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Robust Optimization Model for Spatial Land-Use Allocation Problem in Jatinangor Subdistrict, Indonesia

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ABSTRACT

Land-use planning become an important thing to do because some types of land-use can have an impact to environment and life quality. Land-use planning is generally an activity that involves the allocation of activities in a particular land. Spatial Optimization can be applied in land-use planning activity. This research aims to make Robust Optimization model for spatial land-use allocation problem in Jatinangor. Optimization model for land-use allocation problem aims to determine the percentage of land-use changes that can maximize comprehensive index and compactness index. In land-use planning, there are several uncertainty factors. Therefore, it's needed an approach that can handle uncertainty factor, the approach used in this research is Robust Optimization. The result of Robust Optimization Model for land-use allocation problem which is solved by the box uncertainty set approach is a computationally tractable optimization model.

***Keywords*:** Land-use Allocation, Spatial Optimization, Robust Optimization, Jatinangor Subdistrict

1. INTRODUCTION

Land is an essential component for human living. A different land-use help to fulfill human needs, involving both the manner how land is modified, managed, and maintained and the intended use (Turner *et al*., 1995). Land-use arrangement is a basic for biosfer function

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because several land-uses such as residential, industry, agriculture, and green-land have a huge impact for environment and life quality.

Land use planning commonly involves the allocation of land-use activities to a particular plot of land. Spatial Optimization connecting Geographic Information System (GIS) and mathematical modelling has been increasingly applied to support the evaluation of land use planning activities (Ligmann-Zielinska, Church, & Jankowski, 2008). Church, *et al*. (2009) introduced a basic model for land-use planning problem with knapsack model and threshold model. Then, the model was developed by Yao, *et al*. (2017) in which this model considers the types of land-use. In addition, Li, *et al*. (2009) developed a spatial optimization model for land-use planning that aims to maximize the comprehensive index and density index. State of the art about land-use allocation problem can be seen in Table 1.

Table 1. State of The Art about Land-use Allocation Problem.

No	Title	Year	Author
1	Land-use Plan Design Via Interactive Multiple-Objective Programming	1976	G M Barber
2	A Multiobjective Integer Programming Model for The Land Acquisition Problem	1983	Wright, et al.
3	Integration of Linear Programming and GIS for Land-Use Modelling	1993	Emilio Chuvieco
4	Using Linear Integer Programming for Multi-Site Land-Use Allocation	2003	Aerts, et al.
5	Spatial Optimization as A Generative Technique for Sustainable Multiobjective Land-Use Allocation	2008	Zielinska, et al.
6	A Spatial Decision Support System for Land-use Structure Optimization	2009	Li, et al.
8	A Multiobjective GIS-Based Land Use Planning Algorithm	2014	Stewart, et al.
9	Multi-Objective Land-Use Allocation Considering Landslide Risk Under Climate Change	2017	Yoon, et al.
10	Spatial Optimization for Land-Use Allocation: Accounting for Sustainability Concerns	2017	Yao, et al.

In land-use allocation problem, there are uncertainty factors affecting the allocation of land-use system. Because of that, it is needed an optimization technique considering the uncertainty factors, so optimal solution which is resistant to uncertainty factors can be obtained. One of optimization models which can handle uncertainty factors is Robust Optimization. Robust Optimization was first introduced by Sosyter (1973). In the late 1990s, Robust Optimization reached some significant progress, one of which was made by Ben-Tal and Nemirovski (1999). The development of Robust Optimization happened continuously.

Bertsimas, *et al.* (2009) introduced a methodology for constructing a polyhedral uncertainty set for Robust Optimization in linear programming model. Bental, *et al*. (2015) presents a robust counterpart formulation for non-linear inequalities with uncertain parameters. Gorissen, *et al*. (2015) compiled practical guidelines for Robust Optimization. In Robust Optimization, the problem is solved by considering uncertainty data located in an uncertainty set. There are several types of uncertainty sets i.e. box uncertainty, ellipsoidal uncertainty, and polyhedral uncertainty.

