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Separation Axioms Weaker Than T_1

Khadiga Ali Arwini[1,*], **Mabrouka Mohammed Almrgni**[2]

[1]Department of Mathematics, Tripoli University, Tripoli, Libya

[2]Department of Mathematics, Higher Institute of Science and Technology, Tripoli, Libya

*E-mail address: Kalrawini@yahoo.com

ABSTRACT

The purpose of this paper is to introduce a new type of separation axioms via dense sets, called DT_i-spaces ($i = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, 1$), where a DT_i-space is a topological space which contains a dense T_i-subspace ($i = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, 1$). These new axioms are weaker than the axiom of T_1. We provide the basic properties of DT_i- spaces ($i = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, 1$), and we show that the axioms of $DT_{\frac{1}{4}}$, $DT_{\frac{1}{3}}$, $DT_{\frac{1}{2}}$, $DT_{\frac{3}{4}}$, DT_1 are open hereditary. Moreover, we study the connections between these axioms and the axioms of T_i where ($i = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, 1$).

***Keywords*:** Subspaces, product spaces, lower separation axioms

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1. INTRODUCTION

The concepts of different generalization of open (or closed) sets have been defined; as ʌ-sets, generalized ʌ-sets, λ-sets, b-sets, β-sets, g-closed sets, regular open sets, preopen sets and semi open sets, etc. Some of these generalizations are stronger forms of open (or closed) sets,

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and some are weaker, interrelations between these types of generalized open (or closed) sets were studied in [1-5].

In [6-13] and Arenas, Dontchev, Levine, Dunham and Maki introduced several separation axioms between T_0 and T_1-spaces, in particular, they defined $T_{\frac{1}{4}}$, $T_{\frac{1}{3}}$, $T_{\frac{1}{2}}$ and $T_{\frac{3}{4}}$-spaces by using the concepts of ʌ-sets, generalized ʌ-sets, λ-sets, generalized closed sets and regular open sets. The characterization of these spaces found to be useful in the study of computer science and digital topology.

Several separation axioms have been introduced using various forms of generalization open and closed sets, as in 2007 [14] Caldas, Jafari and Navalagi used the notions of λ-open sets and λ-closed sets to define the operators λ-closure and λ-interior and studied their properties, then they used these operators to defined new separation axioms; namely λ-T_i, where (i = 0, $\frac{1}{2}$, 1, 2), and they proved that λ-T_1 is equivalent to T_0. The axioms μ-$T_{\frac{1}{4}}$, μ-$T_{\frac{3}{8}}$ and μ-$T_{\frac{1}{2}}$ were defined by Sarsak [15] when he used the notions of μ-open sets.

In 2011 [16] the author studied some new separation axioms for topological spaces defined in terms of a new topology, this new idea gave the notion of start-T_i where (i = 1, 2), then he showed that star-T_1 axiom lies between T_0 and T_1. A year later, Hussain and Abd Alatif [17] used bg-closed sets to introduced a new class of spaces namely b-$T_{\frac{1}{2}}$ space, which is strictly between b-T_0 and b-T_1, and they showed that $T_{\frac{1}{2}}$ is b-$T_{\frac{1}{2}}$. More separation axioms between T_0 and T_1 spaces are described in [18] as properties of the space at particular point.

In this paper we introduced a new type of separation axioms, namely dense separation axioms and they are denoted by DT_i-spaces (i = $0,\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{3}{4}$, 1), where a DT_i-space is a topological space which contains a dense T_i-subspace (i = $0,\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{3}{4}$, 1). We provided the properties of these spaces, and proved that every topological space is DT_0-space, then we show that DT_i-space is weaker than T_i-space for any (i = $\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{3}{4}$, 1), moreover, we investigate some of their basic properties, as their subspaces and their continuous images.

Finally, we provide the inter-relations between DT_i-spaces and the classical T_i-spaces; where (i = $\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{3}{4}$, 1).

2. DT₀ – SPACES

In this section we introduce the axiom of DT_0, then we prove that every topological space is DT_0.

Definition 2.1. [19] A topological space (X,T) is T_0-space if whenever x and y are distinct points in X there is an open set containing one and not the other.

Theorem 2.1. [19] A topological space X is T_0-space iff $\overline{\{x\}} \neq \overline{\{y\}}$ for every $x,y \in X$ and $x \neq y$.

Definition 2.2. A topological space X is said to be DT_0-Space if X has a T_0-subspace which is dense in X.

