Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 144 (2020) 243-265 EISSN 2392-2192

Interaction between neutron-proton core and neutron skin region in super heavy nuclei

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ABSTRACT

The core of an atomic nucleus composed of A nucleons is assumed to be composed of equal number of neutrons (N) and protons (Z); and for N>Z, the extra unpaired neutrons (N-Z) are assumed to constitute what is called neutron skin region of the nucleus. It is further assumed that the interaction between the core and the neutron skin results in an average potential V_o in which the neutrons in the skin region move, such that this interaction energy is V_o (A-2Z). The value of V_o is calculated for 254 Super heavy nuclei or elements (SHE or SHN) starting from $^{234}_{93}\mathrm{U}$ to $^{295}_{118}\mathrm{Ei}$, using the recently proposed modified Nuclear Models [1, 2]. The modification has been introduced through the modified coulomb energy term [2]. The calculated values of the interaction potential V_o range from 1345.1283 MeV for $^{234}_{93}\mathrm{U}$ to the value 1878.9611 MeV for $^{295}_{118}\mathrm{Ei}$. However, a very high value of 2047.3124 MeV is noted for $^{267}_{110}\mathrm{Ds}$. Definite variations in V_o are obtained for Isotones, Isotopes and Isobars. For Isotones, the value of V_o increases as the proton number (Z) increases. For isotopes the value of V_o decreases as the number of neutrons in neutron skin region increases. For Isobars, the value of V_o increases as the proton number (Z) increases.

***Keywords*:** Super Heavy Nuclei, Interaction Potential, Isotopes, Isobars, Isotones, Neutron Skin

(Received 15 March 2020; Accepted 03 April 2020; Date of Publication 04 April 2020)

1. INTRODUCTION

J. J. Thomson [3] was the first to discover that the mass of a nucleus is not determined by its charge rather, there exist nuclei of the same charge Z (proton number) but of different masses. Such nuclei are called Isotopes. The mass of each isotope was assumed to be roughly equal to the mass of an integral number of protons. The assumption that a nucleus is composed of protons was contradicted by the fact that in most of the nuclei the mass number A is generally twice or more than twice the proton number Z. Then appeared the discovery of neutron by Chadwick J. [4, 5] and this led Heisenberg [6] to propose the hypothesis that nuclei are composed of neutrons and protons. Hence, nuclear physics, as we know it to-day, began its exposition from the year 1932.

Now the assumption that a nucleus is composed of neutrons (N), protons (Z), and its mass number A = N+Z, is now confirmed experimentally. Protons being positively charged has slightly less mass compared to the mass of the neutron which has no charge. Inside the nucleus, and this nomenclature forces was accepted due to the fact that the nuclear forces between neutrons and protons inside the nucleus are charge independent.

Infact, different types of nuclei have been given different names depending upon whether the proton number is constant (Z constant), and such nuclei are called isotopes (A and N varies). When the neutron number N is constant (A and Z varies), the nuclei are called Isotones. There is another set of nuclei in which Z and N are the same in two or more nuclei, and such nuclei are called Isomers or mirror nuclei. For instance in ${}_{1}^{3}H$ there are two neutrons, whereas in ${}_{2}^{3}He$ there are two protons.

Now the properties of nuclei change drastically as the atomic number Z changes between 2 and 120 or so. A number of nuclear models have been proposed from time to time to explain the properties of nuclei in different regions of mass number A, but no nuclear model can explain all the properties of nuclei.

It is not exactly known as to how many neutrons and protons can get together to form a bound atomic nucleus especially in that region of periodic table when Z is more than 92 to Z=120. By now there are some 3200 isotopes in the nuclear regime [7, 8]. Out of these only 286 primordial nuclei existed in their present form since the creation of the Earth, and these are the stable isotopes. They constitute the valley of stability on the nuclear table of elements. As one adds protons and or neutrons to nuclei, one may move away from the region of stable isotopes, and may enter the region of short-lived radioactive nuclei, and such nuclei may be beta-unstable. At some point when a last nucleon (proton or neutron) is added to the nucleus, the binding fraction f may become zero, and hence the nucleon simply drips off. This stage is called dripline for neutron or proton, and at this stage nuclear existence ends. Even the strong nuclear force can not keep the last nucleon attached to the nucleus. As per some recent theoretical calculations, the number of bound nuclei with atomic number Z between 2 and 120 is of the order of 7000 [9, 10].

By now it is an established fact that one of the most important properties of nuclei is its stability that is directly related to its average binding energy, and binding energy per nucleon. The nuclear binding energy plays a very important role in the study of nuclear mass, decay half-life, nuclear fission, and a very significant nuclear property such as nuclear stability. The limits of nuclear stability are determined by interaction of nucleons in the nucleus. The nature of interactions between the nucleons and the limits of nuclear stability are still not known especially in super heavy nuclei in the so called "Island of Stability".