Jatinangor is a subdistrict located in Sumedang Regency, West Java, Indonesia. Jatinangor is an educational area, this is indicated by the existence of four campuses in Jatinangor. According to the regulation of Sumedang Regency number 2 of 2012 concerning about the spatial plan for Sumedang Regency in 2011 – 2013, Jatinangor is directed to become a residential area and high educational area that support the consolidation of Bandung Metropolitan Area. Because of that concern, there are several land conversion in Jatinangor. Based on the explanation, this becomes an interesting topic to discuss whether there will be change in existing land-use types or not based on certain consideration.

Based on the explanation above, this paper discusses about Robust Optimization model for spatial land-use allocation problem in Jatinangor. In this research, spatial optimization model for land-use allocation by Yao, *et al*. (2017) and Li, *et al.* (2009) will be reformulated and uncertainty data will be considered. The uncertainty set used in this research is box uncertainty.

2. MATERIALS AND METHODS

In this section, it will be discussed about the materials and methods used in this paper.

2. 1. Optimization Model for Land-use Allocation Problem

In this paper, optimization model for land-use allocation is based on the spatial optimization model by Yao, *et al*. (2017) and Li, *et al.* (2009).

Optimization model for land-use allocation introduced by Yao, *et al*. is as follows:

$$\begin{aligned} &\min \sum_i \sum_k c_{ik} x_{ik} \\ &\max \sum_i \sum_k a_{ik} x_{ik} \end{aligned} \tag{1}$$

$$\text{s.t. } \sum_i \overline{c_{ik}} x_{ik} \le \theta_k, \ \forall k \tag{2}$$

$$\sum_i a_{ik} x_{ik} \ge L_k, \ \forall k \tag{3}$$

$$\sum_k x_{ik} = 1, \forall i \tag{4}$$

$$\sum_i s_k x_{ik} \geq LS_k, \ \forall k \tag{5}$$

$$\sum_i s_k x_{ik} \leq US_k, \ \forall k \tag{6}$$

$$x_{ik} = \{0,1\}, \forall i, k. \tag{7}$$

where N is total number of land parcels, K is total number of land-use types, a_{ik} is benefit if land parcel i is used for land-use type k, $\overline{c_{ik}}$ is acquisition cost if land parcel i is used for land-use type k, θ_k is total budget for acquisition of land-use type k, L_k is minimum benefit desired for land-use type k, US_k is upper bound of area for land-use type k, and LS_k is lower bound of area for land-use type k.

The objective functions (equation [1]) are to minimize the total of acquisition cost and to maximize the total of benefit. Constraints (equation [2]) limit the total acquisition cost for each land-use type. Constraints (equation [3]) require a minimum level of benefit for each land-use type. Constraints (equation [4]) restrict only one land-use assigned to each land parcel. Constraints (equation [5] and [6]) impose lower and upper bounds on the total area for each land-use type. Constraints (equation [7]) require the decision variables to be binary.

The decision variable is declared as follows:

$$x_{ik} = \begin{cases} 1, & \text{if land parcel } i \text{ is used for land-use type } k \\ 0, & \text{otherwise.} \end{cases}$$

Optimization model for land-use allocation introduced by Li, *et al*. is as follows:

$$\begin{aligned} \max Z &= \sum_{k=1}^{K} \sum_{i=1}^{n} z_{ik} x_{ik} \\ \max \ R_k &= \sum_{i=1}^{n} r_{ik} x_{ik} \end{aligned} \tag{8}$$

$$s.t.\ B_{1k} \leq \sum_{i=1}^{n} a_i x_{ik} \leq B_{2k}, \forall k = 1, 2, \ldots, K \tag{9}$$

$$\sum_{k=1}^{K} x_{ik} = 1, \forall i = 1, 2, \ldots, n \tag{10}$$

$$x_{ik} = \{0,1\}, \forall i, k \tag{11}$$

where z_{ik} is comprehensive index if the planning unit i is arranged to land-use type k, r_{ik} is density index if the planning unit i is arranged to land-use type k, B_{1k} is lower bound on the total area for land-use type k, B_{2k} is upper bound on the total area for land-use type k, a_i is the area of planning unit i, x_{ik} is decision variables declared as follows:

$$x_{ik} = \begin{cases} 1, & \text{if land-use } i \text{ is arranged to land-use type } k \\ 0, & \text{otherwise} \end{cases}$$

2. 2. Determination of Comprehensive Index and Density Index

The spatial weight matrix is $\mathbf{W}_{n\times n}$ matrix with each element w_{ij} indicating the values of proximity measurement between location i and location j which is observed based on the neighborhood relations between locations. The closeness is determined based on contiguity. If location i is adjacent to or directly adjacent to location j, then the element (i, j) is given a value of 1. If location i is not adjacent to or directly adjacent to location j, then the element (i, j) is given a value of 0. Some methods to determine contiguity according to LeSage (2009) are rook contiguity, bishop contiguity, and queen contiguity. Then, contiguity matrix is standardized by dividing each element in the matrix by the sum of values in its each row.