Theorem 2.2. Every topological space is DT_0.

Proof: Let X be any topological space and define a relation $\sim$ on X by: $x \sim y$ iff $\overline{\{x\}} = \overline{\{y\}}$. Then $\sim$ is an equivalence relation on X. Let $X / \sim$ be the set of all distinct equivalence classes for a relation $\sim$. By the axiom of choice we can choose a set $A \subseteq X$ such that $A \cap [x]$ has exactly one element for x in X. If x and y are distinct points in A, then $x \nsim y$ hence $\overline{\{x\}} \neq \overline{\{y\}}$.
So A is a T_0 – subspace by theorem (2.1.). Now suppose V is a non empty open set in X, then for each $x \in V$ there is $a_x \in A$ such that $x \in [a_x]$, hence $\overline{\{x\}} = \overline{\{a_x\}}$ and since $x \in V$, then $a_x \in V$, i.e $V \cap A \neq \emptyset$, hence A is dense in X. We get X is DT_0-space.

3. $DT_{1/4}$ – SPACES

Arenas, Dontchev and Ganster [7] introduced the notions of λ-closed sets and λ-open sets in topological spaces, and they showed that every ᴧ-set is λ-closed set. They used the concepts of λ-closed sets to introduced the class of $T_{\frac{1}{4}}$ spaces in their study of generalized continuity and λ-closed sets. More details on λ-sets can be found in [8, 1].

Definition 3.1. [7] A topological space (X,T) is called a $T_{\frac{1}{4}}$-space if for every finite subset $F \subseteq X$ and every point $y \notin F$ there exist a subset $A \subseteq X$ such that $F \subseteq A$, $y \notin A$ and A is open or closed.

Theorem 3.1. [7]

1) -Every $T_{\frac{1}{4}}$-space is T_0.

2) -Every subspace of a $T_{\frac{1}{4}}$-space is $T_{\frac{1}{4}}$.

Definition 3.2. A topological space X is said to be $DT_{\frac{1}{4}}$-space if X has a $T_{\frac{1}{4}}$-subspace which is dense in X.

Corollary 3.1. Every $T_{\frac{1}{4}}$-space is $DT_{\frac{1}{4}}$.

Examples 3.1.

1) Let X = {a,b,c,d} and T = {Ø,X,{a,b},{c},{a,b,c},{b,c},{b} }, then $\mathcal{F}$ = {Ø,X,{c,d}, {a,b,d}, {d}, {a,d}, {a,c,d}}. If A={b,c,d}then T_A={Ø, A, {b}, {c}, {c,d}}, and $\mathcal{F}_A = \{Ø, A, \{c,d\}, \{b,d\}, \{d\}\}$ i.e(X,T)is $DT_{\frac{1}{4}}$ since A is $T_{\frac{1}{4}}$-dense subspace, but (X,T) is not $T_{\frac{1}{4}}$-space, since there is not a set F such that $\{b,d\} \subseteq F$, $a \notin F$ and F is open or closed .

2) Let X = ℕ and T = {Ø,ℕ,{2,3,4,…..}, {3,4,5,……},….}, then $\mathcal{F}$ = {Ø, ℕ ,{1}, {1,2} {1,2,3}...}. (X,T) is not $T_{\frac{1}{4}}$–space, since $3 \notin \{1,2,4\}$ but there, is not a set F such that $\{1,2,4\} \subseteq F$, $3 \notin F$ and F is open or closed. Now if A is dense subset in X, then A is infinite set, and T_A is not $T_{\frac{1}{4}}$-space. We have (X,T) is not $DT_{\frac{1}{4}}$. Note that (X,T) is T_0 but not $DT_{\frac{1}{4}}$.

3) X = IR, T = { Ø }∪{U⊆ IR : {1,–1} ⊆ U}, then (X,T) is $DT_{\frac{1}{4}}$–space since {1} is dense subspace. But (X,T) is not T_0-space.

4) Let X be the set of non-negative integers and T = {A⊆X : 0∈A , A‘ inite} ∪ { Ø}, then $\mathcal{F}$ = {F⊆X : F finite, 0∉F } ∪ {X}. (X,T) is $DT_{\frac{1}{4}}$ since A = {0} is $T_{\frac{1}{4}}$-dense subspace but (X,T) is not $T_{\frac{1}{3}}$-space.

Theorem 3.2. Every open subspace of $DT_{\frac{1}{4}}$-space is $DT_{\frac{1}{4}}$.

