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Influence of temperature variation on vibration of rectangular plate of visco-elastic material

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ABSTRACT

This paper deals with free vibration modes and natural frequencies of a thin visco-elastic, non-homogeneous rectangular plate with clamped ends. First two modes of frequency parameter of vibration are derived on the basis of Rayleigh-Ritz technique. Since the effect of temperature decreases the efficiency and durability of the structure, influence of bi-parabolic temperature variations on vibration of rectangular plate is analyzed. Tapering in rectangular plate is assumed bi-exponentially in *x*-and *y*-direction. Effect of non-homogeneity is discussed with exponential variation in density of the plate's material i.e. visco-elastic material "Duralium" which is an alloy of aluminum. Vibration modes of clamped plate are calculated for various values of thermal gradient, taper parameters and non-homogeneity constant. Results are explained with the help of the graphs.

***Keywords*: ** visco-elastic, clamped, frequency parameter, taper parameters

NOMENCLATURE

a	→ Length of rectangular plate	b	→ Breadth of rectangular plate
x, y	→ Coordinate in the plane of the plate	d	→ Thickness of the plate
d_0	→ Thickness of the plate at $x = y = 0$	Y	→ Young's modulus

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Y_0	→ Young's modulus at reference temperature	μ	→ Poisson ratio
$\widetilde{D}$	→ Visco-elastic operator	D_1	→ Flexural Rigidity
ρ	→ Mass density per unit volume of the plate's material	t	→ Time
$T(t)$	→ Time function	α_1	→ Non-homogeneity constant
$w(x, y, t)$	→ Deflection of plate	$\psi(x, y)$	→ Defection function
β_1&β_2	→ Taper parameters in x-& y-directions	α	→ Thermal gradient
τ	→ Temperature excess above the reference temperature at any point on the plate		
τ_0	→ Temperature excess above the reference temperature at $x = y = 0$		

1. INTRODUCTION

In modern technology, scientists and researchers are continuously worked on developing new materials which are necessary for the betterment of machines or systems of structures. In this connection, visco-elastic materials are vigorously used for their characteristics of lesser weight, high strength, high boiling point, more reliability etc. The analysis of free vibration of visco-elastic rectangular plate with clamped ends is the main concern in the field of mechanical, civil, and material engineering. Back in 1969, Leissa published a collection of research papers in his monograph, in which thermal effects were studied on different shapes of plate with various boundary conditions. In this monograph, he discussed vibrational properties for various combinations of clamped, simply-supported and free edge boundary conditions. Sobotka (1978) also discussed the free vibrations of orthotropic visco-elastic plates with various combinations of boundary conditions.

The equation of motion of a plate was usually expressed as a single partial differential equation. This partial differential equation was transformed into two ordinary differential equations that can be solved using Rayleigh-Ritz technique. Rajalingham et al. (1997) analyzed the vibration of rectangular plates by reducing the partial differential equation into simultaneous ordinary differential equations.

To control the noise-vibration of the plate, it is necessary to study the effects of tapering of plates. Cheung, Zhou (1999) studied the free vibration of rectangular plate by tapering of plate using Rayleigh-Ritz technique. Lal (2003) had been studied the transverse vibrations of orthotropic non-uniform rectangular plates with continuously varying density due to the non-homogeneity in the material of plate. Li (2004) analyzed the vibrational properties of rectangular plate with general elastic boundary support.

Khanna, Singhal (2015, 2016) discussed the free vibration of tapered rectangular plate with bi-directional effect of temperature. Hasheminejad and Shakeri (2017) investigated the transient vibro-acoustic response suppression of a piezo-coupled sandwich circular plate backed by a rigid-walled cylindrical acoustic enclosure. Kaur et al. (2018) studied the effect of non-homogeneity of material with the sinusoidal tapering of plate.

Here, authors investigated the effect of various structural parameters i.e. taper parameters, thermal gradient and non-homogeneity constant on vibration of thin isotropic rectangular plate. Tapering is considered bi-exponential and temperature variation is considered bi-parabolic.

Also variation in non-homogeneity is taken exponentially in x-direction. First two modes of frequency parameter at various values of structural parameters for clamped boundary condition are evaluated and presented in the form of graphs.

2. EQUATION OF MOTION

Fourth order differential equation of motion for isotropic rectangular plate in Cartesian coordinate is [10]:

$$\tilde{D}\left[D_1\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial y\,\partial x^2}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}\right)+2(1-\mu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right]/\,\rho d w=-\frac{\partial^2 T}{\partial t^2}/\,\tilde{D}T \quad \ldots (1)$$

Here flexural rigidity D_1 is defined as [12]:

$$D_1=\frac{Y\,d^3}{12(1-\mu^2)}\ .$$

Deflection can be considered as a product of two functions as [11]:

$$w(x,y,t)=\psi(x,y)\times T(t) \quad \ldots(2)$$

where $\psi(x,y)$ is deflection function of the plate & $T(t)$ is a time function.

On using the equation (2) in equation (1), authors obtain:

$$\tilde{D}\left[D_1\left(\frac{\partial^4 \psi}{\partial x^4}+2\frac{\partial^4 \psi}{\partial x^2\partial y^2}+\frac{\partial^4 \psi}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 \psi}{\partial x^3}+\frac{\partial^3 \psi}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 \psi}{\partial y^3}+\frac{\partial^3 \psi}{\partial y\,\partial x^2}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 \psi}{\partial x^2}+\mu\frac{\partial^2 \psi}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 \psi}{\partial y^2}+\mu\frac{\partial^2 \psi}{\partial x^2}\right)+2(1-\mu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 \psi}{\partial x\partial y}\right]/\,\rho d \psi=-\frac{\partial^2 T}{\partial t^2}/\,\tilde{D}T$$

The above equation is satisfied if both of its sides are equal to a constant i.e. p^2, we get:

$$\tilde{D}\left[D_1\left(\frac{\partial^4 \psi}{\partial x^4}+2\frac{\partial^4 \psi}{\partial x^2\partial y^2}+\frac{\partial^4 \psi}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 \psi}{\partial x^3}+\frac{\partial^3 \psi}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 \psi}{\partial y^3}+\frac{\partial^3 \psi}{\partial y\,\partial x^2}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 \psi}{\partial x^2}+\mu\frac{\partial^2 \psi}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 \psi}{\partial y^2}+\mu\frac{\partial^2 \psi}{\partial x^2}\right)+2(1-\mu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 \psi}{\partial x\partial y}\right]-\rho p^2 d\psi=0 \quad \ldots (3)$$

and $\ddot{T}+p^2\tilde{D}T=0$ $\quad \ldots(4)$

These are the differential equations of motion and time function for visco-elastic isotropic rectangular plate respectively.