The comprehensive index and density index contained in objective function are determined by a spatial weight matrix that states the correlation between land locations. A comprehensive index can be determined by determining the spatial weight matrix for land combinations. For example, the spatial weight matrix is stated in the following form:

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & \cdots & w_{mn} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} \tag{12}$$

The vector **z** which represents a comprehensive index of land-use change from type i to type j is determined based on the spatial weight matrix, it can be stated as follows:

$$z^T = \begin{bmatrix} w_1{}^T & w_2{}^T & \cdots & w_m{}^T \end{bmatrix} \tag{13}$$

The density index matrix R represents the incidence matrix where the rows of the matrix represent the transition land planning from type i to type k and the column of the matrix represents type j. So, the elements of the matrix can be determined as follows:

$$R_{((i,k),j)} = \begin{cases} z_{ij}, \text{if } k = j \\ 0, \text{ otherwise} \end{cases} \tag{14}$$

2. 3. Robust Optimization

In this section, it is discussed about Robust Optimization consisted of robust optimization paradigm and solving robust counterpart.

2. 3. 1. Robust Optimization Paradigm

According to Ben-Tal and Nemirovski (2002), Robust Optimization is a method for solving optimization problems with uncertainty data and the data are only known in uncertainty set. Based on Robust Optimization Theory proposed by Ben-Tal & Nemirovski (2002), let $c \in R^n, b \in R^m$, and $A \in R^{m \times n}$ are the parameters of Linear Programming problem:

$$\min_x \left\{ \mathbf{c}^T \mathbf{x} : \mathbf{A}\mathbf{x} \le \mathbf{b} \right\} \tag{15}$$

In Robust Optimization, an uncertain linear programming problem can be stated as follows:

$$\min_x \left\{ \mathbf{c}^T \mathbf{x} : \mathbf{A}\mathbf{x} \le \mathbf{b} \;;\; \mathbf{c}, \mathbf{A}, \mathbf{b} \in \mathrm{U} \right\} \tag{16}$$

where **c, A, b** are uncertainty coefficients and $\mathcal{U}$ is an uncertainty set.

The basic Robust Optimization paradigm is based on these following assumptions (Gorissen, *et al.*, 2015):

A.1 All decision variables represent "here and now" decision: they should get specific numerical values as a result of solving the problem before the actual data "reveals itself"

A.2 The decision maker is fully responsible for consequences of the decisions to be made when, and only when, the actual data is within the prespecified uncertainty set $\mathcal{U}$.

A.3 The constraints of the uncertain problem in question are hard. The decision maker can't tolerate the violations of constraints when data is in uncertainty set $\mathcal{U}$.

According to Gorissen, *et al.* (2015), in addition to basic assumptions of Robust Optimization, we may assume without loss of generality that (1) objective function is certain, (2) the right-hand side is certain, (3) the uncertainty is constraint-wise and $\mathcal{U}$ is a convex and compact set.

2. 3. 2. Solving Robust Counterpart

If it is assumed that $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$ is certain, then the robust counterpart formulation from (16) is as follows (Gorissen, *et al.*, 2015):

$$\min_x \left\{ \mathbf{c}^T \mathbf{x} : \mathbf{A}(\zeta)\mathbf{x} \le \mathbf{b},\ \forall \zeta \in Z \right\} \tag{17}$$

