distribution above. If $\ln(X)$ has normal distribution $X$ has Lognormal distribution. That is, if $X$ is normally distributed $\exp(X)$ is log normally distributed. If $Y$ is normally distributed with mean 0 and variance $\sigma$, then the random variable $X$ defined by the relationship $Y = \log(X)$ is distributed as Lognormal, and is denoted as lognormal $(0,\sigma^2)$. The random variable $X$ has a Lognormal distribution if $\log(X)$ has a normal distribution.

A Lognormal distribution has two parameters $\mu = 0$ and $\sigma^2$, which are the mean and variance of $\log(X)$, not $X$. This is the distribution of a positive variable whose logarithm has a Gaussian distribution.

Those interested in continuous distributions agree that the logarithmic distribution has a complexity that represents the depth of the basic conditions, and given the existence of various software systems it is facilitated by the fact that event rates are determined by a fundamentally multiplier process of tracking the data of the natural distribution skewed to the right.

Logarithmic distribution usually facilitates the tracking of the modeling of life data of units in which the failure and deviation patterns are very similar to the right; the probability density (pdf) and cumulative distribution function (cdf) functions can be formulated as follows:

$$f(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\,\frac{1}{x}\,\exp\left\{-\frac{1}{2}\left(\frac{\ln x}{\sigma}\right)^2\right\} \qquad (1)$$

and

$$F(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^x \frac{1}{t}\exp-\frac{1}{2}\left(\frac{\ln t}{\sigma}\right)^2 dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln x}{\sigma\sqrt{2}}\right)\right] \qquad (1)'$$

In order to accommodate the extent of the logarithmic distribution changes, it is generally defined in terms of zero mean and normal standard deviation logarithmically in general.

The diagram below shows that many of the pdf function diagrams shown in Fig. 1, (a), (b)), and the cdf function of the logarithmic distribution are a traceable amount of data for small parameter value $\sigma$ .

We say that the continuous random variable has an abnormal distribution with the parameters $\mu$ and $\sigma$ if the natural logarithm has a normal distribution and its probability density function is given by the following equation:

$$f(x,\mu,\sigma) = \begin{cases} \frac{1}{\delta\sqrt{2\pi}}\,\frac{1}{x}\,\exp\left\{-\frac{1}{2}\left(\frac{\ln x-\mu}{\delta}\right)^2\right\}; x \succ 0 \\ 0;\ x \le 0 \end{cases} \qquad (2)$$

where $\theta$ is the mean of $\log(X)$ and $\delta$ is the standard deviation of $\log(X)$.

It is possible therefore to write directly the cdf :

$$F(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^x \frac{1}{t}\exp\left[-\frac{1}{2}\left(\frac{\ln t-\mu}{\sigma}\right)^2\right] dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln x-\mu}{\sigma\sqrt{2}}\right)\right] \qquad (2)'$$

The probability density and aggregation functions of the logarithmic distribution are very asymmetric, which is the deviation in particular. This refers to the movement of the mean value by the symmetric standard deviation $\sigma$ to the right, and the most likely positional value is not generally identical (Figure (1, (a), (b)).

