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Further Study on Thermal Performance of Porous Fin with Temperature-Dependent Thermal Conductivity and Internal Heat Generation using Galerkin's method of Weighted Residual

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ABSTRACT

This work is presented as a further study to our previous work, "*Thermal performance analysis of a natural convection porous fin with temperature-dependent thermal conductivity and internal heat" published in "Thermal Science and Engineering Progress. 1 (2017) 39–52*", where it was assumed that the surface convection is negligible and heat is transferred only by natural convection in the porous fin. In this present study, such an assumption has been relaxed. Also, effects of surface convective heat transfer on the thermal performance of porous fin with temperature-dependent thermal conductivity and internal heat generation have been investigated using Galerkin's method of weighted residual. The results of the Galerkin's method of weighted residual show excellent agreement with the results of numerical method using shooting method coupled with Runge-Kutta method and also with the results of homotopy perturbation method. Thereafter, the developed analytical solutions are used to investigate the influences of the thermal model parameters on the thermal performance of the porous fin. It is found as the with the other model parameters that as the convective parameter increases, the rate of heat transfer from the base of the fin increases and consequently, the porous fin efficiency improves. However, increase in the nonlinear thermal conductivity parameter decreases the temperature distribution in the fin. Based on the high accuracy of the Galerkin's method of weighted residual as displayed in this work, it is hoped that the simple analytical solutions given by the approximate analytical method will enhance the analysis of extended surfaces and also assist the designers.

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***Keywords*:** Porous Fin, Surface convective heat transfer, Thermal performance, Temperature-Dependent Thermal Conductivity and Internal Heat Generation, Galerkin's method of weighted residual

1. INTRODUCTION

The vast areas of applications of porous fins as heat transfer enhancers have provoked many research interests in recent times. Indisputably, different studies on thermal performance of porous fins have been carried out in the past few decades following the pioneer work of Kiwan and Al-Nimr [2]. Moreover, numerical methods have been applied to analyze the nonlinear models of heat transfer in porous fin [3-7]. Additionally, the recent developments in semi-analytical methods or approximate analytical methods have been adopted to analyze the thermal performance of porous fin [8-33].

The approximate analytical methods such as Adomian decomposition method (ADM), homotopy analysis method (HAM), least square method (LSM), variational iterative method (VIM), differential transformation method (DTM) etc. have also been used to solve the different linear and nonlinear differential equations in literature. These methods solve nonlinear differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. However, most of the methods have been shown to be semi-analytical in applications as the search for included unknown parameter(s) that will satisfy non-initial boundary condition(s) necessitates the use of numerical methods through the use of computational software.

One angle of overcoming the increased computational cost and time lies in the applications of total analytic methods such as perturbation methods (regular, singular and homotopy), method of weighted residual and Duan-Rach approach. In such methods, the method of weighted residuals provide a very powerful, novel and accurate approximate analytical solution procedure that is applicable to a wide variety of linear and non-linear problems and thus makes it unnecessary to search for included unknown (s) that will satisfy non-initial boundary condition or search for variational formulations in order to apply the finite element method for the problems [28].

The advantages of the method over other known approximate analytical methods and numerical methods can be found in our previous paper [1]. Moreover, in our previous work, it was assumed that the surface convection is negligible and heat is transfer only by natural convection. It was assumed in the work that heat is transferred away from the fin base only through the pores and that there is no convective heat transfer to the surrounding. Actually, such assumption is true in some applications of porous fins. However, there are various other applications of porous fins where the surface convective heat transfer dominates the thermal behaviour of the fins. Therefore, the present work considers the effects of surface convective heat transfer and presents further results on the thermal performance analysis of porous fin with temperature-dependent thermal conductivity and internal heat generation. Approximate analytical solutions are established for the thermal models using Galerkin's method of weighted residual. Also, the influences of the thermal models parameters on the thermal performance of the porous fin are investigated.

2. PROBLEM FORMULATION

Heat is transfer through a straight porous fin of length L and thickness *t* exposed on both faces to a convective environment at temperature T_∞ as shown in Fig. 1.

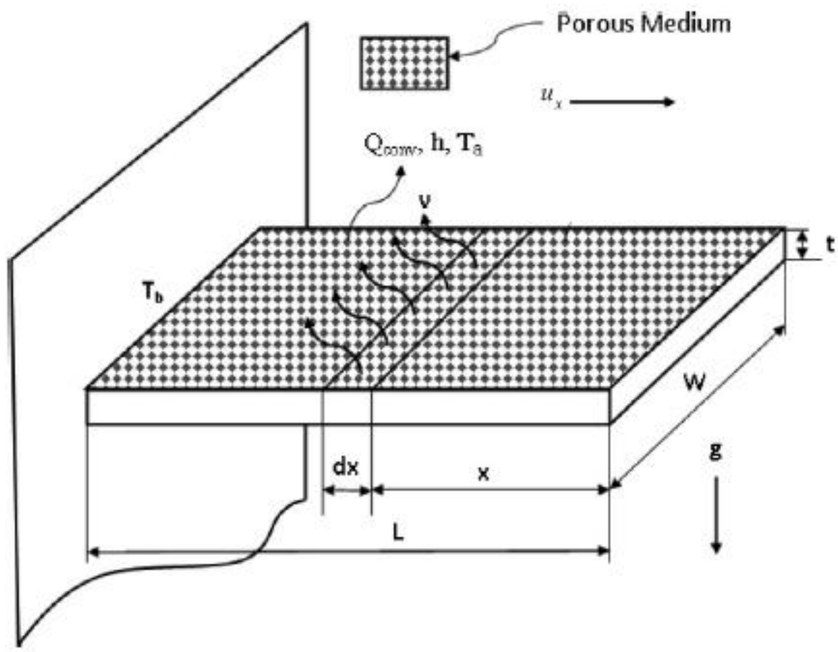


Fig. 1. Schematic of the longitudinal porous fin geometry with the internal heat generation.

There is internal heat generation in the fin. In order to analyze the problem, using the assumptions in our previous paper [1] (except that in this present work, the surface convection is not negligible and heat is transferred away from the fin through the pores and convective surface to the surrounding), the governing equation is given as

$$\frac{d}{dx}\left[k_{eff}(T)\frac{dT}{dx}\right]-\frac{hP(T-T_\infty)}{A}-\frac{\rho c_p g\beta' KP(T-T_\infty)^2}{Av_f}+q_a(T)=0, \tag{1}$$

and the boundary conditions are

$$x=L, \quad T=T_b$$
$$x=0, \quad \frac{dT}{dx}=0. \tag{2}$$

Most of the previous theoretical studies on the thermal analysis of porous fins were based on the assumption that the thermal conductivity is constant [2-19 and 22-28] and there is no internal heat generation in the fin. An excursion into experimental studies has pointed out that in many engineering applications, the effective thermal conductivity of porous fin is temperature-dependent [1, 20 and 21]. Also, there is internal heat generation in the fin which is temperature-dependent. Therefore, the temperature-dependent thermal properties and internal heat generation are given by [1]

$$k_{eff}\left(T\right)=\phi k_f+(1-\phi)k_s=k_{eff,a}[1+\lambda(T-T_\infty)], \tag{4a}$$

$$q_{\text{int}}\left(T\right)=q_a[1+\psi(T-T_\infty)], \tag{4b}$$

On substituting Eqs. (3) and (4) into Eq. (1), one arrives at

$$\frac{d}{dx}\left[[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right]-\frac{h(T-T_\infty)}{k_{eff,a}t}-\frac{\rho c_p\,\mathrm{g}\,K\beta^{'}(T-T_\infty)^2}{k_{eff,a}t\nu_f}+\frac{q_a}{k_{eff,a}}[1+\psi(T-T_\infty)]=0, \tag{5}$$

Introducing the following dimensionless parameters in Eq. (6) into Eq. (5);

$$X=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_b-T_\infty},\quad Ra=Gr.\Pr=\left(\frac{\beta^{'}gT_bt^3}{\nu_f^2}\right)\left(\frac{\rho c_p\nu_f}{k_{eff,a}}\right),\quad Da=\frac{K}{t^2},\quad Q=\frac{q\nu_f t}{\rho\,\mathrm{c}_p\,\beta^{'}\,\mathrm{g}\,K(T_b-T_\infty)^2},\quad M^2=\frac{hL^2}{k_{eff,a}t}$$

$$S_h=\left(\frac{\beta^{'}g(T_b-T_\infty)t^3}{\nu_f^2}\right)\left(\frac{\rho c_p\nu_f K}{k_{eff,a}t^2}\right)\frac{(L/t)^2}{k_{eff,a}}=\frac{RaDa(L/t)^2}{k_{eff,a}},\quad \gamma=\psi(T_b-T_\infty),\quad \beta=\lambda(T_b-T_\infty), \tag{6}$$

one arrives at the dimensionless governing differential Eq. (7) and the boundary conditions

$$\frac{d}{dX}\left[(1+\beta\theta]\frac{d\theta}{dX}\right]-M^2\theta-S_H\theta^2+S_HQ\gamma\theta+S_HQ=0, \tag{7}$$