Proof: Let A be an open subspace of $DT_{\frac{1}{4}}$-space X, then X has $T_{\frac{1}{4}}$-subspace B which is dense in X, hence B∩A≠ Ø so B∩A is $T_{\frac{1}{4}}$-space by theorem (3.1. (2)). Now suppose W is open set in A and since A is open in X, then W is an open in X, hence W∩B ≠ Ø, i.e W∩(B∩A) ≠ Ø, so A∩B is dense subspace of A. Hence A is $DT_{\frac{1}{4}}$-space .

Example 3.2. Let X= ℕ and T = { ℕ, Ø, ℕ/{2}, ℕ/{2,3}, ℕ/{2,3,4},…..}, $\mathcal{F}$ = { ℕ, Ø, {2}, {2,3}, {2,3,4},….}, then (X,T) is $DT_{\frac{1}{4}}$ Since {1} is $T_{\frac{1}{4}}$ − dense subspace, The set A = ℕ/{1} (A,T_A) is not $DT_{\frac{1}{4}}$, since X is not $T_{\frac{1}{4}}$ and any dense set in X is infinite .

4. $DT_{1/3}$ – SPACES

Arenas, Dontchev and Puertas [8] considered the spaces in which compact sets are λ-closed, they are placed between $T_{\frac{1}{2}}$ and $T_{\frac{1}{4}}$, they call them $T_{\frac{1}{3}}$ spaces.

Definition 4.1. [8] A topological space (X,T) is $T_{\frac{1}{3}}$-space if for every compact subset F of X and every y∉F there exists a set A_y containing F and disjoint from {y} such that A_y is either open or closed.

Theorem 4.1. [8]

1) Every $T_{\frac{1}{3}}$-space is $T_{\frac{1}{4}}$.

2) Every subspace of $T_{\frac{1}{3}}$-space is $T_{\frac{1}{3}}$.

Definition 4.2. [8] A topological space (X,T) is called anti-compact if every compact subset of X is finite.

Corollary 4.1. [8] For an anti-compact topological space (X,T) the following conditions are equivalent:

1) X is $T_{\frac{1}{3}}$.

2) X is $T_{\frac{1}{4}}$.

Definition 4.3. A topological space X is said to be $DT_{\frac{1}{3}}$-space if X has a $T_{\frac{1}{3}}$-subspace which is dense in X.

Corollary 4.2.

1) Every $T_{\frac{1}{3}}$-space is $DT_{\frac{1}{3}}$

2) Every $DT_{\frac{1}{3}}$-space is $DT_{\frac{1}{4}}$

Examples 4.1.

1) Let X be the set of non –negative integers and T = $\{A \subseteq X : 0 \in A,\ A^c \text{ finite}\} \cup \{\emptyset\}$, then $\mathcal{F} = \{F \subseteq X : F \text{ finite},\ 0 \notin F\} \cup \{X\}$. X is $DT_{\frac{1}{3}}$-space since{0}is $T_{\frac{1}{3}}$-dense subspace. But (X,T) not $T_{\frac{1}{3}}$-space since ℕ is compact, $0 \notin \mathbb{N}$, but there is not exists a set A such that $\mathbb{N} \subseteq A$ and $0 \notin A$, A is open or closed.

2) Let X=ℕ, T={ ℕ,Ø,{2,3,……},{3,4….},……..}. Then X is not $DT_{\frac{1}{3}}$-space since X is not $T_{\frac{1}{3}}$, and any dense subset A is infinite, so T_A is not $T_{\frac{1}{3}}$-space.

3) Let X = ℕ, T = { ℕ,Ø,{1},{1,2},{1,2,3},…..}, then (X,T) is $DT_{\frac{1}{3}}$-space since{1}is $T_{\frac{1}{3}}$ dense subspace. (X,T) is not $T_{\frac{1}{4}}$, since $2 \notin \{1,3,4\}$ but there is not a set A such that $\{1,3,4\} \subseteq A$, $2 \notin A$ and A is open or closed.

4) Let X = ℕ and T = { ℕ, Ø, ℕ /{2}, ℕ /{2,3}, ℕ /{2,3,4},…..}, then $\mathcal{F}$ = { ℕ,Ø,{2},{2,3},{2,3,4},.. }. The space (X,T) is $DT_{\frac{1}{3}}$– space since {1} is a $T_{\frac{1}{3}}$-dense subspace. (X,T) is not $T_{\frac{1}{4}}$ since $3 \notin \{2,4\}$ but there is not a set A such that $\{2,4\} \subseteq A$, $3 \notin A$ and A is open or closed.