3. ASSUMPTIONS

3. 1. Tapering of plate

Authors assumed that the thickness of the visco-elastic isotropic rectangular plate varies exponentially in both directions [3]:

$$d = d_0.\exp\left(\beta_1 \frac{x}{a}\right).\exp\left(\beta_2 \frac{y}{b}\right) \qquad \dots(5)$$

3. 2. Variations in Temperature

The main emphasize of the present study is to discuss the influence of temperature variation on the vibrational behavior of rectangular plate. Therefore authors assumed that the rectangular plate of visco-elastic material has a steady two dimensional i.e. bi-parabolic temperature variation as [12]:

$$\tau = \tau_0\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right) \qquad \dots(6)$$

The temperature dependence of the modulus of elasticity i.e. Young's modulus (Y) for most of engineering materials can be expressed as [14]:

$$Y = Y_0\left(1-\gamma\tau\right) \qquad \dots(7)$$

where Y_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of variation of Y and τ. Using equation (6), the Young's modulus variation (7) becomes

$$Y = Y_0\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right] \qquad \dots(8)$$

where, $\alpha = \gamma\,\tau_0 \;\left(0 \le \alpha < 1\right)$ is thermal gradient.

3. 3. Non-homogeneity in plate's material

Here authors considered that non-homogeneity is present in the density of the plate's material. It is characterized exponentially in x-direction as:

$$\rho = \rho_0.\exp\left(\alpha_1 \frac{x}{a}\right) \qquad \dots(9)$$

Using the values of Y and d from the equations (5) and (6), the flexural rigidity becomes

$$D_1 = \frac{Y_0\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right] d_0^{\,3}\left(\exp\left(\beta_1\frac{x}{a}\right)\right)^3\left(\exp\left(\beta_2\frac{y}{b}\right)\right)^3}{12\left(1-\mu^2\right)} \qquad \ldots(10)$$

3. 4. Description of a clamped rectangular plate

Boundary conditions are basically, the set of conditions specified for the behavior of the solution to a set of differential equation at the boundary of its domain. In this investigation, plate is assumed to be clamped at all four edges, so the boundary conditions are [4]:

$$\psi = \frac{\partial \psi}{\partial x} = 0 \qquad \text{at } x = 0,\ a$$

&

$$\psi = \frac{\partial \psi}{\partial y} = 0 \qquad \text{at } y = 0,\ b \qquad \ldots(11)$$

4. SOLUTION OF FREQUENCY EQUATIONS

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy (P_E) must be equal to the maximum kinetic energy (K_E). So it is necessary for the problem under consideration that [9]:

$$\delta\left(P_E - K_E\right) = 0 \qquad \ldots(12)$$

for arbitrary variations of ψ satisfying relevant geometrical boundary conditions.

The corresponding two-term deflection function is taken as [1]:

$$\psi = \left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2 \times \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \qquad \ldots(13)$$

where A_1 & A_2 are two arbitrary constants.

In order to make easy and convenient calculation, authors assumed few non-dimensional variables as [17]:

$$X = \frac{x}{a},\ Y = \frac{y}{a},\ \overline{\psi} = \frac{\psi}{a},\ \overline{d} = \frac{d}{a}$$

On using above non-dimensional variables in the expressions for kinetic energy K_E and strain energy P_E [7], authors get:

$$K_E = \frac{1}{2}\rho_0 p^2 \overline{d_0}\, a^3 \int_0^1\int_0^{b/a}\left[\left\{\exp\left(\alpha_1 X\right)\right\}.\exp\left(\beta_2\frac{aY}{b}\right)\overline{\psi}^{\,2}\right] dY\, dX \qquad \ldots(14)$$

and

$$P_E = \frac{Y_0 \overline{d_0}^3 a^3}{24(1-\mu^2)} \int_0^1 \int_0^{b/a} \left(\exp(\beta_1 X)\right)^3 \times \left(\exp\left(\beta_2 \frac{aY}{b}\right)\right)^3 \times \left[\left(\overline{\psi},_{XX}\right)^2 + \left(\overline{\psi},_{YY}\right)^2 + 2\mu \overline{\psi},_{XX} \overline{\psi},_{YY} + 2(1-\mu)\left(\overline{\psi},_{XY}\right)^2 \}\right] dY\, dX \quad \ldots(15)$$

Using equations (14) & (15) in equation (12), one gets

$$\delta(P_E * -\lambda^2 K_E *) = 0 \quad \ldots(16)$$

where

$$P_E * = \int_0^1 \int_0^{b/a} (1+\beta_1 X)^3 \left(1+\beta_2 \frac{aY}{b}\right)^3 \left[\left(\overline{\psi},_{XX}\right)^2 + \left(\overline{\psi},_{YY}\right)^2 + 2\mu\, \overline{\psi},_{XX} \overline{\psi},_{YY} + 2(1-\mu)\left(\overline{\psi},_{XY}\right)^2 \}\right] dY\, dX \quad \ldots(17)$$

and

$$K_E * = \int_0^1 \int_0^{b/a} \left[(1+\beta_1 X)\left(1+\beta_2 \frac{aY}{b}\right)\overline{\mu}^2\right] dY\, dX \quad \ldots(18)$$

Here, $\lambda^2 = \frac{12\rho p^2 (1-\mu^2)}{Y_0 \overline{d_0}^2}$ is a frequency parameter.

Equation (16) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of ψ.