After expansion of Eq. (7), one gets

$$\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-M^2\theta-S_H\theta^2+S_HQ\gamma\theta+S_HQ=0, \tag{8}$$

and the dimensionless boundary conditions are

$$X=1,\quad \theta=1$$
$$X=0,\quad \frac{d\theta}{dX}=0. \tag{9}$$

3. DEVELOPMENT OF APPROXIMATE ANALYTICAL METHOD

Galerkin method of weighted residual is used in this work as applied in [1]. Following the same procedural approach as shown in [1] to Eq. (8) and the corresponding boundary conditions in Eq. (9), one arrives at

$$\theta(X)=1-\frac{1}{4[\beta+(6/7)S_H]}\left\{\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)] \\ -\left[\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]^2 \\ -[20[\beta+((6/7)S_H)][S_H-Q(S_H+\gamma S_H-M^2)]]^{\frac{1}{2}}\end{array}\right]\end{array}\right\}\left(1-X^2\right). \tag{10}$$

3. 1. Heat flux of the Fin and rate of heat transfer per unit area from the porous fin

Following the Fourier's law, the rate of heat transfer at the base of the porous fin is given by Eq. (11)

$$q_b=A_c k(T)\frac{dT}{dx} \tag{11}$$

Applying the dimensionless parameters in Eq. (9) to Eq. (10), one arrives at the dimensionless heat transfer rate at the base of the fin as

$$Q_b=\frac{qL}{k_a A_c(T_b-T_\infty)}=\left[(1+\beta\theta)\frac{d\theta}{dX}\right]_{X=1} \tag{12}$$

On substituting the respective expressions from Eq. (10) into Eq. (12), it can easily be shown that the rate of heat transfer from the fin base is given as

$$Q_b=\left\{\frac{(1+\beta)\left\{\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)] \\ -[[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]]^2-[20S_H[\beta+((6/7)S_H)][1-Q(1+\gamma)+M^2]]]^{\frac{1}{2}}\end{array}\right\}}{2[\beta+(6/7)S_H]}\right\} \tag{13}$$

Also, the rate of heat transfer per unit width as removed by a porous fin may be calculated from

$$Q_{b/w}=k_a(T_b-T_\infty)\left(\frac{t}{L}\right)\left[(1+\beta\theta)\frac{d\theta}{dX}\right]_{X=1} \tag{14}$$

If one substitutes the respective expressions from Eq. (10) into Eq. (14), the rate of heat transfer from the fin base per unit width can be shown to be given by

$$Q_{b/w} = k_a(T_b - T_\infty)\left(\frac{t}{L}\right)\left\{\frac{\left[\begin{array}{l}(1+\beta)\{[5(1+\beta)+2(2\mathrm{S}_H-\gamma Q\mathrm{S}_H)+M^2]- \\ {[[5(1+\beta)+2(2\mathrm{S}_H-\gamma Q\mathrm{S}_H)+M^2]]-[20\mathrm{S}_H[\beta+((6/7)\mathrm{S}_H)][1-Q(1+\gamma)+M^2]]]^{\frac{1}{2}}}\end{array}\right]}{2[\beta+(6/7)\mathrm{S}_H]}\right\} \tag{15}$$

3. 2. Analysis of Heat transfer augmented in porous fin

The thermal performance of the porous fin can be determined when the rate of heat transfer from the porous fin is compared to the rate of heat transfer from a solid fin, the ratio of heat transfer rate between the two fins are given by

$$\frac{q_b}{q_s} = \frac{k_{eff}(T)A_b\left(\frac{dT}{dx}\right)_{x=o}}{hA_s(T_b - T_\infty)} \tag{16}$$

where the denominator represents the maximum possible heat transfer rate obtained using a solid fin. Expressing the above equation in terms of the dimensionless temperature and axial distance, gives

$$\frac{q_b}{q_s} = \frac{A_r}{Nu}\left[(1+\beta\theta)\frac{d\theta}{dX}\right]_{X=1} \tag{17}$$

On substituting the respective expressions from Eq. (10) into Eq. (17), yields

$$\frac{q_b}{q_s} = \frac{A_r}{Nu}\left\{\frac{\left[\begin{array}{l}(1+\beta)\{[5(1+\beta)+2(2\mathrm{S}_H-\gamma Q\mathrm{S}_H)+M^2]- \\ {[[5(1+\beta)+2(2\mathrm{S}_H-\gamma Q\mathrm{S}_H)+M^2]]-[20\mathrm{S}_H[\beta+((6/7)\mathrm{S}_H)][1-Q(1+\gamma)+M^2]]]^{\frac{1}{2}}}\end{array}\right]}{2[\beta+(6/7)\mathrm{S}_H]}\right\} \tag{18}$$

4. FIN EFFICIENCY

The fin efficiency is the ratio of the amount of heat dissipated from entire fin to the maximum heat dissipated that is obtained if the fin base temperature is kept throughout the fin. The computation of the efficiency of the fin is necessary as an indicator for the thermal performance of the fin.

Following the definition, the efficiency of the fin could be expressed mathematically as

$$\eta = \frac{Q_f}{Q_{\max}} \tag{19}$$

The amount of heat dissipated from the entire fin is found by using Newton's law of cooling as

$$Q_f = \int_0^1 \left\{ Ph(T)(T - T_\infty) + \frac{\rho c_p g \beta' KP(T - T_\infty)^2}{\nu_f} \right\} dx \tag{20}$$

The maximum heat dissipated is obtained if the fin base temperature is kept throughout the fin

$$Q_{\max} = Ph_b L(T_b - T_\infty) + \frac{\rho c_p g \beta' KP(T_b - T_\infty)^2 L}{\nu_f} \tag{21}$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by

$$\eta = \frac{Q_f}{Q_{\max}} = \frac{\int_0^L \left\{ Ph(T - T_\infty) + \frac{\rho c_p g \beta' KP(T - T_\infty)^2}{\nu_f} \right\} dx}{Ph_b L(T_b - T_\infty) + \frac{\rho c_p g \beta' KP(T - T_\infty)^2 L}{\nu_f}} \tag{22}$$

Therefore, the fin efficiency in dimensionless variables is given by

$$\eta = \frac{\int_0^1 \left\{ \theta + \frac{S_h}{M^2} \theta^2 \right\} dX}{1 + \frac{S_h}{M^2}} \tag{23}$$

On substituting equations (10) into equation (23), we arrived at

$$\eta = \frac{1}{1 + \frac{S_h}{M^2}} \int_0^1 \left\{ \begin{array}{l} \left\{ 1 - \frac{1}{4[\beta + (6/7)S_H]} \left\{ \begin{array}{l} [5(1+\beta) + 2(2S_H - \gamma QS_H + M^2)] \\ - \left[\begin{array}{l} [5(1+\beta) + 2(2S_H - \gamma QS_H + M^2)]^2 \\ -[20[\beta + ((6/7)S_H)][S_H - Q(S_H + \gamma S_H - M^2)]]^{\frac{1}{2}} \end{array} \right] \end{array} \right\} \left(1 - X^2\right) \right\} \\ + \frac{S_h}{M^2} \left\{ 1 - \frac{1}{4[\beta + (6/7)S_H]} \left\{ \begin{array}{l} [5(1+\beta) + 2(2S_H - \gamma QS_H + M^2)] \\ - \left[\begin{array}{l} [5(1+\beta) + 2(2S_H - \gamma QS_H + M^2)]^2 \\ -[20[\beta + ((6/7)S_H)][S_H - Q(S_H + \gamma S_H - M^2)]]^{\frac{1}{2}} \end{array} \right] \end{array} \right\} \left(1 - X^2\right) \right\}^2 \end{array} \right\} dX \tag{24}$$

$$\eta=\frac{1}{1+\dfrac{S_h}{M^2}}\left\{\begin{array}{l}\left(1-\dfrac{2}{3}\dfrac{1}{4[\beta+(6/7)S_H]}\left\{\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]\\-\left[\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]^2\\-[20[\beta+((6/7)S_H)][S_H-Q(S_H+\gamma S_H-M^2)]]\end{array}\right]^{\frac{1}{2}}\end{array}\right\}\right)\\+\dfrac{S_h}{M^2}\left(\begin{array}{l}1-\dfrac{4}{3}\left\{\dfrac{1}{4[\beta+(6/7)S_H]}\left\{\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]\\-\left[\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]^2\\-[20[\beta+((6/7)S_H)][S_H-Q(S_H+\gamma S_H-M^2)]]\end{array}\right]^{\frac{1}{2}}\end{array}\right\}\right\}\\+\dfrac{8}{15}\left\{\dfrac{1}{4[\beta+(6/7)S_H]}\left\{\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]\\-\left[\begin{array}{l}[5(1+\beta)+2(2S_H-\gamma QS_H+M^2)]^2\\-[20[\beta+((6/7)S_H)][S_H-Q(S_H+\gamma S_H-M^2)]]\end{array}\right]^{\frac{1}{2}}\end{array}\right\}\right\}^2\end{array}\right)\end{array}\right\} \quad (25)$$