However, what is certain is that the super heavy nuclei (SHN) are at the limits of coulomb stability [11]. We must also understand that the "Island of Stability" is a term from nuclear physics that describes the possibility of elements with particularly stable "magic Numbers" of protons and neutrons. Some isotopes of transuranic elements are far more stable than others. The idea of the island of stability was first proposed in the 1960s, through developments by Vilen Strutinsky [12] and others. The understanding of the nuclear structure of lighter nuclei led to predictions by Adam Sobiczewski [13] and his collaborators on new nuclear shell closures. William [14] and Seaborg [15] independently predicted the existence of heavy nuclei that would occupy a so-called island of stability. Since that time, the concept of an island of stability has dominated the physics of Super Heavy Nuclei (SHN) [16].

The atomic nucleus is built up in "shells" in a manner similar to the electron shells in atoms. Energy levels from quantum states in two different shells will be separated by a relatively large energy gap. So when the number of neutrons and protons completely fill the energy levels of a given shell in the nucleus, the binding energy per nucleon will reach a local minimum such that a particular configuration will have a longer life time than the nearby isotopes that do not have filled shells [16, 17]. In the shell-model representation, those nuclei with closed shell Z or N are called "magic" and also when both Z and N numbers are magic, the nucleus is called "double magic" [18]. Several models have been presented to study of binding energy of SHE [18-22]. For the study of SHN mostly macro-micro approach is used [23-26] and some mass formulas were proposed that combine the liquid-drop ideology with the shell-model corrections of Strutinsky [12] and Mayers [14]. In order to improve the agreement with experimental results, Different corrections were introduced in the mass formula [27, 28]. Three different ways for description of binding energy of super heavy nuclei have been in use. First, one can consider SHN as a part of a whole system of nuclei for which a global mass formula is found [29].

Another way is the detailed local description of energy of SHN taking into account the effects of shells and subshells. The third way of description, applied for nuclei in the region limited by principal magic numbers, is attached to the beta-stability line [30]. Although the SHN are at the limits of Coulomb stability, shell stabilization lowers the ground-state energy, creates a fission barrier, and thereby enables the SHE to exist [11]. Thus in this manuscript we have included the role of modified Coulomb interaction [2] on the binding energy of the nucleus in the region of SHN. Simultaneously the nucleus is assumed to be composed of a core region which is composed of equal number of protons (Z) and neutron (N), and this core is surrounded by what is called the neutron skin which contains (N-Z) neutrons only. The neutrons in the neutron skin region move in an average potential V_o created by the interaction of the core with the neutrons in the neutron skin region.

2. DERIVATION

Bethe and Weizsäcker [31-33], first proposed a formula for the calculation of binding energy B of the nucleus and assuming similarities between the atomic nucleus and the liquid drop, it is written as:

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c Z(Z-1)A^{-\frac{1}{3}} - a_a(\frac{A}{2} - Z)^2 A^{-1} + a_p \delta A^{-\frac{1}{2}}$$

where a_v, a_s , a_c , a_a and a_p are volume, surface, coulomb, asymmetry and pairing coefficients, respectively. Over the years this formula has been modified to include different parameters.

There are many choices for these coefficients [18, 22]. The formula proposed recently looks like [1]:

$$B_{tot}(A, Z) = 2a_v Z - \epsilon_\rho \; 2Z(2Z-1) - 0.72\frac{Z^2}{2Z^{1/3}} - V_0(A-2Z) \tag{1}$$

where the coefficient a_v and ϵ_ρ are the volume term and the pairing energy term. This equation gives the formula for the binding energy of the nucleus, in this nuclear model it is assumed that when N>Z, the core of the nucleus is composed of equal number of neutrons and protons that form neutron-proton pairs, and the unpaired neutrons reside in the surface region of the nucleus which is also called neutron skin.

Thus for a nucleus of mass number A = Z+N, the core of the nucleus will be composed of Z proton-neutron pairs and the unpaired neutrons equal to, N-Z, will constitute the surface region of the nucleus. If ϵ_ρ is the pairing energy between a neutron-proton pair in the core, then the total energy of the core of the nucleons will be $\varepsilon_p\left({}^{2Z}C_2\right) = \varepsilon_p 2Z(2Z-1)$ since there are 2Z nucleons in the core. It will be assumed that the interaction between the core and the neutrons in the surface region leads to an average potential in which each neutron in the surface region can move. Coulomb energy is one of the major parameters in determining the binding energy as shown in different binding energy formula [18, 34-37].

Coulomb energy has continuously been corrected in order to understand nuclear correlations, charge-dependence, nuclear force, Coulomb perturbation and nuclear rearrangement energy [2, 38, 39].

The semi-empirical mass formula (SEMF) gives the Coulomb energy term as $a_c Z(Z-1)A^{-\frac{1}{3}}$, where $a_c = 0.71\text{MeV}$. This term is applied for calculating binding energy of light and medium nuclei, The Coulomb energy term changes drastically when it is calculated for heavy and super heavy nuclei, where the term Z(Z-1) is replaced by Z^2 [40]. Using a modified Coulomb energy equation from [2], we can write an expression for E_c as,

$$E_c = \frac{3}{5}\frac{Z_1 Z_2 e^2}{4\Pi\varepsilon_o R_o} e^{\frac{R_o^n}{nR^n}} \tag{2}$$

