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Spatial-Temporal Analysis of Rainfall West Java Indonesia Using Empirical Orthogonal Function based on Singular Value Decomposition

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ABSTRACT

Rainfall is one of the climate variables that have a significant influence, especially in supporting the activities of various sectors in tropical countries. Climate change is causing rainfall variability in Indonesia. However, the analysis of climate variable patterns is difficult because of the formation of a large matrix. Empirical Orthogonal Function (EOF) analysis can be used to reduce the dimensions of large data by maintaining as much variation as possible from the original data set. The method used in this study is through the Singular Value Decomposition (SVD) approach. The analysis shows that 98.50% of the total rainfall variance can be represented by four EOF modes. Analysis of the spatial pattern of EOF1 shows that rainfall is below average, while the other EOF modes show variations in rainfall.

***Keywords*:** Empirical Orthogonal Function, Rainfall, Singular Value Decomposition, Spatial-Temporal

1. INTRODUCTION

The Indonesian archipelago is an atmospheric heat center that makes an important contribution to understanding the world climate system [4]. Besides, Indonesia is a country that is crossed by the equator surrounded by two oceans and two continents [3]. This makes Indonesia a tropical country with a high variation in rainfall, temperature, and humidity [2, 9].

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Indonesia has two seasons, namely the rainy season and the dry season. At the moment the rainfall in the rainy season is unpredictable. This is the impact of global climate change on Indonesia [5, 20, 33]. Extreme climate change has the potential to cause catastrophic impacts on society. Climate variability can affect extreme rainfall and drought which has an impact on the environment. Heavy rainfall can cause extensive flooding while prolonged dry conditions can trigger severe drought [30]. Climate change in Indonesia is a global concern because Indonesia is the third-largest emitter of greenhouse gases in the world. Greenhouse gas emissions are the cause of global climate change. Indonesia's greenhouse gas emissions are largely the result of forest fires and environmental degradation.

Indonesia's climate change impacts the country's environment, people and development [16]. Climate information is important in supporting activities in various sectors, including agriculture, fisheries, water resources, industry and other sectors [8, 11, 28, 35]. However, climate variables, especially rainfall are atmospheric parameters that are difficult to predict because of their high spatial and temporal diversity. Rainfall forecast information is an important requirement in supporting water resource management [21]. Analyzing climate data is an interesting challenge. Caraka et al [26] analyzing climate data can be done by analyzing the relationship between two climate measurements and climate models. Cheng et al [18], David [22], and Naveau et al [24] and use statistical methods to analyze climate change.

Empirical Orthogonal Function (EOF) is widely used to analyze climate variables, several studies have been done. Aldrian and Djamil [9] analyzed monthly rainfall at 40 stations in 1955-2005. Liang et al [34] used EOF analysis to construct a nitrate map. Yu and Chu [13] and Yu and Lin [14] analyze the spatial and temporal patterns of rainfall and groundwater relationships using the EOF method. . Raghavan et al [29] analyzed the variability of spatial and temporal patterns of rainfall in Southeast Asia.

In addition to direct analysis of rainfall, the Empirical Orthogonal Function can also be applied to rainfall analysis as an indicator of drought. Drought is characterized by rainfall below normal for several months to years. Tatli and Turkes [12] conducted an EOF analysis for the drought index and compared spatial patterns. Climate data can be arranged in a three-dimensional matrix and transform into a two-dimensional matrix, then an EOF analysis is performed based on the eigenvalue problem. Nurdiati et al [27] conducted a multivariate analysis of forest fires in Indonesia as a result of drought by using EOF based on Singular Value Decomposition (SVD). Next Setiawan et al [23] analyzed the variables that influence the pattern of forest fires in Indonesia using SVD based on EOF.

In this study, an analysis of rainfall in West Java Indonesia was conducted using Empirical Orthogonal Function (EOF) based on Singular Value Decomposition. The data used in this study is satellite data of the Tropical Rainfall Measuring Mission (TRMM). According to Prasetia et al [25], TRMM satellite data has a pattern that is very similar to observational data, this is indicated by high correlation values.

