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New product differentiation rule for paired scalar reciprocal functions

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ABSTRACT

When integral kernel of an integral transform is being formed, it should be the outcome of scalar product differentiation rule if the kernel is supposed to be eventually used as an integrand in a prospective integration. Yet it has already been shown that despite ensuing from properly performed differentiation, the resulting integral kernel contains, beside the covariant differential that is suitable for integration, also a certain contravariant term, which is not appropriate for integration in the same space as the covariant differential. But the contravariant term also can be turned into proper, though multiplicatively inverse covariant differential, if placed within a space that is reciprocal to the given primary space in which the first, covariant differential, is represented naturally. This uncharacteristic conversion of the contravariant expression from the primary space into the reciprocal covariant differential in the dual reciprocal space that is paired with the given primary space, can be considered as indirect proof that pairing of mutually dual reciprocal spaces is necessary in order to properly form operationally legitimate and geometrically valid differential structures. Consequently, the pairing of an infinitesimal descending singularity of the 2D domain of complex numbers with an infinitely ascending singularity deployed in the 1D domain of real numbers requires certain dual reciprocal spatial or quasispatial structures, for the downward transition from 2D descending complex singularities to the 1D ascending "real" singularities to be meaningfully/unambiguously implemented. Furthermore, just as integration by parts formula is a counterpart of the regular product differentiation rule, a new multispatial scalar product integration rule is proposed as a counterpart to the singlespatial product differentiation rule, and introduced by analogy to the latter, "regular" product integration rule.

***Keywords*:** Scalar product differentiation rules, scalar product integration rules, single-space reality paradigm, multispatial reality paradigm, paired dual reciprocal space, integral kernels, singularity

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1. INTRODUCTION

Many problems encountered in traditional mathematics could be traced down to the most basic – yet sometimes quite arbitrary and even deliberately frivolous – assumptions about the character of the abstract mathematical reality. One of them is the unwarranted presumption that mathematics should deal with single-space reality (SSR), which became the tacitly accepted and thus unmentioned and therefore, never challenged paradigm of traditional mathematics. Nevertheless, my analyses of certain formerly unanticipated and previously irreconciled curious experimental results strongly suggested that perhaps both the abstract mathematical and the – corresponding to it – physical reality demand a new approach, which proposes that multispatial reality (MSR) paradigm should be espoused instead of the previous SSR paradigm. This suggestion is also supported by some unquestionable operational laws and rules of differential calculus. The present paper demonstrates that this is indeed the case.

Since mathematical laws must be abided by and abstract operational rules of mathematics are supposed to be dutifully obeyed no matter what, then why is that some unquestionable as they may seem at first glance, and rigorously proven – yes: you read it right: PROVEN mathematical laws and rules can give some ostensibly illegitimate or even unlawful though legitimately obtained results? Let me narrow the scope of this question just in order not to keep the reader in suspense.

Since differentials can serve as integrands in prospective integrations, which – as antiderivatives – are expected to yield the compounded function (from the prior differentiation of which the differential emerged) then why some compound differentials apparently yield quite unacceptable (operationally) expressions even though the differentiation was performed in accordance with the operational rules that admittedly govern the process of composite differentiation [1]?

The selfimposing – even if not always decisively compelling – answer to this question is, to the best of my knowledge, of course, threefold: (1.) the lack of operationally viable and structurally sound notion of the ascending infinity, (2.) whose absence – in turn – defeats the operational handling of the neverending, ascending *ad infinitum* singularities, and (3.) the silly, if not totally absurd, prohibition of division by zero. Although numerous nonsensical deductions follow these three, they are not of interest to this presentation at this time.

There are two basic operationally different and conceptually distinct types of singularity: the fairly nice behaving complex infinitesimal (descending) singularities and the troublesome infinite (ascending) singularities.

While the descending infinitesimal singularity is handled pretty well in the domain of complex numbers, the ascending infinity frequently popping up in the domain of real numbers can lead to most obvious nonsenses, such as 1=0 [2], when operations on the infinity and zero are performed the traditional way [3], [4] p.65, i.e. as if the never ending ascending infinity were a kind of bottomless trash bin into which anything could be thrown and just disappear without leaving a trace.

