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Peristaltic flow of a third grade fluid accounting Joule heating and Magnetic field. Effects in an asymmetric channel

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ABSTRACT

In this article, we have analysed the MHD third grade fluid flow induced by a peristaltic wave. The flow is analysed using the lubrication approximations. The reduced equations are solved by Adomian Decomposition Method (ADM) and the expressions for stream function, velocity, pressure gradient and frictional force are obtained. The effect of pertinent parameters such as Brinkmann number, magnetic field, Deborah number and phase difference are analysed and illustrated graphically. The results shown that the rate of conduction of heat enhances by supplying heat to the channel. Also it is noticed that by increasing magnetic field the Lorentz forces reduces the velocity. This study finds application in various practical devices like electric power generators, heaters and conductors.

***Keywords*:** Peristaltic flow, MHD third grade fluid, Joule heating, asymmetric channel, ADM

1. INTRODUCTION

The joule heating or ohmic heating effect is named after the famous amateur scientist James Prescott Joule, who first analysed this heating effect in the year 1840. It is found that one suitable method to apply heat to the channel wall is to use Joule heating, which is a mechanism of generating heat by passing electric field through an electrically conducting fluid. The joule heating principle finds application in practical devices like electric stoves, electric heaters and

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conductors, fuses and power lines etc. Keeping all such aspects in mind many researches in the past have analysed peristaltic flow with joule heating. Abbasi and Hayat [1] studied the MHD joule heating peristaltic flow of non-Newtonian fluids. Hayat et.al [2] discussed the peristaltic flow of Powell–Eyring liquid with hall and joule heating effects. Ranjit et al. [3] discussed the zeta potential impact on peristaltic blood flow with joule heating in a porous vessel. Bhatti and Rashidi [4] analysed the hemodynamic peristaltic flow under the impacts of joule and Hall heating. Peristaltic flow with magnetic field has created considerable interest due to its engineering, biomedical and industrial applications. Some devices which need magnetic field are electric power generators, electrostatic precipitation, purifying of metallic substances from non-metallic impurities, biomedical devices like MRI (Magnetic Resonance Imaging) and NMR (Nuclear Magnetic Resonance) use intense magnetic field for treatment of diseases. Prasanth and Subba [5] investigated the MHD third grade fluid with peristalsis. Hayat and Mehmood [6] studied the MHD planar channel flow of a third grade fluid with slip effects.

Peristalsis is a series of muscular relaxation and contraction of vessel walls which pumps the material forward through wave-like motion that pushes fluids in tubes without direct contact with the pump components. Few examples of peristaltic motion include transportation of food through the digestive tracts including the oesophagus, stomach and small intestine, blood flow through the capillaries, the veins, and the arteries, urine transport from kidney to bladder etc. Peristaltic phenomenon finds application in carrying malignant liquid in nuclear industries, roller and finger pumps, sanitary fluid transport etc. Peristaltic pumps are also used to pump blood in heart lung machine during surgeries. Latham [7] initially investigated the peristaltic pump behaviour. Later Shapiro et al. [8] studied the viscous fluid flow with peristalsis. Since then several researchers have analysed the peristaltic transport phenomenon under different assumptions (see few studies [9-13]). Recently, some physiologists claim that the intra-uterine fluid flow appearing in non-symmetric and symmetric channels represents peristaltic mechanism. The investigation on peristaltic flow of Newtonian and non-Newtonian fluids finds application in industrial, medical and technological fields. In nature, most of the fluids are found to be non-Newtonian. Some examples of such fluids include tomato ketchup, blood, mud, honey etc. Raju and Devanathan [14] initially investigated the non-Newtonian fluids. Mekheimer [15] analysed the flow of blood in non-uniform channels. Due to the non-linearity and complex nature of non-Newtonian fluids it becomes hard to find solutions to these type of flow problems. Third grade fluid belongs to one special subclass of fluids characterized by a truncation error of order three. These fluids belong to a family of those fluids which exhibit third order complexity. Amrouche and Cioranescu [16] were the first to study the results on third grade fluids mathematically. Few studies regarding the third grade fluid are given in references [17-18]. Later, Hayat et al. [19] analysed the grade three fluids flow in a tube. Haroun [20] studied the third order fluid under the effects of Deborah number. Prakash et.al [21] investigated the third grade fluid flow in a tapered channel. Amin [22] analysed the unsteady thin film flow of third grade fluid. Among the many models which report the behaviour of non-Newtonian fluids, fluids of differential type are given major attraction. Dunn and Rajagopal [23] have reported a detailed review on the differential type fluid flows.