2. MATRIX DATA

Climate data are usually presented in an array. Vertical coordinates are composed of two spatial dimensions, i.e. longitude $a_i, i = 1,..., o$ and latitude $b_j, j = 1,..., p$ combined with the total data grid so that horizontal coordinates become $s_l = s(a_{i,} b_j)$ where $l = 1, 2, ..., m$ with $m = o \times p$. Horizontal coordinates are arranged based on the time variable, $t_k, k = 1,..., n$. With the data grid

s_l and the time variable t_k denoted $\bar{x}_{lk}$, where $\bar{x}_{lk}$ is the average data in the area l for time k. The data fields to be analyzed can be represented in the $m \le n$ data matrix as follows:

$$A_{m\times n} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1(n-1)} & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2(n-1)} & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3(n-1)} & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{m(n-1)} & x_{mn} \end{bmatrix} \tag{1}$$

3. SINGULAR VALUE DECOMPOSITION

Let A be the real matrix $m \times n$, then

$$A = USV^T \tag{2}$$

where $U^TU = V^TV = VV^T = I_n$ dan $S = diag(\sigma_{1,} \ldots, \sigma_n)$

The U matrix consists of n orthonormal eigenvectors that correspond to the eigenvalue AA^T and the V matrix consists of orthonormal eigenvectors from $A^T A$. The diagonal element of A is the nonnegative square root of $A^T A$ and is called the singular value [10].

Singular Value Decomposition Theorem [1, 6, 15, 19, 31]. If A is a real matrix $m \times n$, then there are orthogonal matrices $U_{m\times m}$ and $V_{n\times n}$ so

$$U^T AV = diag(\sigma_1, \ldots, \sigma_q), \qquad q = \min(m, n)$$

where $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_r \succ \sigma_{r+1} = \ldots = \ldots = \sigma_q = 0$ and $r = rank(A)$.
μ_i called the singular value of A and the set $\sigma(A) = \{\sigma_1, \ldots, \sigma_q\}$.

4. EMPIRICAL ORTHOGONAL FUNCTION

Empirical Orthogonal Function was first introduced by a geophysicist, E. Lorenz. Empirical Orthogonal Function (EOF) analysis can be used to reduce the dimensions of large data by maintaining as much variation as possible from the original data set. EOF breaks down data records into a set of individual models consisting of a single spatial pattern (Loading Vectors, LVs) and an appropriate amplitude time series (Principal Component Time Series) [7, 17, 32].

Note the Singular Value Decomposition (SVD) matrix in equation (2) Matrix V is the EOF matrix and US is the score matrix of the main components. The variance of the main i components is obtained from [21]:

$$\mu_i = \frac{\sigma_i^2}{\sum_{i=1}^{r}\sigma_i^2} \tag{3}$$

The EOF1 mode or the first main component is the data with the largest variance. The score of the main component or EOF mode obtained is a score that shows the contribution of each major component to each observation unit. The value of the score of the main component is positive gives a large contribution to the unit of observation, while the value of the score of the main component of a positive component makes a large contribution to the unit of observation. Spatial patterns are the result of visualizing the scores of the main components of each mode. Singular vector shows plot time-series data from EOF analysis.

5. RESULTS AND DISCUSSION

5. 1. Data Extraction

5. 1. 1. Determine the regional domain and period

The object of research in this study is West Java Province which is located at longitude $104°48'\mathrm{EL}$ to $108°48'\mathrm{EL}$ and $5°50'\mathrm{SL}$ to $7°50'\mathrm{SL}$. The time span of research is monthly in 2010-2018. The climate variable analyzed is rainfall. Data was obtained from Tropical Rainfall Measuring Mission (TRMM) 3B43.