These two constants are to be determined as follows

$$\frac{\partial}{\partial A_n}\left(P_E * -\lambda^2 K_E *\right) = 0, \qquad n = 1, 2 \quad \ldots(19)$$

On simplifying (19), one gets

$$b_{n1} A_1 + b_{n2} A_2 = 0, \qquad n = 1, 2 \quad \ldots(20)$$

where b_{n1} & b_{n2} $(n = 1, 2)$ involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (20) must be zero. So one gets:

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \quad \ldots(21)$$

With the help of equation (21), one can obtain a quadratic equation in λ^2 from which the two values of frequency parameter for both the modes of vibration can be calculated easily.

5. RESULTS AND DISCUSSION

For calculating the values of frequency parameter (λ) for a rectangular plate with different values of thermal gradient (α), non-homogeneity constant (α_1), taper parameters (β_1 and β_2) and aspect ratio (a/b) for first two modes of vibrations, authors considered that the rectangular plate is made up of a visco-elastic material i.e. Duralumin which is for an alloy of aluminum and following material parameters are used [13]:

$Y_0 = 7.08*10^{10}\ N/M^2$, $\rho_0 = 2.80*10^3\ Kg/M^3$, $\mu = 0.345$ and $d_0 = 0.01M$.

Results for both the modes of frequency parameters are calculated at various values of non-homogeneity constant, thermal gradient, taper constants and aspect ratio. These numeric results for frequency parameter are shown and explained with the help of the graphs as follows:

5. 1. Frequency Parameter Vs Non-homogeneity constant (α_1)

Figure 1(a) and 1(b) show the behavior of frequency parameters with non-homogeneity constant (α_1) for fixed values of taper parameters ($\beta_1 = \beta_2 = 0.0$), aspect ratio ($a/b = 1.5$) at different values of thermal gradient *i.e*. (*i*) $\alpha = 0.0$, (*ii*) $\alpha = 0.3$ and (*iii*) $\alpha = 0.6$.

From these figures, it is clear that frequency parameter for both the modes of vibration are decreasing continuously as non-homogeneity constant α_1 increases from 0.0 to 0.2. Also, it is observed that frequency parameter for both the modes of vibration decrease rapidly with increasing values of thermal gradient.

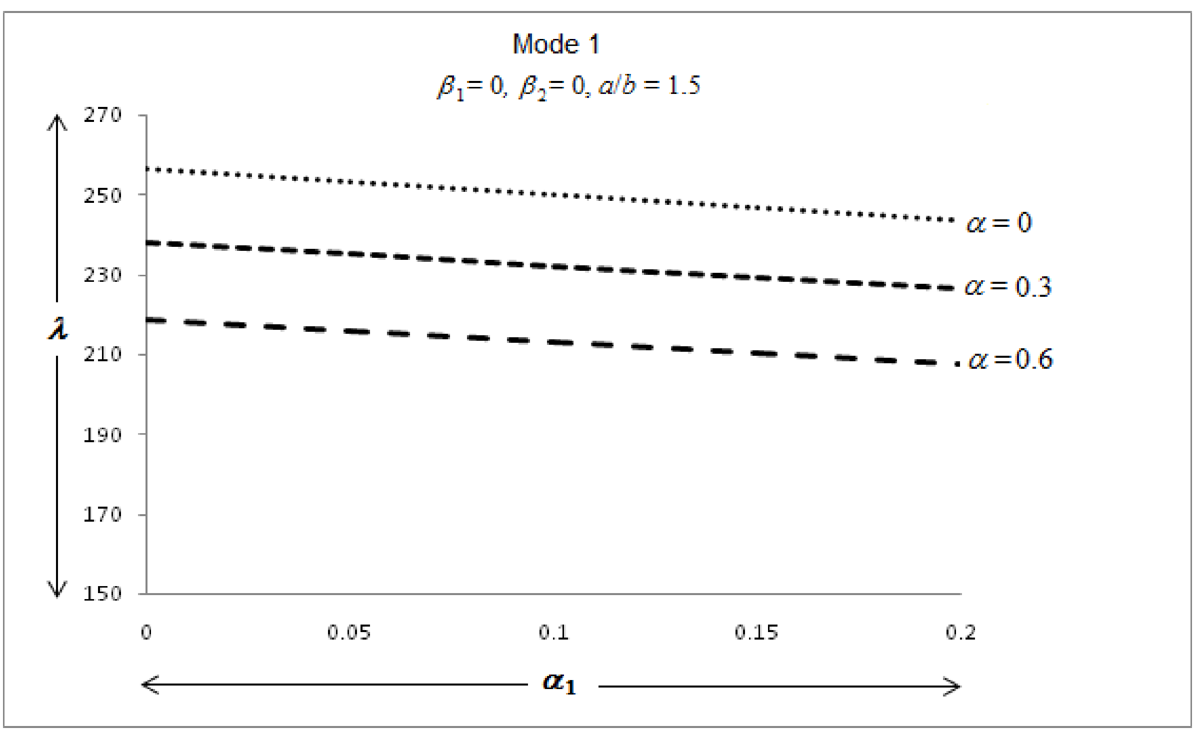


Figure 1(a). Frequency Parameter (Mode 1) vs Non-homogeneity Constant (α_1)

