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SHORT COMMUNICATION

Geometric Programming in the Design of Standard Laboratory for Students' Practical Work

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ABSTRACT

In this paper, we applied Geometric programming in the design of a standard laboratory for students' practical work in the Federal University of Technology Owerri. The laboratory has a capacity of 1,357 seats thereby containing 1357 students with the following dimensions: design length is 644.3m; design width is 337.8m and design height is 217.7m. The laboratory would cost the university authority a minimum of 915,875.2 naira to construct.

***Keywords*:** Geometric programming, Design length, Design width, Design height, Optimal objective function

1. INTRODUCTION

Geometric programming (Gp) belongs to a class of non-linear optimization problem characterized by objective and constraint functions having the same form. The basic approach

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in Gp modeling is to attempt to express a practical problem, such as an engineering analysis or design problems in Gp format. GP modeling involves some knowledge, as well as creativity, to be done effectively, see [1]. Geometric Programming derives its name from the role arithmetic-geometric inequality played at the early development of the program,[2]. Posynomial is a polynomial with both positive cost coefficients and primal decision variables, when the posynomial is optimized, we have Gp. When the number of term in the Gp is one, it is called monomial and when the number of terms in the Gp is more than one, it is called posynomial. Hence, another name for Geometric programming is Posynomial programming.

The function $f(x)=\sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{a_{ij}}$ is called an unconstrained posynomial, where $C_j > 0$; $x_i > 0$; $a_{ij} \in R$; n is the total number of terms in the posynomial; m is the total number of variables in the posynomial; C_j is the cost coefficient of the posynomial; x_i is the variables in the posynomial; but when the function is optimized, x_i becomes the primal decision variables; a_{ij} is the exponent matrix. Geometric (posynomial) programming broadens the scope of linear programming applications and it is suited to model several types of important non-linear systems in physical sciences and engineering. Geometric programming is very flexible and has been extended to different fields different from its original conceptions.

Equation (1) is the unconstrained Geometric programming model

$$\text{Minimize } f(x)=\sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{a_{ij}} \tag{1}$$

where

$C_j > 0$; $x_i > 0$; $a_{ij} \in R$
n = number of terms in the geometric program
m = number of variables in the geometric program
C_j = coefficient of the geometric program
x_i = primal decision variables in the geometric program
a_{ij} = elements of exponent matrix in the geometric program whose entries are real number.

The dual form of Geometric programming is given as:

$$\text{Maximize } f(y)=\prod_{k=0}^{m}\prod_{j=1}^{n_k}\left(\frac{C_{kj}}{y_{kj}}\sum_{j=1}^{N_k} y_{kj}\right)^{y_k} \tag{2}$$

Subject to the normality and orthogonality conditions given as:

$$\sum_{j=1}^{n_0} y_{0j} = 1 \tag{3}$$

$$\sum_{k=0}^{m}\sum_{j=1}^{n_k} a_{kij}\, y_{kj} = 0 \tag{4}$$

In geometric programming, the quantity K = (n - m+1) is termed the degree of difficulty. The degree of difficulty is the measure of computational complexity of geometric program. If n = m+1, the problem is said to have a zero degree of difficulty. In this case, the unknown y_j (j=1, 2,…,n) can be determined uniquely from the orthogonality and normality condition and the problem is said to have a unique solution. We maximize the dual program whenever the degree of difficulty if greater than zero, that is, when the solution is not unique. But one can either maximize the dual or minimize the primal function since at optimality both yield the same optimal solution, [3]. A set of non-homogenous system of linear equations are obtained from the necessary and sufficient conditions for optimality. These conditions are called orthogonality and normality conditions. Whenever the degree of difficulty is zero n = (m+1)), the solution becomes easier because the resulting matrix from the orthogonality and normality condition is a square and can be invertible. But in the case where the degree of difficulty is greater than zero (n > (m+1)), the resulting matrix is rectangular and cannot ordinarily be invertible; hence the dual decision variable is no longer unique. Here, the dual objective function of the geometric programming is maximized subject to orthogonality and normality conditions (linear constraints), [4]. Geometric programming (Gp) has wide applications, even in marketing, [5]. This shows the extent to which Gp can be applied in different fields of study.

2. MATERIAL

Several engineering problems were modeled using geometric programming, [1]. Some other authors worked on reversed constraint of geometric programming. They introduced signum function in the dual geometric program; see [6]. The signum function takes care of reversed constraint as well as the conventional constraints in geometric programming. The signum function assumes positive one (+1) if the constraint equation is bounded above by unity, but assumes negative one (-1) if the constraint equations are bounded below by unity. This allows the model to accommodate many more physical and engineering problems.

Geometric programming was applied in the modeling of solid waste product by some authors, see [3]. From their study, they were able to obtain the optimal monthly expenditure on solid waste disposal in Enugu metropolis by the government. They also obtained the contribution of different classes of solid waste products to the optimal objective function. A work was also done on geometric programming problems whose cost coefficients are allowed to take negative values. With complementary program, such problem can be resolved, see [4].