Theorem 4.2.

Every open subspace of $DT_{\frac{1}{3}}$-space is $DT_{\frac{1}{3}}$.

Proof: Let A be an open subspace of $DT_{\frac{1}{3}}$-space. Then X has a $T_{\frac{1}{3}}$-subspace B which is dense, hence $B \cap A \neq \emptyset$, and $B \cap A$ is $T_{\frac{1}{3}}$-space by theorem (4.1. (2)). Now suppose W is an open set in A, and since A is an open in X, then W is open in X, and $W \cap B \neq \emptyset$, i.e $W \cap (B \cap A) \neq \emptyset$, so $A \cap B$ is dense subspace of A. Hence A is $DT_{\frac{1}{3}}$-space.

Example 4.2. The topological space (ℕ,T) in (3.2.) is $DT_{\frac{1}{3}}$ but the subspace A is not $DT_{\frac{1}{3}}$.

Corollary 4.3.

For an anti-compact topological space (X,T) the following conditions are equivalent:

1) X is $DT_{\frac{1}{3}}$.

2) X is $DT_{\frac{1}{4}}$

5. $DT_{1/2}$ – SPACES

Levine [12] introduced the concepts of generalized closed sets of a topological space, and a class of topological spaces called $T_{\frac{1}{2}}$-space, when he proved that $T_{\frac{1}{2}}$-space is properly placed between T_0-space and T_1-space. After that, the authors in [7] characterized $T_{\frac{1}{2}}$-spaces as those spaces where every subset is λ-closed. Dunham [11] showed that a space is $T_{\frac{1}{2}}$ if and only if each singletons is open or closed. See [13], [2].

Definition 5.1. [12] A topological space (X ,T) is $T_{\frac{1}{2}}$-space if every g-closed subset of X is closed.

Theorem 5.1. [12]

1) Every $T_{\frac{1}{2}}$-space is $T_{\frac{1}{3}}$.

2) Every subspace of $T_{\frac{1}{2}}$-space is $T_{\frac{1}{2}}$.

Definition 5.2. A topological space X is said to be $DT_{\frac{1}{2}}$-space if X has a $T_{\frac{1}{2}}$-subspace which is dense in X.

Corollary 5.1.

1) Every $T_{\frac{1}{2}}$-space is $DT_{\frac{1}{2}}$.

2) Every $DT_{\frac{1}{2}}$-space is $DT_{\frac{1}{3}}$.

Examples 5.1.

1) Let X ={a,b,c,d} and T={Ø,X,{b},{a,b},{b,c},{a,b,c}}, then $\mathcal{F}$ = {Ø,X,{a,c,d}, {c,d}, {a,d}, {d}}. (X,T) is $DT_{\frac{1}{2}}$ -space since A = {b} is $T_{\frac{1}{2}}$–dense subspace, but (X,T) is not $T_{\frac{1}{2}}$ since {a} is not open and not closed.

2) Let X be the set of non–negative integers and T = {A⊆X : 0∈A, A' finite}∪{Ø}, then $\mathcal{F}$ = {F⊆X : F finite, 0∉F } ∪ {X}. (X,T) is $DT_{\frac{1}{2}}$–space since A = {0} is $T_{\frac{1}{2}}$-dense subspace, but (X,T) is not $T_{\frac{1}{3}}$-space.

Theorem 5.2. Every open subspace of $DT_{\frac{1}{2}}$-space is $DT_{\frac{1}{2}}$

Proof: Let A be an open subspace of $DT_{\frac{1}{2}}$-space then X. Then has a $T_{\frac{1}{2}}$ subspace B which is dense, hence B∩A ≠ Ø, and B is $T_{\frac{1}{2}}$-space, so B∩A is $T_{\frac{1}{2}}$-space by theorem (5.1. (2)). We need to prove A∩B is dense. Now suppose W is an open set in A, and since A is an open in X, then W an open in X, hence W∩B ≠ Ø, i.e W∩(B∩A) ≠ Ø, hence A∩B is dense subspace of A. So A is $DT_{\frac{1}{2}}$-space.

Example 5.2. The topological space $(\mathbb{N},T)$ in (3.2.) is $DT_{\frac{1}{2}}$ but the subspace A is not $DT_{\frac{1}{2}}$.