To the best of authors knowledge is considered no study has ever been done considering the joule heating effect on peristaltic flow of a third grade fluid in an asymmetric channel. In view of the above discussion, the present article aims to investigate the effect of joule heating on peristaltic flow of a third grade fluid. The equations governing the model for third grade fluid are defined and reduced applying the assumptions of long wavelength and low Reynolds number and are solved by using the Adomian decomposition method (ADM). The effect of pertinent parameters are analysed and plotted graphically.

2. MATHEMATICAL FORMULATION

We consider the incompressible, electrically conducting flow of a third order fluid in a two -dimensional channel. Let B_0 be the magnetic field which is uniform, applied transverse to the direction of flow. A rectangular co-ordinate system $(X^{'}, Y^{'})$ is considered in which $X^{'}$-axis is taken along the centreline and $Y^{'}$-axis is perpendicular to it. A schematic diagram of the channel is represented in Figure 1.

The equations of the wall deformation are represented as,

$$h_1^{'}\left(X^{'},t^{'}\right)=y_1=d_1^{'}+a_1^{'}\cos\left[\frac{2\pi}{\lambda}(X^{'}-ct^{'})\right], \tag{1}$$

$$h_2^{'}\left(X^{'},t^{'}\right)=y_2=-d_2^{'}-a_2^{'}\cos\left[\frac{2\pi}{\lambda}(X^{'}-ct^{'})+\phi\right], \tag{2}$$

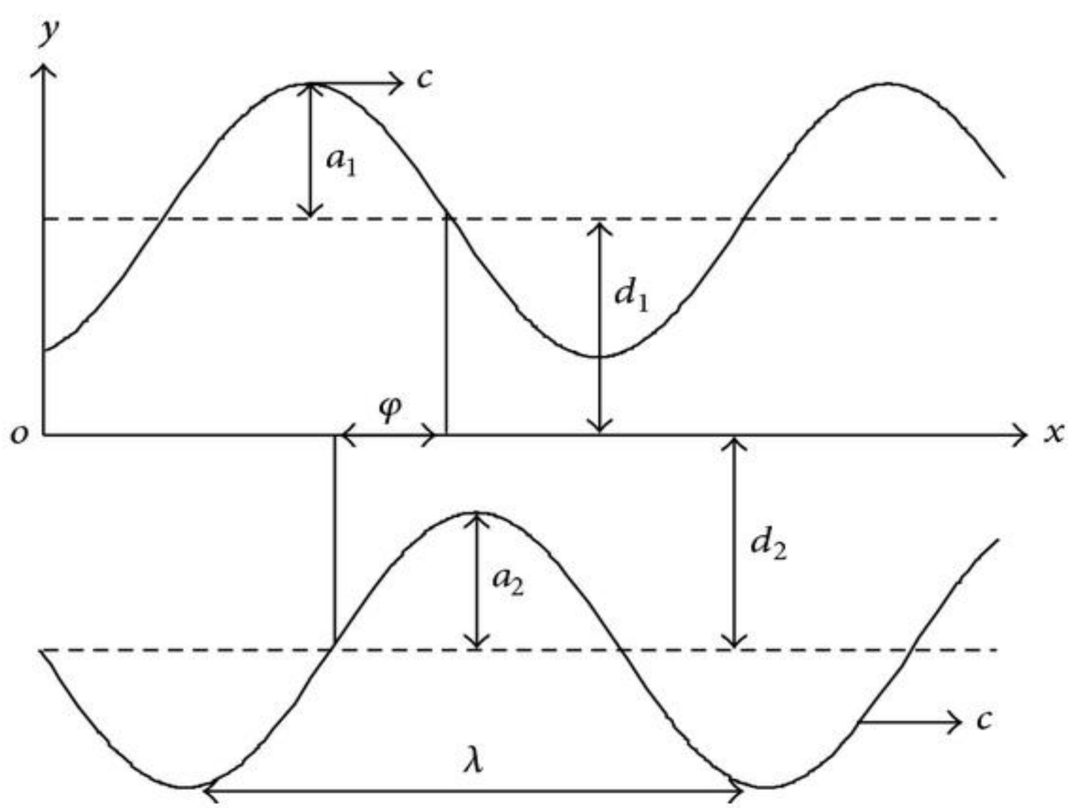

Figure 1. A rough sketch of asymmetric channel

here $a_1^{'}, a_2^{'}$, $d_1^{'}$ and $d_2^{'}$ are the upper wall and lower wall amplitudes, half-width of the upper and lower wall respectively, $\phi\left(0\le\phi\le\pi\right)$ is the phase difference and $\bar{t}$ is the time. Let us denote the velocity V as $V=\left[U^{'}\left(X^{'},Y^{'},t^{'}\right),V^{'}\left(X^{'},Y^{'},t^{'}\right),0\right]$ where $U^{'}$ and $V^{'}$ are the velocity components along the $X^{'}$ and $Y^{'}$ direction in the fixed frame.