where the terms Z_1, Z_2 are the proton nuclear charges, e is the electron charge, ε_o is the permittivity of free space, R_o is the radius of the core, R is the nuclear radius and n is an integer. The correction terms are the exponential terms of the ratio of the powers of the nuclear core radius (R_o) to the powers of the effective nuclear radius (R). As the power of their radii increases from n = 1 to n > 21, the ratio of $\frac{R_o^n}{nR^n}$ tends to the value zero and the exponential correction term goes to one. Therefore Eq (2) can be rewritten as:

$$E_c = \frac{3}{5}\frac{Z_1 Z_2 e^2}{4\Pi\varepsilon_o R_o} \tag{3}$$

Eq (3) is inserted to Eq (1) to replace the Coulomb energy term. The terms are re-arranged to get value of V_o such that,

$$V_o = \frac{2a_v Z - B_{tot}(A,Z) - \varepsilon_p 2Z(2Z-1) - \frac{3}{5}\frac{Z_1 Z_2 e^2}{4\Pi\varepsilon_o R_o}}{A-2Z} \tag{4}$$

Following values are used to calculate V_o, I.e

$a_v = 15.8$ MeV [41, 42]

$\epsilon_\rho = 2$MeV

$B_{tot}(A,Z)$ is obtained from the experimental binding energy values given in [43,44]

Using the respective binding energy values and mass number we calculate the values of V_o for 254 SHN.

3. RESULTS

Table 1. Values of V_o for Super Heavy Nuclei.

SYMBOL	B(Exp) MeV	A	Z	N	A-2Z	V₀ (MeV)
U	1778.565438	234	92	142	50	1345.128398
U	1783.86291	235	92	143	51	1318.857203
U	1790.408336	236	92	144	52	1293.620438
U	1801.68856	238	92	146	54	1245.917463
U	1795.534278	237	92	145	53	1269.309221
U	1806.494884	239	92	147	55	1223.351806
U	1812.42384	240	92	148	56	1201.612112
U	1816.899	241	92	149	57	1180.60971
U	1822.744	242	92	150	58	1160.355146
U	1826.874	243	92	151	59	1140.758109
Np	1741.7969	229	93	136	43	1597.281501

Np	1756.08972	231	93	138	45	1526.60883
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_0 (MeV)**
Np	1769.90761	233	93	140	47	1461.94075
Np	1775.97342	234	93	141	48	1431.610022
Np	1783.14945	235	93	142	49	1402.53994
Np	1788.69356	236	93	143	50	1374.600024
Np	1795.270497	237	93	144	51	1347.776042
Np	1800.758694	238	93	145	52	1321.962814
Np	1806.974079	239	93	146	53	1297.137391
Np	1812.42384	240	93	147	54	1273.217249
Pu	1762.504	232	94	138	44	1595.077336
Pu	1767.02307	233	94	139	45	1559.731597
Pu	1774.797804	234	94	140	46	1525.993404
Pu	1781.03445	235	94	141	47	1493.658154
Pu	1788.387468	236	94	142	48	1462.693463
Pu	1794.268224	237	94	143	49	1432.962592
Pu	1801.268014	238	94	144	50	1404.443336
Pu	1806.91409	239	94	145	51	1377.015939
Pu	1813.4484	240	94	146	52	1350.660523
Pu	1818.689871	241	94	147	53	1325.275258
Pu	1824.999682	242	94	148	54	1300.849971
Pu	1836.053884	244	94	150	56	1254.58844
Am	1791.957	237	95	142	47	1525.714562
Am	1798.22804	238	95	143	48	1494.059488

Am	1805.329759	239	95	144	49	1463.713411
Am	1811.2824	240	95	145	50	1434.558196
Am	1817.928311	241	95	146	51	1406.559916
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_o (MeV)**
Am	1823.465886	242	95	147	52	1379.617179
Am	1829.830824	243	95	148	53	1353.706759
Cm	1796.424	238	96	142	46	1591.846022
Cm	1802.79134	239	96	143	47	1558.112433
Cm	1810.2852	240	96	144	48	1525.807879
Cm	1816.378681	241	96	145	49	1494.7933
Cm	1823.348032	242	96	146	50	1465.036821
Cm	1829.041074	243	96	147	51	1436.422237
Cm	1835.842336	244	96	148	52	1408.929526
Cm	1841.362425	245	96	149	53	1382.450102
Cm	1847.820144	246	96	150	54	1356.968762
Cm	1852.975722	247	96	151	55	1332.39034
Cm	1859.1878	248	96	152	56	1308.708586
Cm	1869.7325	250	96	154	58	1263.762509
Bk	1790.95	238	97	141	44	1698.805571
Bk	1798.953	239	97	142	45	1661.23218
Bk	1839.769435	245	97	148	51	1466.593422
Bk	1845.6888	246	97	149	52	1438.503537
Bk	1850.84263	247	97	150	53	1411.459203
Bk	1857.768	248	97	151	54	1385.449317

