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A New Notion of Star Open Sets in Soft Topological Spaces

P. Gnanachandra[1], M. Lellis Thivagar[2]

[1]Centre for Research and Post Graduate Studies in Mathematics,
Ayya Nadar Janaki Ammal College (Autonomous), Sivakasi - 626 124, Tamil Nadu, India

[2]School of Mathematics, Madurai Kamaraj University, Madurai - 625 021, Tamil Nadu, India

[1,2]E-mail address: pgchandra07@gmail.com , mlthivagar@yahoo.co.in

ABSTRACT

In our day-to-day life, we look out problems with unreliabilities. To handle the lack of unreliability and to solve the problems related to uncertainty, a short time ago numberless theories have been developed like Rough Sets [Pawlak 1982], Fuzzy Sets [Zadeh, 1965], Vague Sets [Gau and Buehrer, 1993]. However, these methodologies have their own risks. To circumvent these difficulties, Molodtsov [1999] developed Soft set theory to deal with unreliability. The development of Soft Set theory is whistle stop now-a-days. In 2013, Gnanambal et al. made a study of soft topology via soft pre-open sets. Quite recently, the authors defined soft semi*-open sets using semi*-open sets. This paper aims at developing a soft topology via soft pre*-open sets. Further, we study the fundamental properties of soft pre*-open and soft pre*-closed sets, soft pre*-interior and soft pre*-closure operators.

***Keywords*:** Soft pre*-open sets, soft pre*-closed sets, soft pre*-interior, soft pre*-closure

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1. INTRODUCTION

Molodtsov [13] initiated the study of soft sets in 1999. Soft topology was introduced by Muhammad Shabir [14] et al. in 2011. Now many researchers are working on various properties and types of soft topological spaces. Sabir Hussain [16] et al. defined and discussed properties of soft interior, soft exterior and soft boundary. Samanta [15] et al introduced mappings in soft topological spaces. Gnanambal Ilango and Mrudula Ravindran [9] made a study of soft topology via pre-open sets, which is a motivation and base for this paper. Kannan [11] studied on soft g-closed sets. Andeijevic [1, 2, 5] and Mashhour gave many results on pre-open sets in general topology. In this paper, we introduce soft pre*-open sets and obtain some basic properties.

Let us recall the following definitions that are used in sequel.

Definition 1.1. Let U be an initial universal set and E be the set of parameters. Let P(U) denote the power set of U and let $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

The collection of soft sets (F, A) over a universe U and the parameter set A is a family soft sets denoted by $SS(U)_A$.

Definition 1.2. Let τ be a collection of soft sets over a universe U with a fixed set A of parameters, then $\tau \subseteq SS(U)_A$ is called a soft topology on U with a fixed set A if,

(i) Φ_A, U_A belong to τ.

(ii) The union of any number of soft sets in τ belongs to τ.

(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (U, A, τ) is called a soft topological space over U. The members of τ are called soft open sets in U and complements of them are called soft closed sets in U.

Soft operations are denoted by usual set theoretical operations with '~' symbol above. Soft interior and soft closure are denoted by $\tilde{s}$ int and $\tilde{s}$ cl respectively.

Theorem 1.3. Arbitrary union of soft open sets is soft open and finite intersection of soft closed sets is soft closed.

Definition 1.4. Let (U, A, τ) be a soft topological space and let (G, A) be a soft set.

Then

(i) The soft closure of (G, A) is the soft set

$\tilde{s}$ cl(G, A) = $\tilde{\cap}$ { (S, A): (S, A) is soft closed and (G, A) $\tilde{\subseteq}$ (S, A)}

(ii) The soft interior of (G, A) is the soft set

$\tilde{s}$ int(G, A) = $\tilde{\cup}$ { (S, A): (S, A) is soft open and (S, A) $\tilde{\subseteq}$ (G, A)}

$\tilde{s}$ cl(G, A) is the smallest soft closed set containing (G, A) and $\tilde{s}$ int(G, A) is the largest soft open set contained in (G, A).

Definition 1.5. In a soft topological space (U, A, τ), a soft set

(i) (G, A) is said to be soft pre-open set if (G, A) $\tilde{\subseteq}$ $\tilde{s}$ int($\tilde{s}$ cl(G, A)).