5. 1. 2. Data matrix

Based on West Java Province monthly rainfall data from January 2010 - August 2019, the following data matrix is obtained:

$$A_{32\times116} = \begin{bmatrix} 543.30 & 179.30 & 360.22 & \cdots & 0.58 & 0.00 \\ 459.22 & 207.10 & 421.32 & \cdots & 3.02 & 2.63 \\ 350.09 & 198.55 & 349.45 & \cdots & 2.02 & 2.82 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 646.69 & 387.68 & 361.70 & \cdots & 13.61 & 0.62 \end{bmatrix} \tag{4}$$

5. 2. Data Analysis

5. 2. 1. EOF analysis based on SVD

Perform Singular Value Decomposition decomposition on matrix A, so the following matrix is obtained:

$$A_{32\times116} = U_{32\times32} S_{32\times116} V_{116\times116}$$

where

$$U_{32\times32} = \begin{bmatrix} -0.03 & -0.20 & -0.21 & \cdots & -0.14 \\ 0.03 & -0.44 & -0.42 & \cdots & 0.12 \\ -0.09 & 0.05 & 0.01 & \cdots & 0.36 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.00 & 0.02 & -0.02 & \cdots & 0.32 \end{bmatrix}$$

$$S_{32\times116} = \begin{bmatrix} 16535.26 & 0.00 & \cdots & 0.00 & 0.00 & \cdots & 0.00 & 0.00 \\ 0.00 & 2773.06 & \cdots & 0.00 & 0.00 & \cdots & 0.00 & 0.00 \\ 0.00 & 0.00 & \cdots & 0.00 & 0.00 & \cdots & 0.00 & 0.00 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & 0.00 & 0.00 \\ 0.00 & 0.00 & \cdots & 52.00 & 0.00 & \cdots & 0.00 & 0.00 \end{bmatrix}$$

$$V_{116\times116} = \begin{bmatrix} -0.13 & -0.09 & -0.13 & -0.06 & \cdots & -0.01 \\ -0.02 & 0.11 & 0.03 & 0.08 & \cdots & 0.03 \\ 0.20 & 0.00 & -0.03 & 0.00 & \cdots & -0.07 \\ -0.02 & 0.05 & 0.14 & 0.18 & \cdots & -0.03 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -0.11 & 0.13 & 0.07 & 0.04 & \cdots & 0.92 \end{bmatrix}$$

Singular values, variance and total variance are presented in Table 1.

Table 1. Singular value, variance and total variance.

EOF	Singular Value	Variance	
		Individual	Cumulative
1	16535.26	0.9388	0.9388
2	2773.06	0.0264	0.9652
3	1801.22	0.0111	0.9764
4	1584.54	0.0086	0.9850
5	1052.06	0.0038	0.9888
6	842.17	0.0024	0.9912
7	735.58	0.0019	0.9931
8	622.28	0.0013	0.9944
9	584.94	0.0012	0.9956

10	527.70	0.0010	0.9966
11	487.03	0.0008	0.9974
12	414.26	0.0006	0.9980
13	314.52	0.0003	0.9983
14	302.77	0.0003	0.9986
15	275.28	0.0003	0.9989
16	264.54	0.0002	0.9991
17	234.63	0.0002	0.9993
18	208.28	0.0001	0.9994
19	165.14	0.0001	0.9995
20	150.75	0.0001	0.9996
21	144.18	0.0001	0.9997
22	131.38	0.0001	0.9997
23	121.53	0.0001	0.9998
24	110.10	0.0000	0.9998
25	106.55	0.0000	0.9999
26	88.50	0.0000	0.9999
27	81.49	0.0000	0.9999
28	76.33	0.0000	1.0000
29	67.10	0.0000	1.0000
30	61.72	0.0000	1.0000
31	57.23	0.0000	1.0000
32	52.00	0.0000	1.0000

5. 2. 2. Data analysis using Empirical Orthogonal Function based on Singular Value Decomposition

Spatial analysis is done by paying attention to the matrix of the main component scores, namely $U_{32\times 32}V_{116\times 116}$. Based on Table 1, if the first four EOF modes are selected, the total variance is 98.50% of the total variance of all rainfall data.