Define the uncertainty parameter as follows:

$$\mathbf{a}(\zeta) = \bar{\mathbf{a}} + \mathbf{P}\zeta \tag{18}$$

where $\bar{\mathbf{a}} \in \mathbb{R}^n$ is nominal values vector and $\mathbf{P} \in \mathbb{R}^{n \times L}$ is perturbation matrix. Substitute equation (18) to equation (17), so we get the new robust counterpart formulation as follows:

$$\left(\bar{\mathbf{a}}+\mathbf{P}\zeta\right)^{T}\mathbf{x}\le\mathbf{b},\ \forall\zeta\in Z \tag{19}$$

The challenge of Robust Optimization is to find an uncertainty set $\mathcal{U}$ that can be formulated into a computationally tractable problem and determine an approximation that has already been proven to be computationally tractable. According to Ben-Tal and Nemirovski (2002) and Chaerani & Ross (2013), computationally tractable can be analyzed by representing robust counterpart into one of three classes of optimization problems i.e. Linear Programming, Conic Quadratic Programming, or semi-definite Optimization.

Table 2. Tractable Reformulation for Different Types of Uncertainty Sets (Gorissen, *et al*., 2015)

Uncertainty Set	*Z*	*Robust Counterpart*	*Tractability*
Box	$\lVert\zeta\rVert_\infty\le 1$	$\mathbf{a}^T\mathbf{x}+\lVert\mathbf{P}^T\mathbf{x}\rVert_1\le\mathbf{b}$	LP
Ellipsoidal	$\lVert\zeta\rVert_2\le 1$	$\mathbf{a}^T\mathbf{x}+\lVert\mathbf{P}^T\mathbf{x}\rVert_2\le\mathbf{b}$	CQP
Polyhedral	$\mathbf{D}\zeta+\mathbf{q}\ge 0$	$\begin{cases}\mathbf{a}^T\mathbf{x}+\mathbf{q}^T\mathbf{y}\le\mathbf{b}\\ \mathbf{D}^T\mathbf{y}=-\mathbf{P}^T\mathbf{x}\\ \mathbf{y}\ge\mathbf{0}\end{cases}$	LP

3. RESULT

In this section, it is discussed about the result of this research.

3. 1. Formulation of Optimization Model for Land-use Allocation Problem (OMLAP)

Optimization model for land-use allocation problem used in this research is a reformulation from optimization model for land-use allocation problem introduced by Yao, *et al*. and Li, *et al*. The objective function is based on optimization model from Li, *et al.*, the constraints are based on optimization problem from Yao, *et al.*, and the decision variables are assumed as the proportion in which the values are defined between 0 and 1. The optimization model for land-use allocation problem is as follows:

$$\begin{aligned}\max Z &= \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik}x_{ik}\\ \max R_k &= \sum_{i=1}^{N} r_{ik}x_{ik}\end{aligned} \tag{20}$$

$$s.t \sum_i s_i x_{ik} \geq LS_k, \ \forall k \tag{21}$$

$$\sum_i s_i x_{ik} \leq US_k, \ \forall k \tag{22}$$

$$\sum_k x_{ik} = 1, \forall i \tag{23}$$

$$\sum_i \overline{c_{ik}} x_{ik} \leq \theta_k, \ \forall k \tag{24}$$

$$\sum_i a_{ik} x_{ik} \geq L_k, \ \forall k \tag{25}$$

$$0 \leq x_{ik} \leq 1, \forall i, k \tag{26}$$

where z_{ik} is comprehensive index, r_{ik} is density index, N is total number of planning unit, K is total number of land-use types, a_{ik} is benefit if planning unit i is changed to land-use type k, $\overline{c_{ik}}$ is acquisition cost if planning unit i is changed to land-use type k, θ_k is total budget for acquisition of land-use type k, L_k is minimum benefit desired for land-use type k, US_k is upper bound of area for land-use type k, and LS_k is lower bound of area for land-use type k.

The objective functions (equation [17]) are to maximize the comprehensive index and density index. Constraints (equation [18] and [19]) impose lower and upper bounds on the total area for each land-use type. Constraints (equation [20]) restrict the sum of percentage change from planning unit i to land-use type k equal to 1. Constraints (equation [21]) limit the total acquisition cost for each land-use type.

Constraints (equation [22]) require a minimum level of benefit for each land-use type. Constraints (equation [23]) require the range values of decision variables between 0 and 1. The decision variables declare the percentage change from planning unit i to land-use type k.