(ii) (F, A) is said to be soft pre-closed set if (F, A) $\tilde{\supseteq}$ $\tilde{s}$ cl($\tilde{s}$ int(F, A)).

A soft pre-closed set is nothing but the complement of a soft pre-open set.

Theorem 1.6. Let (U, A, τ) be a soft topological space and let (G, A) and (F, A) be a soft sets over U. Then,

(i) (F, A) is soft closed if and only if (F, A) = $\tilde{s}$ cl(F, A).

(ii) (G, A) is soft open if and only if (G, A) = $\tilde{s}$ int(G, A).

Lemma 1.7. If $\{(G, A)_\alpha / \alpha \in I\}$ is a collection of soft sets, then

$$\tilde{\cup}\ \tilde{s}\ \mathrm{int}(G, A)_\alpha \tilde{\subseteq}\ \tilde{s}\ \mathrm{int}(\tilde{\cup}\ (G, A)_\alpha)$$

$$\tilde{\cup}\ \tilde{s}\ \mathrm{cl}(G, A)_\alpha \tilde{\subseteq}\ \tilde{s}\ \mathrm{cl}(\tilde{\cup}\ (G, A)_\alpha)$$

Definition 1.8. A soft set (A, E) is called a soft generalized closed (soft g-closed) in a soft topological space (X, τ, E) if $\overline{(A, E)}$ $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) is soft open in X.A soft set (A, E) is called a soft generalized open (soft g-open) in a soft topological space (X, τ,. E) if the relative complement (A, E)′ is soft g-closed in X.

Definition 1.9. A subset of a topological space (X, τ) is called pre*-open if A⊆int*(cl(A)).

Definition 1.10. Let A be a subset of X. The generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by cl*(A). The generalized interior of A is defined as the union of all g-open sets contained in A and is denoted by int*(A).

2. SOFT PRE*-OPEN SETS

In this section, we define a new notion of star version of pre-open sets in soft sets namely soft pre*-open sets and study some of their basic properties

Definition 2.1. In a soft topological space (U, A, τ), a soft set

(i) (G, A) is said to be soft pre*-open set if (G, A) $\tilde{\subseteq}$ $\tilde{s}$ int*($\tilde{s}$ cl(G, A))

(ii) (F, A) is said to be soft pre*-closed set if (F, A) $\tilde{\supseteq}$ $\tilde{s}$ cl*($\tilde{s}$ int(F, A))

A soft pre*-closed set is the complement of a soft pre*-open set.

Example 2.2. Let U = {a, b}, A = {e_1, e_2} Define

$(F, A)_1 = \{(e_1, \Phi), (e_2, \Phi)\}$, $(F, A)_2 = \{(e_1, \Phi), (e_2, \{a\})\}$,

$(F, A)_3 = \{(e_1, \Phi), (e_2, \{b\})\}$, $(F, A)_4 = \{ (e_1, \Phi), (e_2, \{a, b\})\}$,

$(F, A)_5 = \{(e_1, \{a\}), (e_2, \Phi)\}$, $(F, A)_6 = \{ (e_1, \{a\}), (e_2, \{a\})\}$,

$(F, A)_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F, A)_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$,

$(F, A)_9 = \{(e_1, \{b\}), (e_2, \Phi)\}$, $(F, A)_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$,

$(F, A)_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F, A)_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$,

$(F, A)_{13} = \{(e_1, \{a, b\}), (e_2, \Phi)\}$, $(F, A)_{14} = \{(e_1, \{a, b\}), (e_2, \{a\})\}$,

$(F, A)_{15} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$, $(F, A)_{16} = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$,

are all soft sets on universal set U under the parameter set A.

τ = { $(F, A)_1$, $(F, A)_5$, $(F, A)_7$, $(F, A)_8$, $(F, A)_{16}$ } is a soft topology over U.