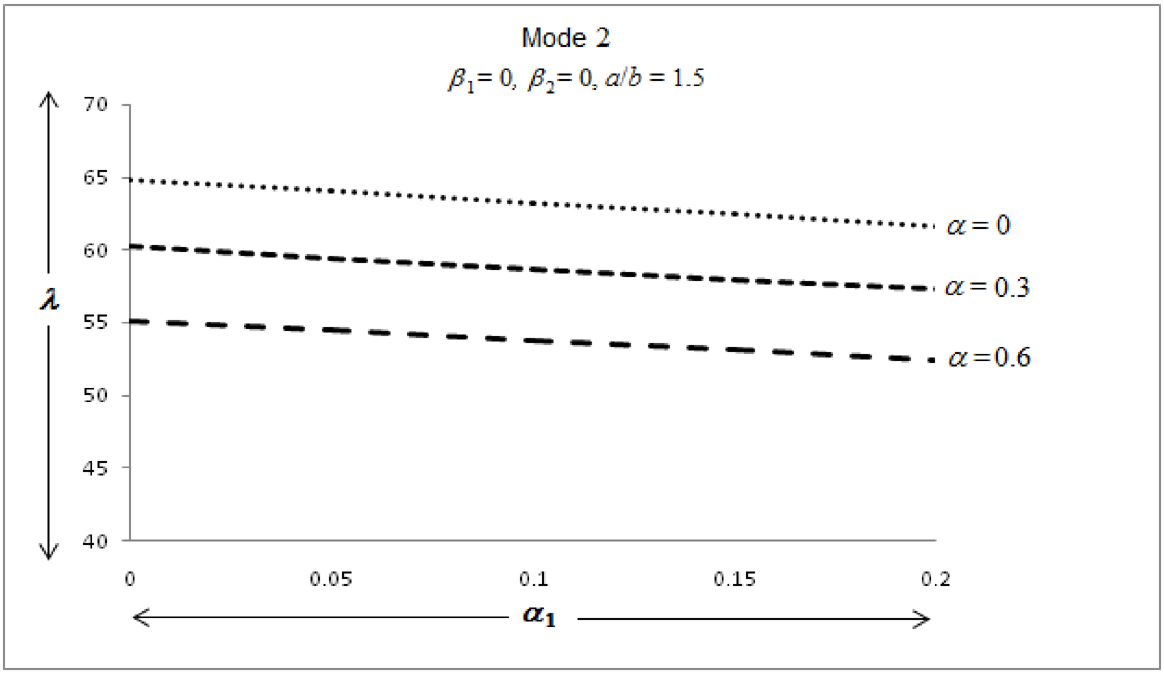


Figure 1(b). Frequency Parameter (Mode 2) vs Non-homogeneity Constant (α_1)

5. 2. Frequency Parameter Vs Taper parameters (β_1 & β_2)

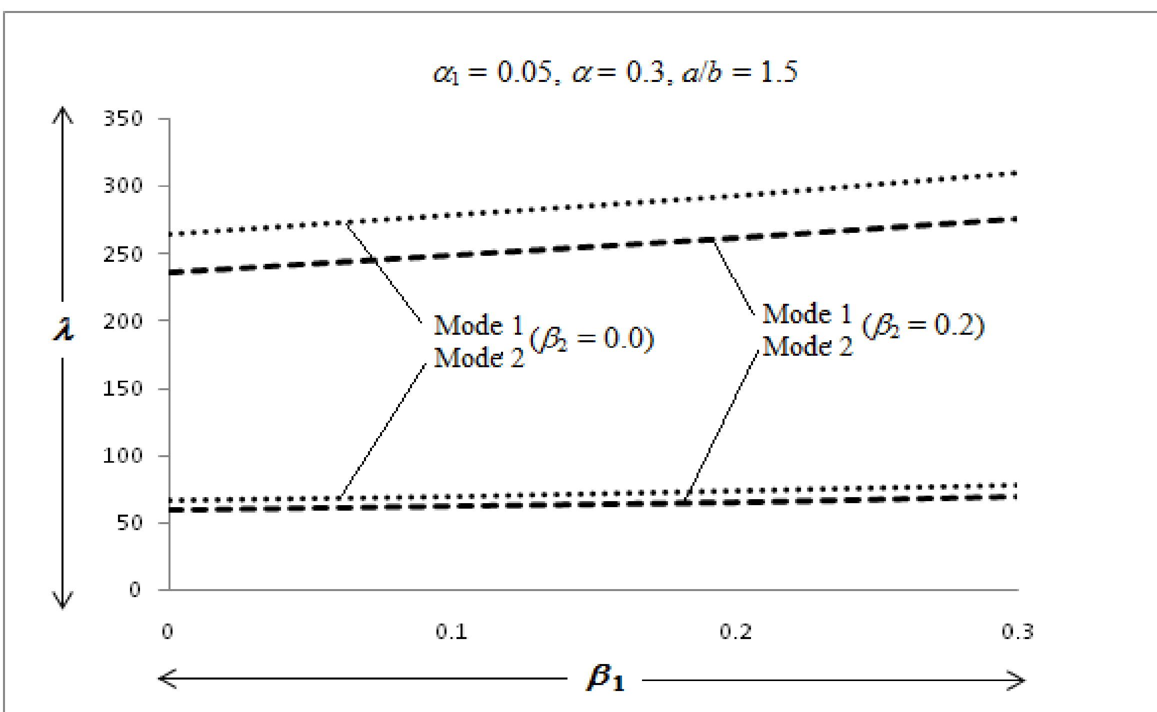


Figure 2(*a*). Frequency Parameter (λ) versus Taper Constant (β_1)