Bk	1864.019727	249	97	152	55	1360.372997
Bk	1868.99025	250	97	147	56	1336.169381
Cf	1802.4552	240	98	142	44	1734.161101
Cf	1809.187	241	98	143	45	1695.773783
Cf	1817.2022	242	98	144	46	1659.083379
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_o (MeV)**
Cf	1823.715	243	98	145	47	1623.922303
Cf	1831.251964	244	98	146	48	1590.247609
Cf	1837.41572	245	98	147	49	1557.919367
Cf	1844.782044	246	98	148	50	1526.908306
Cf	1857.77668	248	98	150	52	1468.43096
Cf	1863.362118	249	98	151	53	1440.830101
Cf	1869.98725	250	98	152	54	1414.27075
Cf	1875.094245	251	98	153	55	1388.649591
Cf	1881.266688	252	98	154	56	1363.962499
Cf	1886.070978	253	98	155	57	1340.117618
Cf	1892.10188	254	98	156	58	1317.116123
Es	1795.44	240	99	141	42	1853.722546
Es	1803.885	241	99	142	43	1810.809115
Es	1810.886	242	99	143	44	1769.813476
Es	1819.098	243	99	144	45	1730.666777
Es	1833.58	245	99	146	47	1657.32951
Es	1840.08	246	99	147	48	1622.937228
Es	1847.5847	247	99	148	49	1589.969217

Es	1854.048	248	99	149	50	1558.299099
Es	1861.026	249	99	150	51	1527.881038
Es	1873.934625	251	99	152	53	1470.468709
Es	1879.22448	252	99	153	54	1443.335767
Es	1885.576363	253	99	154	55	1417.208787
Es	1890.670082	254	99	155	56	1391.992447
Es	1896.6441	255	99	156	57	1367.676334
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_0 (MeV)**
Fm	1797.86	241	100	141	41	1937.443311
Fm	1806.53	242	100	142	42	1891.520137
Fm	1813.752	243	100	143	43	1847.69925
Fm	1822.192	244	100	144	44	1805.897903
Fm	1829.17	245	100	145	45	1765.921906
Fm	1837.12062	246	100	146	46	1727.705139
Fm	1843.608	247	100	147	47	1691.083484
Fm	1876.35312	248	100	148	48	1656.534768
Fm	1858.002891	249	100	149	49	1622.353442
Fm	1865.5225	250	100	150	50	1590.056765
Fm	1871.71202	251	100	151	51	1559.000545
Fm	1878.919812	252	100	152	52	1529.158376
Fm	1884.459621	253	100	153	53	1500.410856
Fm	1890.975644	254	100	154	54	1472.746137
Fm	1896.150165	255	100	155	55	1446.063017
Fm	1902.535168	256	100	156	56	1420.354481

Fm	1907.503087	257	100	157	57	1395.523138
Fm	1913.844	258	100	158	58	1371.57172
Fm	1918.413	259	100	159	59	1348.402182
Fm	1924.52	260	100	160	60	1326.030596
Md	1845.616	248	101	147	46	1762.510331
Md	1874.376	252	101	151	50	1622.084704
Md	1906.314462	257	101	156	55	1475.203158
Md	1911.694344	258	101	157	56	1448.956313
Md	1922.96	260	101	159	58	1399.186538
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_0 (MeV)**
No	1856.5	250	102	148	46	1797.718193
No	1863.173	251	102	149	47	1759.610848
No	1871.3016	252	102	150	48	1723.121634
No	1877.883898	253	102	151	49	1688.090219
No	1885.58932	254	102	152	50	1654.482524
No	1891.5798	255	102	153	51	1622.15915
No	1898.63424	256	102	154	52	1591.099444
No	1904.278765	257	102	155	53	1561.1852
No	1911.006	258	102	156	54	1532.398942
No	1916.6	259	102	157	55	1504.638852
No	1923.22	260	102	158	56	1477.888515
No	1934.87	262	102	160	58	1427.127704
Lr	1864.548	252	103	149	46	1833.215136
Lr	1872.959	253	103	150	47	1794.389516

Lr	1879.6	254	103	151	48	1757.144756
Lr	1887.6579	255	103	152	49	1721.449105
Lr	1890.67008	256	103	153	50	1687.080367
Lr	1901.029	257	103	154	51	1654.203476
Lr	1907.136	258	103	155	52	1622.509313
Lr	1914.01	259	103	156	53	1592.025628
Lr	1919.58	260	103	157	54	1562.64682
Lr	1926.441	261	103	158	55	1534.359805
Lr	1931.988	262	103	159	56	1507.059576
Rf	1867.14	253	104	149	45	1910.476289
Rf	1875.536	254	104	150	46	1869.126717
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_0 (MeV)**
Rf	1882.41	255	104	151	47	1829.504319
Rf	1890.67008	256	104	152	48	1791.561731
Rf	1897.0969	257	104	153	49	1755.130406
Rf	1904.69274	258	104	154	50	1720.179715
Rf	1910.643	259	104	155	51	1686.567372
Rf	1918.02	260	104	156	52	1654.27525
Rf	1923.92757	261	104	157	53	1623.173973
Rf	1930.94	262	104	158	54	1593.245055
Rf	1936.732	263	104	159	55	1564.382273
Rf	1960.314	267	104	163	59	1458.722152
Db	1883.648	256	105	151	46	1905.326492
Db	1892.034	257	105	152	47	1864.966056