Soft open sets are $(F, A)_1$, $(F, A)_5$, $(F, A)_7$, $(F, A)_8$, $(F, A)_{16}$

Soft closed sets are $(F, A)_1$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{12}$, $(F, A)_{16}$

Soft g-open sets are $(F, A)_1$, $(F, A)_4$, $(F, A)_5$, $(F, A)_6$,$(F, A)_7$, $(F, A)_8$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{11}$, $(F, A)_{12}$, $(F, A)_{13}$, $(F, A)_{14}$,$(F, A)_{15}$, $(F, A)_{16}$

Soft g-closed sets are $(F, A)_1$, $(F, A)_2$, $(F, A)_3$, $(F, A)_4$, $(F, A)_5$, $(F, A)_6$, $(F, A)_7$, $(F, A)_8$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{11}$, $(F, A)_{12}$, $(F, A)_{13}$, $(F, A)_{16}$

Soft pre-open sets are $(F, A)_1$, $(F, A)_5$, $(F, A)_6$, $(F, A)_7$, $(F, A)_8$, $(F, A)_9$, $(F, A)_{13}$, $(F, A)_{14}$, $(F, A)_{15}$, $(F, A)_{16}$

Soft pre-closed sets are $(F, A)_1$, $(F, A)_2$, $(F, A)_3$, $(F, A)_4$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{11}$, $(F, A)_{12}$, $(F, A)_{16}$

Soft pre* -open sets are $(F, A)_1$, $(F, A)_2$, $(F, A)_3$, $(F, A)_4$, $(F, A)_5$, $(F, A)_6$, $(F, A)_7$, $(F, A)_8$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{11}$, $(F, A)_{12}$, $(F, A)_{13}$, $(F, A)_{14}$, $(F, A)_{15}$, $(F, A)_{16}$

Soft pre* -closed sets are $(F, A)_1$, $(F, A)_2$, $(F, A)_3$, $(F, A)_4$, $(F, A)_5$, $(F, A)_6$, $(F, A)_7$, $(F, A)_8$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{11}$, $(F, A)_{12}$, $(F, A)_{13}$, $(F, A)_{14}$, $(F, A)_{15}$, $(F, A)_{16}$

Remark 2.3. (Φ, A) and (U, A) are always soft pre*-open and soft pre* -closed set.

Theorem 2.4. Every soft open set is a soft pre* -open set.

Proof: Let (G, A) be a soft open set. Then $(G, A) = \tilde{s}\,\mathrm{int}\,(G, A)$. Since $\mathrm{int}\,A \subseteq \mathrm{int}^*A$, $(G, A) \,\tilde{\subseteq}\, \tilde{s}\,\mathrm{int}^*(G, A)$. Clearly $(G, A) \,\tilde{\subseteq}\, \tilde{s}\,\mathrm{cl}(G, A)$.Therefore $(G, A) \,\tilde{\subseteq}\, \tilde{s}\,\mathrm{int}^*(\tilde{s}\,\mathrm{cl}\,(G, A))$ and hence (G, A) is a soft pre*-open set.

Theorem 2.5. Every soft closed set is a soft pre* -closed set.

Proof: Let (F, A) be a soft closed set. Then $(F, A) = \tilde{s}\,\mathrm{cl}(F, A)$. Since $\mathrm{cl}\,A \supseteq \mathrm{cl}^*\,A$, $(F, A) \,\tilde{\supseteq}\, \tilde{s}\,\mathrm{cl}^*(F, A)$. Clearly $(F, A) \,\tilde{\supseteq}\, \tilde{s}\,\mathrm{int}(F, A)$.Therefore $(F, A) \,\tilde{\supseteq}\, \tilde{s}\,\mathrm{cl}^*(\tilde{s}\,\mathrm{int}\,(F, A))$ and hence (F, A) is a soft pre*-closed set.

But the converse of the above two theorems need not be true. In example 2.2, $(F, A)_6$ is a soft pre*-closed set but not soft closed and $(F, A)_2$ is a soft pre*-open set but not soft open.

Theorem 2.6. Arbitrary union of soft pre*-open sets is a soft pre*-open set.