This shows that the four EOF modes can represent rainfall data from January 2010 - August 2019.Spatial analysis is done by visualizing the column vector results of matrix multiplication $U_{32\times 32}V_{116\times 116}$. Furthermore, each color indicates the variability of rainfall in West Java at a time. In the column vector with the greatest variance, the maximum and minimum elements are determined. Next, seven value intervals are made. The element with the largest value is indicated by the dark red color, the element located at the first interval is indicated by red, then the orange, yellow, light green, green, light blue, blue sequentially indicate the element that is located at the third, fourth largest value interval, fifth, sixth, seventh and dark blue indicate the elements that are located at the smallest value intervals.

Figure 1 is the result of EOF1 mode visualization, the largest singular value is -540.61 colored dark red and the smallest singular value -3759.97 colored dark blue. Based on the EOF1 mode score matrix, the main components are grouped into seven intervals, i.e. $(-540.61, -1000.52]$ represented by red, $(-1000.52, -1460.43]$ represented by orange, $(-1460.43, -1920.34]$ represented by yellow, $(-1920.34, -2380.24]$ represented by green, $(-2380.24, -2840.15]$ represented by light blue, $(-2840.15, -3300.06]$ represented by blue, and $(-3300.06, -3759.97]$ represented by dark blue. Based on the value of the EOF1 mode score matrix, it is known that 93.88% of rainfall has negative variability. The variation in rainfall is relatively below average with the greatest value in the southwest.

Figure 1. Spatial pattern EOF1 mode.

The visualization of EOF2 mode is presented in Figure 2. In the same way as EOF1 mode visualization, the main component of the EOF2 mode score matrix is divided into seven intervals with the highest value of 454.76 represented by dark red and the lowest value of -1220.48 represented by dark blue. The first interval is represented by red, the second interval by orange, the third interval by yellow, the fourth interval by green, the fifth interval by light blue, the sixth interval by blue, and the seventh interval which is the interval with the smallest value represented by color dark blue. The value of the main components of the EOF2 mode score matrix has positive and negative variability. Based on Figure 2, the visualization of EOF2 mode is relatively positive.

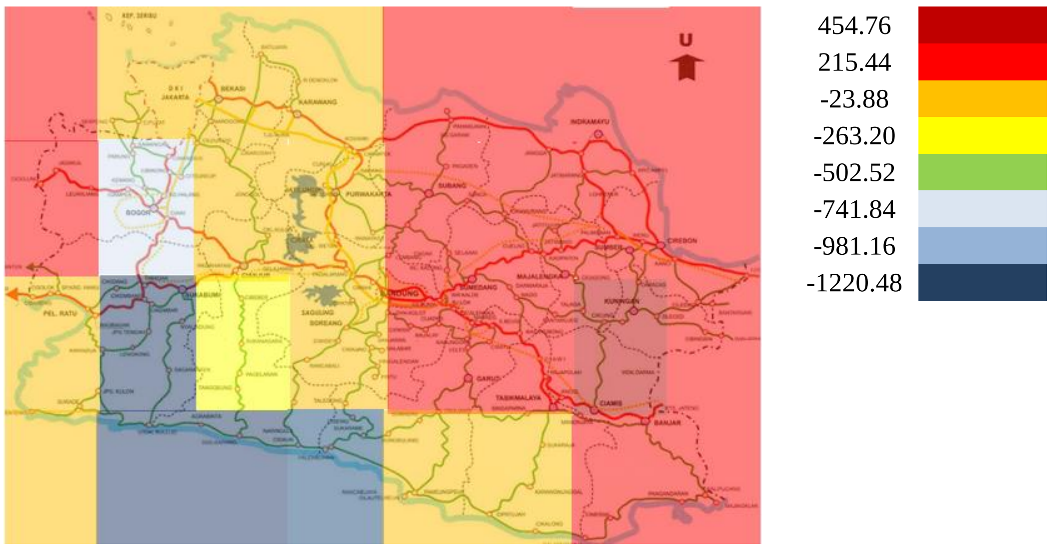

Figure 2. Spatial pattern EOF2 mode.