5. RESULTS AND DISCUSSION

Effects of convective or thermo-geometric parameter on the performance of the fin are shown in Figs. 2a and 2b. The figures depict that with increase in the convective parameter, the rate of heat transfer through the fin increases. This is shown as the temperature along the fin, especially at the tip of the fin decreases faster as the thermo-geometric parameter, M, increases. However, the thermal performance of the fin has been shown to be favoured at low values of thermogeometric parameter [1].

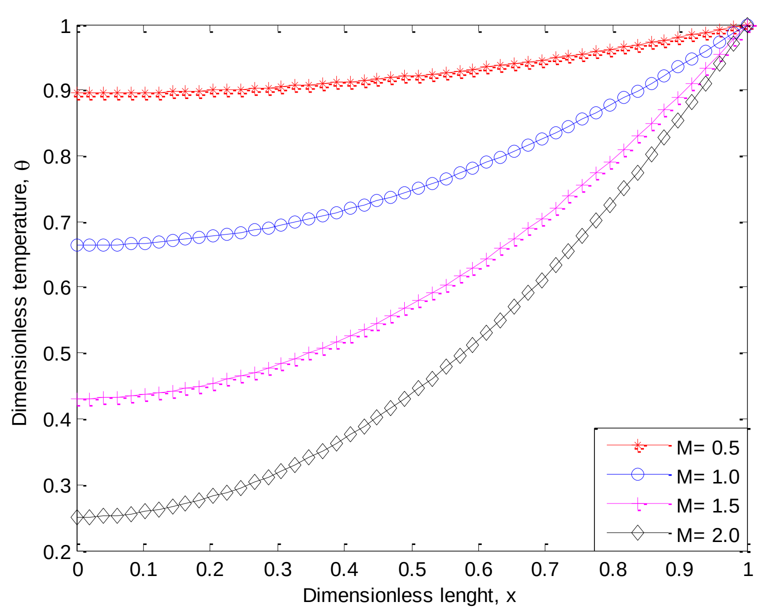


Fig. 2a. Effects of thermo-geometric parameter on the temperature distribution in the fin when β = 0.1, $S_h = 0$.

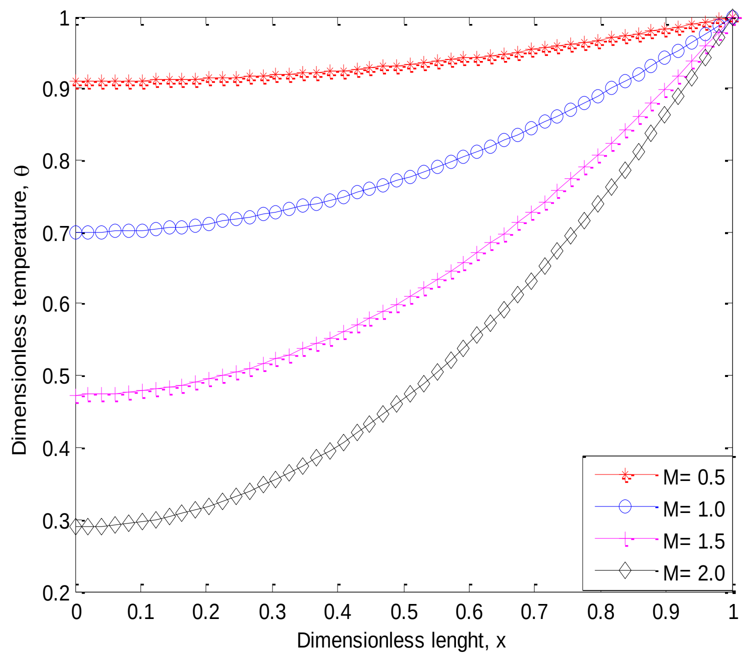
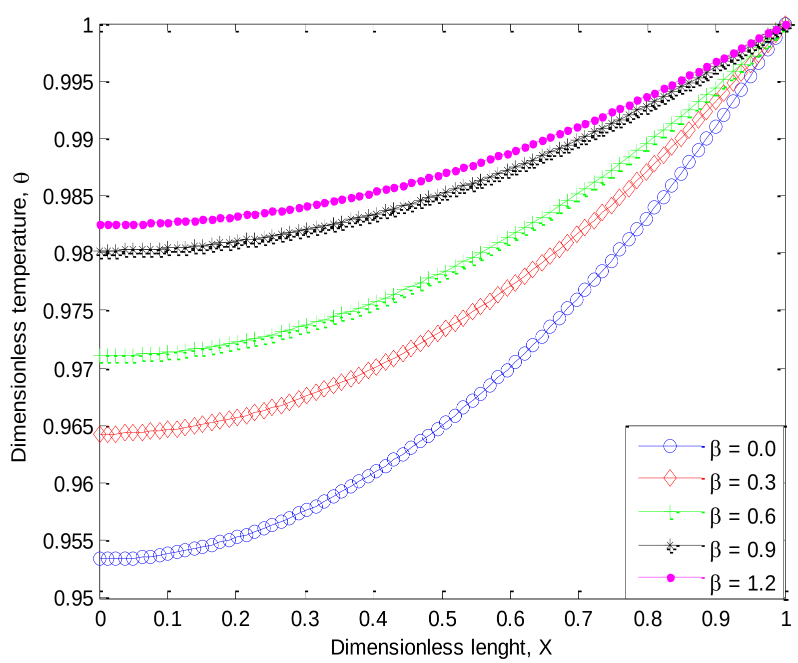
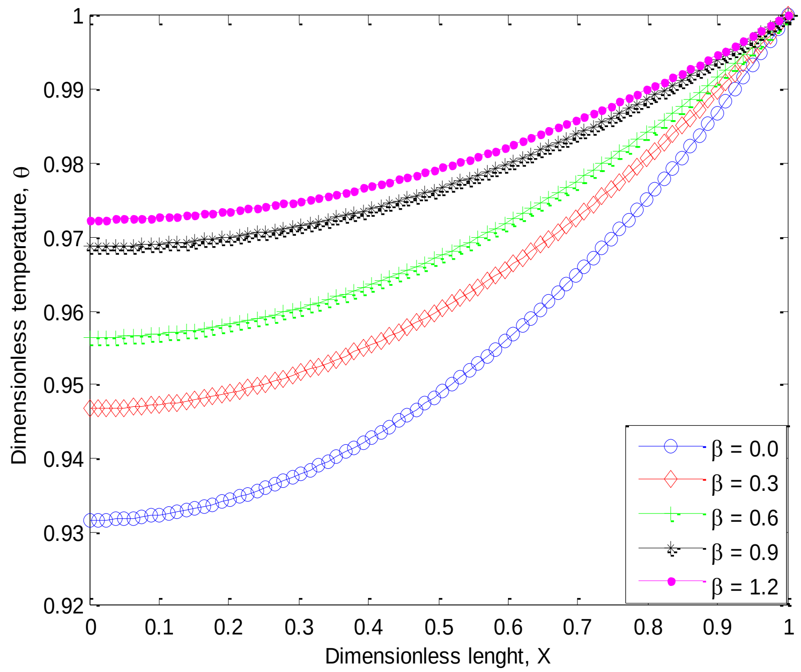


Fig. 2b. Effects of thermo-geometric parameter on the temperature distribution in the fin when β = 0.3, S_h = 0.