It is because some mathematical operations demand the presence of quite unambiguously defined (and respected both operationally and structurally) infinity, which is homeless and thus nowhere to be found in the traditional mathematics, that the traditionally mandated operations involving infinity (and even zero – surprise, surprise!) imply tacitly veiled yet not really concealed from conscientious mathematicians – such as Dr. Eugenia Cheng who exposed some of them in her book [2], for instance – nonsenses.

Some authors advised students that the pesky singularities, especially those which could yield the ascending infinity, should be detoured [5]; see also the subsection discussing integrals around singularities [6]. Notice the word ‘around’ instead of the expected phrase ‘emerging from within’ the feared singularity, if that were possible. Others advise that expressions involving infinity should be avoided for they can allegedly occur only as abbreviations for limiting processes, in otherwise excellent textbook [7].

Clearly something is not quite right when it comes to revealing the – mathematically indispensable yet shunned – infinity, not to mention operating on it. And why is that tacit cover-up of issues pertinent to singularities and suppression of views of those who may have something more tangible to say about a mathematically viable infinity than those who insist on erasing it even at the (routinely unmentioned) cost of generating even more nonsenses?

2. TRADITIONAL SCALAR PRODUCT DIFFERENTIATION RULE

The traditional, regular scalar product differentiation rules (RSPDR1) for compounded scalar-valued functions u(x) and v(x) that depend on the same independent variable x reads:

$$(uv)' = uv' + vu' \quad \Leftrightarrow \quad d(uv) = udv + vdu \tag{1}$$

where the primed terms u' and v' denote derivatives of the functions u(x) and v(x), respectively, and – as usual – the letter d signifies differential of the varying function that follows it [S-B] p.134. The RSPDR1 on the left-hand side (LHS) of the equivalence formula (1), which is expressed in terms of derivatives, is conceptually equivalent to the RSPDR1 formula on the RHS of (1) that is displayed in terms of differentials. The relation between the differential du(x)=u'(x)dx and the derivative u'(x), which is commonly understood as limit, is simply explained in [8] on p.228.

The RSPDR formulas are based upon binomial evaluation of compounded functions [9] p.87 commonly attributed to Leibniz; compare also [10] p.275. The meaning of the RSPDR formulas is that the differential – i.e. the rate of change – of one function is compounded with the other function unchanged and the compounded terms are then added together. The number 1 in the moniker RSPDR1 means that it refers only to first order derivatives or differentials, which alone are of interest to this presentation.

Although the RSPDR1 formula was indeed useful in most mathematical analyses, which usually relied upon the abstract topological treatment of functions that is based on the set-theoretical approach to spaces that are commonly identified with mere sets and oftentimes just assumed as being commutative by default, it failed when the commutativity cannot be ascertained, such as is the case of handling paired dual reciprocal spaces [1].

Nevertheless, since reciprocity is usually interchanged with inversion, it is important to realize that reciprocity is actually multiplicative inversion. For traditional treatment of inverse functions see [11], for instance. Even multiplication of distributions, which are viewed as generalizations of regular functions, apparently cannot be defined so that that the operation be associative, see the remark in [4] p.726.

In the traditional mathematics that is being suffocated under the unwarranted but never questioned in the past (insofar as I know) – by most well-trained (or well indoctrinated, if you will) mathematicians – SSR paradigm, the RSPDR1 formula implies the regular scalar product integration rule (RSPIR1) that is also called integration by parts formula

$$\int udv = uv - \int vdu \tag{2}$$

which is well explained in [12]. Poisson has already regarded it as legitimate and thus universally valid formula [13]. However, it will be shown below that a more subtle multispatial rule lurks behind it; but this fact was not recognized in the traditional mathematics.

3. MULTISPATIAL RULES FOR DIFFERENTIATION AND INTEGRATION OF COMPOUNDED PRODUCTS OF SCALAR FUNCTIONS

By analogy to the RSPDR1 I propose multispatial scalar differentiation rule (MSPDR1)

$$\{u(t) \cdot v(t,b)\}'_t = u(t) \cdot v'_t(t,[b]) \oplus \frac{1}{u'_t(t) \cdot v'_t(t,[b])} \tag{3}$$

and the corresponding to it multispatial scalar product integration rule (MSPIR1)

$$\int_0^\infty d\{u(t) \cdot v(t,[b])\} = \int_1^\infty u(t) \cdot v'(t,[b])dt \oplus \int_0^1 \frac{1}{u'(t) \cdot v'(t,[b])dt} \tag{4}$$

which are intended for operations performed on paired dual reciprocal spatial or quasispatial structures. Only the variable t is actively varying therein whereas the set of variables codetermining the integral transform v(t,b) denoted by the functional [b] remains unchanged during the differentiation processes.