The basic equations of motion are,

$$divV^{'}=0, \tag{3}$$

$$\rho\frac{dV^{'}}{dt^{'}}=divT^{'}+J^{'}\times B^{'}=0, \tag{4}$$

$$\rho c_p\frac{dT^{'}}{dt^{'}}=\kappa\nabla^2T^{'}+\mu\nabla^2V^{'}. \tag{5}$$

Let us consider the displacement currents to be negligible and define the Maxwell equations and Ohm's law as,

$$divB^{'}=0,\ curlB^{'}=\mu_e J^{'},\ J^{'}=\sigma\left(E^{'}+V^{'}\times B^{'}\right), curlE^{'}=-\frac{\partial B^{'}}{\partial t^{'}}, \tag{6}$$

where μ_e is the magnetic permeability and σ is the electric conductivity. $J^{'}\times B^{'}$ under the assumptions of low Reynolds magnetic number reduces to $J^{'}\times B^{'}=-\sigma\mu_e^{2}B_0^{2}U^{'}$.

The stress tensor $T^{'}$ for fluid model is

$$T^{'}=-p^{'}I^{'}+S^{'}, \tag{7}$$

The extra stress tensor $S^{'}$ is represented by,

$$S^{'}=A_1^{'}\mu+A_2^{'}\alpha_1+A_1^{'2}\alpha_2+A_3^{'2}\beta_1+\left(A_2^{'}A_1^{'}+A_1^{'}A_2^{'}\right)\beta_2+\left(trA_1^{'2}\right)A_1^{'}\beta_3, \tag{8}$$

where μ, α_1, α_2, β_1, β_2 and β_3 are the material constants which satisfy the relation $\mu\ge 0,\alpha_1\ge 0,\left|\alpha_1+\alpha_2\right|\le\sqrt{24\mu\beta_3},\beta_1+\beta_2=0,\beta_2\ge 0$.

Here $A_n^{'}$ represents the Rivlin-Erickson tensor defined as,

$$A_1^{'}=\left(gradV^{'}\right)^T+\left(gradV^{'}\right), \tag{9}$$

$$A_n^{'}=\frac{d}{dt^{'}}\left(A_{n-1}^{'}\right)+\left[\left(gradV^{'}\right)+\left(gradV^{'}\right)^T\right]\left(A_{n-1}^{'}\right),\ n\ge 1. \tag{10}$$

Using the above equation the equations (3) to (6) reduce to,

$$\frac{\partial U'}{\partial X'}+\frac{\partial V'}{\partial Y'}=0, \tag{11}$$

$$\rho\left(\frac{\partial}{\partial t'}+U'\frac{\partial}{\partial X'}+V'\frac{\partial}{\partial Y'}\right)U'=-\frac{\partial P'}{\partial X'}+\frac{\partial S'_{xx}}{\partial X'}+\frac{\partial S'_{xy}}{\partial Y'}-\sigma\mu_e^{\,2}B_0^{\,2}U', \tag{12}$$

$$\rho\left(\frac{\partial}{\partial t'}+U'\frac{\partial}{\partial X'}+V'\frac{\partial}{\partial Y'}\right)V'=-\frac{\partial P'}{\partial Y'}+\frac{\partial S'_{yx}}{\partial X'}+\frac{\partial S'_{yy}}{\partial Y'}, \tag{13}$$

$$\rho C_p\left(\frac{\partial}{\partial t'}+U'\frac{\partial}{\partial X'}+V'\frac{\partial}{\partial Y'}\right)T'=\kappa\left(\frac{\partial^2 T'}{\partial X'^2}+\frac{\partial^2 T'}{\partial Y'^2}\right)+\mu\left[2\left(\frac{\partial U'}{\partial X'}\right)^2+\left(\frac{\partial V'}{\partial Y'}\right)^2+\left(\frac{\partial U'}{\partial Y'}+\frac{\partial V'}{\partial X'}\right)^2\right]+\sigma B_0^{\,2}U'^2 \tag{14}$$