(a)


Dimensionless temperature, θ
Dimensionless lenght, X
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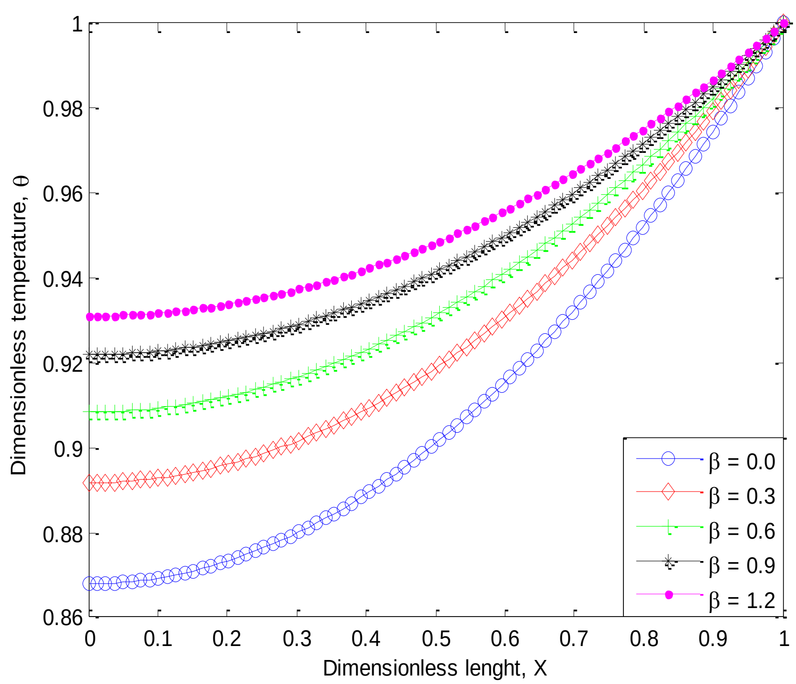
β = 0.3
β = 0.6
β = 0.9
β = 1.2

(b)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Dimensionless lenght, X
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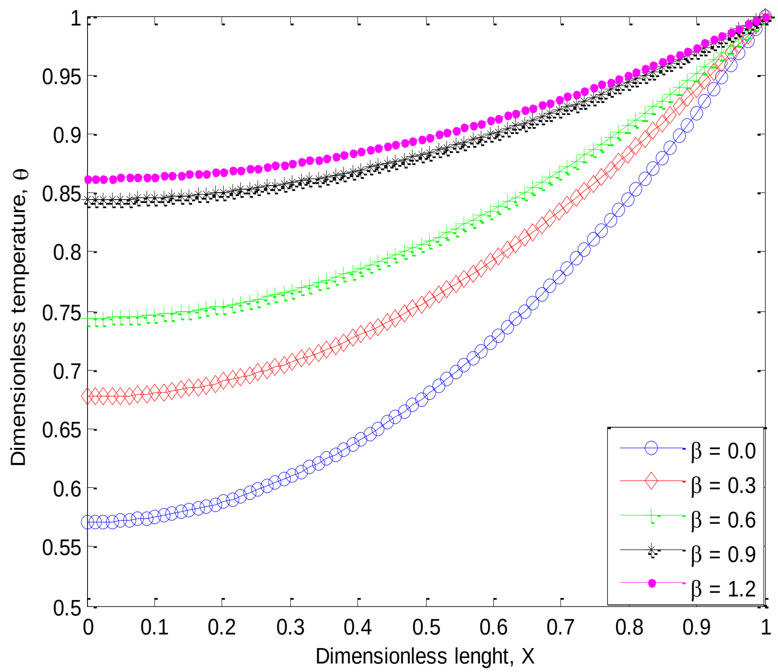
β = 0.3
β = 0.6
β = 0.9
β = 1.2

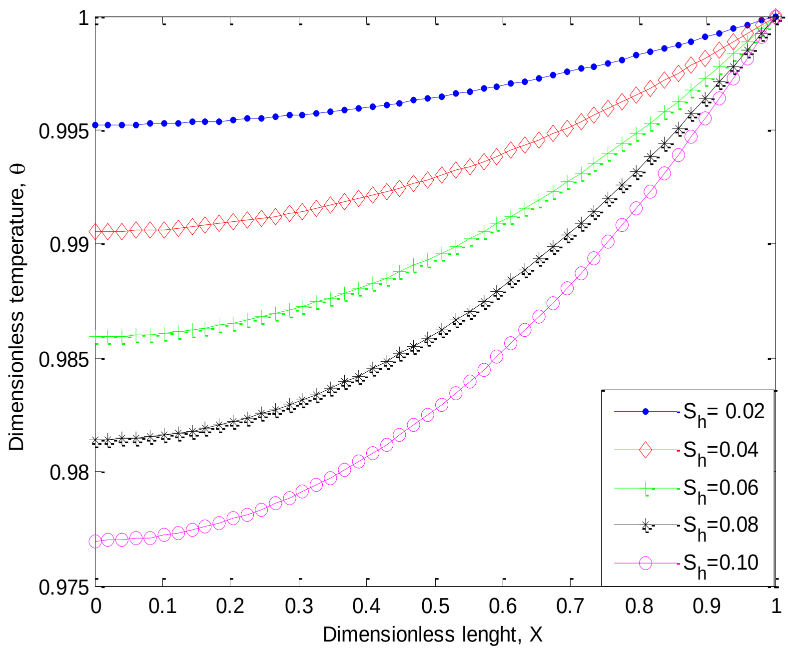
(c)



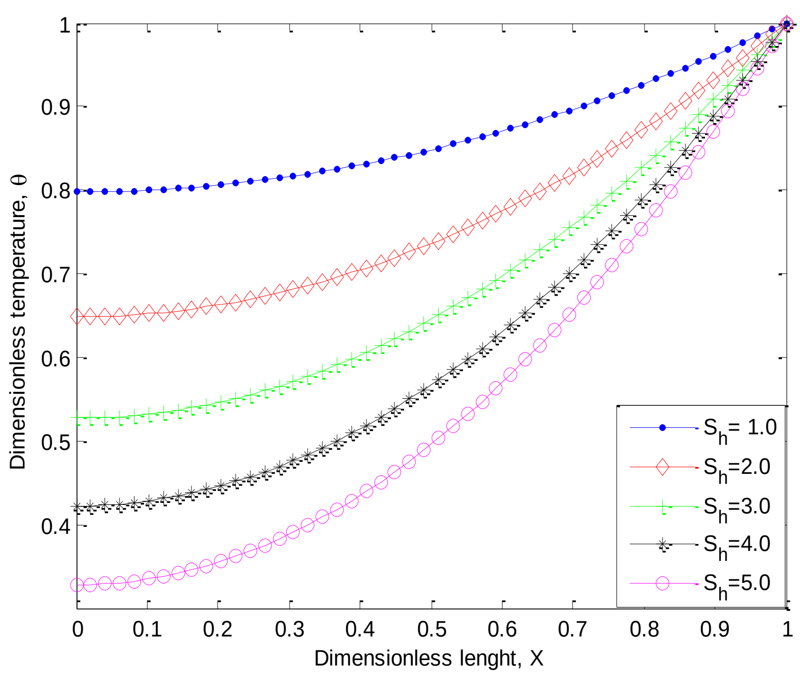
(d)

Fig. 3(a-d). Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) M = 0.3, S_h = 0, Q = 0.4, γ = 0.2, (b) M = 0.3, S_h = 0.1, Q = 0.4, γ = 0.2, (c) S_h = 0.5, M = 0.3, Q = 0.4, γ = 0.2, (d) M = 0.8, S_h = 0.1, Q = 0.4, γ = 0.2

Most of the previous theoretical studies on the thermal analysis of porous fins were based on the assumption that the thermal conductivity is constant [2-19 and 22-28] and there is no internal heat generation in the fin. An excursion into experimental studies has pointed out that in many engineering applications, the effective thermal conductivity of porous fin is temperature-dependent [1, 20 and 21]. The considerations of the temperature-dependent thermal conductivity lead to the development of nonlinear differential terms in the thermal model as shown in Eq. (1) and consequently, in the dimensionless form, there is an accompanying nonlinear thermal conductivity term in the differential term as shown in Eq. (8). Therefore, Fig. 3 presents the effects of non-linear thermal conductivity parameters on the temperature distribution in the porous fin. From the figure, it is depicted that the fin temperature distribution decreases as the non-linear thermal conductivity parameter increases. The influence of porous parameter or porosity on the dimensionless temperature distribution in the porous fin is shown in Fig. 4a and 4b. The temperature along the porous fin drops faster as the porosity parameter increases. The rapid decrease in fin temperature due to increase in the porosity parameter is because the increase in the porosity of the fin causes an increase in the permeability of the porous fin which consequently increases the ability of the working fluid to penetrate more through the fin pores and increases the buoyancy effect and as a result, the rate of heat transfer from the fin is enhanced and the thermal performance of the fin is increased.