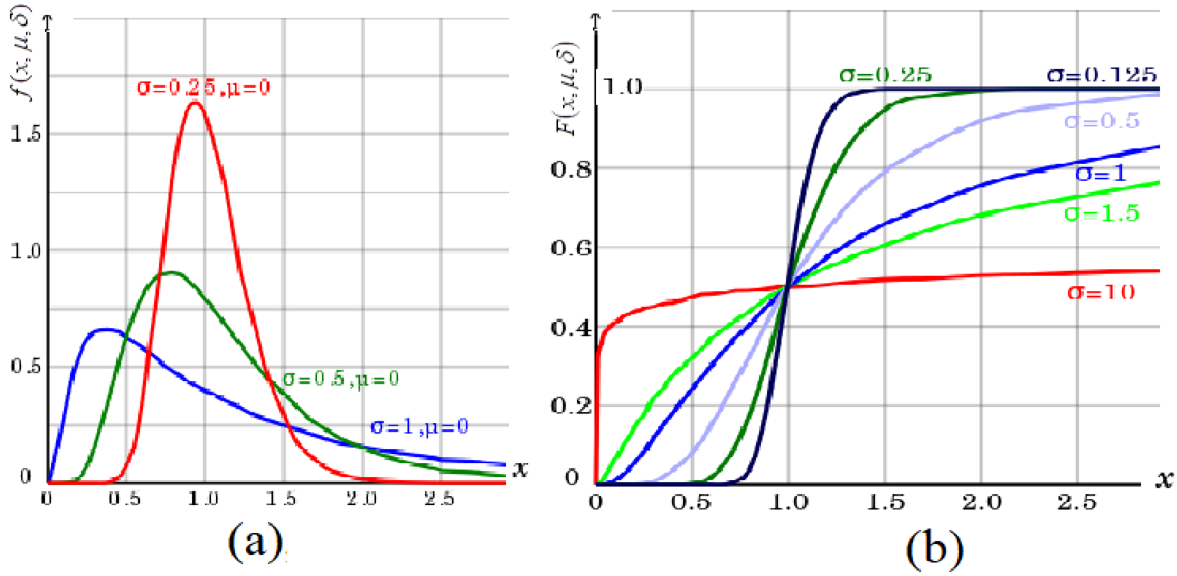


**Figure 1(a,b).** Graphs of $f_X(x, \sigma)$ and $F_X(x, \sigma)$ for various values of $\sigma$.

The values of the probability density function start at zero, then continue to increase to the point of position, then decrease significantly as the degree of deviation increases as the data value increases. Through this continuation of the increase is formed a curve that resembles the upper tail of the logarithmic distribution closely, which generally resembles the distribution of Pareto. You can search for a formula that represents the percentages of the area underneath the natural curve, where it increases from left to right. This change represents each standard deviation at a constant rate, the $100p^{th}$ percentile: $P(X \leq X_p) = p$ can be given by:

$$p = P(X \leq X_p) = \int_0^{X_p} \frac{1}{\sqrt{2\pi}\sigma t} e^{-(\ln(t)-\mu)^2/2\sigma^2} dt$$

## 3. CHARACTERISTICS OF THE EXTENDED LOGNORMAL DISTRIBUTION

For the values of the random variable following the logarithmic distribution that greatly increases 1, the probability density function rises very sharply at first, that is, for very small values near zero, and when increasing and sequencing on the x-axis in the usual axis, its image values decrease. Slightly early, then decreases sharply as in the exponential distribution, and in the weibull distribution, when adjusted using the parameter range separator in the period (0, 1) [4].

The natural logarithmic distribution is often found around the propagation form of the data, especially for the quantities of data associated with either the level of force, field strength or time domain, so the natural logarithmic distribution is explicitly used because the quasi-natural variable is the one that mutates to become The exact representative of the data is the logarithmic distribution, which means that the numerical values of the variable are the result of the work of many causes of simple individual significance that are multiplied individually at the end of the range of the random variable.

When tracking the distribution phenomenon it is clear that in some cases, the distribution of the random variable can be considered a result of mixture of two distributions, the first is the logarithmic distribution of long changes n and the second is for the short-term changes so the mixture is formed and the logarithmic distribution is shown.

It is also evident that the logarithmic distribution is continuously skewed to the right, thus a good companion to the Weibull distribution and Rayleigh distribution. Generally, the normal logarithm distribution data variable is assimilated after estimating the parameters based on the sample mean statistic and the standard deviation statistic.