The corresponding dimensional boundary conditions,

$$\left.\begin{aligned}
&\psi'=\frac{q'}{2},\ \frac{\partial\psi'}{\partial y'}=0,\ T'-T_0=0 \text{ at } y=y_1=h_1\\
&\psi'=-\frac{q'}{2},\ \frac{\partial\psi'}{\partial y'}=0,\ T'-T_0=1 \text{ at } y=y_2=h_2
\end{aligned}\right\} \tag{15}$$

where

$$\begin{aligned}
S'_{xx}&=2U'_{X'}\mu+\left(2U'_{xt}+2U'_{xx}U'+2U'_{xy}U'_{xy}V'+4U'^2_X+2V'^2_X+2V'_XU'_XU'_Y\right)\alpha_1+\alpha_2\left(4U'^2_X+U'^2_Y+V'^2_X+2V'_XU'_Y\right)\\
&+2\beta_1\begin{pmatrix}U'_{Xtt}+U'_{xx}U'_t+2U'_{XXt}U'+U'_{XY}V'_t+2U'_{XYt}V'+6U'_{xt}U'_t+3V'_{xt}V'_X+V'_{xt}U'_y+2V'_{xt}U'_y\\
+7U'_{XX}U'_XV'+6V'U'_{XY}U'_X+4U'^3_X+3U'_{XY}V'_X+3V'_{XY}V'_XV'+2U'_{YY}V'_XV'+3V'_{XY}V'_XU'+U'_{XYt}U'^2_XV'\\
+U'_{Xtt}U'V'+U'_{XX}U'_YV'+U'_{XYt}V'^2_X+U'_{XY}V'_XV'+U'_{Y'}V'_{XY}U'+2U'_XU'_YV'_X+U'_YU'V'_{XX}+2U'_{XYt}U'V'\end{pmatrix}\\
&+2\beta_2\begin{pmatrix}4U'_{Xt}U'_X+4U'_{XX}V'_XU'+4U'_{XY}U'_XV'+8U'^2_X+U'_YV'_{Xt}+V'_{Xt}V'_X+V'_{XX}U'_YU'\\
+V'_{XX}V'_XU'+V'_{YY}U'_YV'+V'_{XY}V'_XV'+2U'^2_XV'_X+2U'^2_YU'_X+4V'_XU'_XU'_Y+U'_{Yt}V'_X+\end{pmatrix}\\
&+2\beta_3\left(U'^3_X+2U'_XU'^2_Y+4U'_XV'^2_Y+2U'_XV'^2_X+4U'_XU'_YV'_X\right)
\end{aligned}$$

$$\begin{aligned}
S'_{xy}&=\left(V'_X+U'_Y\right)\mu+\left(U'_{Yt}+V'_{Xt}+U'_{XX}U'+V'_{XY}V'+U'_{YY}V'+U'_{XX}U'+2U'_YU'_X+2V'_YV'_X\right)\alpha_1+\left(V'_XU'_X+V'_XV'_Y\right)2\alpha_2\\
&+\beta_1\begin{pmatrix}U'_{Yt}+U'_{XX}V'_{Xt}+U'_t+U'_{XYt}U'+V'_{XY}V'_t+2V'_{XYt}U'+2U'_{XYt}V'+U'_{XY}V'_t+V'_{XX}U'_t\\
+2U'_{XXt}V'_tU'+2V'_{Xt}V'_Y+4U'_{Yt}V'_X+V'_{Yt}V'_Y+4U'_{Xt}U'_Y+3U'_{Yt}U'_X+5U'_{XY}U'_YV'+U'_{XYt}U'^2_X+U'_{XXt}V'_tU'+2U'_{XXt}V'_{XX}U'_Y\end{pmatrix}\\
&+2\beta_2\begin{pmatrix}U'^2U'_{Yt}U'_X+U'_{XY}U'_XU'+V'_{XY}U'_XV'+V'_{XY}V'_t+V'_{YY}U'_XV'+V'_{YY}V'_XV'+U'_{YY}V'_XU'+V'_{XX}V'_YV'\\
+3U'^2_XV'_X+U'_{Yt}V'_Y+V'_{Yt}V'_Y+V'_{YY}V'_XV'+U'_{XY}V'_YV'+U'_{YY}V'_YV'+U'^2_XU'_Y+V'^3_X+3U'^2_YU'_YV'_X+2V'_XV'^2_Y+2U'_{Yt}U'_Y\end{pmatrix}\\
&+\beta_3\left(U'^2_XU'_Y+2U'^3_Y+6U'_YV'_X+4V'^2_XU'_Y+6U'^2_YV'_X+U'^2_XV'_X+2V'^3_X+4V'_XV'^2_Y\right)
\end{aligned}$$