6. $DT_{3/4}$ – SPACES

Via regular open sets, Levine [12] produced some new separation axiom which lies between T_0 and T_1, called $T_{\frac{3}{4}}$-space. Regular open (or closed) sets in a topological space used as a generalizations for algebraic openings and closings in a complete lattice, see [3] and [20].

Definition 6.1. [12] Atopological space (X,T) is called $T_{\frac{3}{4}}$ -space if every singleton is closed or regular open.

Theorem 6.1. [12] Every $T_{\frac{3}{4}}$-space is $T_{\frac{1}{2}}$.

Definition 6.2. A topological space X is said to be $DT_{\frac{3}{4}}$-space if X has a $T_{\frac{3}{4}}$-subspace which is dense in X.

Corollary 6.1.

1) Every $T_{\frac{3}{4}}$-space is $DT_{\frac{3}{4}}$

2) Every $DT_{\frac{3}{4}}$-space is $DT_{\frac{1}{2}}$

Examples 6.1.

1) Let X = {a,b,c} and T = {Ø,X,{a},{b,c}}, then X is $DT_{\frac{3}{4}}$-space since A = {a,c} is $T_{\frac{3}{4}}$-dense subspace, but (X,T) is not $T_{\frac{3}{4}}$-space since {b} is not closed nor regular open. Note that X is not T_1 space.

2) The topological space $(\mathbb{N},T)$ in (3.2.) is $DT_{\frac{3}{4}}$ but the subspace A is not $DT_{\frac{3}{4}}$.

7. DT_1 – SPACES

In this section we introduce the axiom of DT_1, and we study its properties; as its relations with the classical separation axioms, its hereditary property, its continuous images, and its product spaces

Definition 7.1. [19] A topological space (X,T) is T_1-space if for any x ,yϵX, $x \neq y$ there exist two open sets U and V such that $x\epsilon U$, $y\notin U$ and $y\epsilon V$, $x\notin V$.

Theorem 7.1. [19]

1) Every T_1-space is $T_{\frac{3}{4}}$.

2) Every subspace of T_1-space is T_1.

3) The closed image of T_1-space is T_1.

4) A non empty product space is T_1-space iff each factor is T_1 .

Definition 7.2. [19] A topological space (X,T) is regular-space if whenever A is closed in X and $x \notin A$, then there are two disjoint open sets U and V such that $x \in U$, $A \subseteq V$. A regular T_1-space is called T_3 .

Theorem 7.2. [19]

1) Every regular T_0-space is T_3 .

2) Every subspace of regular-space is regular.

Definition 7.3. A topological space X is said to be DT_1-space if X has a T_1-subspace which is dense in X.

Corollay 7.1.

1) Every T_1-space is DT_1.

2) Every DT_1-space is $DT_{\frac{3}{4}}$.

Examples 7.1.

1) Let X be the set of positive integers and τ consists all sets V such that V = {n, n+1,…..} for some n in X. Let A be a T_1-subspace of X, then A has exactly one element so A is not dense in X, hence X can not be DT_1.

2) Let X = IR, and T = { Ø,IR,{0}}∪{A⊆ IR: 0∈A , A' finite}, then $\mathcal{F}$ = { Ø,IR,{0}'}∪ {{F⊆IR : 0∉F, F finite}∪{X}. The space (X,T) is DT_1-space since {0} is a T_1-dense subspace, but (X,T) is not T_0 since $0 \neq 1$ and there is not open set contain 1 and not contain 0.

Theorem 7.3. Every regular-space is DT_1.

Proof: Let X be a regular-space, then X is DT_0 by theorem (2.2.), i.e X has a dense T_0-subspace A, hence A is a regular T_0 -subspace by (7.2. (2)), then A is rgular T_1-subspace from (7.2. (1)), hence X is DT_1.

Example 7.2. The topological space (IR,T) in (7.1. (2)) is DT_1 but not regular.

Theorem 7.4. Every open subspace of DT_1-space is DT_1.

Proof: Let A be an open subspace of DT_1-space, then X has T_1 subspace B which is dense, hence $B \cap A \neq \emptyset$ B is T_1-space, so $B \cap A$ is T_1-space by theorem (7.1. (2)). Now suppose W is an open set in A, since A is an open in X, then W is an open in X, hence $W \cap B \neq \emptyset$, i.e $W \cap (B \cap A) \neq \emptyset$. Hence $A \cap B$ is dense subspace of A.
So A is DT_1-space.

Example 7.3. The topological space (ℕ,T) in (3.2.) is DT_1 but the subspace A is not DT_1.