Db	1898.364	258	105	153	48	1826.244471
Db	1906.33324	259	105	154	49	1789.136813
Db	1912.82	260	105	155	50	1753.483812
Db	1920.177	261	105	156	51	1719.246032
Db	1926.224	262	105	157	52	1686.299896
Db	1933.576	263	105	158	53	1654.621634
Db	1952.174	266	105	161	56	1566.313297
Db	1958.712	267	105	162	57	1538.948818
Db	1963.904	268	105	163	58	1512.504735
Db	1969.887	269	105	164	59	1486.970468
Db	1974.78	270	105	165	60	1462.269177
Sg	1894.236	258	106	152	46	1941.930798
Sg	1901.06	259	106	153	47	1900.758313
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_o (MeV)**
Sg	1909.0656	260	106	154	48	1861.325964
Sg	1915.67997	261	106	155	49	1823.474707
Sg	1923.38916	262	106	156	50	1787.159397
Sg	1929.631	263	106	157	51	1752.239445
Sg	1937.232	264	106	158	52	1718.688706
Sg	1943.245	265	106	159	53	1686.37407
Sg	1950.312	266	106	160	54	1655.275791
Sg	1956.309	267	106	161	55	1625.288904
Sg	1979.655	271	106	165	59	1515.49552
Sg	1985.872	272	106	166	60	1490.340878

Sg	1990.443	273	106	167	61	1465.983995
Bh	1901.38	260	107	153	46	1978.810931
Bh	1909.737	261	107	154	47	1936.886379
Bh	1916.53	262	107	155	48	1896.6761
Bh	1924.634	263	107	156	49	1858.133813
Bh	1931.16	264	107	157	50	1821.101656
Bh	1938.74	265	107	158	51	1785.542408
Bh	1945.258	266	107	159	52	1751.3304
Bh	1952.571	267	107	160	53	1718.424412
Bh	1971.27	270	107	163	56	1626.699872
Bh	1982.88	272	107	165	58	1570.806945
Bh	1989.078	273	107	166	59	1544.288149
Bh	1994.172	274	107	167	60	1518.634914
Bh	2000.075	275	107	168	61	1493.83603
Hs	1918.585	263	108	155	47	1973.36131
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_o (MeV)**
Hs	1926.76968	264	108	156	48	1932.42013
Hs	1933.5036	265	108	157	49	1893.120412
Hs	1941.33982	266	108	158	50	1855.414728
Hs	1947.765	267	108	159	51	1819.160031
Hs	1962.086	269	108	161	53	1750.78269
Hs	1969.65	270	108	162	54	1718.500862
Hs	1975.048	271	108	163	55	1687.353538
Hs	1998.425	275	108	167	59	1573.352908

Hs	2009.635	277	108	169	61	1521.951337
Mt	1933.82	266	109	157	48	1968.433782
Mt	1948.628	268	109	159	50	1889.992591
Mt	1963.17	270	109	161	52	1817.580222
Mt	1989.24	274	109	165	56	1688.218599
Mt	1995.675	275	109	166	57	1658.713624
Mt	2001.276	276	109	167	58	1630.211682
Mt	2007.696	277	109	168	59	1602.689789
Mt	2012.998	278	109	169	60	1576.066659
Ds	1934.949	267	110	157	47	2047.312475
Ds	1950.25	269	110	159	49	1964.060966
Ds	1958.5179	270	110	160	50	1924.945105
Ds	1965.292	271	110	161	51	1887.333908
Ds	1973.36	272	110	162	52	1851.194179
Ds	1978.977	273	110	163	53	1816.371968
Ds	1986.226	274	110	164	54	1782.869691
Ds	2016.612	279	110	169	59	1632.294056
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_0 (MeV)**
Ds	2028.82	281	110	171	61	1578.97635
Rg	1965.744	272	111	161	50	1960.166951
Rg	1980.198	274	111	163	52	1885.053875
Rg	2006.604	278	111	167	56	1750.878706
Rg	2013.543	279	111	168	57	1720.283272
Rg	2019.36	280	111	169	58	1690.723509

Rg	2026.291	281	111	170	59	1662.184653
Rg	2031.81	282	111	171	60	1634.573559
Cn	1995.785	277	112	165	53	1883.177071
Cn	2016.56	280	112	168	56	1782.663567
Cn	2022.357	281	112	169	57	1751.490469
Cn	2029.554	282	112	170	58	1721.416444
Cn	2035.053	283	112	171	59	1692.333097
Cn	2042.528	284	112	172	60	1664.252129
Cn	2047.725	285	112	173	61	1637.054504
Ed	1996.596	278	113	165	52	1953.756184
Ed	2024.196	282	113	169	56	1814.695028
Ed	2031.374	283	113	170	57	1782.984203
Ed	2037.416	284	113	171	58	1752.347269
Ed	2044.59	285	113	172	59	1722.768061
Ed	2050.334	286	113	173	60	1694.150993
Ed	2057.216	287	113	174	61	1666.490846
Fl	2047.474	286	114	172	58	1783.594046
Fl	2053.198	287	114	173	59	1753.460656
Fl	2060.64	288	114	174	60	1724.360345
SYMBOL	**B(Exp) MeV**	**A**	**Z**	**N**	**A-2Z**	**V_0 (MeV)**
Fl	2066.061	289	114	175	61	1696.181011
Ef	2048.893	287	115	172	57	1846.811537
Ef	2055.168	288	115	173	58	1815.078149
Ef	2062.304	289	115	174	59	1784.435062