Proof: Let $\{(G, A)_\alpha \,/\, \alpha \in I\}$ be a collection of soft pre*-open sets of a soft topological space (U, A, τ). Then for each α, $(G, A)_\alpha \,\tilde{\subseteq}\, \tilde{s}\,int^*(\tilde{s}\,cl\,(G, A)_\alpha)$. Since $(G, A)_\alpha \,\tilde{\subseteq}\, \tilde{s}\,int^*(\tilde{s}\,cl\,(G, A)_\alpha)$, $\tilde{\cup}\,(G, A)_\alpha \,\tilde{\subseteq}\, \tilde{\cup}\,\tilde{s}\,int^*(\tilde{s}\,cl\,(G, A)_\alpha)$

$$\tilde{\subseteq}\, \tilde{s}\,int^*(\tilde{\cup}\,\tilde{s}\,cl\,(G, A)_\alpha)$$

$$\tilde{\subseteq}\, \tilde{s}\,int^*(\tilde{s}\,cl\,(\tilde{\cup}\,(G, A)_\alpha))$$

Therefore $\tilde{\cup}\,(G, A)_\alpha \,\tilde{\subseteq}\, \tilde{s}\,int^*(\tilde{s}\,cl\,(\tilde{\cup}\,(G, A)_\alpha))$ and hence arbitrary union of soft pre*-open sets is a soft pre*-open set.

Theorem 2.7. Arbitrary intersection of soft pre*-closed sets is a soft pre*-closed set.

Proof: Let $\{(F, A)_\alpha \,/\, \alpha \in I\}$ be a collection of soft pre*-closed sets of a soft topological space (U, A, τ).Then for each α, $(F, A)_\alpha \,\tilde{\supseteq}\, \tilde{s}\,cl^*(\tilde{s}\,int(F, A))$. Since $(F, A)_\alpha \,\tilde{\supseteq}\, \tilde{s}\,cl^*(\tilde{s}\,int(F, A))$, $\tilde{\cap}\,(F, A)_\alpha \,\tilde{\supseteq}\, \tilde{\cap}(\tilde{s}\,cl^*(\tilde{s}\,int(F, A))) \,\tilde{\supseteq}\, \tilde{s}\,cl^*(\tilde{\cap}\,\tilde{s}\,int(F, A)) \,\tilde{\supseteq}\, \tilde{s}\,cl^*(\tilde{s}\,int(\tilde{\cap}(F, A)))$.

Therefore $\tilde{\cap}\,(F, A)_\alpha \,\tilde{\supseteq}\, \tilde{s}\,cl^*(\tilde{s}\,int(\tilde{\cap}(F, A)))$ and hence arbitrary intersection of soft pre*-closed sets is a soft pre*-closed set.

Finite intersection of soft pre*-open sets need not be a soft pre*-open set as shown by the following example.

Example 2.8. Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \subseteq E$

$F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$.

$\tau = \{F_\Phi, F_A, (e_1, \{h_1\}), (e_2, \{h_1, h_2\}), ((e_1, \{h_1\}), (e_2, \{h_1, h_2\}))\}$.

$\tau^c = \{F_\Phi, F_A, (e_1, \{h_2\}), (e_2, \{h_1, h_2\}), ((e_1, \{h_1, h_2\}), (e_2, \{h_2\}))\}$.

Here $\{(e_1, \{h_1, h_2\}), (e_2, \{h_2\})\}$ and $\{((e_1, \{h_1, h_2\}), (e_2, \{h_1\})\}$ are soft pre*-open sets.

Now, $\{(e_1, \{h_1, h_2\}), (e_2, \{h_2\})\} \tilde{\cap} \{((e_1, \{h_1, h_2\}), (e_2, \{h_1\})\} = \{((e_1, \{h_1, h_2\}), (e_2, \Phi)\}$ is not soft pre*-open.

Theorem 2.9. If (G, A) is a soft pre*-open set such that $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}\,cl(H, A)$ soft pre*-open.

Proof: Given that $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}\,cl(H, A)$ and (G, A) is a soft pre*-open set. Since (G, A) is a soft pre*-open set, $(G, A) \tilde{\subseteq} \tilde{s}\,int^*(\tilde{s}\,cl(G, A))$. Now $\tilde{s}\,cl(G, A) \tilde{\subseteq} \tilde{s}\,cl(H, A)$. This implies $\tilde{s}\,int^*(\tilde{s}\,cl(G, A)) \tilde{\subseteq} \tilde{s}\,int^*(\tilde{s}\,cl(H, A))$.Also $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}\,int^*(\tilde{s}\,cl(G, A)) \tilde{\subseteq} \tilde{s}\,int^*(\tilde{s}\,cl(H, A))$.