Although the inverse integrand appears as significantly different from what would be expected from the traditional pattern encoded in the RSPDR1, the multiplicatively inverse reciprocal integrand actually complies with the traditional pattern, as you shall see below. Notice formwise resemblance of the MSPDR1 to the sum of incomplete elliptic integrals of the second and first kind, respectively, which are not solvable in general – see [1]. One can see that the variables t and [b] are most likely to be either noncommutative or anticommutative and as such are not really representable both within the same space. Yet, to the best of my knowledge, traditional mathematics was not always concerned with quite possible adverse consequences of that incompatibility. Recall that the integral transform v(t,b) sometimes is also called influence function [14] and for a good reason, I surmise. Recall that even in the SSR setting the diffeomorphism f: $M_1 \to M_2$ between smooth manifolds is a smooth map that has a smooth inverse [15], which fact surely signifies the importance of reciprocity. The reciprocity viewed as multiplicative inverse is not new. It is the blind adherence to the unwarranted SSR paradigm that the issue of reciprocity was largely ignored. I prefer to retain the multiplicative compounding sign ▫because at this stage it would be rather difficult to determine *a priori* whether or not an extra conversion would be necessary to keep the formal patterns in operationally admissible shape and their corresponding spatial or quasispatial structures in compliance with the operational procedure (3). The relation between the integral kernel and Green's function is concisely explained in [16]. Comprehensive discussion of integral equations with integral kernels can be found in [17]. Compare also [18].

If an alert reader may wonder why the reciprocal integral contains two derivatives and the formal differential dt, the implication (7) shows the reason why. For it is actually one single differential whereas the other differential emerges from integration as the price paid for the conversion of the contravariant expression into the inverted covariant differential – compare also [1] for more details. The formula (3) is still formwise equivalent to the RHS pattern of RSPDR1 in formula (1); it is split, however, between mutually dual reciprocal spaces joined by the extra sign $\oplus$ of interspatial addition, and therefore the sums in (3) and (4) can be regarded as an abstract expression pertaining to a pair of spaces or a paired multispatial structure. This abstract issue shall be further elaborated elsewhere.

4. QUEST FOR PREDICTIVE ROLE OF CAUCHY'S INTEGRAL FORMULA

Fritz John once wrote: << It is good to remember that mathematics is not only concerned with solving problems, but with studying the structure and behavior of the objects it creates. One of the best examples is the classical theory of functions of complex variable. It, incidentally, does solve problems as in the Riemann mapping theorem. But much of its beauty lies in statements that can hardly be considered as "solving" anything, like the calculus of residues, or Pickard's theorem, or [the following] Cauchy's [integral] formula

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(\xi)}{\xi - z} d\xi \qquad \text{(5=CIF)}$$

The only "problem" solved by (5=CIF) is the improper one of determining f from its values on C, which generally has no solution. Formula (5=CIF) is not only strikingly beautiful but extremely useful. It shows immediately that analytic [function] f can be differentiated infinitely often and can be represented by convergent power series. >> [19].

Since – as Fritz John has said – CIF can be differentiated infinitely, then why there was no – meaningfully defined and unambiguously represented – definitely spatial structure (i.e. a geometric or quasigeometric space, if you will) for the obviously operationally necessary infinity to dwell in? I have shown that infinity should be housed in the dual reciprocal space that is paired with the primary space, if the infinity should be made operational [20, 21].

After the theory of limits became firmly established, the use of infinitesimally small and of the notion of infinitely large quantities in mathematical analysis became discredited and survived only as a matter of speaking [35]. In absence of operationally sound and structurally discernible notion of infinity some theoretical results of mathematics involving both spatial and temporal variables may appear almost surreal. Fritz John has also remarked that "It is as if looking into a crystal ball of radius ε over a very long time can tell you everything about the outside world." [19] p.82. As ingenious as the CIF surely is, it will certainly need the MSPIR1 for the envisioned by Fritz John CIF's predictive role to be implemented.