Therefore, the logarithmic distribution becomes the probability distribution, so that it forms the natural record of the sample values at alignment, which has a natural distribution for the logarithmic variable, and its probability density function is defined as:

$$f(x,\alpha,\theta)=\frac{1}{x\beta\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln x-\theta}{\beta}\right)^2\right]$$

where $\theta$ and $\beta$ are the mean and the standard deviation of $\ln(X)$ respectly. These are related to the mean and standard deviation of random variable $x$ (μ and σ respectively) as follows:

1). $\mu=\exp\left(\theta+\frac{1}{2}\beta^2\right)$,

2). $\sigma=\sqrt{\exp[2\theta+\beta^2](\exp[\beta^2]-1)}$,

3). $\theta=\ln\mu-\frac{1}{2}\ln\left[\left(\frac{\sigma}{\mu}\right)^2+1\right]$,

4). $\beta=\sqrt{\ln\left[\left(\frac{\sigma}{\mu}\right)^2+1\right]}$ .

note that $P(x\le X)=F\left(\frac{\ln X-\theta}{\beta}\right)$ where $P$ represent the $F(x)$ of the log normal distribution, which is

$$F(x) = P(x \le X) = \int_0^X \frac{1}{x\beta\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \theta}{\beta}\right)^2\right] dx$$

**Proposition (1):** Let $\log(X)$ be random variable with lognormal distribution $Lognormal(\mu, \delta)$ Then the mean and standard deviation of random variable $X \sim (\mu, \sigma^2)$ are:

1). $\mu = \exp\left(\theta + \frac{1}{2}\delta^2\right)$

2). $\sigma = \sqrt{\left(e^{2\mu+\delta^2}\right)\left(e^{\delta^2} - 1\right)}$

**Proof:** The result follows by using the transformation technique, the proof of this is simple and straightforward.

**Proposition (2):** The median value and root mean square value of random variable $X \sim (\mu, \sigma^2)$ are:

1). $Median = \exp(\theta)$

2). $Root\ mean\ square\ value = \exp(\theta + \sigma^2)$

**Proof:** The characteristic quantities of the variable $x$ can be derived without difficulty.

## 4. SOME PROPERTIES OF LOGNORMAL DISTRIBUTION

The abnormal random variable $X$ is considered to be a Lognormal distribution specified with its mean $\mu$ and variance $\sigma^2$. If the logarithm of a random variable is a normal distribution. This section presents the important characteristics of the lognormal distribution, which we hope will achieve positive results when applied in tracking data with the same distribution. We can only hope to achieve favorable outcomes if we examine the change in distribution of the things that we measure.

**Property (1):** Let $F_X(x,0,\sigma)$ and $f_X(x,0,\sigma)$ be respectively the cumulative probability distribution function and the probability density function of the $N(0,1)$ distribution.

**Proof:** The result follows by using the definition

$$f_X(x,0,\sigma) = \frac{d}{dx} P(X \le x) = \frac{d}{dx} P(\ln X \le \ln x) = \frac{d}{dx} F\left(\frac{\ln x - \mu}{\sigma}\right)$$

$$= f\left(\frac{\ln x - \mu}{\sigma}\right) \frac{d}{dx}\left(\frac{\ln x - \mu}{\sigma}\right) = f\left(\frac{\ln x - \mu}{\sigma}\right)\left(\frac{1}{\sigma\, x}\right)$$

$$= \frac{1}{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\}$$

This completes the proof.

**Property (2):** A positive random variable $X$ is Lognormal distributed if the logarithm of $X$ is normally distributed, $\ln(X) \sim N(\mu, \sigma^2)$.

**Proof:** The result follows by using the transformation technique; the proof of this is simple and straightforward.

**Property (3):** The mean, expected square, variance and the standard deviation of the random variable following the lognormal distribution $X$ are defined respectively by the following equations:

1). $E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$

2). $E(X^2) = \exp(2\mu + 2\sigma^2)$

3). $V(X) = \left(e^{\sigma^2} - 1\right)\exp(2\mu + \sigma^2)$

4). $SD(X) = \sqrt{\left(e^{\sigma^2} - 1\right)}\exp\left(\mu + \frac{\sigma^2}{2}\right)$

**Proof:** The result follows by using the transformation technique. The proof of this is simple and straightforward.

**Property (4):** Let $X$ be random variable with lognormal distribution. Then

1). If $X \sim \log normal(\mu, \sigma^2)$, then $X + c$ is said to have a shifted log-normal distribution.

2). $\mathrm{E}(\mathrm{X} + \mathrm{c}) = E(X) + c$.