The EOF3 mode shows the state of rainfall in West Java at one time with the third largest variance. Rainfall variability has a scale that ranges from -508.44 to 674.91. The largest value is represented by the dark red color and the smallest value is represented by the dark blue color. Visualization of the main components of the EOF3 mode score matrix is done in the same way as the EOF1 mode and EOF2 mode. Based on Figure 3, it is known that rainfall has positive variability and negative variability. Rainfall variability in the north of West Java Indonesia is relatively positive, while in the south it is relatively negative.

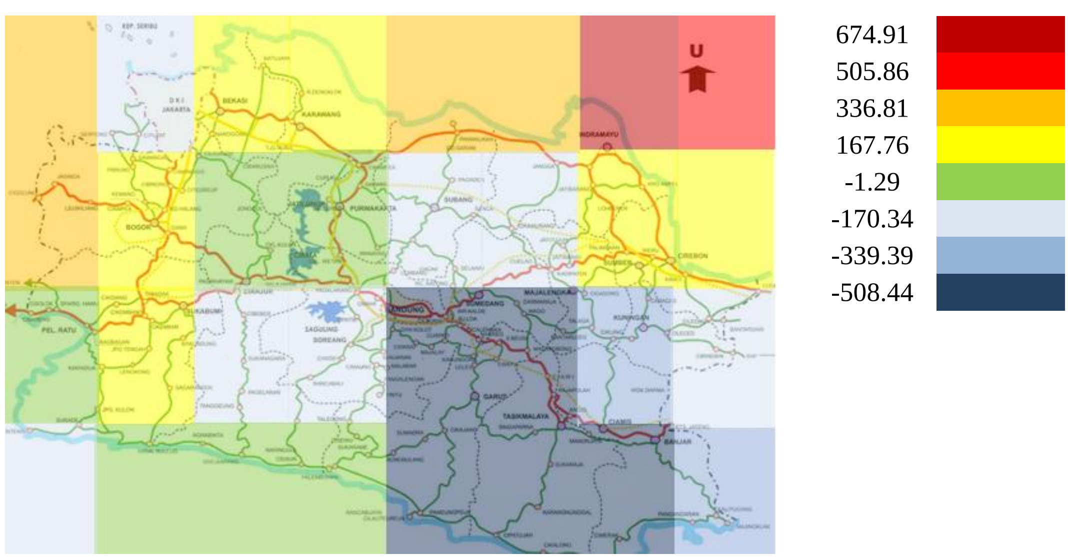

Figure 3. Spatial pattern EOF3 mode.

The EOF4 mode shows the rainfall in West Java at one time with the fourth largest variance. Rainfall variability has a scale ranging from -349.21 to 833.03. Based on Figure 4, Indonesian West Java rainfall with EOF4 mode shows relatively small results and has negative variability.