Ef	2068.28	290	115	175	60	1754.794077
Ef	2075.121	291	115	176	61	1726.139109
Lv	2057.391	289	116	173	57	1879.145054
Lv	2064.8	290	116	174	58	1846.873743
Lv	2070.756	291	116	175	59	1815.671747
Lv	2078.164	292	116	176	60	1785.534018
Lv	2083.523	293	116	177	61	1756.350821
Eh	2072.032	292	117	175	58	1878.906028
Eh	2079.421	293	117	176	59	1847.1854
Eh	2085.342	294	117	177	60	1816.49766
Ei	2073.561	293	118	175	57	1944.646437
Ei	2081.52	294	118	176	58	1911.255274
Ei	2087.42	295	118	177	59	1878.961117

4. DISCUSSIONS

4. 1. Isotopes

Table 2. Calculated values of V_o for Isotopes of ${}^{234-238}_{92}\mathrm{U}$, ${}^{238-242}_{94}\mathrm{Pu}$ and ${}^{270-274}_{110}\mathrm{Ds}$.

A-2Z	**V_o (MeV)**		
	${}^{234-238}_{92}\mathrm{U}$	**${}^{238-242}_{94}\mathrm{Pu}$**	**${}^{270-274}_{110}\mathrm{Ds}$**
50	1345.1284	1404.4433	1924.9451
51	1318.8572	1377.0159	1887.3339
52	1293.6204	1350.6605	1851.1941
53	1269.3092	1325.2752	1816.3719
54	1245.9174	1300.8499	1782.8696

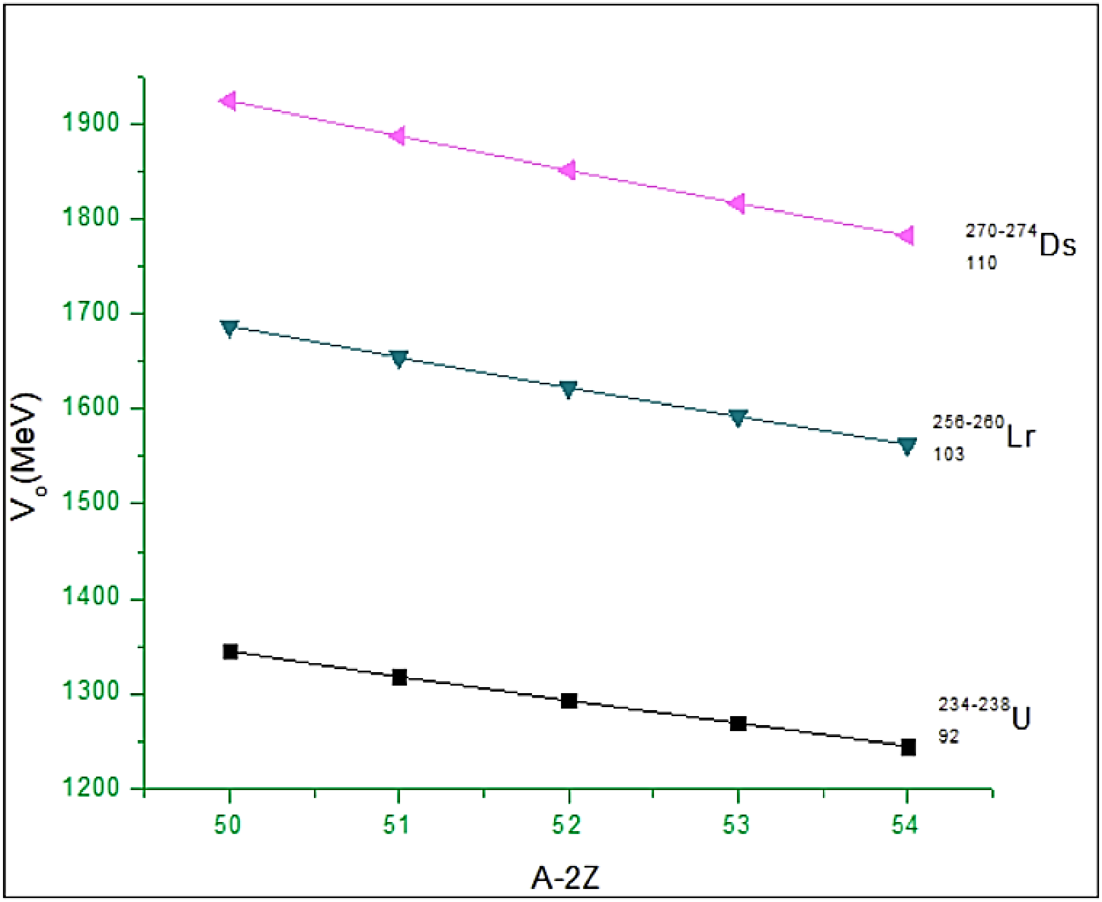

Figure 1. Graph showing variation of V_o against neutrons in the neutron skin region for isotopes.