3). $X \sim \log normal(\mu, \sigma^2)$, then $a\, X \sim \log normal(\mu + \ln a, \sigma^2)$.

4). $X \sim \log normal(\mu, \sigma^2)$, then $\frac{1}{X} \sim \log normal(-\mu, \sigma^2)$.

5). $X \sim \log normal\left(\mu, \sigma^2\right)$, then $X^a \sim \log normal\left(a\mu, a^2\sigma^2\right)$ for $a \neq 0$

**Proof:** The result follows by using the transformation technique. The proof of this is simple and straightforward.

**Property (5):** Let $X$ be random variable with lognormal distribution. Then

1). If $X \sim normal\left(\mu, \sigma^2\right)$, then $\exp(X) \sim \log normal\left(\mu, \sigma^2\right)$.

2). If $\left(X_i\right)_{i=1}^{n}$ are independent Lognormal n random variables where $X_i \sim \log normal\left(\mu_i, \sigma_i^2\right)$, for $i = 1,.n$, then $\prod_{i=1}^{n} X_1$ is distributed log-normally with $\prod_{i=1}^{n} X_i \sim \log normal\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$.

**Proof:** The result follows by using the transformation technique. The proof of this is simple and straightforward.

**Property (6):** The values of skewness, kurtosis and mode are given by

$$Skewness(X) = \sqrt{(e^{\sigma^2} - 1)} \exp\left(\sigma^2 + 2\right)$$

and

$$kurtosis(X) = \exp\left(4\sigma^2\right) + 2\exp\left(3\sigma^2\right) + 3\exp\left(2\sigma^2\right) - 6$$

**Proof:** The result follows by using the transformation technique. The proof of this is simple and straightforward.

**Property (7):** Let $X$ be random variable with lognormal distribution. Then

1). The Lognormal Reliable Life Function is given by:

$$R(t) = \int_{\ln(t)}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx$$

2). The Lognormal conditional reliability function is given by:

$$R(t/T) = \frac{R(T+t)}{R(T)} = \frac{\int_{\ln(T+t)}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx}{\int_{\ln(T)}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx}$$

3). The Lognormal Failure Rate Function is given by:

$$\lambda(t)=\frac{f(t)}{R(t)}=\frac{\dfrac{1}{t\sigma\sqrt{2\pi}}\exp\left\{-\dfrac{1}{2}\left(\dfrac{t-\mu}{\sigma}\right)^2\right\}}{\int_t^{+\infty}\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right\}dx}$$

**Proof:** The result follows by using the transformation technique. The proof of this is simple and straightforward.

**Property (8):** Let $X$ be random variable with Lognormal distribution. Then $X$ is not symmetric and achieves the following results:

1). The Median of the log-normal distribution is $Median(X)=\exp(\mu)$. .

2). The Mode of the log-normal distribution is $Mode(X)=\exp(\mu-\sigma^2)$. .

**Proof:** This property was proved in [16].

## 5. RECENT MODIFICATIONS OF EXTENDED FAMILY OF LOGNORMAL DISTRIBUTIONS

The Lognormal distribution formula can be generalized by three parameters with a new formula in which the position parameter is inserted, this allows for a wider expansion of the random variable and control of its range, so that the formula for the generalized probability density function of the random variable of the proposed Lognormal distribution in terms of three parameters becomes:

$$f(x,\sigma,m,\theta)=\begin{cases}\dfrac{1}{\sigma\sqrt{2\pi}}\dfrac{1}{(x-\theta)}\exp\left\{-\dfrac{\left(\frac{\ln(x-\theta)}{m}\right)^2}{2\sigma^2}\right\}; x\succ 0\\ 0;\ x\leq 0\end{cases}$$

where $\sigma$ the shape parameter is $\theta$ is the location parameter and $m$ is the scale parameter. The case where $\theta$ equals zero is called the two-parameter lognormal distribution. For new parameterization if $\mu=\ln(m)$ then the general formula for the (pdf) of the lognormal distribution with three parameters is:

$$f(x,\sigma,\mu,\theta)=\begin{cases}\dfrac{1}{\sigma\sqrt{2\pi}}\dfrac{1}{(x-\theta)}\exp\left\{-\dfrac{((\ln x-\theta)-\mu)^2}{2\sigma^2}\right\}; x\succ 0\\ 0;\ x\leq 0\end{cases}$$