5. ATYPICAL YIELD OF SCALAR PRODUCT DIFFERENTIATION RULE WHEN IT IS APPLIED TO COMPOUNDING OF INTEGRAL KERNELS

As reciprocal to covariant representations, contravariant depictions of scalar compounded functions can be expressed in terms of inverse differential operators [21-23]. Hence in the equation of integral kernel $g(x) = f(t) \cdot K(t,x)$ of an integral transform K(t,x), when f(t)=t for the sake of simplicity, the g(x) is usually expressed as compounded scalar function – compare [1]. Lichnerowicz dubbed integral kernels integral operators and the integral transform K(s,t) he called kernel but the nomenclature is immaterial [AL] p.256. He extensively discussed integral kernels, mostly in the context of Fredholm integral equations.

The integral kernel g(x) should be properly evaluated according to the RSPDR1 rule as:

$$\{f(t) \cdot K(t,x)\}'_t := t \cdot \frac{dK(t,[x])}{dt} \oplus \frac{1}{K([t],[x])} \cdot \frac{1}{dt} = tK'(t,[x])dt \oplus \frac{1}{K([t],[x]) \cdot dt} \tag{6}$$

for if the integral kernel is supposed to be eventually integrated scalarly then it should be formed as proper differential that is legitimately evaluated according to the RSPDR1 rule. Although the latter requirement is not always explicitly enforced, in physically realistic cases it would not be appropriate to actually integrate the integral kernel if it had not resulted from properly performed – according to the RSPDR1 or its equivalent – operationally legitimate product differentiation rule. Most authors assume that any compounding multiplication does form legitimate differential expression by default, provided it complies formwise with (1). Nothing is further from the truth, however. For the traditional mathematics that is taught at universities throughout the world contains tacitly veiled yet clearly embarrassing nonsenses.

This mistaken assumption is presumably due to lack of the extra symbol▫in the traditional mathematics, which routinely neglects to use any explicit symbol as indicator of scalar multiplication/compounding, probably because its presence might have revealed their careless indolence and thus cause discomfiture. The traditional mathematics' notational conventions are full of such tacitly veiled cover-ups for they apparently prefer to allow utter confusion while pretending to omit all those allegedly unnecessary symbols for the sake of simplicity. The deliberate notational and conceptual confusion then generates some difficult to recognize, or tacitly veiled imperceptible baloneys and thus discourages alert honest students from asking questions, likely for understandable fear of reprisals.

The special symbol $\oplus$ clearly emphasizes the inappropriateness of adding the inverse contravariant expression standing on the far RHS of (6) – even though it was obtained in mathematically legitimate manner – as the expression is not an operationally valid covariant derivative/differential. For the dual reciprocal contravariant expression – as it appears <u>in the formula</u> (6) – does not really represent an inverted covariant differential. Thus we have: K([t],[x])▫dt $\not\Leftrightarrow$ K'(t,[x])dt. Since the term K([t],[x])▫dt is not properly formed covariant differential, it cannot be legitimately used as prospective (operand or integrand) even though it was formed in the legitimate differentiation that had created it. Hence it must be replaced by an operationally admissible inverted/reciprocal differential. Recall that variables in square brackets are functionals, i.e. as if frozen variables or functions, not actively varying variables or functions. The multiplicative compounding symbol▫is supposed to make us aware of the fact that the contravariant expression K([t],[x])▫dt is not equivalent to the covariant differential K'(t,[x])dt, which fact was not always perceived as being problematic in the traditional mathematical notation that did not use the extra symbol ▫. Without the extra symbol▫it was easy to miss the distinction without realizing the notational inappropriateness that can generate various nonsenses or just some imperceptibly anomalous results.