Theorem 7.5. Every finite topological space X has a discrete subspace which is dense subset in X.

Proof: Let A be the collection of all nonempty open sets in X, then A is partially ordered set by the inclusion (⊆). Let B be the set of all minimal element of A, since X is a finite set, hence B≠Ø. Choose $x_v \in V$ for each v∈B, and define D = {x_v : v∈B}, then D is a nonempty set, and for any $x_v \in D$ there is an open set V∈B such that V∩D = { x_v }, Then D is a discrete subspace. Now let a non empty open set in X then there is an open set V∈B such that V⊆W, so $x_v \in V \subseteq W$, then W∩D ≠ Ø, hence D is dense set in X.

Corollary 7.2. Every finite topological space is DT_1-space.

Proof: By the theorem above, any finite topological space has a discrete subspace which is dense, and since every discrete space is T_1.

Theorem 7.6. If a space X has a DT_1-subspace which is dense subspace in X, then it is DT_1.

Proof: Let A be a DT_1 subspace which is dense subset in X, then A has a subspace B which is dense in A, hence B is a T_1-subspace which is dense in X, so X is DT_1.

Note that: The above theorem is correct for any DT_i -spaces, where (i = $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$).

Theorem 7.7. The closed continuous image of a DT_1-space is DT_1.

Proof: Suppose X is DT_1-space and f is a closed continuous map of X onto a space Y. Then X has a dense T_1-subspace A, since f is closed continuous map, f(A) is dense T_1-subspace of Y, hence Y is DT_1 from (7.1. (3)).

Theorem 7.8. If a space X_α is DT_1-space for each α∈I, then the product space πX_α is DT_1.

Proof: Since X_α is DT_1-space, then for each α∈I, X_α has a dense T_1-subspace A_α, and form (7.1. (4)) we get πA_α is a dense T_1-subspace of the product space πX_α .

Example 7.5. Let X= ℕ, and τ_1 be topology on ℕ in example (7.1. (1)), and let τ_2 be the cofinite topology on ℕ. Then the diagonal Δ = {(n, n): n ∈ ℕ} is a dense T_1-subspace of the product space, hence $\tau_1 \times \tau_2$ is DT_1-space, but τ_1 can not be DT_1.

8. CONCLUSIONS

We used the concepts of dense sets to define a new class of separation axioms, called DT_i (i = 0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1), where a DT_i-space is a topological space which contains a dense T_i-subspace, (i = 0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1).

The implications of these axioms among themselves and with the known axioms T_i are investigated.

Here we give a brief summary of the main results of this paper:

- ✓ Every topological space is DT_0-space.
- ✓ Every T_i-space is DT_i-space (i = 0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1), but not conversely.
- ✓ All the spaces DT_i (i = $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1) are weaker than the space T_1, and they are weakly ordered as: $DT_{\frac{1}{4}}$, $DT_{\frac{1}{3}}$, $DT_{\frac{1}{2}}$, $DT_{\frac{3}{4}}$, DT_1.
- ✓ No general relations between T_0 space and DT_i-spaces, (i = $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1).
- ✓ DT_i–spaces (i = $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1) do not satisfy the hereditary property, but any open subspace of DT_i-space is DT_i, where (i = $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 1).
- ✓ Every finite topological space X has a discrete subspace which is dense subset in X.
- ✓ Every finite topological space is DT_1-space
- ✓ A space that contains a dense DT_i -subspace is DT_i, (i = $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1).
- ✓ Every regular-space is DT_1.
- ✓ The image of DT_1-space under a closed continuous mapping is DT_1.
- ✓ The product space of DT_1-spaces is DT_1, but not conversely.

This diagram shows the relations between T_i-spaces and DT_i-spaces, where (i = 0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, 1):

$$
\begin{array}{ccccccccccccc}
 & & T_1 & \Rightarrow & T_{\frac{3}{4}} & \Rightarrow & T_{\frac{1}{2}} & \Rightarrow & T_{\frac{1}{3}} & \Rightarrow & T_{\frac{1}{4}} & \Rightarrow & T_0 \\
 & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \\
\text{Regular} & \Rightarrow & DT_1 & \Rightarrow & DT_{\frac{3}{4}} & \Rightarrow & DT_{\frac{1}{2}} & \Rightarrow & DT_{\frac{1}{3}} & \Rightarrow & DT_{\frac{1}{4}} & &
\end{array}
$$

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