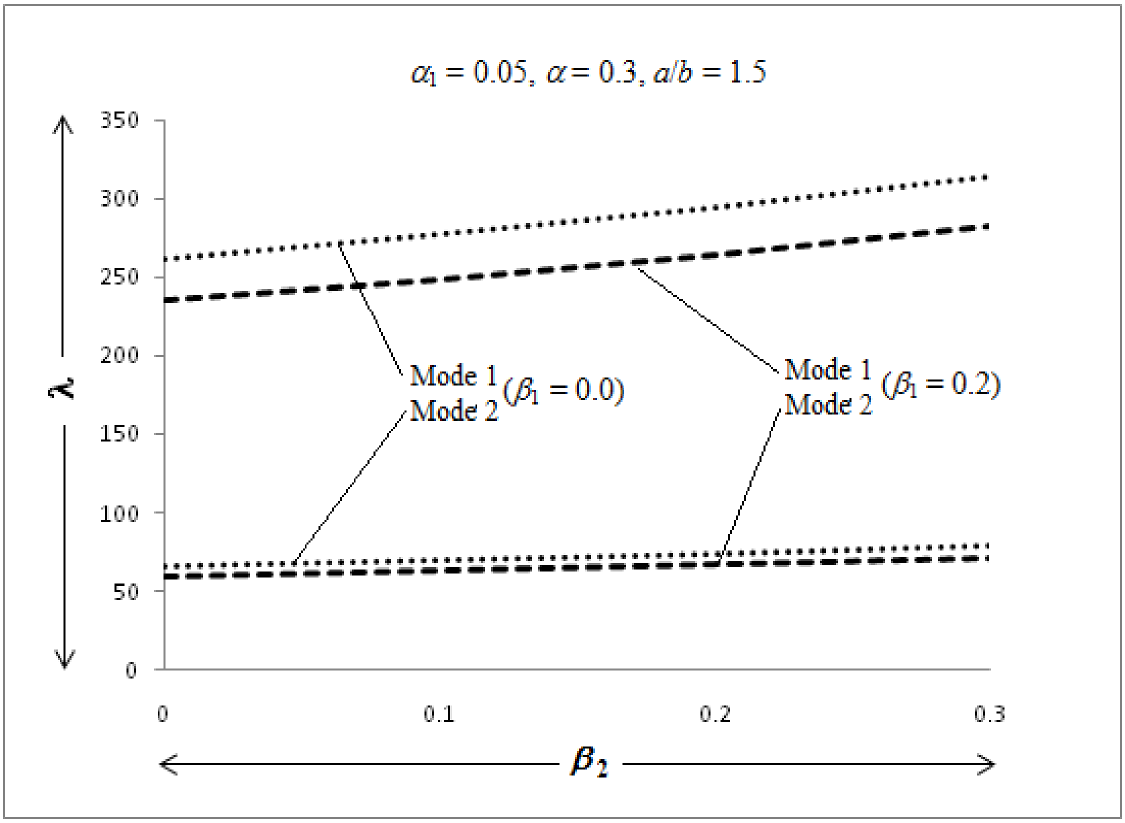


Figure 2(*b*). Frequency Parameter (λ) versus Taper Constant (β_2)

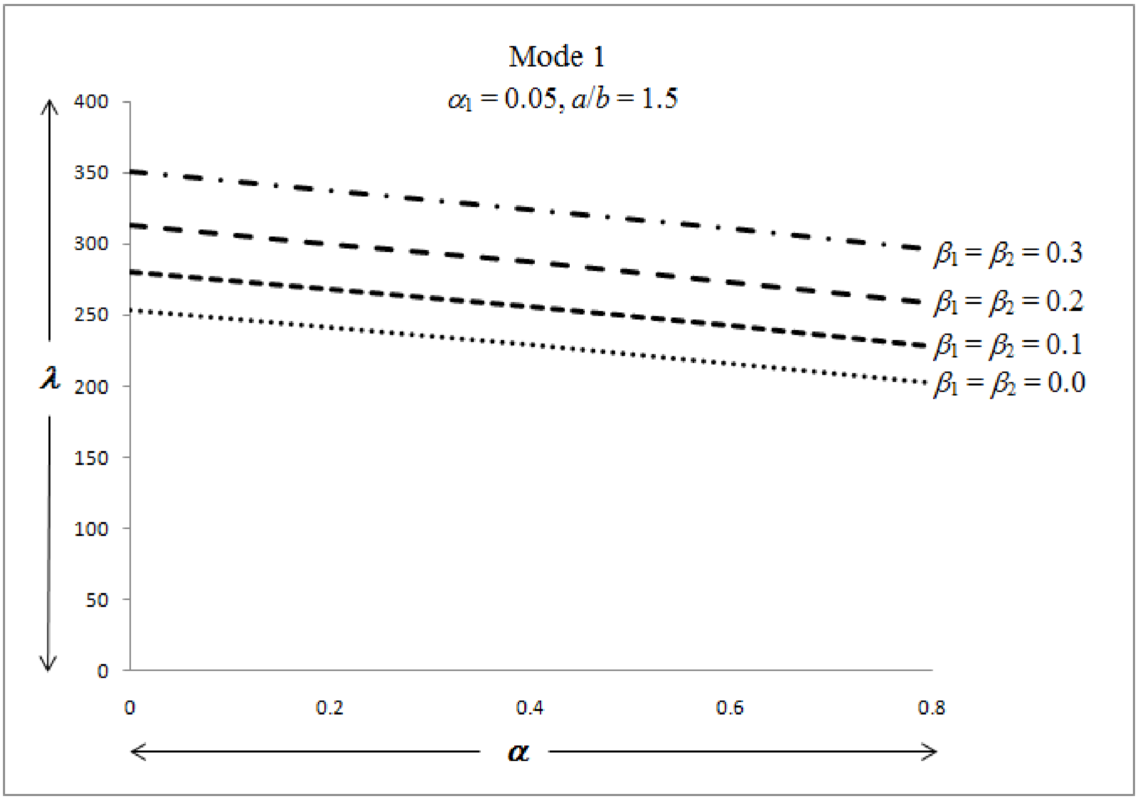
Figures 2(a) and 2(b) represent the variations in frequency parameters with respect to β_1 and β_2 respectively. From Figures 2(a) and 2(b), authors conclude that frequency parameter increases continuously as taper parameters β_1 and β_2 increases from 0.0 to 0.3 respectively for the fixed values of thermal gradient, non-homogeneity constant and aspect ratio ($\alpha_1 = 0.05$, $\alpha = 0.3$ & $a/b = 1.5$).

Also values of frequency parameters for zero taper parameters are higher than non-zero taper parameters. It shows that how varying thickness may affects the vibration of plates or structures.