1
0.995
0.99
0.985
0.98
0.975
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Dimensionless temperature, θ
Dimensionless lenght, X
S_h= 0.02
S_h=0.04
S_h=0.06
S_h=0.08
S_h=0.10

(a)


1
0.9
0.8
0.7
0.6
0.5
0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Dimensionless temperature, θ
Dimensionless lenght, X
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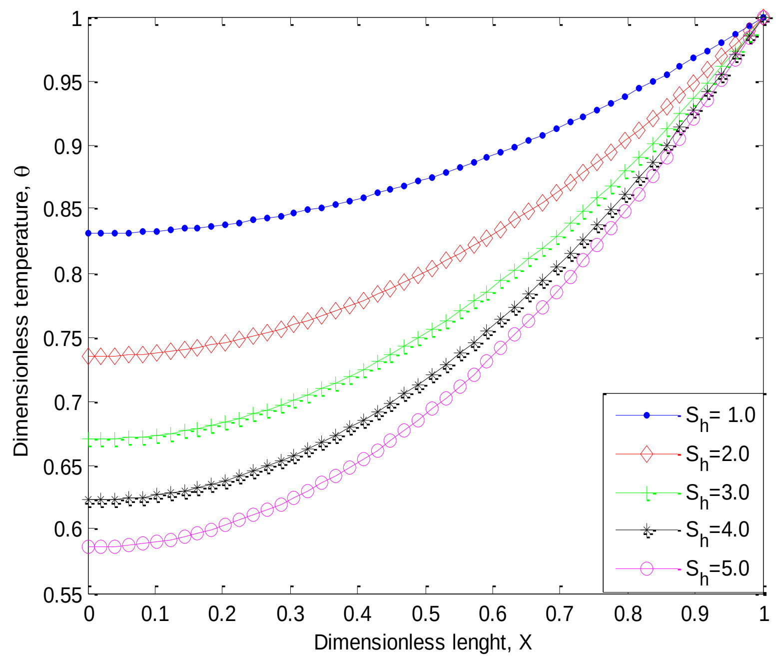
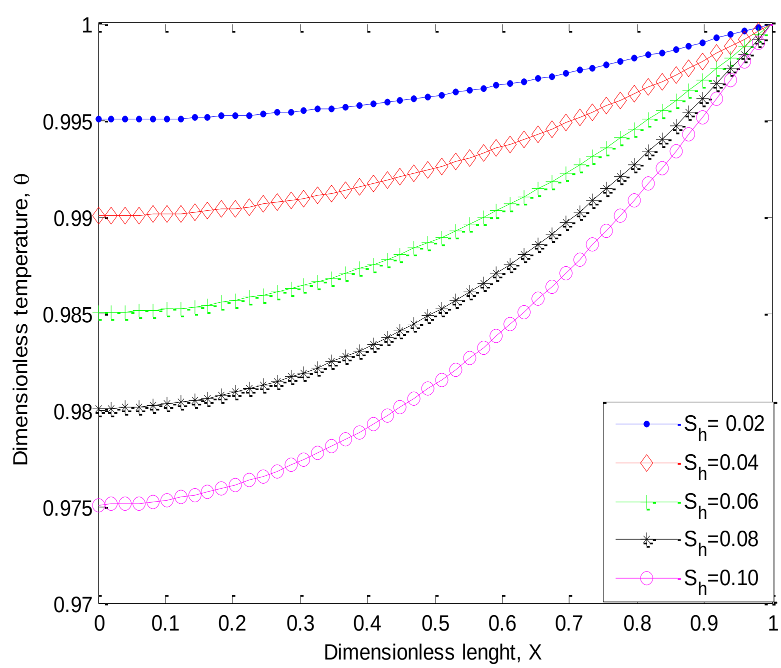
S_h=2.0
S_h=3.0
S_h=4.0
S_h=5.0

(b)



(c)



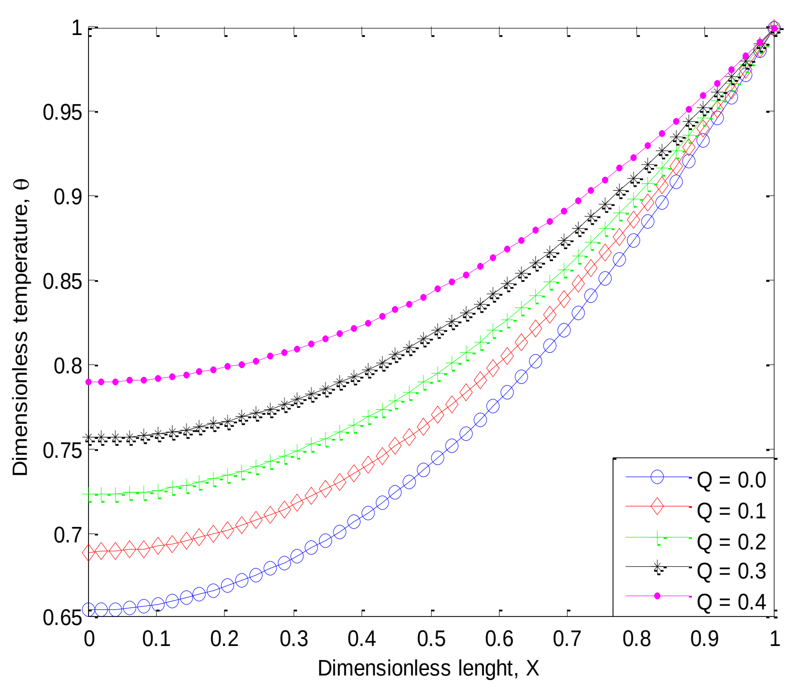
(d)

Fig. 4. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) β = 0.5, M = 0.5, Q = 0.2, γ = 0.4, (b) β = 0.5, M = 1.0, Q = 0.2, γ = 0.4, (c) β = 0.5, M = 2.0; Q = 0.2, γ = 0.4, (d) β = 0.5, M = 10, Q = 0.2, γ = 0.4

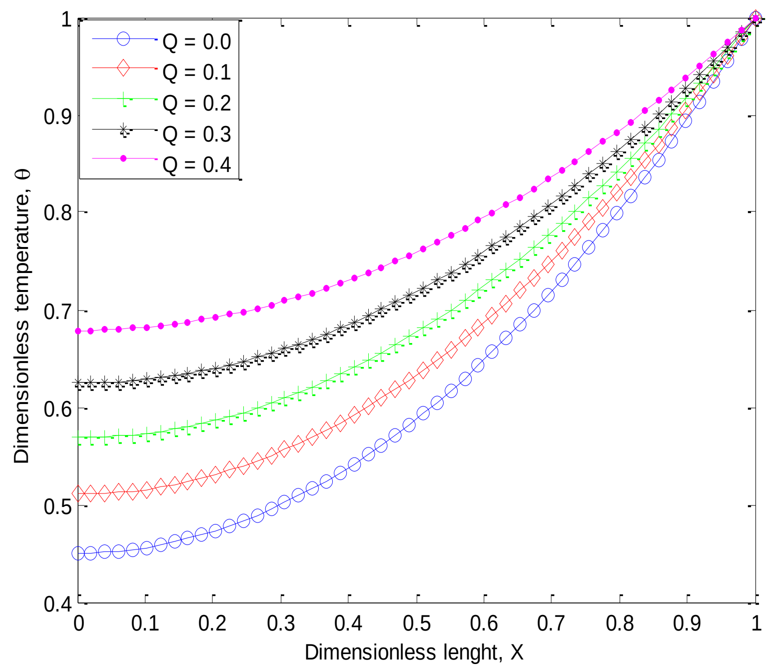
Figs. 5a-d show the effects of the dimensionless internal heat generation parameter on the thermal performance and stability of the porous fin while Figs. 6a and 6b depict the effects of dimensionless temperature-dependent internal heat generation on the thermal performance of the porous fin.