3. 2. Uncertainty Model for OMLAP

In this research, the uncertainty factors are the benefit and acquisition cost if a land planning is changed to a land-use type.

Therefore, the uncertainty parameters in this model are benefit if planning unit i is changed to land-use type k or a_{ik} and acquisition cost if planning unit i is changed to land-use type k or $\overline{c_{ik}}$.

So, the uncertainty parameter $\overline{c_{ik}}$ is defined as follows:

$$\overline{c_{ik}} = \overline{\overline{c_{ik}}} + P_{ik}\zeta, \forall \zeta \in Z \tag{27}$$

where $\overline{\overline{c_{ik}}} \in \mathbb{R}^n$ is nominal value vector for acquisition cost, $P_{ik} \in \mathbb{R}^{n \times L}$ is perturbation matrix, and $\zeta \in \mathbb{R}^L$ is primitive uncertainty vector. Then, the uncertainty parameter a_{ik} is defined as follows:

$$a_{ik} = \overline{a_{ik}} + P1_{ik}\zeta, \forall \zeta \in Z \tag{28}$$

where $\overline{a_{ik}} \in \mathbb{R}^n$ is nominal value vector for acquisition cost, $P_{ik} \in \mathbb{R}^{n \times L}$ is perturbation matrix, and $\zeta \in \mathbb{R}^L$ is primitive uncertainty vector.

The uncertainty parameters are in the first constrain and the second constrain of the model, so the objective function is certain. $\overline{c_{ik}}$ and a_{ik} are in the left-hand side of the model, so the certain right-hand side assumption is fulfilled. Substitute (27) and (28) to OMLAP, so the uncertain model for OMLAP is defined as follows:

$$\begin{aligned}
&\max Z = \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik} x_{ik} \\
&\max R_k = \sum_{i=1}^{N} r_{ik} x_{ik} \\
&\text{s.t} \quad \sum_{i=1}^{N} s_i x_{ik} \geq LS_k, \ \forall k \\
&\qquad \sum_{i=1}^{N} s_i x_{ik} \leq US_k, \ \forall k \\
&\qquad \sum_{k=1}^{K} x_{ik} = 1, \ \forall i \\
&\qquad \sum_{i=1}^{N} (\overline{\overline{c_{ik}}} + P_{ik}\zeta) x_{ik} \leq \theta_k, \ \forall k \\
&\qquad \sum_{i=1}^{N} (\overline{a_{ik}} + P1_{ik}\zeta) x_{ik} \geq L_k, \ \forall k \\
&\qquad 0 \leq x_{ik} \leq 1, \forall i, k, \forall \zeta \in Z.
\end{aligned} \tag{29}$$

3. 3. Robust Counterpart Formulation with Box Uncertainty Set for OMLAP

It is known that box uncertainty set is defined as follows:

$$Z = \{\zeta : \|\zeta\|_\infty \leq 1\}$$

Robust counterpart formulation for box uncertainty set is as follows:

$$\overline{\mathbf{a}}^T \mathbf{x} + \left\|\mathbf{P}^T \mathbf{x}\right\|_1 \leq \mathbf{b}$$

where $\bar{a} \in \mathbb{R}^n$ is nominal value vector, $P \in \mathbb{R}^{n\times L}$ is perturbation matrix, and $\zeta \in \mathbb{R}^L$ is uncertainty primitive vector. Assume that the uncertainty parameters located in box uncertainty set, so we get the formulation of robust counterpart with box uncertainty set for land-use allocation problem as follows:

$$\max Z = \sum_{k=1}^{K}\sum_{i=1}^{N} z_{ik} x_{ik}$$

$$\max R_k = \sum_{i=1}^{N} r_{ik} x_{ik}$$

$$\text{s.t. } \sum_{i=1}^{N} s_i x_{ik} \geq LS_k, \ \forall k$$

$$\begin{aligned} &\sum_{i=1}^{N} s_i x_{ik} \leq US_k, \ \forall k \\ &\sum_{k=1}^{K} x_{ik} = 1, \ \forall i \\ &\sum_{i=1}^{N} \overline{\overline{c}}_{ik} x_{ik} + \left\| \sum_{i=1}^{N} P_{ik} x_{ik} \right\|_1 \leq \theta_k, \ \forall k \\ &\sum_{i=1}^{N} \overline{a}_{ik} x_{ik} + \left\| \sum_{i=1}^{N} P1_{ik} x_{ik} \right\|_1 \geq L_k, \ \forall k \\ &0 \leq x_{ik} \leq 1, \forall i, k. \end{aligned} \tag{30}$$