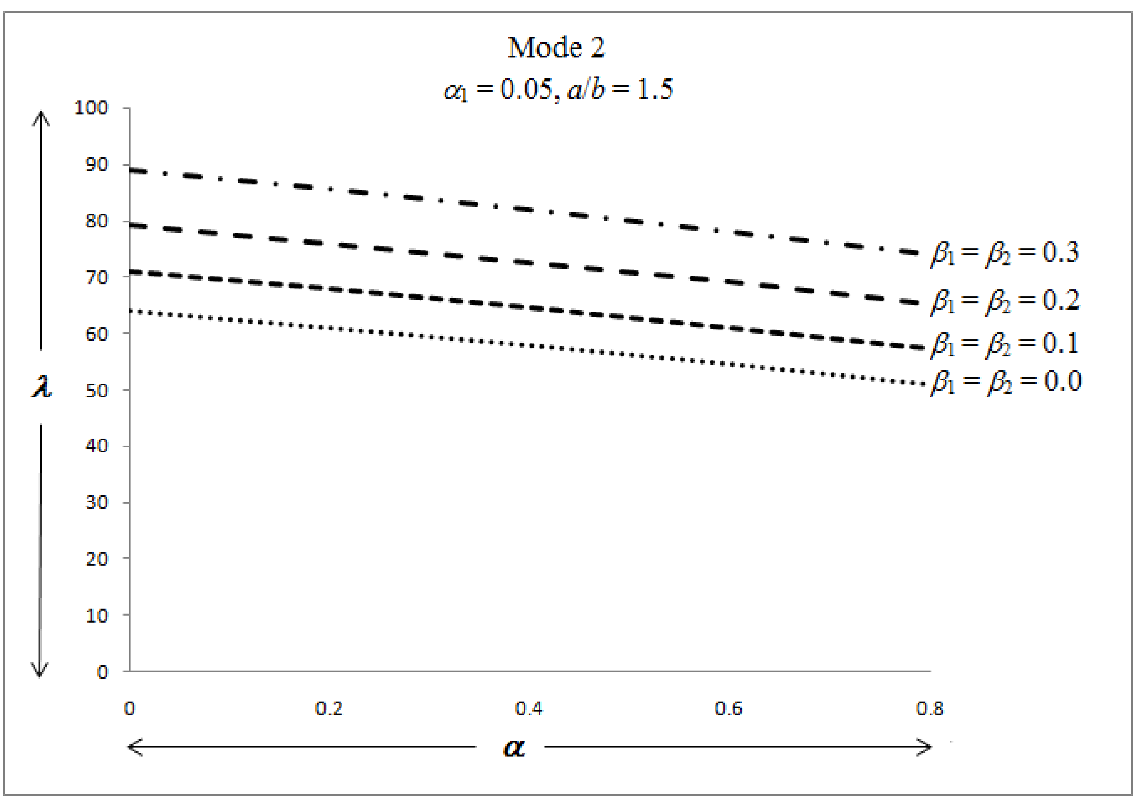
5. 3. Frequency Vs Thermal gradient (α)

Figure 3 [(a), (b), (c) and (d)] shows the numeric results of frequency parameters with thermal gradient for different combinations of taper parameters *i.e.* (i) $\beta_1 = \beta_2 = 0.0$ (ii) $\beta_1 = \beta_2 = 0.1$, (iii) $\beta_1 = \beta_2 = 0.2$, (iv) $\beta_1 = \beta_2 = 0.3$ and two different combinations of non-homogeneity constant (i) $\alpha_1 = 0.05$ and (ii) $\alpha_1 = 0.1$.

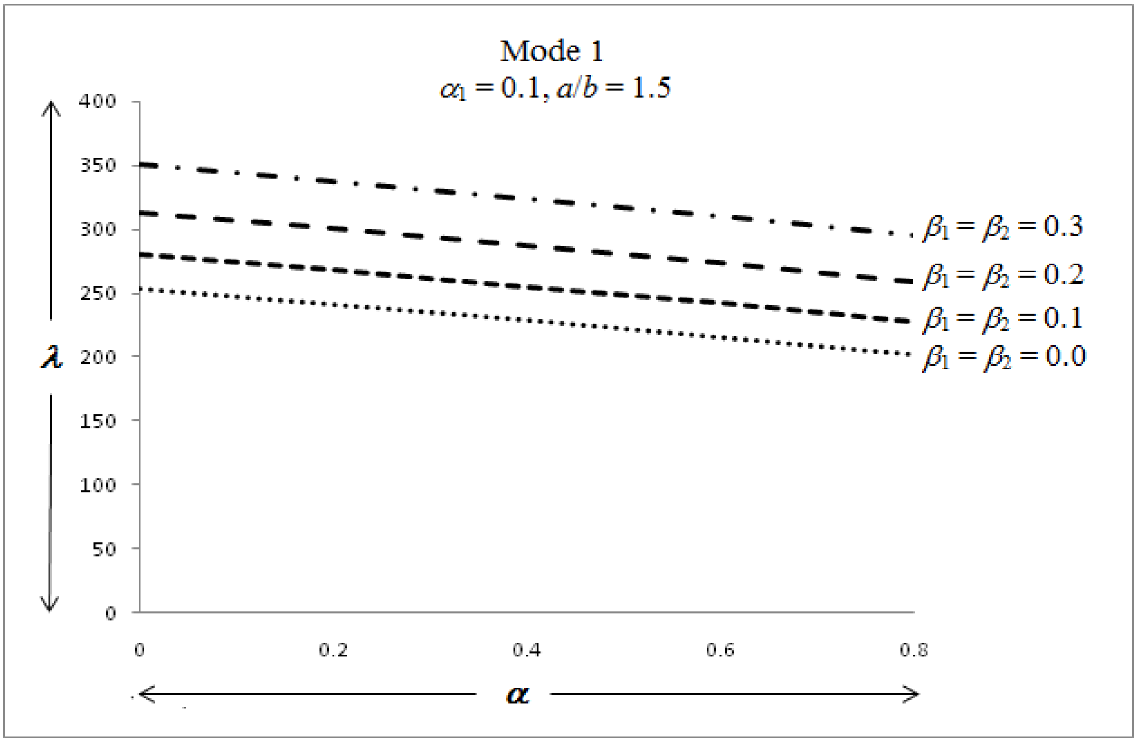
Authors can easily conclude that frequency continuously decreases as thermal gradient α increase from 0.0 to 0.8 for first two modes of vibrations at each paired values of taper parameters. Also it is clearly seen that frequency parameter (both modes) has less value for uniform thickness as compared to non-uniform thickness of the rectangular plate.