From the figures, it could be inferred that the dimensionless temperature distribution decreases as the porous parameter increases. However, the fin dimensionless tip temperature becomes negative (which contradicts the assumption made in the development of the thermal model and the boundary conditions) when the thermo-geometric, porous, nonlinear thermal conductivity, internal heat generation and temperature-dependent internal heat generation parameters are 2.0, 5.0, 0.5, 0.4 and 0.2, respectively. Such behavior of thermal instability is not peculiar to these set of values. It occurs when the thermo-geometric and porous parameters of the fin exceed some certain values of thermal stability [1, 29]. Moreover, it is established as shown in Fig. 5c that the value of porosity parameter for the thermal stability increases with nonlinear thermal conductivity and internal heat generation parameters.

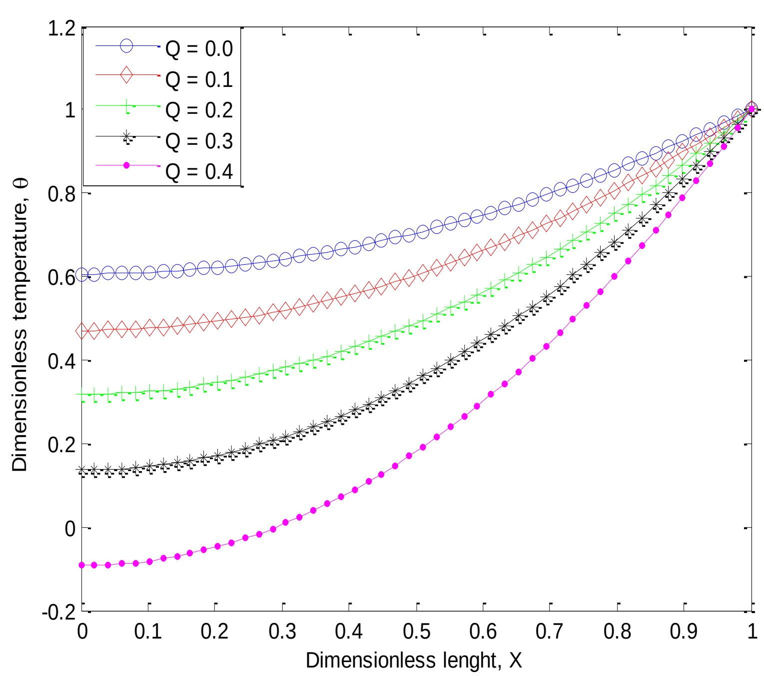
On the evaluation of the rate of heat transfer through the fin thermal performance under some certain conditions, Figs.7a-b shows the effect of temperature-dependent internal heat generation on the rate of heat transfer in the porous fin. It is shown that an increase in the temperature-dependent internal heat generation parameter leads to decrease in the temperature gradient. Consequently, the rate of heat transfer in the fin decreases as the temperature-dependent internal heat generation decreases. However, the rates of heat transfer at the base of the fin increases as the porous parameter or porosity increases.



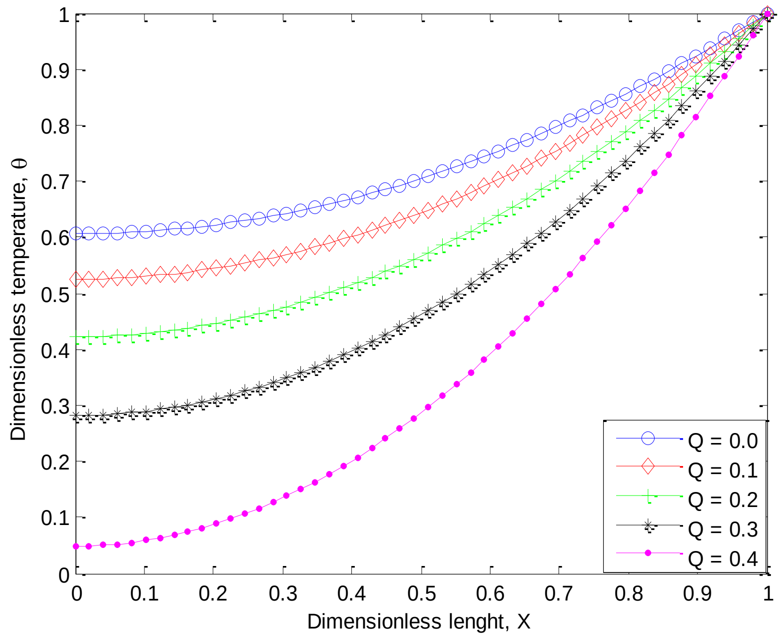
(a)


Q = 0.0
Q = 0.1
Q = 0.2
Q = 0.3
Q = 0.4
Dimensionless temperature, θ
Dimensionless lenght, X
1
0.9
0.8
0.7
0.6
0.5
0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

(b)

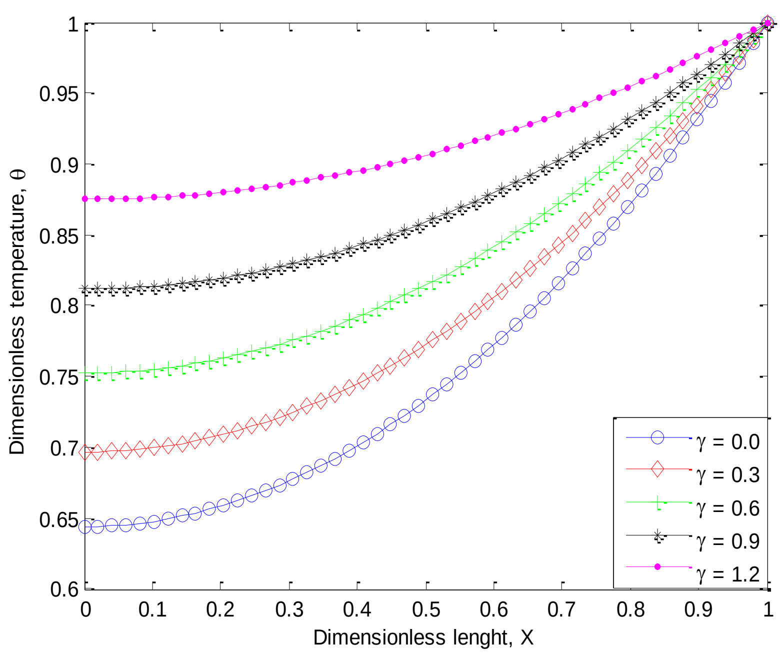

Q = 0.0
Q = 0.1
Q = 0.2
Q = 0.3
Q = 0.4
Dimensionless temperature, θ
Dimensionless lenght, X
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

(c)

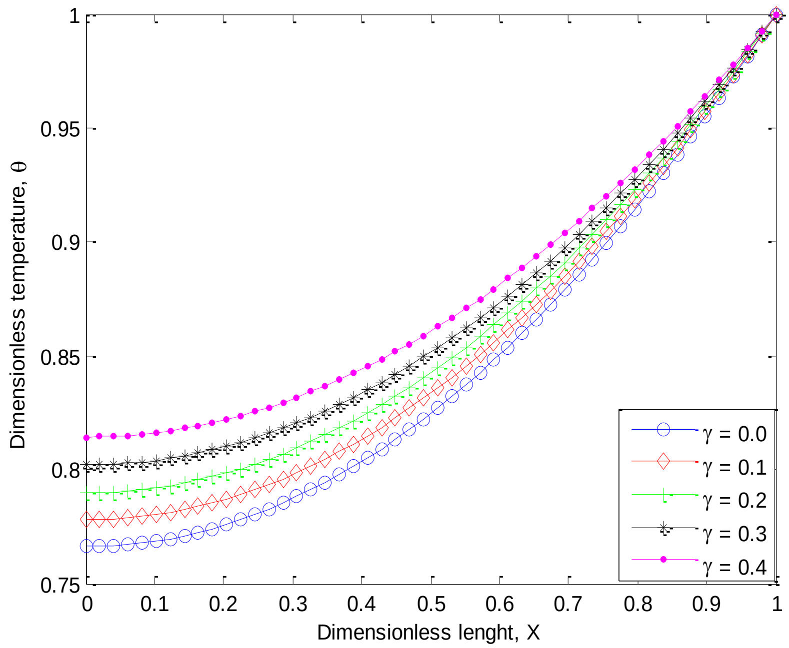


(d)

Fig. 5. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) M = 0.5, S_h = 2.0, β = 0.5, γ = 0.2, (b) M = 0.5, S_h = 5.0, β, = 0.5, γ = 0.2, (c) M = 2.0, S_h = 5.0, β = 0.5, γ = 0.2, (d) M = 2.0, S_h = 5.0, β = 0.5, γ = 2.0.