For the special case $m = 1$, the value of parameter $\mu = \ln(1) = 0$. So, the formula of (pdf) of the two-parameter lognormal distribution is:

$$f(x,\sigma,\theta) = \begin{cases} \dfrac{1}{\sigma(x-\theta)\sqrt{2\pi}} \dfrac{1}{} \exp\left\{-\dfrac{(\ln x-\theta)^2}{2\sigma^2}\right\}; x \succ 0 \\ 0;\ x \leq 0 \end{cases}$$

where $\theta$ is the location parameter and $\sigma$ is the shape parameter. Also the standard deviation for the Lognormal, affects the general shape of the distribution. Usually, these parameters are known from historical data. The graph below shows various lognormal distribution (pdf) illustrated in Figure (2, (a), (b)).

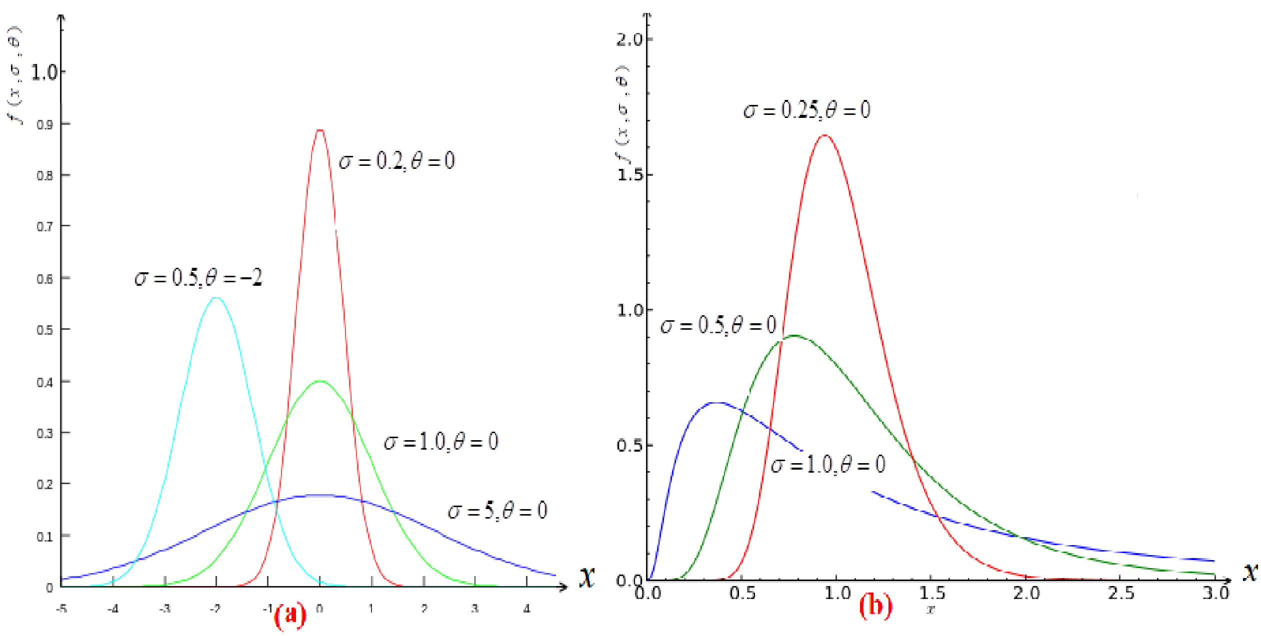


**Figure 2(a,b).** Graphs of $f(x,\sigma,\theta)$ for various values of $\theta$ and $\sigma$.

## 6. MIXTURE AND FITTING OF FAMILY OF LOGNORMAL DISTRIBUTIONS

The idea of discussing the problem of the representation of the sums of lognormal distributed random variables is discussed mix began in the early 1930s. This method was used extensively during the application of natural logistic distribution to the population distribution data in general of lognormal distributions correctly. Some techniques for estimating parameters have been widely applied when tracking crop yield distributions and outstanding processing, where the subject was studied (e.g., Jung and Ramirez, 1999 [15]; Stokes, 2000 [21]), as well as a mixture of several marginal distributions from (Goodwin and Ker, 2002) [12].