Mode 1
α1 = 0.05, a/b = 1.5
400
350
300
250
λ
200
150
100
50
0
0
0.2
0.4
0.6
0.8
α
β1 = β2 = 0.3
β1 = β2 = 0.2
β1 = β2 = 0.1
β1 = β2 = 0.0

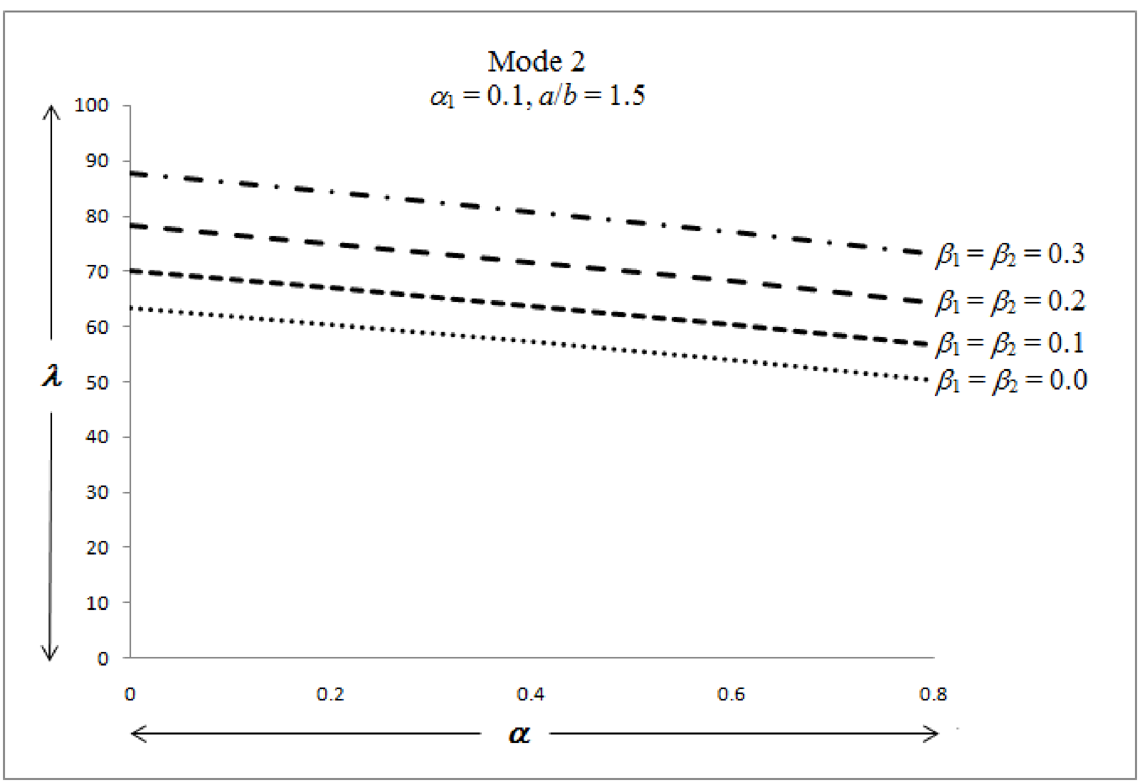
(a)


Mode 2
α1 = 0.05, a/b = 1.5
100
90
80
70
60
λ
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
α
β1 = β2 = 0.3
β1 = β2 = 0.2
β1 = β2 = 0.1
β1 = β2 = 0.0

(b)



(*c*)



(*d*)

Figure 3(a-d). Frequency Parameter (λ) versus Thermal Gradient (α)

5. 4. Frequency Vs Aspect ratio (*a*/*b*)

From Figure 4(*a*) and 4(*b*), one can clearly observe that frequency increases continuously as aspect ratio increases from 0.5 to 2.5 for different values of thermal gradient α, non-homogeneity constant α_1 and taper parameters β_1 & β_2 for both the first two modes of vibrations for the following cases:

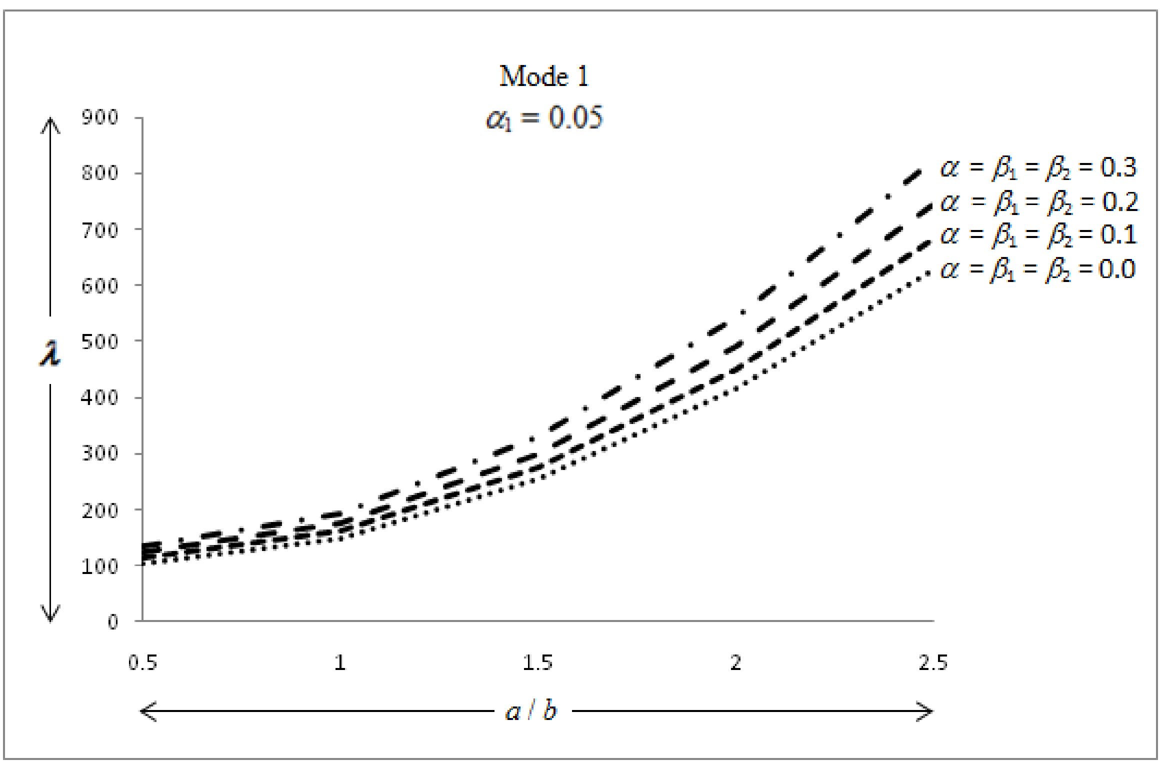
(i) $\alpha = \beta_1 = \beta_2 = 0.0$, $\alpha_1 = 0.05$