Having said that, I should mention that the expressions standing on the LHS of (6) can be admitted as equivalent: $t \cdot \frac{dK(t,[x])}{dt} \Leftrightarrow tK'(t,[x])dt$ for although the differential dt has

emerged from differentiation of the function K(t,[x]) with respect to the (actively varying) variable t, it is coincidentally also the differential of the function f(t): f '(t)=dt. Here I am truly using the function f(t)=t for the sake of simplicity of this presentation.
Yet I can turn the contravariant term into legitimate though inverse covariant differential

$$\frac{1}{K([t],[x])} \, ▫ \, \frac{1}{dt} \Rightarrow \int\{1 \,/\, \frac{\partial K(t,[x])}{\partial t}\} \, ▫ \, \frac{1}{dt} = \int\{\frac{\partial t}{K'(t,[x])dt} \, ▫ \, \frac{1}{dt}\} = \int\{\frac{1}{K'(t,[x])dt}\} \tag{7}$$

which yields quite legitimate reciprocal (i.e. multiplicative inverse) yet covariant differential suitable for prospective integration, at least in principle. Now, with the simple function f(t)=t, the compound function g(x) in (6) of the integral kernel can now be properly rewritten as:

$$g(x) = K(t,x) \, ▫ \, f(t) = \int_0^\infty K(t,x) \, ▫ \, f(t) \, ▫ \, dt \Rightarrow \int_1^\infty tK'(t,x)dt \oplus \int_0^1 \frac{1}{K'(t,x)dt} \tag{8}$$

where the term K'(t,x)dt in the integrands of both integrals is legitimately obtained and properly formed covariant differential of the integral transformation K() with respect to the formally independent variable t. Now the absence of the extra multiplicative compounding separator ▫ between the derivative K'(t,x) and the differential dt can be tolerated as it does not confuse us anymore on the RHS of the formula (8). Notice, however, that the inverse term standing on the far RHS in (8) is definitely reciprocal, and therefore, it must not be integrated within the same primary space as the regular term tK'(t,x)dt that stands on the LHS of the resulting implication (8).

The attained results conform to the inevitability of pairing of dual reciprocal spaces [25] and with the finding that differential operators appear as acting simultaneously on objects represented within paired reciprocal spaces [26]. The results also comply with prospective expansions of differential calculus [27] that I envisaged.

Nevertheless, the inverted integral on the far RHS of the formula (8) belongs in dual reciprocal space paired with the given primary space in which the first integral standing on the LHS of the final expression dwells, and therefore, the interspatial sum symbol ⊕ is still necessary in order to avoid possible conceptual confusion. The inverted integral is obviously infinitesimal and thus refers to the infinitesimal descending infinity, which belongs to interval (0,1[rather than to (0,∞); nor could it belong in the interval [1,∞) to which the first integral belongs. Inversions are fairly well discussed in simple geometric terms in [28].

One can see that the curious result (8) is extremely significant and thus it could have considerable theoretical impact on mathematical methods applied to problems encountered in physical sciences. This realization demands more comprehensive explanations pertaining to topics on differential geometry and those involving the traditional approach to tensor calculus, especially the apparent vindication of formerly undesirable contravariant derivatives, which now are acceptable, though only when they are considered within dual reciprocal spaces. The latter issues shall be discussed elsewhere. The rudimentary 1D primary space in this case is identified with the interval [1,∞) whereas the simplified rudimentary reciprocal space is identified with the interval (0,1[when they both are considered within the real numbers domain. Although on 1D line segment we can certainly write (0,∞) = (0,1[+ [1,∞), for higher-dimensional spaces the regular plus sign should be replaced with the encircled plus sign ⊕ just in order to make us aware of the significantly more complicated summation process of primary spaces or subspaces with their paired dual reciprocal counterparts.

Lichnerowicz has also recognized the reciprocity of solutions of the Fredholm integral equation he investigated [24] p.275. Yet because he worked under the old SSR paradigm, he considered the kernel as degenerate [24] pp.260, 279. The formula (8) can be regarded as being degenerate from the SSR point of view, because it requires two distinct mutually reciprocal spaces paired together whereas in the SSR framework there is only one single space and thus anything that does not fit into the single space may appear as degenerate.

Just because the MSR paradigm was unknown until I proposed it, even great mathematicians fought nonexistent phantoms, and trying to explain why even legitimate operations produce partly inconvenient results blowing up their single-space reality assumption, they blamed the undesirable results on some sort of degeneracy presumably for lack of a better word. In any case, the degeneracy pertains only to the former thinking under the unwarranted SSR paradigm. I do not see the degeneracy under the MSR paradigm.