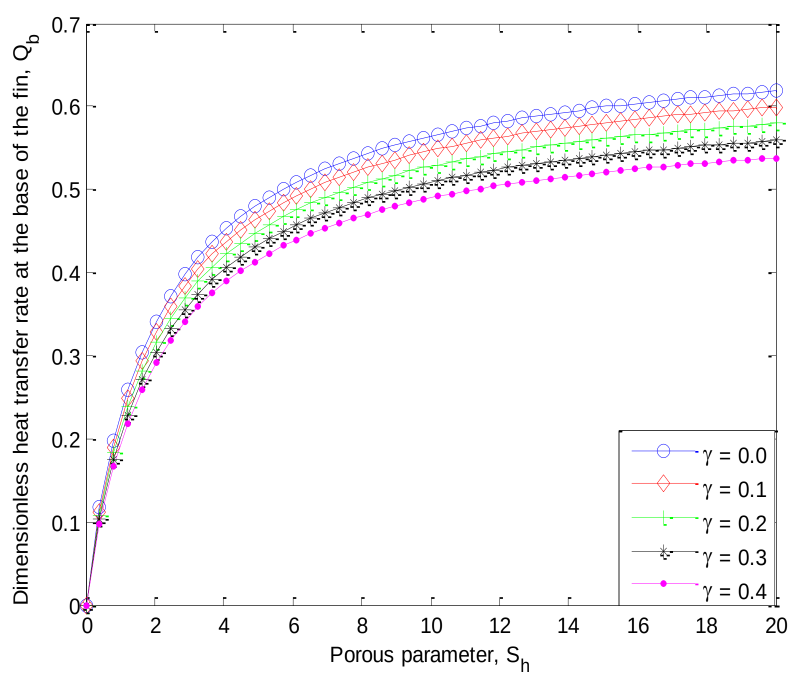


(a)

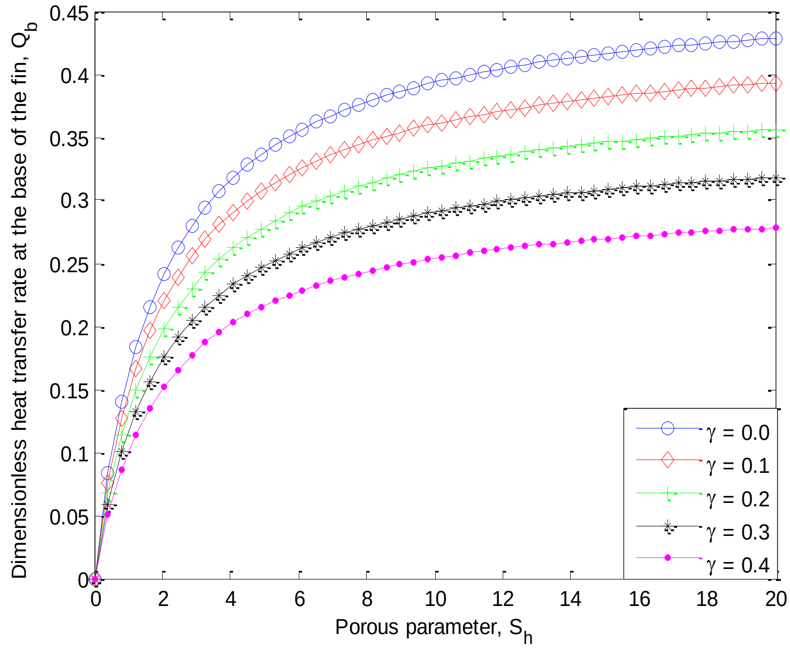


(b)

Fig. 6. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) $S_h = 5.0$, $M = 0.5$, $\beta = 0.5$, $Q = 0.4$, $\gamma = 0.2$, (b) $S_h = 5.0$, $\beta = 0.5$, $M = 0.5$, $Q = 0.4$, $\gamma = 0.2$

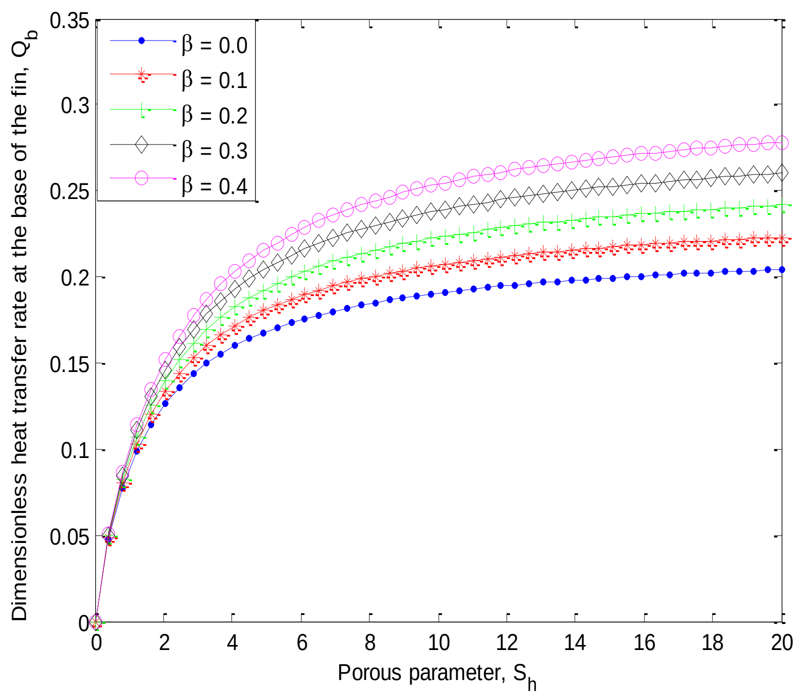


(c)

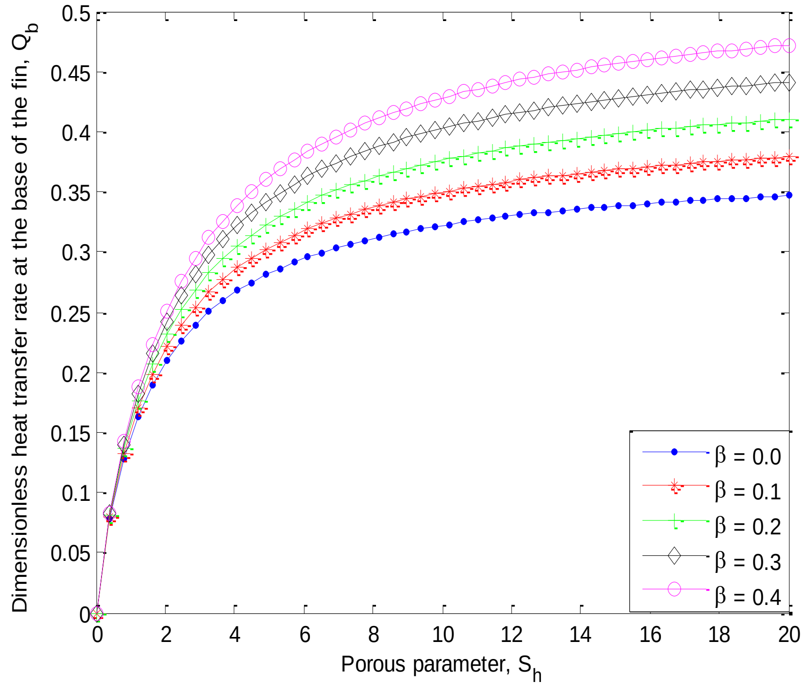


(d)