(ii) $\alpha = \beta_1 = \beta_2 = 0.1$, $\alpha_1 = 0.05$

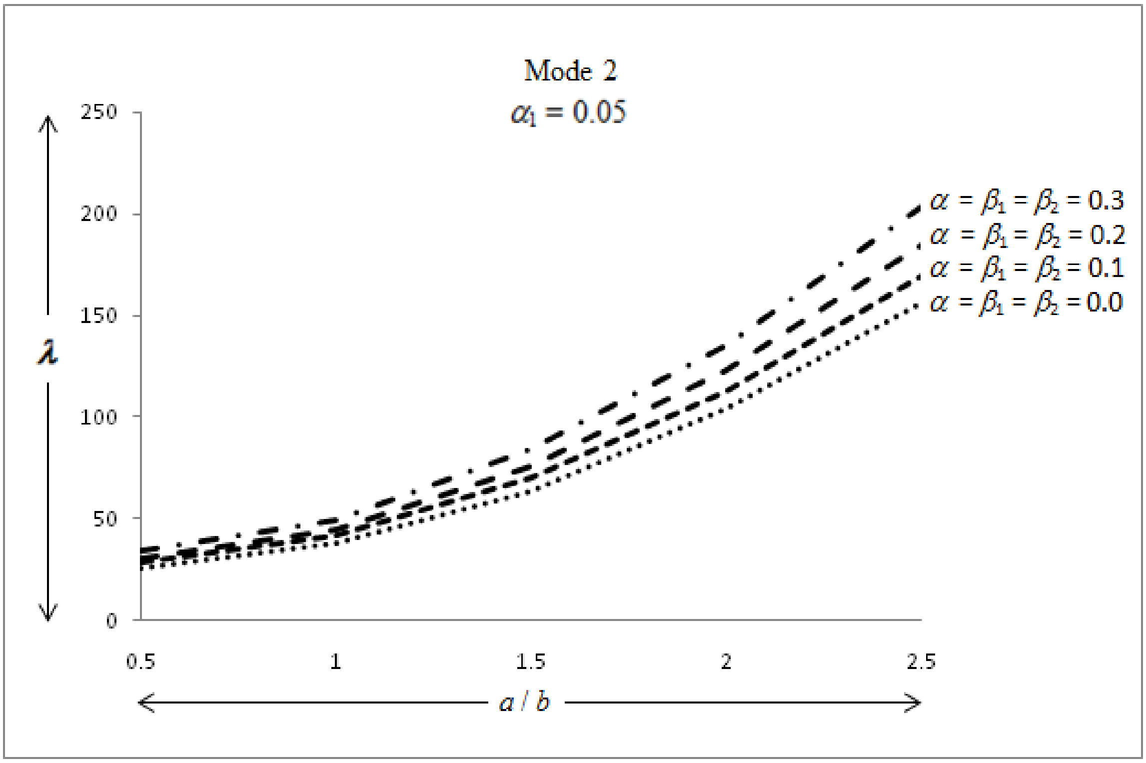
(iii) $\alpha = \beta_1 = \beta_2 = 0.2$, $\alpha_1 = 0.05$

(iv) $\alpha = \beta_1 = \beta_2 = 0.3$, $\alpha_1 = 0.05$

It can be observed that as the combined values of β_1 & β_2 increases from 0.0 to 0.3 for fixed value of $\alpha_1 = 0.05$, both the modes of frequency parameters increase rapidly. It proves that variation in aspect ratio changes the vibrational properties of the plates.



(*a*)



(b)

Figure 4(a,b). Frequency Parameter (λ) versus Aspect Ratio (a/b)

6. COMPARISON & CONCLUSIONS

Authors compared the influence of temperature variations i.e. bi-parabolic in present paper with [12] i.e. linear in x-direction and parabolic in y-direction. For zero taper parameters and zero non-homogeneity constant, it is observed that frequency for both the modes of vibration are lesser in the present paper than [12]. It proves that influence of temperature, even in one direction, may directly affect the vibrational characteristics of plate.

On the behalf of above results and discussion, authors concluded the findings of present paper as follows:

i) For fixed values of structural parameters, frequency parameter (both the modes) decreases as thermal gradient increases.

ii) Frequency parameter (both the modes) is maximum for zero value of thermal gradient i.e. when there is no influence of temperature, which shows that it is necessary to investigate the influence of temperature for getting authentic data so that scientists and researchers may use the data for making authentic and better structures or machines.

iii) It is evident that frequency parameter (both the modes) continuously increases with increasing values of taper parameters for corresponding values of structural parameters. It indicates that practitioners or mechanical engineers may control the frequency or vibrational properties of the structure by appropriate tapering.

iv) Non-homogeneity of the plate's material directly affects the vibration of the plate. As non-homogeneity constant increases, frequency parameters decrease continuously for corresponding values of other parameters.

v) As aspect ratio increases from 0.5 to 2.5, frequency parameters for both the modes of vibration rapidly increase as shown in figure 5.4 which directly shows that size of the rectangular plate i.e. aspect ratio matters for active vibration control of the plate.

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