3. 4. Numerical Experiments

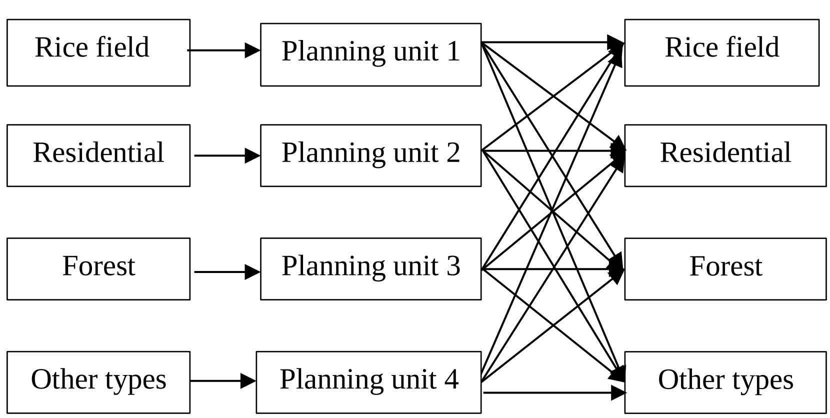

Figure 1. Land-use Flow for Land-use Allocation Problem in Jatinangor

Case study used for numerical experiment is spatial planning in Jatinangor Subdistrict. The data used in this paper are secondary data from Indonesian Statistics 2018, observational data, and illustrative data. According to the data, there are four land-use types in Jatinangor i.e. rice field, residential, forest, and other types. Other types are used for educational area, industry and warehousing, trade area, governmental area, water catchment area, and green open area. Land-use allocation problem discussed in this paper is to determine whether there will be change in existing land-use types or not in order to maximize the comprehensive index and density index. An illustration for the problem of land-use allocation in Jatinangor can be seen in Figure 1.