An augmented program which converts greater than zero degrees of difficulty program to a zero degree of difficulty problem was founded by [7]. The zero degree of difficulty problem has a unique solution. The principle method applied by this researcher was to expand the objective function and introduce monomial in the objective function and augmenting the inverse of the monomial in the constraint equation. This method has contributed immensely to the development and solution to complex Gp problems.

An author did some work on geometric programming problem with one degree of difficulty, see [8]. He applied the exact method to determine the optimal dual decision variables. This method was able to solve a one degree of difficulty geometric programming problem directly without transformation. Others worked on the primal geometric programming, see [9]. They converted primal geometric program to a linear program. They used transformed problem to derive optimality condition and with this technique, the solution to a greater than zero degree of difficulty geometric programming problem could be approximated as closely as possible. Another group of authors worked on the dual geometric programming and they used separable programming, see [10] to arrive at optimal solution. Their approach was intended to overcome the computational difficulties arising from a greater than zero degree of difficulty problem in Gp problems.

3. METHOD

Geometric programming will be used to develop a model for construction of standard laboratory for students' practical work in the Federal University of Technology Owerri. We shall determine the optimal dual decision (design) variables, the optimal objective (cost) function and the contribution of each of the primal decision (design) variables to the optimal objective function. In this paper, our interest is to determine the dimension of the building and the minimum cost required to construct it.

A. Determination of the Optimal Weights of the Dual Decision Variables y*.

We shall apply the Moore (1935) [11] and Penrose (1955) [12], put together as the Moore-Penrose generalized inverse to determine the positive optimal weights, y*, of the dual decision variables of the geometric program. The following steps shall be followed, from the orthogonality and normality conditions, see equations (3) and (4), we have:

$$A_{mxn}.y_{nx1} = B_{mx1} \tag{5}$$

$$y^{*} = A^{-}B \tag{6}$$

where y^{*} is the vector of optimal weight of the dual decision variables, A^{-} is the Moore-Penrose generalized inverse and B is the vector of constants.

B. Determination of the Optimal Objective function of the Geometric Program (f*(x)).

We shall apply the determined optimal weights of the dual decision variables y^{*} to determine the optimal value of the objective function from the stationery point thus;

$$f^{*}(x) = f^{*}(y) = \prod_{k=0}^{m}\prod_{j=1}^{N_k}\left(\frac{C_{kj}}{y_{kj}^{*}}\sum_{i=1}^{N_k} y_{kj}^{*}\right)^{y_{kj}^{*}} \tag{7}$$

where $f^{*}(x)$ is the optimal value of the objective function.

C. Determination of the optimal weight of the primal decision variables (x*)

We shall apply the optimal value of the objective function and the optimal weights of the dual decision variables with the relationship that exist between them and the original geometric program to determine the optimal weights of the primal decision variables as follows:

$$C_j \prod_{i=1}^{m} (x_i)^{a_{ij}} = y^*{}_j f^*(x) . \quad (8)$$

$$\frac{y_j * f * (x)}{C_j} = (x_1 *)^{a_{1j}} (x_2 *)^{a_{2j}} \ldots (x_n *)^{a_{nj}} .$$

$$\ln\left[\frac{y_j * f * (x)}{C_j}\right] = a_{1j} \ln x_1 * + a_{2j} \ln x_2 * + \ldots + a_{nj} \ln x_n *$$

$we\ let\ w_i = \ln x_1 *$

we have

$$\ln\left[\frac{y_j * f * (x)}{C_j}\right] = a_{1j} w_1 + a_{2j} w_2 + \ldots + a_{nj}$$

$$x_i * = e^{w_i} \quad ; i = 1, 2, \ldots, m \ ; \quad w_i \in R \quad (9)$$

where $x_i *$ are the optimal weights of the primal decision variables of the geometric program.

4. METHOD OF DATA COLLECTION

In this paper we used both primary and secondary method of data collection. We used secondary method in collecting data for the floor, wall length and width. We used primary method in collecting data on the standard spacing for each student, which was 4m × 4m.

5. DATA PRESENTATION AND ANALYSIS

A. Data Presentation

Table 1. Estimated Cost of Constructing the Floor

S/N0	ITEMS	PRICE
1	Cement	400,000
2	Block	200,000
3	Labour	400,000
4	Rods	280,000
	TOTAL	1,780,000

Table 1 presents the estimated cost of constructing the floor of the laboratory, Table 2 presents the estimated cost of constructing the wall length and Table 3 presents the estimated cost of constructing the wall width. Tables 1 - 3 present the data for constructing the floor and the walls.

Table 2. Estimated Cost of Constructing Wall Length.