Therefore $(H, A) \tilde{\subseteq} \tilde{s}\,int^*(\tilde{s}\,cl(H, A))$ and hence (H, A) is a soft pre*-open set.

We shall denote the family of all soft pre*-open sets [resp. soft pre*-closed sets] of a soft topological space (U, A, τ) by $P^*OSS(U)_A$ [resp. $P^*CSS(U)_A$].

Definition 2.10. Let (U, A, τ) be a soft topological space and (G, A) be a soft set over U.

(i) The soft pre*-closure of (G, A) is a soft set, $\tilde{s}\,p^*cl(G, A) = \tilde{\cap}\{(S, A): (G, A) \tilde{\subseteq} (S, A)$ and $(S, A) \in P^*CSS(U)_A\}$

(ii) The soft pre*-interior of (G, A) is a soft set, $\tilde{s}\,p^*int(G, A) = \tilde{\cup}\{(S, A): (S, A) \tilde{\subseteq} (G, A)$ and $(S, A) \in P^*OSS(U)_A\}$

Note that, $\tilde{s}\,p^*cl(G, A)$ is the smallest soft pre*-closed set containing (G, A) and $\tilde{s}\,p^*int(G, A)$ is the largest soft pre*-open set contained in(G, A).

Theorem 2.11. Let (U, A, τ) be a soft topological space and (G, A) be a soft set over U. Then

(i) $(G, A) \in P^*CSS(U)_A$ if and only if $(G, A) = \tilde{s}\,p^*cl(G, A)$

(ii) $(G, A) \in P^*OSS(U)_A$ if and only if $(G, A) = \tilde{s}\,p^*int(G, A)$

(iii) $\tilde{s}\,p^*cl(\Phi, A) = (\Phi, A)$ and $\tilde{s}\,p^*cl(U, A) = (U, A)$

(iv) $\tilde{s}\,p^*int(\Phi, A) = (\Phi, A)$ and $\tilde{s}\,p^*int(U, A) = (U, A)$

(v) $\widetilde{s}$ p*cl($\widetilde{s}$ p*cl (G, A)) = $\widetilde{s}$ p*cl(G, A)

(vi) $\widetilde{s}$ p*int($\widetilde{s}$ p*int(G, A)) = $\widetilde{s}$ p*int(G, A)

(vii) ($\widetilde{s}$ p*cl (G, A))^c = $\widetilde{s}$ p*int (G^c, A)

(viii) ($\widetilde{s}$ p*int (G, A))^c = $\widetilde{s}$ p*cl (G^c, A)

Proof: Let (G, A) be a soft set over U.

(i) Let (G, A) be a soft pre*-closed set. Then it is the smallest soft pre*-closed set containing itself, and hence (G, A) = $\widetilde{s}$ p*cl(G, A). Conversely, let (G, A) = $\widetilde{s}$ p*cl(G, A), $\widetilde{s}$ p*cl(G, A) being the intersection of soft pre*-closed sets is soft pre*-closed so $\widetilde{s}$ p*cl(G, A) $\in$ P*CSS(U)$_A$ implies that (G, A)$\in$P*CSS(U)$_A$.

(ii) Let (G, A) be a soft pre*-open set. Then it is the smallest soft pre*-open set contained in (G, A), and hence (G, A) = $\widetilde{s}$ p*int(G, A). Conversely, let (G, A) = $\widetilde{s}$ p*int(G, A), $\widetilde{s}$ p*int(G, A) being the union of soft pre*-open sets is soft pre*-open so $\widetilde{s}$ p*in(G, A) $\in$ P*OSS(U)$_A$ implies that (G, A)$\in$P*OSS(U)$_A$.