By analogy to the regular scalar product integration rule RSPIR1 that is also known as the integration by parts rule (2), one can envision also its multispatial counterpart, namely the corresponding to it Multispatial Scalar Integration By Parts Rule (MSIBPR1)

$$\int_1^\infty u(t) \bullet v'(t,[b])dt = u(t) \bullet v(t,[b]) \ominus \int_0^1 \frac{1}{u'(t) \bullet v'(t,[b])dt} \qquad (9)$$

where the symbol $\ominus$ means minus signifies interspatial subtraction of the integral object housed within the reciprocal space that is dual to the given primary space in which the integral on the LHS is natively represented. The symbols $\oplus$ and $\ominus$ signify interspatial addition and subtraction, respectively. Although the analogy is compelling, the rule (9) is not identical to the regular one shown in (2).

6. CAN WE KNOW THAT THE MULTISPATIAL RULES ARE DECENT?

I was tempted to ask if we could know whether the multispatial rules are admissible, but because at this stage of development of the theory of paired reciprocal structures the answer to this question cannot be precisely determined from the facts that are available.

Therefore, I decided to settle for now on something lesser but unquestionable, namely to show that the multispatial rules look pretty decently and thus may be acceptable in absence of some better alternatives. Nevertheless, at present I can only use some examples for demonstrating that.

The whole quest can be divided into four interdependent subquestions:

1) Do the split boundaries of integration in (8) make operational sense, and if so, then
2) Is there really reciprocity also enshrined in the integral formulas also at the level of variables, not just the reciprocity found at the level of the functions involved, and
3) Is there really a transition from the ascending neverending singularity to a descending (i.e. drill-down) singularity making thus the split integrals equivalent, and in particular
4) Is there really a spatial pivot between zero and infinity, whose presence could suggest also the realistic possibility of a tripartite multispatial pairing rather than just mutually inverted sets (and cosets) of variables?

6. 1. Do the split boundaries of integration make sense?

The fact that within the traditional framework of classical mathematical analysis the split of boundaries of integration involves inverse/reciprocal variable in the infinitesimal integral

$$\int_0^{\infty} df(x) = \int_1^{\infty} df(x) + \int_0^{1} d\{1 \,/\, F\left(t = \frac{1}{x}\right)\} \tag{10}$$

where x and t are the so-called dummy variables of the integration used in each of the split integrands, is well known – compare [29] p.14. Although Nahin identifies the function f() with our inverse function 1/F() in his rendition of the formula (10), which is presumably due to the fact that he still operates in the traditional SSR setting, everything else is the same as in our formula (10), see ibid., for very concise but also more comprehensive – as a whole – presentation of the issues. While the dummy variable x remains unchanged in the infinite integral $\int_1^{\infty} df(x)$, the actual dummy variable in the infinitesimal integral $\int_0^{1} d\{\,1/F(t = \frac{1}{x})\}$ must be inverse of the original variable: t=1/x. Thus, the answer to this question is tentative Yes, given that the multispatial formulas have been derived under auspices of the MSR paradigm. However, Nahin did not use the letter d to signify differentials in the integrands, that omission is perhaps just a minor inconvenience. His intentions were clear to me.

6. 2. Do the reciprocal functions involve inverted variables?

To answer this particular inquiry, let us consider the following chain of derivations:

$$\int_0^{1} \frac{\ln(x)}{1+x^2} dx = \int_{\infty}^{1} \frac{\ln(\frac{1}{t})}{1+\frac{1}{t^2}} \left(-\frac{1}{t^2} dt\right) = -\int_{\infty}^{1} \frac{\ln(\frac{1}{t})}{t^2+1} dt = \int_{1}^{\infty} \frac{\ln(\frac{1}{t})}{t^2+1} dt = -\int_{1}^{\infty} \frac{\ln(t)}{t^2+1} dt \tag{11}$$

where dx= – dt/t^2 and inverse zero is equated with infinity: (1/0)=∞ compare with much more explanatory derivation in [29] on p.15. The LHS integral of the formula (11) is formwise equivalent to the RHS integral in (11) with the boundaries of integration reversed and zero replaced with infinity, i.e. its natural reciprocal. Thus, the answer to this question is definite Yes. Notice that the conventional division by zero (1/0)=∞ is essential to the derivation.