$$
\begin{aligned}
S_{yy} &= 2V_Y^{'}\mu + \left(V_{Yt}^{'}U^{'} + V_{XY}^{'} + V_{YY}^{'}V^{'} + U_Y^{'2} + 2V_Y^{'2} + V_X^{'} + U_Y^{'}\right)\alpha_1 + \left(U_Y^{'2} + 4V_Y^{'2} + V_X^{'2} + 2U_Y^{'}V_X^{'}\right)\alpha_2 \\
&+2\beta_1\begin{pmatrix} V_{Yt}^{'} + U_t^{'}U^{'}V_{XY}^{'} + 2V_{XYt}^{'}U^{'} + V_t^{'}V_{YY}^{'} + 2U_{YYt}^{'}V^{'} + V_{Xt}^{'}U_Y^{'} + U_{Yt}^{'}V_X^{'} + V_{Xt}^{'}U_Y^{'} + 3U_Y^{'}V_{Yt}^{'} + 6V_{Yt}^{'}V_Y^{'} \\ +2V_{XX}^{'}U_Y^{'}U^{'} + 3V_{XY}^{'}U_Y^{'}V^{'} + 8V_Y^{'3} + 6U_{XY}^{'}U_Y^{'}V^{'} + 4U_{YY}^{'}U_Y^{'}V^{'} + 6V_{XY}^{'}V_Y^{'}U^{'} + 7V_{YY}^{'}V_Y^{'}V^{'} + V_{XYY}^{'}U_Y^{'}V^{'}U^{'2} + 2U_X^{'}V_{XXY}^{'}U^{'} + V_{YY}^{'}V_X^{'}U^{'} \\ +V_{YYY}^{'}V^{'2} + U_{YY}^{'}V_X^{'}U^{'} + 4U_X^{'}V_Y^{'}U_Y^{'} + V_X^{'}U_{XY}^{'}U^{'} \end{pmatrix} \\
&+2\beta_2\begin{pmatrix} 4V_{Yt}^{'}V_Y^{'} + 4V_{XY}^{'}V_Y^{'}U^{'} + 2V_{YY}^{'}V_Y^{'}V^{'} + 4V_Y^{'3} + V_{Xt}^{'}U_Y^{'} + U_{Yt}^{'} + V_{Xt}^{'}V_X^{'} + V_{XY}^{'}U_Y^{'}V^{'} + V_{XX}^{'}V_X^{'}U^{'} + U_Y^{'}V_{XX}^{'}U^{'} \\ +U_{YY}^{'}V_X^{'}V^{'} + U_{XY}^{'}V_X^{'}U^{'} + U_{XY}^{'}U_Y^{'}U^{'} + U_{XY}^{'}U_Y^{'}V^{'} + 2U_Y^{'2}V_Y^{'} + 2V_X^{'2}V_Y^{'} + 4V_Y^{'}U_Y^{'}U_X^{'} + V_{XY}^{'}V_X^{'}V^{'} + 2V_X^{'}U_{Yt}^{'} \end{pmatrix} \\
&+2\beta_3 4\left(V_Y^{'3} + U_X^{'2}V_Y^{'} + 2V_X^{'2}V_Y^{'} + 2U_Y^{'2}V_Y^{'} + 4U_X^{'}U_Y^{'}V_Y^{'}\right)
\end{aligned}
$$

Consideringthe wave frame $(x^{'}, y^{'})$ travelling with a velocity $'c'$ away from the fixed frame $(X^{'}, Y^{'})$ as the steady and unsteady motion respectively. The following transformations are done between the two frames,

$$x^{'} = X^{'} - ct^{'}, y^{'} = Y^{'}, u^{'} = U^{'} - c, v^{'} = V^{'}, p^{'} = P^{'}. \tag{16}$$

Introducing the non-dimensional variables

$$
\left.\begin{aligned}
& x = \frac{x^{'}}{\lambda}, y = \frac{y^{'}}{d_1}, u = \frac{u^{'}}{c}, v = \frac{\lambda v^{'}}{d_1 c}, Re = \frac{\rho d_1 c}{\mu}, \delta = \frac{d_1^{'}}{\lambda}, h_1 = \frac{h_1^{'}}{d_1}, h_2 = \frac{h_2^{'}}{d_2}\, p = \frac{p^{'} d_1^{'2}}{\lambda\mu c}, t = \frac{ct^{'}}{\lambda}, \\
& \psi = \frac{\psi^{'}}{cd_1}, F = \frac{q^{'}}{cd_1}\, a = \frac{a_1^{'}}{d_1}, b = \frac{a_2^{'}}{d_1},\ d = \frac{d_2^{'}}{d_1}, \theta = \frac{T^{'} - T_0}{T_1 - T_0}, \upsilon = \frac{\mu}{\rho}, S = \frac{d}{\mu c} S^{'}\left(x^{'}\right), M = \sqrt{\frac{\sigma}{\mu}} B_0 d_1, \\
& \Pr = \frac{\mu C_p}{\kappa}, E = \frac{c^2}{C_p\left(T_1 - T_0\right)}, Br = \Pr E, u = \frac{\partial\psi}{\partial y}\, v = -\delta\frac{\partial\psi}{\partial x}, \gamma_2 = \frac{\beta_2 c^2}{d_1^{\,2}\mu}, \gamma_3 = \frac{\beta_3 c^2}{d_1^{\,2}\mu}.
\end{aligned}\right\} \tag{17}
$$