(iii) Since (Φ, A) and (U, A) are soft pre*-closed sets, so by (i) $\widetilde{s}$ p*cl(Φ, A) = (Φ, A) and $\widetilde{s}$ p*cl(U, A) = (U, A)

(iv) Since (Φ, A) and (U, A) are soft pre*-open sets, so by (i) $\widetilde{s}$ p*int(Φ, A) = (Φ, A) and $\widetilde{s}$ p*int(U, A) = (U, A)

(v) Since $\widetilde{s}$ p*cl(G, A) $\in$ P*CSS(U)$_A$. By (i), (G, A) $\in$ P*CSS(U)$_A$ iff (G, A) = $\widetilde{s}$ p*cl(G, A).

Therefore, $\widetilde{s}$ p*cl($\widetilde{s}$ p*cl (G, A)) = $\widetilde{s}$ p*cl(G, A)

(vi) Since $\widetilde{s}$ p*int(G, A) $\in$ P*OSS(U)A. By(ii), (G, A)$\in$P*OSS(U)$_A$ iff (G, A) = $\widetilde{s}$ p*int(G, A)

Therefore, $\widetilde{s}$ p*int($\widetilde{s}$ p*int (G, A)) = $\widetilde{s}$ p*int(G, A)

(vii) ($\widetilde{s}$ p*cl (G, A))^c = $\widetilde{\cap}$ {(S, A): (G, A) $\widetilde{\subseteq}$ (S, A) and (S, A)$\in$P*CSS(U)$_A$}^{c}

= $\widetilde{\cup}$ {(S, A)^c: (S, A)^c $\widetilde{\subseteq}$ (G, A)^c and (S, A)^c$\in$P*OSS(U)$_A$}

$= \tilde{\cup}\ \{(S^c, A): (S^c, A) \tilde{\subseteq} (G^c, A)$ and $(S^c, A) \in P^*OSS(U)_A\}$

$= \tilde{s}\ p^*int\ (G^c, A)$

(viii) $(\tilde{s}\ p^*int(G, A))^c = \tilde{\cup}\ \{(S, A): (S, A) \tilde{\subseteq} (G, A)$ and $(S, A) \in P^*OSS(U)_A\}^c$

$= \tilde{\cap}\ \{(S, A)^c: (G, A)^c \tilde{\subseteq} (S, A)^c$ and $(S, A)^c \in P^*CSS(U)_A\}$

$= \tilde{\cap}\ \{(S^c, A): (G^c, A) \tilde{\subseteq} (S^c, A)$ and $(S^c, A) \in P^*CSS(U)_A\}$

$= \tilde{s}\ p^*cl(G^c, A)$

Theorem 2.12. Let (U, A, τ) be a soft topological space and (G, A) and (K, A) be two soft sets over U. Then,

(i) $(G, A) \tilde{\subseteq} (K, A) \Rightarrow \tilde{s}\ p^*int(G, A) \tilde{\subseteq} \tilde{s}\ p^*int(K, A)$

(ii) $(G, A) \tilde{\subseteq} (K, A) \Rightarrow \tilde{s}\ p^*cl(G, A) \tilde{\subseteq} \tilde{s}\ p^*cl(K, A)$

(iii) $\tilde{s}\ p^*cl((G, A) \tilde{\cup} (K, A)) = \tilde{s}\ p^*cl(G, A) \tilde{\cup} \tilde{s}\ p^*cl(K, A)$

(iv) $\tilde{s}\ p^*int((G, A) \tilde{\cap} (K, A)) = \tilde{s}\ p^*int(G, A) \tilde{\cap} \tilde{s}\ p^*int(K, A)$

(v) $\tilde{s}\ p^*cl((G, A) \tilde{\cap} (K, A)) \tilde{\subseteq} \tilde{s}\ p^*cl(G, A) \tilde{\cap} \tilde{s}\ p^*cl(K, A)$

(vi) $\tilde{s}\ p^*int((G, A) \tilde{\cup} (K, A)) \tilde{\supseteq} \tilde{s}\ p^*int(G, A) \tilde{\cup} \tilde{s}\ p^*int(K, A)$

Proof:

(i) By definition, $\tilde{s}\ p^*int(G, A) = \tilde{\cup}\ \{(S, A): (S, A) \tilde{\subseteq} (G, A)$ and $(S, A) \in P^*OSS(U)_A\}$ and $\tilde{s}\ p^*int(K, A) = \tilde{\cup}\ \{(T, A): (T, A) \tilde{\subseteq} (K, A)$ and $(T, A) \in P^*OSS(U)_A\}$. Now, $\tilde{s}\ p^*int(G, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} (K, A) \Rightarrow \tilde{s}\ p^*int(G, A) \tilde{\subseteq} (K, A)$. Since $\tilde{s}\ p^*int(K, A)$ is the largest soft pre*-open set contained in (K, A). Therefore $\tilde{s}\ p^*int(G, A) \tilde{\subseteq} \tilde{s}\ p^*int(K, A)$.

(ii) By definition, $\tilde{s}\ p^*cl(G, A) = \tilde{\cap}\ \{(S, A): (G, A) \tilde{\subseteq} (S, A)$ and $(S, A) \in P^*CSS(U)_A\}$ $\tilde{s}\ p^*cl(K, A) = \tilde{\cap}\ \{(T, A): (K, A) \tilde{\subseteq} (T, A)$ and $(T, A) \in P^*CSS(U)_A\}$. Since $(G, A) \tilde{\subseteq} (K, A)$ and $(K, A) \tilde{\subseteq} \tilde{s}\ p^*cl(K, A)$, $(G, A) \tilde{\subseteq} \tilde{s}\ p^*cl(K, A)$.

(iii) We have, (G, A) $\tilde{\subseteq}$(G, A) $\tilde{\cup}$ (K, A) and (K, A) $\tilde{\subseteq}$(G, A) $\tilde{\cup}$ (K, A). By (ii), (G, A) $\tilde{\subseteq}$ (K,A), $\tilde{s}$ p*cl(G, A) $\tilde{\subseteq}$ $\tilde{s}$ p*cl(K, A), $\tilde{s}$ p*cl (G, A) $\tilde{\subseteq}$ $\tilde{s}$ p*cl ((G, A) $\tilde{\cup}$(K, A)) and $\tilde{s}$ p*cl (K, A) $\tilde{\subseteq}$ $\tilde{s}$ p*cl ((G, A) $\tilde{\cup}$ (K, A)), $\tilde{s}$ p*cl (G, A) $\tilde{\cup}$ $\tilde{s}$ p*cl (K, A)$\tilde{\subseteq}$ $\tilde{s}$ p*cl ((G, A) $\tilde{\cup}$(K, A)) → (1). Now, $\tilde{s}$ p*cl (G, A), $\tilde{s}$ p*cl (K, A) ∊ P*CSS(U)$_A$, then (G, A) $\tilde{\subseteq}$ $\tilde{s}$ p*cl(G, A) and (K, A) $\tilde{\subseteq}$ $\tilde{s}$ p*cl(K, A). This implies (G, A) $\tilde{\cup}$(K, A) $\tilde{\subseteq}$ $\tilde{s}$ p*cl (G, A) $\tilde{\cup}$ $\tilde{s}$ p*cl (K, A), $\tilde{s}$ p*cl (G, A) $\tilde{\cup}$ $\tilde{s}$ p*cl (K, A) is a soft pre*-closed set containing (G, A) $\tilde{\cup}$(K, A). But, $\tilde{s}$ p*cl ((G, A) $\tilde{\cup}$ (K, A)) is the smallest soft pre*-closed set containing G, A) $\tilde{\cup}$(K, A). Hence $\tilde{s}$ p*cl ((G, A) $\tilde{\cup}$ (K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*cl (G, A) $\tilde{\cup}$ $\tilde{s}$ p*cl (K, A) →(2). From (1) and (2), $\tilde{s}$ p*cl((G, A) $\tilde{\cup}$(K, A)) = $\tilde{s}$ p*cl(G, A) $\tilde{\cup}$ $\tilde{s}$ p*cl(K, A)