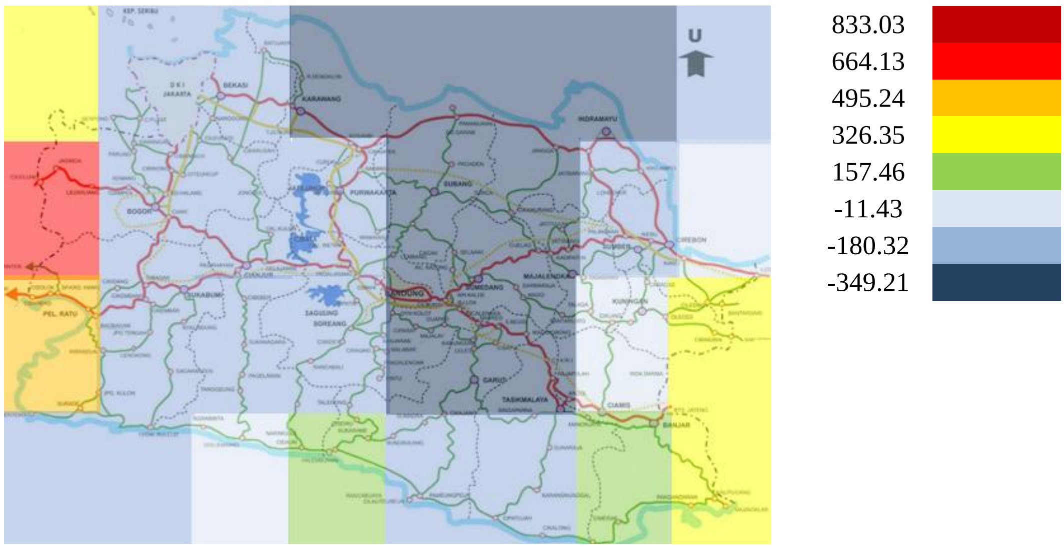

Figure 4. Spatial pattern EOF4 mode.

The temporal pattern of rainfall in West Java Indonesia can be done by analyzing the EOF matrix or matrix $V_{116\times116}$. Visualization of temporal patterns is presented in graphical form with the X-axis showing the period of the moon and the Y-axis showing the singular vector elements. Based on EOF1, EOF2, EOF3, and EOF4 modes, the value of the singular vector element is at intervals $[-0.30, 0.30]$.

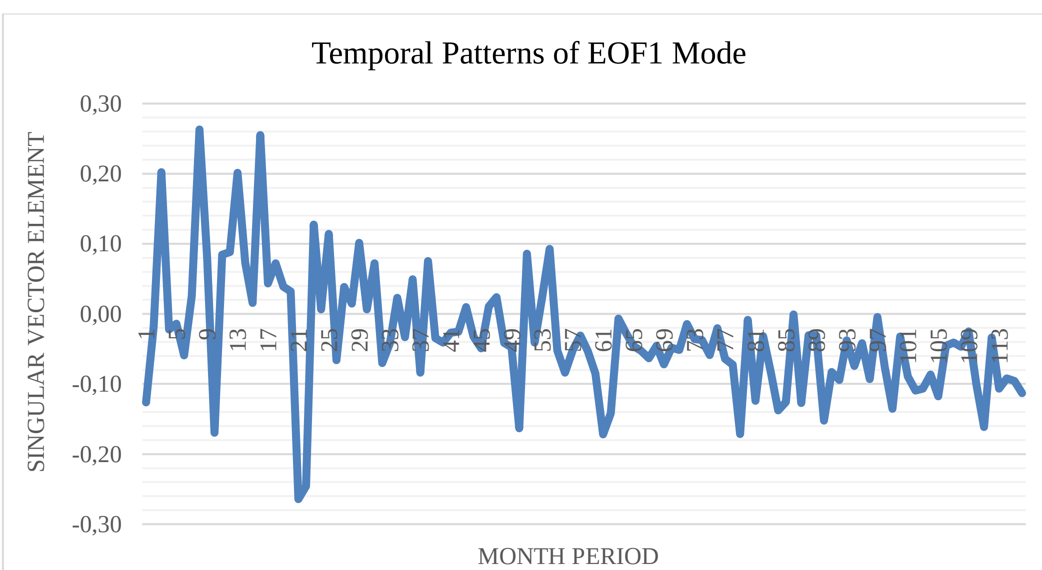

Figure 5. Temporal pattern EOF1 mode.