where $\Pr$ is the Prandtl number, κ is the thermal conductivity, C_p is the specific heat, F is the dimensionless mean flow, E is the Eckert number and Br is the Brinkman number. Using the above defined non-dimensional variables in Equations (11) – (14) and employing the lubrication approach we get,

$$\frac{\partial p}{\partial x} = \frac{\partial^3\psi}{\partial y^3} + 2\Gamma\frac{\partial}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2}\right)^3 - M^2\left(\frac{\partial\psi}{\partial y} + 1\right), \tag{18}$$

$$\frac{\partial p}{\partial y} = 0, \tag{19}$$

$$\frac{\partial^2\theta}{\partial y^2} + Br\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + BrM^2\left(\frac{\partial\psi}{\partial y} + 1\right)^2 = 0, \tag{20}$$

With the following non-dimensional boundary conditions

$$\left.\begin{array}{l}\psi = \dfrac{F}{2}, \dfrac{\partial \psi}{\partial y} = 0, \theta = 0, \text{ at } y = y_1 = h_1 = 1 + a\cos(2\pi x) \\ \psi = -\dfrac{F}{2}, \dfrac{\partial \psi}{\partial y} = 0, \theta = 1, \text{ at } y = y_2 = h_2 = -d - b\cos(2\pi x + \phi)\end{array}\right\} \tag{21}$$

where $\Gamma = \gamma_2 + \gamma_3$ is the Deborah number.

3. SOLUTION OF THE PROBLEM

Double integrating equation (20) we get the expression for temperature θ as

$$\begin{aligned}\theta = &-Br\left\{\frac{F_1 n^3}{12}(\cosh(ny))^4 + \frac{F_2{}^2 n^3}{12}(\sinh(ny))^4 + \left(\frac{\partial p}{\partial x} + n^2\right)\frac{y^2}{2} + 2\left\{\frac{F_1 F_2 n^3}{6}\frac{(\sinh(ny))^3}{(\cosh(ny))} - F_2 \sinh(ny)\left(\frac{\partial p}{\partial x} + n^2\right) + F_1 \cosh(ny)\left(\frac{\partial p}{\partial x} + n^2\right)\right\}\right\} \\ &+ BrM^2\left\{\frac{F_1}{12}(\sinh(ny))^4 - \frac{F_2}{12}(\cosh(ny))^4\left(\frac{\partial p}{\partial x} + n^2\right)\frac{y^2}{2} + \frac{y^2}{2} + 2\left\{\begin{array}{l}-\dfrac{F_1 F_2 n}{6}(\sinh(ny))^3 - \dfrac{F_2}{n}(\cosh(ny))\left(\dfrac{\partial p}{\partial x} + n^2\right) + \dfrac{F_1}{n}(\sinh(ny))\left(\dfrac{\partial p}{\partial x} + n^2\right) \\ + \dfrac{F_1}{n}(\sinh(ny)) - \dfrac{F_2}{n}(\cosh(ny)) + \left(\dfrac{\partial p}{\partial x} + n^2\right)\dfrac{y^2}{2}\end{array}\right\}\right\} \\ &+ a_1 y + a_2\end{aligned} \tag{22}$$

where

$$\begin{aligned}a_1 = &-\frac{1}{Br}\left\{\begin{array}{l}\dfrac{F_1 n^3}{12}(\cosh(n(y_1 - y_2)))^4 + \dfrac{F_2{}^2 n^3}{12}(\sinh(n(y_1 - y_2)))^4 + \left(\dfrac{\partial p}{\partial x} + n^2\right)\dfrac{(y_1 - y_2)^2}{2} \\ +2\left\{\dfrac{F_1 F_2 n^3}{6}(\tanh(n(y_1 - y_2)))^3 - F_2 \sinh(n(y_1 - y_2)) + F_1 \cosh(n(y_1 - y_2))\right\}\end{array}\right\} + \\ &\frac{1}{BrM^2}\left\{\begin{array}{l}\dfrac{F_1}{12}(\sinh(n(y_1 - y_2)))^4 - \dfrac{F_2}{12}(\cosh(n(y_1 - y_2)))^4\left(\dfrac{\partial p}{\partial x} + n^2\right)\dfrac{(y_1 - y_2)^2}{2} + \dfrac{(y_1 - y_2)^2}{2} \\ +2\left\{\begin{array}{l}-\dfrac{F_1 F_2 n}{6}(\sinh(n(y_1 - y_2)))^3 - \dfrac{F_2}{n}(\cosh(n(y_1 - y_2)))\left(\dfrac{\partial p}{\partial x} + n^2\right) + \dfrac{F_1}{n}(\sinh(n(y_1 - y_2))) \\ \left(\dfrac{\partial p}{\partial x} + n^2\right) + \dfrac{F_1}{n}(\sinh(n(y_1 - y_2))) - \dfrac{F_2}{n}(\cosh(n(y_1 - y_2))) + \left(\dfrac{\partial p}{\partial x} + n^2\right)\dfrac{(y_1 - y_2)^2}{2}\end{array}\right\}\end{array}\right\}\end{aligned}$$