(iv) We have, (G, A)$\tilde{\cap}$(K, A) $\tilde{\subseteq}$(G, A) and (G, A)$\tilde{\cap}$(K, A) $\tilde{\subseteq}$(K, A). By (i), (G, A) $\tilde{\subseteq}$ (K, A), $\tilde{s}$ p*int(G, A) $\tilde{\subseteq}$ $\tilde{s}$ p*int(K, A), $\tilde{s}$ p*int((G, A)$\tilde{\cap}$(K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*int(G, A) and $\tilde{s}$ p*int((G, A)$\tilde{\cap}$(K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*int (K, A). This implies $\tilde{s}$ p*int((G, A) $\tilde{\cap}$ (K, A))$\tilde{\subseteq}$ $\tilde{s}$ p*int(G, A) $\tilde{\cap}$ $\tilde{s}$ p*int(K, A)) → (3). Now, $\tilde{s}$ p*int(G, A), $\tilde{s}$ p*int(K, A) ∊ P*OSS(U)$_A$. This implies $\tilde{s}$ p*int (G, A) $\tilde{\cap}$ $\tilde{s}$ p*int (K, A)) ∊ P*OSS(U)$_A$. Then $\tilde{s}$ p*int(G, A) $\tilde{\subseteq}$ (G, A) and $\tilde{s}$ p*int(K, A) $\tilde{\subseteq}$ (K, A), $\tilde{s}$ p*int(G, A) $\tilde{\cap}$ $\tilde{s}$ p*int(K, A) $\tilde{\subseteq}$ (G, A) $\tilde{\cap}$ (K, A). That is $\tilde{s}$ p*int(G, A) $\tilde{\cap}$ $\tilde{s}$ p*int (K, A) is a soft pre*-open set contained (G, A) $\tilde{\cap}$ (K, A). But, $\tilde{s}$ p*int((G, A) $\tilde{\cap}$ (K, A)) is the largest soft pre*-open set contained in G, A) $\tilde{\cap}$ (K, A). Hence $\tilde{s}$ p*int(G, A) $\tilde{\cap}$ $\tilde{s}$ p*int(K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*int((G, A) $\tilde{\cap}$ (K, A)) → (4). From (3) and (4), $\tilde{s}$ p*int((G, A) $\tilde{\cap}$ (K, A)) = $\tilde{s}$ p*int(G, A) $\tilde{\cap}$ $\tilde{s}$ p*int(K, A).

(v) We have, (G, A)$\tilde{\cap}$(K, A) $\tilde{\subseteq}$(G, A) and (G, A)$\tilde{\cap}$(K, A) $\tilde{\subseteq}$(K, A), $\tilde{s}$ p*cl((G, A)$\tilde{\cap}$(K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*cl (G, A) and $\tilde{s}$ p*cl((G, A)$\tilde{\cap}$(K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*cl (K, A), $\tilde{s}$ p*cl((G, A) $\tilde{\cap}$ (K, A)) $\tilde{\subseteq}$ $\tilde{s}$ p*cl(G, A) $\tilde{\cap}$ $\tilde{s}$ p*cl(K, A).

(vi) We have, (G, A) $\tilde{\subseteq}$(G, A) $\tilde{\cup}$ (K, A) and (K, A) $\tilde{\subseteq}$(G, A) $\tilde{\cup}$ (K, A). Then $\tilde{s}$ p*int(G, A) $\tilde{\subseteq}$ $\tilde{s}$ p*int((G, A) $\tilde{\cup}$ (K, A)) and $\tilde{s}$ p*int (K, A) $\tilde{\subseteq}$ $\tilde{s}$ p*int((G, A) $\tilde{\cup}$ (K, A)), $\tilde{s}$ p*int(G, A) $\tilde{\cup}$ $\tilde{s}$ p*int (K, A) $\tilde{\subseteq}$ $\tilde{s}$ p*int((G, A) $\tilde{\cup}$ (K, A)).

3. CONCLUSION

In this paper, we introduced the concept of soft pre*-open sets and soft pre*-closed sets in soft topological spaces and some of their properties are studied. We also introduced soft pre*-closure and soft pre*-interior operators and have established several interesting properties. We hope that this paper is just a beginning of a new structure. It will inspire many to contribute to the cultivation of Soft topology in the field of mathematics. Also, we are further motivated to study the relation between soft separation axioms and soft pre* open sets.

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