By the way, for some functions such as the one evaluated in (11), the value of interspatial addition ⊕ can become effectively negative, which is yet another reason for distinguishing the symbol from the usual scalar addition signified by the regular algebraic plus sign.

6. 3. Are the primary and inverted integrals equivalent?

Given the formula (10) and the result already established by Euler – see [29] p.15, that

$$\int_0^{\infty} \frac{\ln(x)}{1+x^2} dx = 0 \tag{12}$$

it is clear that values of the split integrals in (10) must be equal and opposite in order for their sum to become zero. Thus, the answer to this question is definite Yes.

6. 4. Can a spatial pivot between zero and infinity exist?

In the European tradition zero and infinity are regarded as being naturally reciprocal: $(1/0)=\infty$. It follows from the formula that $0\cdot\infty=1$, which I ascribe to without reservations. To me, the implication $0\cdot\infty=1$ is not something to be evaded, but mathematical hint that former mathematics is incomplete. But the question to consider is: can this multiplicatively inverse operational relationship also be carried over onto the prospective geometric spatial structure that is supposed to hold – or just host – the structural geometric entity that should be representing infinity? Since the inverse tangent function evaluates to nonzero value

$$tan^{-1}(x) = \int_0^\infty \frac{dx}{1+x^2} \quad \Rightarrow \quad tan^{-1}(\infty) - tan^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2} \tag{13}$$

– compare [29] p.15 for instance, then the outcome of the formula (13) implies that there can be a certain pivot indeed. Thus, the answer to this question is definite Yes.

While reciprocity by itself ensures the possibility that 4D spacetime can be overlaid operationally with the – operationally corresponding to it – 4D timespace structure [30], the need of pivoting these dual reciprocal structures virtually guarantees the structural necessity of restricting the highest number of unique dimensions of abstract quasispatial structures to four, in agreement with the theorems of Abel and Galois. It is also in agreement with the curious behavior of Lagrange's resolvents, which instead of reducing the polynomials of degree five to ones of degree four, as it was expected by Lagrange, paradoxically increased the polynomial's degree to six [31]. This apparent failure indicates to me that traditional mathematics is totally wrong when it comes to the issues of dimensionality and spatiality and thus should replace the SSR paradigm with the MSR paradigm in an unavoidable paradigm shift.

Notice that inconvenient but unquestionably valid theorems of Abel and Galois, as well as the aforesaid curious behavior of Lagrange's resolvents, not just I, contradict the notion of dimension proposed by Grassmann, further developed by Riemann, and their followers, and more recently axiomatized by topologists. On one hand we have proven theorems and on the other unproven and frivolously proposed (by Riemann) concept of higher dimensions, and yet the traditional mathematics endorsed and accepted the unfounded, artificial artlike proposals while ignoring the proven ones and tacitly suppressing any mention of the ignored ones.

This stance of traditional mathematics sounds to me like it is compliant with textbook definition of insanity by any standard. I do not mind artlike creations, but the simultaneous tacit suppression of other ideas indicates subversion and sabotage of unquestionable but inconvenient achievements of Abel, Galois and Lagrange, to name just a few great minds. Why would some scientists try to undermine the sanity and relevance of mathematics (to both the abstract and the physical reality), is beyond my comprehension.

7. HOW CAN PROVEN RULE PRODUCE UNACCEPTABLE EXPRESSION?

Although the Leibniz's rule that underlies the RSPDR1 formula has been rigorously proven – see [32], for instance, and therefore it was not faulty or incorrectly stated, yet it has produced operationally unacceptable expression. The RSPDR1 is perfectly well conceived and quite legitimate mathematical formula, from the traditional operational point of view, even though its traditional definition was paradigmwise incomplete. And so was the RSPIR1 rule.

However, the curious failure raises very disturbing theoretical question. For if the proof was correct, then how the failure happened? What is the reason of the failure?