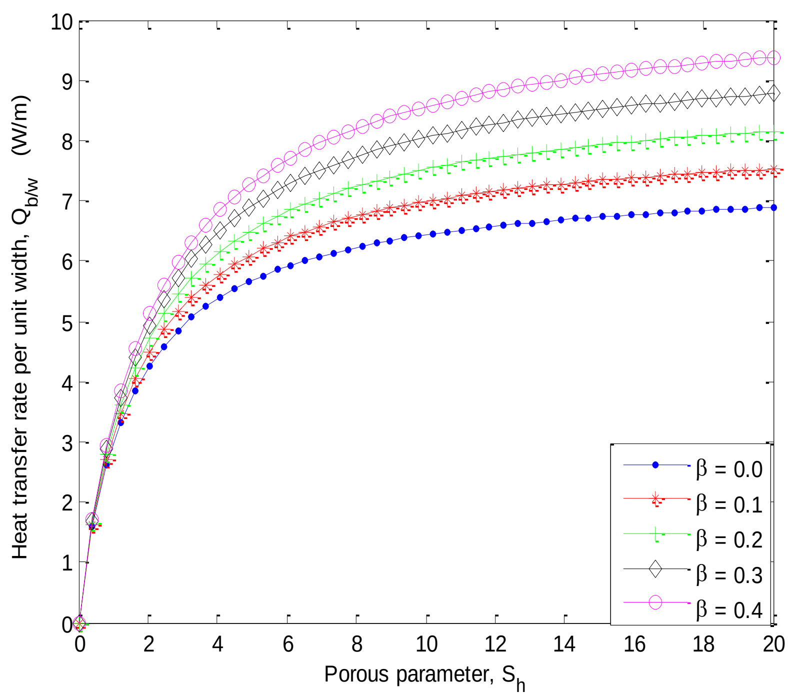
Fig. 7. Effects of temperature-dependent internal heat generation parameter on the dimensionless heat transfer rate in the fin when (c) M = 0, β = 0.4, Q = 0.3, (d) M = 0, β = 0.4, Q = 0.3



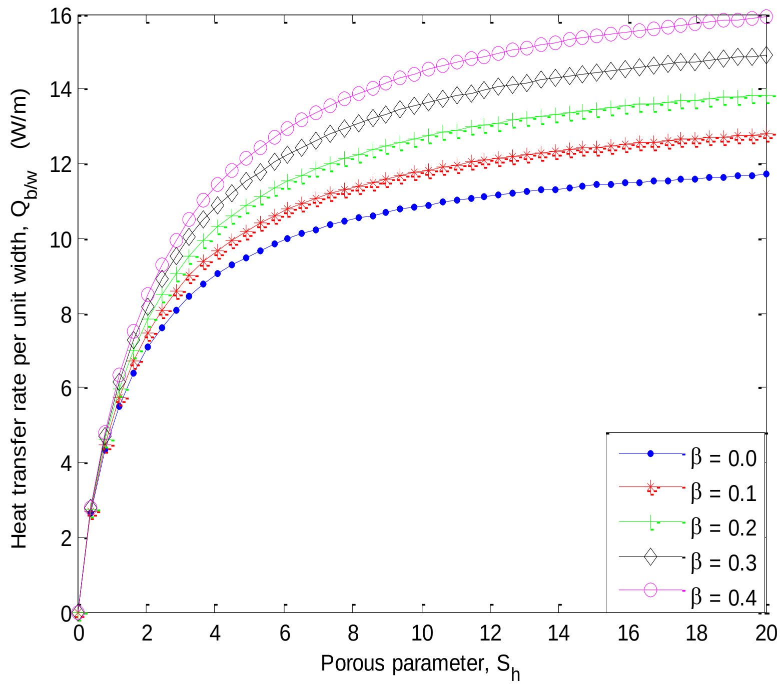
(a)


Dimensionless heat transfer rate at the base of the fin, Q_b
Porous parameter, S_h
β = 0.0
β = 0.1
β = 0.2
β = 0.3
β = 0.4

(b)


Heat transfer rate per unit width, Q_b/w (W/m)
Porous parameter, S_h
β = 0.0
β = 0.1
β = 0.2
β = 0.3
β = 0.4

(c)



(d)

Fig. 8. Effects of temperature-dependent thermal conductivity parameter and fin thickness-lenght ratio on the dimensionless heat transfer rate at the base of the fin when (a) γ = -0.4, Q = 0.5, M = 0, (b) γ = 0.7, Q = 0.3, (c) *t/L* = 1/1000, k = 45 W/mK; Tb = 373 K; Ta = 298 K; γ = 0.4, Q = 0.5, M = 0, (d) *t/L* = 1/1000, k = 45 W/mK; Tb = 373 K; Ta = 298 K; γ = -0.4, Q = 0.5, γ = 0.7, Q = 0.3, M = 0

The rate of heat transfer at the base of the fin has been used as an indicator for determining the thermal performance of the fin. Therefore, Figs. 8a-d present the effect of nonlinear thermal conductivity parameter and fin thickness-length ratio, *t/L*, on the dimensionless heat transfer rate at the base of the fin. The figures show that the rate of heat transfer at the base of the fin is significantly affected by the nonlinear thermal conductivity parameter, porosity and fin thickness ratio. It is presented from the figure that rate of heat transfer increases as the dimensionless thickness parameter (fin thickness-length ratio) increases.

In order to verify the results of the Galerkin's method of weighted residual as applied in this present work, the developed thermal model was solved numerically using shooting method coupled with fourth-order Runge-Kutta method.

Table 1 shows comparison of the results of the numerical method and approximate analytical method used in this work. Also, the results were compared with the previous results in literature where homotopy perturbation method is applied to analyze the thermal model of the porous fin. From the table, it could be inferred that the Galerkin's method is highly accurate and displays excellent agreement with the results of the numerical and homotopy perturbation methods.

Table 1. Comparison of results

X	NM	HPM [27]	GWRM (The Present study)
0.0	0.9581	0.9581	0.9581
0.1	0.9585	0.9585	0.9585
0.2	0.9597	0.9597	0.9597
0.3	0.9618	0.9618	0.9618
0.4	0.9647	0.9647	0.9647
0.5	0.9685	0.9685	0.9685
0.6	0.9730	0.9730	0.9730
0.7	0.9785	0.9785	0.9785
0.8	0.9846	0.9846	0.9848
0.9	0.9919	0.9919	0.9919
1.0	1.0000	1.0000	1.0000

6. CONCLUSIONS

As a further study to our previous work, "*Thermal performance analysis of a natural convection porous fin with temperature-dependent thermal conductivity and internal heat" published in "Thermal Science and Engineering Progress. 1 (2017) 39–52*", where it was assumed that the surface convection is negligible and heat is transferred only by natural convection, In this work, the effects of surface convective heat transfer on the thermal performance analysis of porous fin with temperature-dependent thermal conductivity and internal heat generation have been included, investigated and analyzed. Approximate analytical solutions were established for the developed thermal models using Galerkin's method of weighted residual. Also, the effects of the thermal models parameters on the thermal performance of the porous fin were investigated. It was established that as the with the other model parameters (except thermal conductivity parameter) that as the convective parameter increases, the rate of heat transfer from the base of the fin increases and consequently, the porous fin efficiency improves. Excellent agreements were established between the results of second-order approximation of the Galerkin's method of weighted residual and results of numerical method using shooting method coupled with Runge-Kutta method and also with the results of homotopy perturbation method.

Nomenclature

a_r aspect ratio ratio of the porous fin base area to the surface area
A cross sectional area of the fins, m^2
A_b porous fin base area
A_s porous fin surface area
Bi Biot number
h heat transfer coefficient, $Wm^{-2}k^{-1}$
h_b heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
c_p specific heat of the fluid passing through porous fin (J/kg-K)
Da Darcy number
g gravity constant(m/s^2)
h heat transfer coefficient over the fin surface (W/m^2K)
H dimensionless heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
k thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
k_b thermal conductivity of the fin material at the base of the fin, $Wm^{-1}k^{-1}$
k_{eff} effective thermal conductivity ratio
K permeability of the porous fin (m^2)
L Length of the fin, m
M dimensionless thermo-geometric parameter
m mass flow rate of fluid passing through porous fin(kg/s)
Nu Nusselt number
P perimeter of the fin (m)
Q dimensionless internal heat generation paramter
q_b heat transfer rate per unit area at the base (W/m^2)
Q_b dimensionless heat transfer rate the base in porous fin
Q_s dimensionless heat transfer rate the base in solid fin
Ra Rayleigh number
S_h Porosity parameter
t thickness of the fin
T_b base temperature (K)
T fin temperature (K)
T_a ambient temperature, K
T_b Temperature at the base of the fin, K
v average velocity of fluid passing through porous fin (m/s)
x axial length measured from fin tip (m)
X dimensionless length of the fin
w width of the fin
q internal heat generation in W/m^3

Greek Symbols

β thermal conductivity parameter or non-linear parameter
δ thickness of the fin, m
δ_b fin thickness at its base.
γ dimensionless temperature-dependent internal heat generation parameter
θ dimensionless temperature

θ_b dimensionless temperature at the base of the fin
η efficiency of the fin
ε effectiveness of the fin
β' coefficient of thermal expansion (K^{-1})
ε porosity or void ratio
υ kinematic viscosity (m^2/s)
ρ density of the fluid (kg/m^3)

Subscripts

s solid properties
f fluid properties
eff effective porous propertie

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