The previous results were applied to the study of river flooding using abnormal distribution when flooding is repeated or loss of the amount of accident forecasting, since the data follow abnormal distribution frequently. Continuous studies on the limited mix models of the abnormal mixture of distributions have continued to receive increasing interest over the years in practice and theory. Al-Hussaini and Sultan mentioned some important results obtained in their article [3]. The asymmetric mixture model of the Lognormal family of distributions was used to predict too little to predict the possibility of some possible and recurring floods with the only relatively common and frequently occurring sources as disasters in which thousands of lives are at risk of one recurring event.

The method of guessing the mixture of continuous distributions and precisely logarithmic distributions has become very common in astronomy and meteorological applications; implicitly, it is not necessary to know that the probability distribution that generates data values (and errors) was to apply a non-parametric test.

The family of lognormal distributions mix has an important biological property in many areas where other distributions do not know the possibility of adjusting values, which is most common for extended Weibull family distributions, which are multi-application techniques and analyzes, especially when applying time-series analysis [4]. Logarithmic distributions are characterized by twisting (or homogeneity, or blurring) of values derived from the image through a useful response to the alignment data, as well as noise in which some image values are at the boundary (which may arise based on the application prior to the first flux or enhancement in Weather and weather images, such as in radio astronomy interferometers).

After interference with radar tracking of atmospheric targets, or in opaque optical telescopes), and due to recent modifications in the representation of the family of lognormal distributions of some mixtures of conditions such as Rayleigh and gamma, it must be considered to characterize the results of dividend mixture family of lognormal distributions which adds to it a wide range of options for the distribution of representation, and to clarify the ideas are advised to see the article in the reference [5, 24].

## 7. APPLICATIONS

### A) Observation data

A series of flood discharges of Greater – Soumam River at Bejaia over a period of (18) years (2000 – 2017) as shown in Table (1):

**Table 1.** Maximum water discharge for Greater Soumam River at Bejaia Over a period of (18) years.

| Years | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|---|---|---|---|---|---|---|---|---|---|
| Discharge ( $m^3$ / sec ) | 459 | 398 | 477 | 440 | 480 | 399 | 440 | 426 | 450 |
| Years | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
| Discharge ( $m^3$ / sec) | 366 | 435 | 480 | 420 | 440 | 450 | 320 | 450 | 380 |

### B) Discussion and Conclusions

To track the case (Predicted flood magnitudes), the study of the floods was used for the statistical model of logarithmic distribution. For the statistical model "Lognormal" was used to estimate the flood magnitude over various return periods. A computer program (SPSS) is used to compute the parameters of the distributions. These parameters were estimated by the method of the moment, and this value is given in Table (2).

**Table 2.** Parameter estimation for peak flood data

| | | | | |
|---|---|---|---|---|
| Origin Data | Mean( $\bar{x}$ ) | Standard deviation | Skewness | Kurtosis |
| | 428.3333 | 42.23325 | -1.07593 | 1.165629 |
| Logarithm of Data $(\sigma, \mu, \theta)$ | Mean | Standard Deviation | Skewness | Kurtosis |
| | 425 | 43.5 | -1 | 1.15 |

This program gives also the magnitude of floods for various return periods. The upper limit & lower limit are close to the Kurtosis that is computed from the data table (1), and so lognormal distribution could be accepted as the best. The skewness of the logarithm of data should be greater than zero for Lognormal, so it could not be accepted as the best. As with lognormal distribution, so this distribution could not be regarded as the best.

Through the results obtained and through the extrapolation, it is clear that the analysis of frequency for flood and after analysis, the results by comparison of fits, using measures statistical model Lognormal and goodness of fit, from these methods the lognormal model could be regarded as the best for flood data for the Soumam River at Bejaia.

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