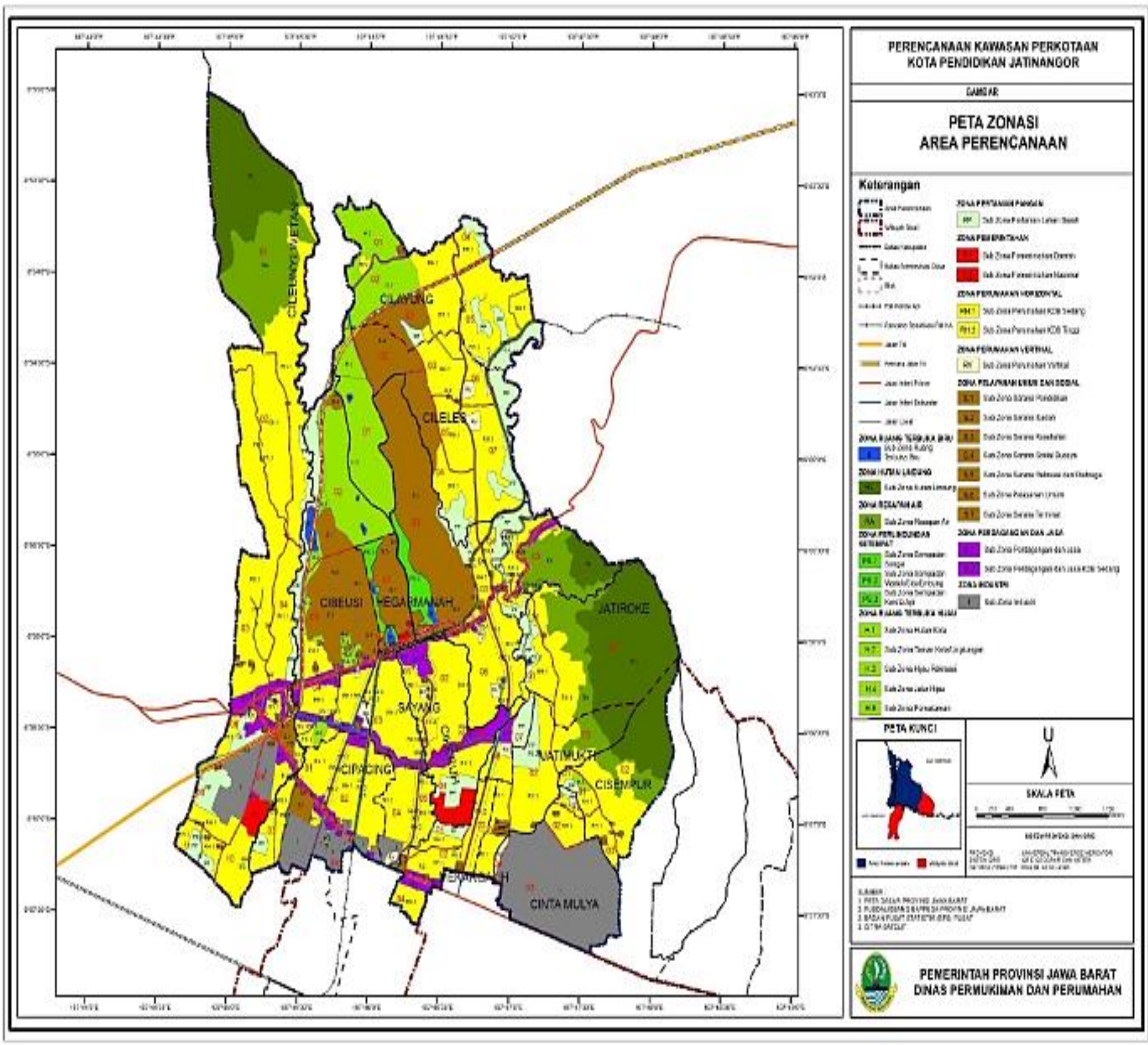

Figure 2. Spatial Planning in Jatianangor

Illustrative data about land area, lower and upper bound for each land use are shown in Table 3. The lower and upper bound are hypothetically determined according to the conditions.

Table 3. Illustrative Data about Land Area, Lower, and Upper Bound for Each Land-use

Land-use type	Land area (Ha)	Lower bound (Ha)	Upper bound (Ha)
Rice field	371	0	371,65
Residential	1.168	1.000	1.200
Forest	755	754,3	756
Other types	326	325	1.000

Data about the benefit for each land-use type are observational data which are obtained based on the amount of the Land and Building Tax and considering the land area of each land-use type. The benefits for land-use type are shown in Table 4.

Table 4. Illustrative Data about The Benefit for Each Land-use Type

Land-use type	Benefit (millions IDR)
Rice field	33.398,88
Residential	110.620
Forest	27.000
Other types	29.340

Data about acquisition costs for each land-use type is hypothetical data considering the land area and the 2018 Regional Budget of Sumedang Regency. The acquisition cost for each land-use type are shown in Table 5.

Table 5. Illustrative Data about The Acquisition Cost for Each Land-use Type

Land-use type	Acquisition cost (millions IDR)
Rice field	1.518,5
Residential	1.917
Forest	1.710,5
Other types	1.496

The spatial weight matrix can be obtained using the position between land-use type as shown in Figure 1. So, we get the spatial weight matrix as follows:

$$\mathbf{W} = \begin{bmatrix} 0 & 0,5 & 0 & 0,5 \\ 0,5 & 0 & 0 & 0,5 \\ 0 & 0 & 0 & 1 \\ 0,33 & 0,33 & 0,33 & 0 \end{bmatrix} \tag{31}$$

The determination of comprehensive index and density index is based on spatial weight matrix. The comprehensive index and density index are shown in equation (26) and equation (27), respectively.