EOF1 mode is the mode with the greatest variance, which is 93.88%. Analysis of the temporal pattern of rainfall in West Java based on the EOF1 mode is presented in Figure 5. The variability of the temporal pattern is relatively down. The largest EOF1 singular mode vector element occurred in September 2010 and after the 57th month or September 2014, the temporal pattern is negative. Figure 6 shows the temporal pattern of EOF2 mode with the second largest variance. The EOF2 mode illustrates rainfall in West Java Indonesia of 2.64% with a total variance of 96.52%. Variations not saved on EOF1 are saved on EOF2.

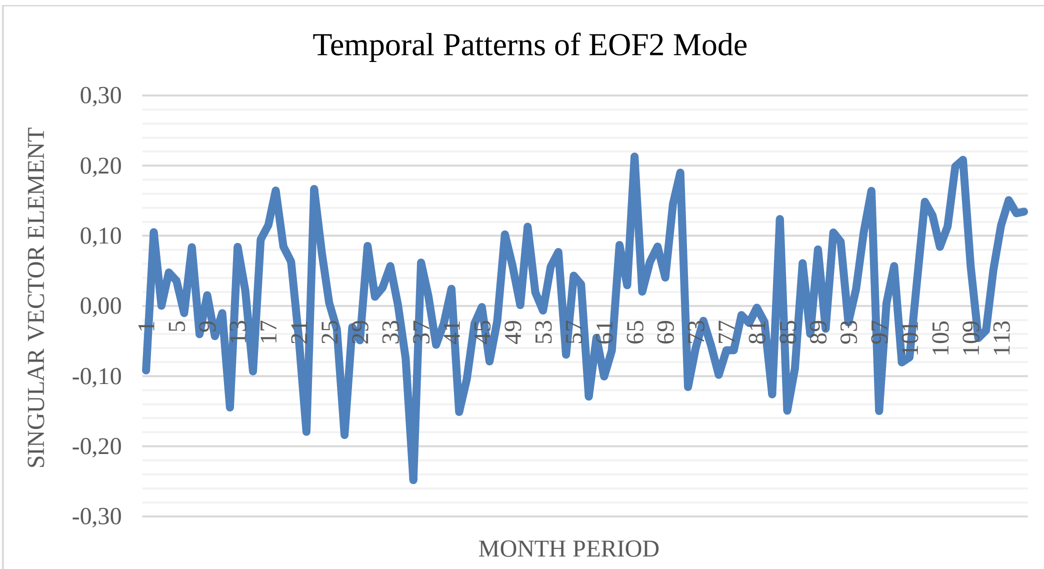

Figure 6. Temporal pattern EOF2 mode.

Figure 7 shows the temporal pattern of EOF3 mode with the third largest variance. The EOF3 mode illustrates rainfall in West Java Indonesia of 1.11% with a total variance of 97.64%. Variations that are not saved in EOF1 and EOF2 modes are saved in EOF3.