From Table 2, as the Neutron number (N) increases at constant Proton number (Z), the value of V_o reduces for the respective isotopes as the neutron number increases by unity. The reduction in the value of V_o is small when Neutron number increases by unity.

4. 2. Isotones

Table 3. Values of V_o for Isotones with Neutron number N = 142.

Nuclei	A	N	Z	A-2Z	V_o (MeV)
U	234	142	92	50	1345.128
Np	235	142	93	49	1402.539
Pu	236	142	94	48	1462.693

Am	237	142	95	47	1525.714
Cm	238	142	96	46	1591.846
Bk	239	142	97	45	1661.232
Cf	240	142	98	44	1734.161
Es	241	142	99	43	1810.809
Fm	242	142	100	42	1891.520

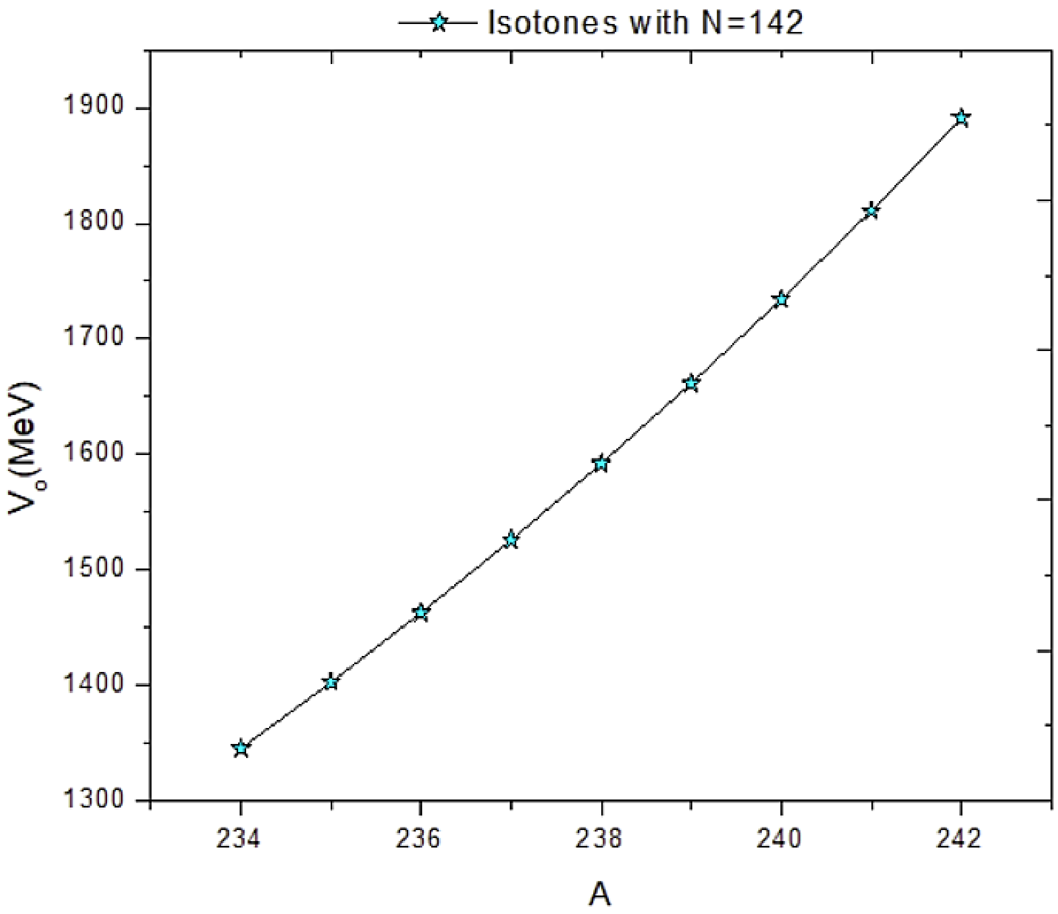

Figure 2. Graph showing variation of V_o against A for Isotones with neutron number N = 142.

From Table 3, as the proton number (Z) increases with constant Neutron number (N), the value of V_o increases. This means that the excess neutrons on the surface region reduces by moving to the core region and forming more N-Z pairs, this increases the interaction between the core region and surface region.

4. 3. Isobars

Table 4. Values of V_o for Isobars with mass number A = 238.

Nuclei	**A**	**N**	**Z**	**V_o (MeV)**
U	238	146	92	1245.917
Np	238	145	93	1321.9628
Pn	238	144	94	1404.443
Am	238	143	95	1494.059
Cm	238	142	96	1591.846
Bk	238	141	97	1698.805