$$\begin{aligned} &\mathbf{z}^T = [\mathbf{z}_1 \quad \mathbf{z}_2 \quad \mathbf{z}_3 \quad \mathbf{z}_4] \\ &\mathbf{z}_1 = [0 \quad 0,5 \quad 0 \quad 0,5];\ \mathbf{z}_2 = [0,5 \quad 0 \quad 0 \quad 0,5] \\ &\mathbf{z}_3 = [0 \quad 0 \quad 0 \quad 1];\ \mathbf{z}_4 = [0,33 \quad 0,33 \quad 0,33 \quad 0] \end{aligned} \tag{32}$$

$$\begin{aligned} &\mathbf{R}^T = [\mathbf{R}_1 \quad \mathbf{R}_2 \quad \mathbf{R}_3 \quad \mathbf{R}_4] \\ &\mathbf{R}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0,5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,5 \end{bmatrix};\ \mathbf{R}_2 = \begin{bmatrix} 0,5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,5 \end{bmatrix} \\ &\mathbf{R}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{R}_4 = \begin{bmatrix} 0,33 & 0 & 0 & 0 \\ 0 & 0,33 & 0 & 0 \\ 0 & 0 & 0,33 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{33}$$

Assume that the minimum desired benefits for land-use types consisting of rice fields, residential, forests, and other types are respectively 0, 16,500, 11,000, and 5,500 (in millions IDR). Then, assume the available budget for each land-use type are 1,500, 2,000, 1,720 and 10,000, respectively (in millions IDR).

Table 6. The result of Numerical Experiment.

No	Deterministic Model for Land-use Allocation Problem			Robust Counterpart with Box Uncertainty Set for Land-use Allocation Problem		
	Decision Variables		Objective Function	Decision Variables		Objective Function
1	x_{11}	0	$Z^* = 1,3654$ $R_1^* = 0,2397$ $R_2^* = 0,1972$ $R_3^* = 0,0903$ $R_4^* = 0,8382$	x_{11}	0	$Z^* = 1,3517$ $R_1^* = 0,2444$ $R_2^* = 0,1896$ $R_3^* = 0,0856$ $R_4^* = 0,8321$
2	x_{12}	0,3944		x_{12}	0,3791	
3	x_{13}	0		x_{13}	0	

4	x_{14}	0,6056		x_{14}	0,6209	
5	x_{21}	0		x_{21}	0	
6	x_{22}	0,7309		x_{22}	0,7357	
7	x_{23}	0,2691		x_{23}	0,2643	
8	x_{24}	0		x_{24}	0	
9	x_{31}	0		x_{31}	0	
10	x_{32}	0		x_{32}	0	
11	x_{33}	0,4646		x_{33}	0,4783	
12	x_{34}	0,5354		x_{34}	0,5217	
13	x_{41}	0,7263		x_{41}	0,7407	
14	x_{42}	0		x_{42}	0	
15	x_{43}	0,2737		x_{43}	0,2593	
16	x_{44}	0		x_{44}	0	

The result of optimization model for land-use allocation problem and RC model with box uncertainty for land-use allocation problem are obtained by using software Maple 15. The result consisted the objective function and decision variables is shown in Table 6.

From the result of numerical experiment, it can be determined the total of land area converted to another land-use type. The total of land area converted to another land-use type for each land-use type is shown in Table 7.

Table 7. The Total of Land Area Converted to another Land-use Type (Ha).

Deterministic Model for Land-use Allocation Problem				
Land-use type	Rice field	Residential	Forest	Other types
Rice field	0	146,3224	0	224,6776
Residential	0	853,6912	314,3088	0
Forest	0	0	350,773	625,3472
Other types	236,7738	0	89,2262	0

Robust Counterpart with Box Uncertainty Set for Land-use Allocation Problem				
Land-use type	Rice field	Residential	Forest	Other types
Rice field	0	140,6461	0	230,3539
Residential	0	859,2976	308,7024	0
Forest	0	0	361,1165	393,8835
Other types	241,4682	0	84,5318	0

It can be seen that the form of robust counterpart formulation with box uncertainty for land-use allocation problem is linear programming, so robust counterpart with box uncertainty for land-use allocation problem is a computationally tractable optimization model. The solution from RC with box uncertainty for allocation problem smaller than the deterministic model, but the RC model has been made by considering the uncertainty factors that represent the worst possibility that might occur.

4. CONCLUSIONS

Robust Optimization Model for land-use allocation problem which is solved by the box uncertainty set approach is a computationally tractable optimization model. Numerical experiment in study case shows that Robust Optimization model for land-use allocation using box uncertainty set obtained optimal robust solution. The optimal solution for the Robust optimization model with box uncertainty set is smaller than the optimal solution from the deterministic model for land use allocation problem.

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