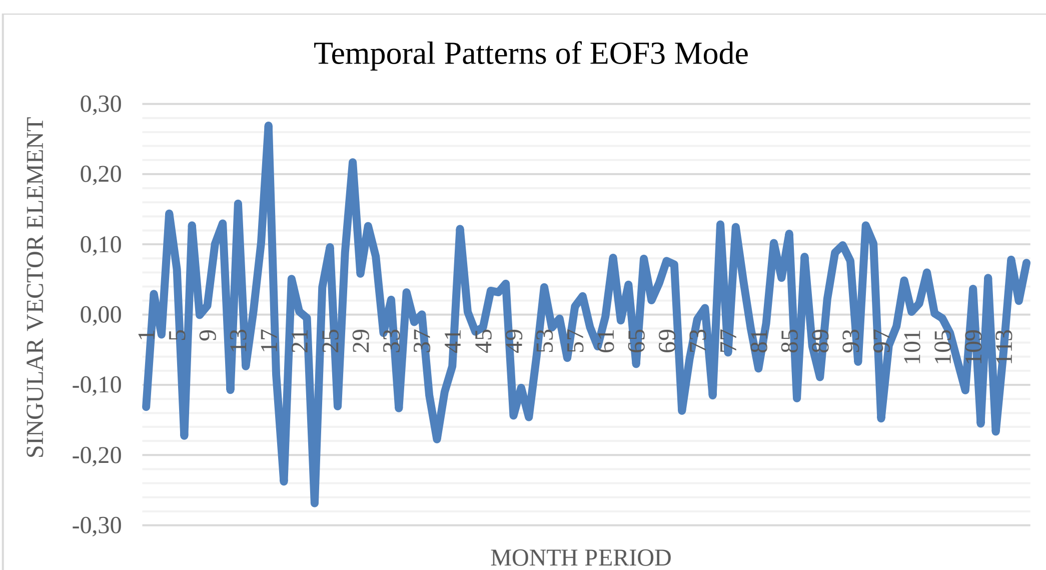

Figure 7. Spatial pattern EOF3 mode.

Figure 8 shows the temporal pattern of EOF4 mode with the fourth largest variance. The EOF4 mode illustrates rainfall in West Java Indonesia of 0.86% with a total variance of 98.50%. Variations that are not saved in EOF1, EOF2 and EOF3 modes are saved in EOF4, and so on.

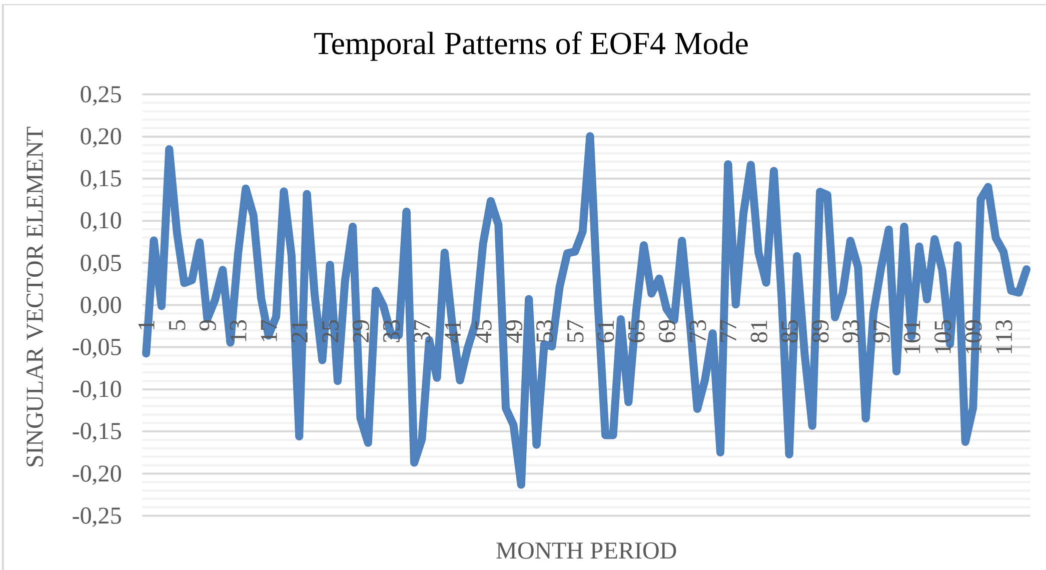

Figure 8. Temporal pattern EOF4 mode.

The total variance explained by the four main components or EOF modes is 98.50%. This value is a high achievement value. Based on the analysis results, it can be seen that the analysis of the four EOF modes can represent the actual data matrix.

6. CONCLUSIONS

Empirical Orthogonal Function Analysis based on Singular Value Decomposition of TRMM 3B43 rainfall data in West Java Indonesia produces thirty-two EOF modes with four dominant EOF modes. The four EO modes can explain 98.50% of the total variance. Spatial patterns can be obtained by analyzing the main component scores, while temporal patterns are obtained from analyzes of singular vectors. Analysis of the spatial pattern of EOF1 shows that rainfall in January 2010 - August 2019 is below average, while the other EOF modes show variations in rainfall.

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