The problem is, however, that proofs rely on axioms and primitive notions, both of which in turn, depend on the paradigms espoused by the proofs' creators. Since paradigms can change without notice, mathematical proofs may become meaningless often without us realizing that. That is why I have proposed the new synthetic approach to mathematical and physical sciences, which demands matching of operational procedures to the – corresponding to them – geometric or abstract quasigeometric structures. For it is clear to me that inventing operational procedures that do not have any truly constructible realistic structures over which to perform the operations inscribed in the procedures, is an exercise in futility. And so is also the creation of abstract geometric structures that cannot be operated on. If an abstract structure does not have an operational procedure fitting the structure, then perhaps the structure is either nonexistent within the framework upon which the operational procedure has been proposed or is impossible to be constructed therein. Therefore, syntheses – not proofs – should become the proverbial king of conceptually meaningful mathematics.

The splendid Euclid's dream (of proving everything based upon some old, often artificial and arbitrarily guessed, assumptions) is certainly not dead but mathematics should have awakened from its childishly unconcerned sleep and start dressing up for support of the actual physical reality that is revealed afresh in formerly unanticipated experimental results. Enough of snorting on unanticipated physical phenomena, which were left irreconciled and/or unexplained. For mathematics to be truly realistic it has to be guided by the curious yet unbiased results of physical experiments. Axiomatization can create virtually unreal mathematical reality, which is impossible to be implemented in the physical reality whose character is exposed in actually conducted physical experiments.

8. SOME HINTS FROM PHYSICS IN FAVOR OF MULTISPATIALITY

It seems to me that the generalized product differentiation rule MSPDR1, which emerged from matching partly covariant and partly contravariant evaluation of an integral kernel according to the – proven and operationally mandatory – regular scalar product differentiation rule RSPDR1, can also be compared to certain equations specifying conditions for entropy

$$\frac{\partial U(x,t)}{\partial t}+\frac{\partial}{\partial x}F(U)=0 \quad where\ U(x,0)=V(x) \tag{14}$$

which have been concisely discussed in [33]. This particular venue is of interest to physical applications and thus shall be further explored elsewhere.

It has been found that the behavior of eigenfunctions within the domain of complex eigenvalues is very unusual, for the logarithm of the wave function at different coordinates fluctuates strongly just like the position of Brownian particle fluctuates in time, even though the wave functions are strongly localized [34]. This kind of behavior might virtually suggest the real possibility that space and time domains can actually be considered as composing a paired dual reciprocal structure.

Hence the 4D spacetime may be regarded as virtual quasispatial structure in which elapsing time plays the role of hinge/pivot connecting the two naturally reciprocal geometric domains when they are viewed as quasigeometric spaces. Similarly, in the 4D timespace the spatial trajectory curve may be virtually pivoting whole temporal reciprocal space. The issue shall be further discussed elsewhere.

9. CONCLUSIONS

It has been shown that combining certain scalar functions to form integral kernels can result in legitimately obtained – i.e. gained in accordance with scalar product differentiation rule – parts of the outcome that could not be immediately used as proper integrands for prospective integrations within the same given primary space. This is because the improper parts appear as contravariant expressions that are multiplicative inverses of both functions involved. The contravariant terms are form wise inappropriate to be directly integrated.

Therefore, in order to form covariant differential from the contravariant expression, the latter should be housed within the dual reciprocal space that is paired with the given primary space and reformulated accordingly. Thus, certain new multispatial operational rules for scalar differentiation and integration of compounded functions, which resemble integral kernels, have been proposed for use in paired dual reciprocal spaces.

Although the failures exposed in this paper are not nonsensical *per se*, their emergence in the – viewed as complete and thus unquestionably valid – differential and integral calculus, indicate need to rework both the real and the complex mathematical analysis upon multispatial reality paradigm and modified principles. Deference to obsolete axioms is inadmissible.

The traditional mathematics that irrationally defended some indefensible assumptions while providing tacitly veiled cover-up for covert failures is unacceptable conceptually and disastrous theoretically, which is especially visible in some physical applications. If it has to be realistic, mathematics should be discovered, not invented from scratch upon frivolously concocted axioms, primitive notions and principles having no relevance to previously unanticipated yet curious experimental results. If the traditional mathematics generates nonsenses then perhaps experimental physics should guide its methods. Suppression of new mathematical methods that have already been able to explain some previously unexplained – and sometimes deemed as unexplainable by accepted physical theories – did not remove entrenched mathematical nonsenses, but makes the mathematics despicable despite its numerous successes, for even the valid past successes can deteriorate fast and thus should be replaced with better evaluations based upon more comprehensive ideas and principles.

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