S/N0	ITEMS	PRICE
1	Cement	200,000
2	Block	300,000
3	Rods/Labour	200,000
	TOTAL	700,000

Table 3. Estimated Cost of Constructing Wall Width

S/N0	ITEMS	PRICE
1	Cement	120,000
2	Block	250,000
3	Rods/Labour	50,000
	TOTAL	312,000

B. Data Analysis

Here, we used the data in tables 1 to 3 to estimate the dimensions of the desired standard laboratory for the student' practical work in FUTO. Let the dimensions be x_1, x_2 and x_3 respectively and f(x) be the objective function. Our interest is to minimize the cost of constructing the building subject to linear constraints. Hence, we have

$$\min f(x) = c_1 \text{floor} + c_2 \text{Sides} + c_3 \text{Ends}$$

$$\text{Subject to } \; Ay = B$$

That is

$$\min f(x) = c_1[x_1 * x_2] + c_2[2(x_1 * x_3)] + c_3[2(x_2 * x_3)]$$
$$\min f(x) = 1{,}780{,}000\; [x_1 * x_2] + 700{,}000\; [2(x_1 * x_3)] + 312{,}000\; [2(x_2 * x_3)]$$

and

$U_1(x) = 1{,}780{,}000x\ _1x_2;\quad U_2(x) = 1{,}400{,}000x\ _1x_3;\quad U_3(x) = 624{,}000x\ _2x_3$

$C_j > 0;\ x_1, x_2, x_3 > 0$

Hence, the model becomes

$$\text{Minimize}\ \ f(x) = 1{,}780{,}000x\ _1x_2 + 1{,}400{,}000x\ _1x_3 + 624{,}000x\ _2x_3 \tag{10}$$

The model in equation (10) is the same as the model in equation (1).

Since at optimality, minimum of f(x) = maximum of f(y), we maximize the dual objective function of equation (2), subject to orthogonality and normality conditions of equation (5).

Therefore, we have

$$f^*(x) = f(y^*) = \prod_{k=0}^{m}\prod_{j=1}^{n_k}\left(\frac{C_{kj}}{y_{kj}^*}\sum_{i=1}^{n_k} y_{kj}^*\right)^{y_{kj}^*}$$

Subject to $A_{mxn}.y_{nx1} = B_{mx1}$

where f*(x) = f*(y) are the optimal objective functions respectively.

Forming orthogonality and normality conditions from the exponent matrix of equation (10) and writing it in the form of equation (5), we have

$y_1 + y_2 + 0y_3 = 0$

$y_1 + 0y_2 + y_3 = 0$

$0y_1 + y_2 + y_3 = 0$

$y_1 + y_2 + y_3 = 1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

From equation (6), we have

```
>> A = [1, 1, 0; 1, 0, 1; 0,1,1; 1, 1, 1];
>> B = [0; 0; 0; 1];
y* = Pinv(A)*B
```

$y^* = y_1 = y_2 = y_3 = 142.9$

Solving for equation (10), we have

$$f^*(y)=\left[\left(\frac{1,780,000}{142.9}\right)^{\wedge}142.9\right]*\left[\left(\frac{1,400,000}{142.9}\right)^{\wedge}142.9\right]*\left[\left(\frac{624,000}{142.9}\right)^{\wedge}142.9\right]=915875.2$$

From equation (8), we solve for optimal primal decision (design) variables

$$C_j\prod_{i=1}^{m}(x_i)^{a_{ij}}=y_j^*f^*(x)$$
$$142.9*915875.2=1,780,000x_1x_2$$
$$142.9*915875.2=1,400,000x_1x_3$$
$$142.9*915875.2=624,000x_2x_3$$
$$0.07353=x_1x_2$$
$$0.093485=x_1x_3$$
$$0.20974=x_2x_3$$

Taking ln of both sides, we have

$$-2.6100=\ln x_1+\ln x_2$$
$$-2.3670=\ln x_1+\ln x_3$$
$$-1.5619=\ln x_2+\ln x_3$$

From equation 5, we have

$$\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix}=\begin{bmatrix}-2.6100\\ -2.3670\\ -1.5619\end{bmatrix}$$

```
>> A = [1, 1, 0; 1, 0, 1; 1, 0, 1];
>> B = [-2.6100; -2.3670; -1.5619];
W* = Pinv(A)*B
```

$$W^*=\begin{bmatrix}-1.5248\\ -1.0852\\ -0.4396\end{bmatrix}$$

From equation (9), we have

$$x^*(km)=\exp(w^*)=\begin{bmatrix}0.2177\\ 0.3378\\ 0.6443\end{bmatrix}$$

Hence, the dimensions of the building are: length = x_1 = 644.3m; width = x_2 = 337.8m and height = x_3 = 217.7m. But the spaces between one sit to another is 4m by 4m, then one seat will occupy a floor area: Area = a = 4^2 = $16m^2$. Again, the laboratory has a floor area of 644.3m × 337.8m = A = $21712.91m^2$

Therefore, the hall has a capacity of:

$$\text{Capacity} = \frac{A}{a} = \frac{21712.91m^2}{16m^2} = 1{,}357 \text{ students}$$

The hall has a capacity of 1357 seats and therefore will contain 1357 students.

6. DISCUSSION AND CONCLUSION

In this paper, we apply Geometric Programming in the design of a standard laboratory for students' practical work in the Federal University of Technology Owerri (FUTO). We designed a standard laboratory of 1357 capacity with the following dimensions: length = x_1 = 644.3m; width = x_2 = 337.8m and height = x_3 = 217.7m. The laboratory will cost the university authority a minimum of 915,875.2 naira to construct. Our model did not take care of the roofing of the building.

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