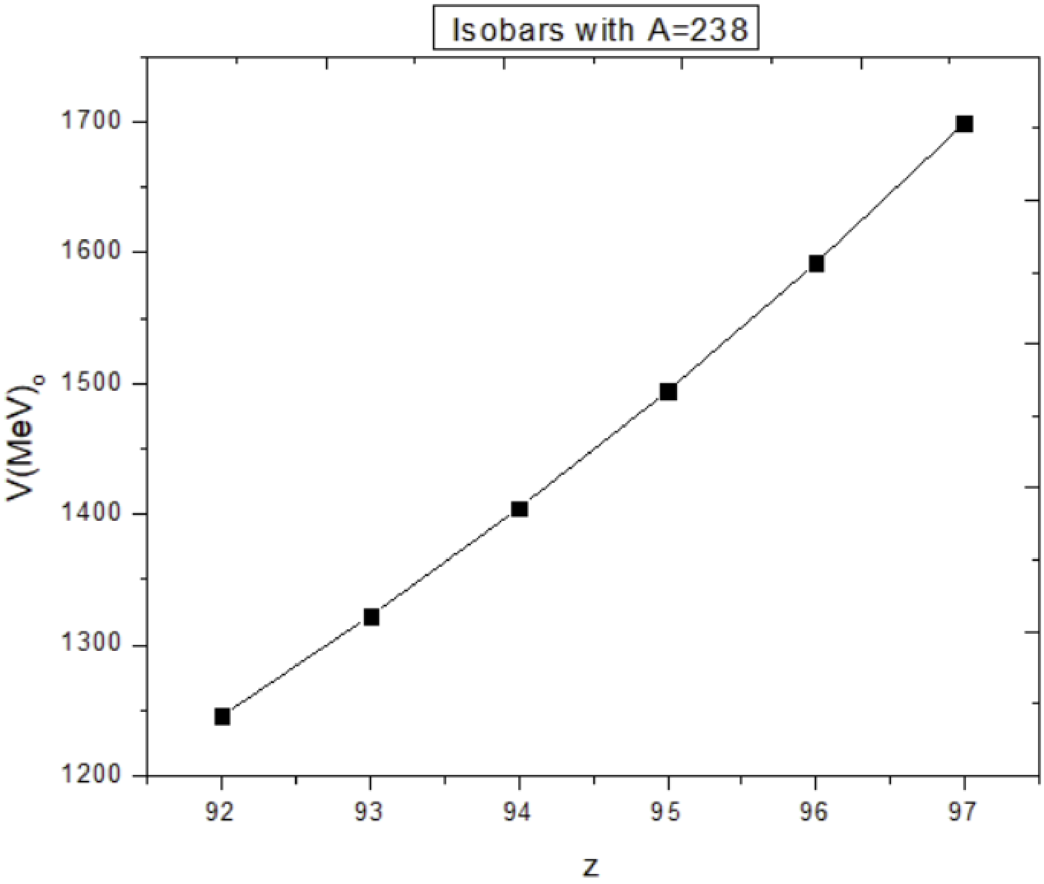

Figure 3. Graph showing variation of V_o (MeV) against A for Isotones with neutron number N = 142.

From Table 4, as the proton number (Z) increases with reducing neutron number (N) at constant value of mass number, the value of V_o increases, this indicates that the ratio of N:Z decreases with an increase in proton number (Z), meaning that the interaction between the core region and the surface region increases, this is attributed to the increase in repulsion energy.

4. CONCLUSIONS

From $^{234}_{93}\mathrm{U}$ onwards as the neutron is added one by one, the value of V_o decreases, and this indicates the march towards neutron dripline as more neutrons are added keeping Z constant. This means that adding a neutron to the nucleus keeping Z constant results in the reduction of the value of the average potential V_o in which the neutron in the neutron skin region moves. This trend is maintained for all the nuclei and their isotopes as we go from $^{234}_{93}\mathrm{U}$ to $^{295}_{118}\mathrm{Ei}$ (See Table 1, 2).

In the case of Isotones when the neutron number is kept constant at N=142, the value of Z is increased from Z = 92 to Z = 100. Such a change will lead to different nuclei, namely from U to Fm (See table 3). The value of V_o increases as Z increases. In this model, the added proton will enter the core region, and to maintain the N = Z condition in the core, a neutron will shift from the neutron skin region to the core resulting in the reduction of the neutrons in the neutron surface or skin region. Number of N-P pairs in the core will increase resulting in the increase of the core size, and the number of neutrons in the neutron skin will decrease, Hence the average potential, V_o , created by the core will increase.

In the case of Isobars, A is kept constant at A = 238. To keep A constant, N and Z will have to be changed. When we go from U to Bk, Z increases from 92 to 97, and consequently N decreases from 146 to 141. This situation is somewhat similar to what we have in Isotones except that in Isotones N is constant and Z increases. The crucial variation is the value of Z that decides the Coulomb interaction which determines the Coulomb stability for super heavy nuclei (SHN) in the “Island of Stability” of super heavy elements (SHE). The validity of our concepts and calculations is confirmed by the fact that as Z increases both in Isotones and isobars, V_o increases.

Figure 1 shows that V_o decreases linearly for all the isotopes, but its value is large for higher A and Z values.

Figure 2 shows that the value of V_o increases linearly for the Isotones for which A = 238. This means that even in super heavy nuclei, the variation in interaction between the nucleons is governed by the laws that lead to stability of the super heavy nuclei.

ACKNOWLEDGEMENT

Thanks are due to Dr. L.S. Chelimo of the department of physics, University of Eldoret, Eldoret, Kenya, for very helpful suggestions and critical remarks during the preparation of the manuscript.

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