and $a_2 = 0$

Employing the Adomian decomposition method (ADM),equation (18) takes the form,

$$L_{yyy}[\psi] - n^2[\psi_y + 1] = \frac{\partial p}{\partial x} + n^2 - 2\Gamma\frac{\partial^2}{\partial y^2}[\psi_{yyy}]^3, \tag{23}$$

where $n^2 = M^2$

Applying the inverse operator $L_{yyy}^{-1} = \iiint [.] dydydy$ Eq. (23) can be written as,

$$\psi = c_1 + c_2 y + c_3 \frac{y^2}{2!} + L_{yyy}^{-1}\left[\frac{\partial p}{\partial x} + n^2\right] + L_{yyy}^{-1}\left[n^2 \psi_y\right] - L_{yyy}^{-1}\left[2\Gamma \frac{\partial^2}{\partial y^2}\left[\psi_{yy}\right]^3\right], \tag{24}$$

Now we decompose ψ as $\psi = \sum_{n=0}^{\infty} \psi_n$

$$\psi_0 = c_1 + c_2 y + c_3 \frac{y^2}{2!} + \left[\frac{\partial p}{\partial x} + n^2\right]\frac{y^3}{3!}, \tag{25}$$

$$\psi_{n+1} = \iiint \left[n^2 (\psi_n)\right]_y dydydy - 2\Gamma \iiint \frac{\partial^2}{\partial y^2}\left[(\psi_n)_{yy}\right]^3$$

where $n \geq 0$.

Therefore,

$$\psi_1 = n^2\left(c_2 \frac{y^3}{3!} + c_3 \frac{y^4}{4!} + \left(\frac{\partial p}{\partial x} + n^2\right)\frac{y^5}{5!}\right) - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(c_3 \frac{y^4}{4!} + \frac{y^5}{5!}\right)\right],$$

$$\psi_2 = n^4\left(c_2 \frac{y^5}{5!} + c_3 \frac{y^6}{6!} + \left(\frac{\partial p}{\partial x} + n^2\right)\frac{y^7}{7!}\right) - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(c_3 \frac{y^4}{4!} + \frac{y^5}{5!}\right)\right],$$

$$\psi_n = n^{2n}\left(c_2 \frac{y^{2n+1}}{(2n+1)!} + c_3 \frac{y^{2n+2}}{(2n+2)!} + \left(\frac{\partial p}{\partial x} + n^2\right)\frac{y^{2n+3}}{(2n+3)!}\right) - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(c_3 \frac{y^4}{4!} + \frac{y^5}{5!}\right)\right],$$

According to (24) the solution of ψ can be written as,

$$\psi = A_0 + \sinh(ny) + \left(\frac{A_1}{n} + \frac{1}{n^3}\left(\frac{\partial p}{\partial x} + n^2\right)\right) + \frac{A_2}{n^2}\left(\cosh(ny) - 1\right) - \frac{1}{n^2}\left(\frac{\partial p}{\partial x} + n^2\right) y - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(c_3 \frac{y^4}{4!} + \frac{y^5}{5!}\right)\right],$$

which can be simplified as,

$$\psi = C_0 + C_1 \cosh(ny) + C_2 \sinh(ny) + \left(\frac{\partial p}{\partial x} + n^2\right) y - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(C_3 \frac{y^4}{4!} + \frac{y^5}{5!}\right)\right], \tag{26}$$

where

$$C_0 = \frac{Fn(y_2 + y_1) + \tanh\left[n\left(\frac{y_1 - y_2}{2}\right)\right]\left(2(y_2 + y_1) + \frac{Fn^2 (y_2 + y_1)}{1 + \Gamma}\right)}{2n(y_2 - y_1) + 2\tanh\left[n\left(\frac{y_1 - y_2}{2}\right)\right]\left(2 + \frac{n^2 (y_2 - y_1)}{1 + \Gamma}\right)} - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(\frac{(y_1 - y_2)^4}{4!} + \frac{(y_1 - y_2)^5}{5!}\right)\right],$$

$$C_1 = \frac{F + (y_1 - y_2)\sinh\left[n\left(\frac{y_1+y_2}{2}\right)\right]\operatorname{sech}\left[n\left(\frac{y_1-y_2}{2}\right)\right]}{(y_1-y_2)n - 2\tanh\left[n\left(\frac{h_1-h_2}{2}\right)\right]\left(2+\frac{n^2(y_2-y_1)}{1+\Gamma}\right)} - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(\frac{(y_1-y_2)^4}{4!} + \frac{(y_1-y_2)^5}{5!}\right)\right],$$

$$C_2 = \frac{F + (y_1 - y_2)\cosh\left[n\left(\frac{y_1+y_2}{2}\right)\right]\operatorname{sech}\left[n\left(\frac{y_1-y_2}{2}\right)\right]}{(y_1-y_2)n - \tanh\left[n\left(\frac{y_1-y_2}{2}\right)\right]\left(2+\frac{N^2(y_2-y_1)}{1+\Gamma}\right)} - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(\frac{(y_1-y_2)^4}{4!} + \frac{(y_1-y_2)^5}{5!}\right)\right],$$

$$, C_3 = \frac{\left(2n^2\left(\left(\frac{\partial p}{\partial x}+n^2\right)+1\right) + \frac{N^4\left(F - y_1\left(\frac{\partial p}{\partial x}+n^2\right) + \left(\frac{\partial p}{\partial x}+n^2\right)y_2\right)}{1+\Gamma}\right)\tanh\left[n\left(\frac{y_1-y_2}{2}\right)\right]}{(y_1-y_2)n - \tanh\left[n\left(\frac{y_1-y_2}{2}\right)\right]\left(2+\frac{n^2(y_2-y_1)}{1+\Gamma}\right)}$$

$$-12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(\frac{(y_1-y_2)^4}{4!} + \frac{(y_1-y_2)^5}{5!}\right)\right]$$

The expression for velocity u is given by,

$$u = -C_1 n\sinh(ny) + C_2 n\cosh(ny) + \left(\frac{\partial p}{\partial x} + n^2\right) - 12\Gamma\left[\frac{\partial p}{\partial x} + n^2\left(C_3\frac{y^3}{3!} + \frac{y^4}{4!}\right)\right], \tag{27}$$

The volumetric rate of flow is defined by,

$$Q'(x,t) = q + c_1 d_1 + c_2 d_2, \tag{28}$$

The instantaneous volumetric flow rate is defined by,

$$Q = \int_{H_2}^{H_1}\left[U(x,y,t)\right]dy = \int_{h_2}^{h_1}\left[u(x,y) + c\right]dy = q + ch_1 - ch_2, \tag{29}$$

The mean flow rate is given as,

$$Q'(x,t) = \frac{1}{T}\int_0^T Q(x,y)dt \cdot \tag{30}$$

Using (29) and (30) we obtain,

$$Q'(x,t) = q + c_1 d_1 + c_2 d_2, \tag{31}$$

The mean flow in dimensionless form is,

$$\Theta = \frac{Q'}{cd_1}, \; F = \frac{q}{cd_1}. \tag{32}$$

Using (31) and (32) we obtain,

$$\Theta = F + d + 1, \tag{33}$$

in which

$$F = \int_{h_2}^{h_1} u(x, y)dy \cdot \tag{34}$$

The pressure gradient is defined as,

$$\frac{\partial p}{\partial x} = \frac{\partial^3 \psi}{\partial y^3} + 2\Gamma \frac{\partial}{\partial y}\left(\frac{\partial^2 \psi}{\partial y^2}\right)^3 - M^2\left(\frac{\partial \psi}{\partial y} + 1\right) \cdot \tag{35}$$

Integrating over one wavelength we get,

$$\Delta p_\lambda = \int_0^1 \frac{\partial p}{\partial x} dx \cdot \tag{36}$$

The expression for force of frictionis defined as,

$$F_{\lambda 1} = \int_0^1 h_1^2 \left(-\frac{\partial p}{\partial x}\right) dx \cdot \tag{37}$$

4. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results of the problem under consideration are discussed through graphs. Figure (2) shows the temperature plot for different values of Brinkmann number (Br). The plot shows that the Brinkmann number (Br) enhances the temperature values this is because Brinkman number refers to the rate of conduction of heat from a wall to a viscous fluid. With an enhancement in the Brinkman number, the forces of viscosity dominates over the irreversibility due to heat transfer to total heat transfer and fluid friction irreversibility and thereby leads to an increase in temperature (θ). Figures (3) and (4) are depicting the velocity plot for various values of magnetic field (M) and Deborah number (Γ). We can see from Figure (3) that the magnetic field (M) decreases the magnitude of velocity. This is due to the fact that a strong Lorentz force was perceived due to the magnetic field, hindering the movement of the fluid flow. Thus, the reduction in velocity components was noticed. The velocity is found to be maximum at the centre of the channel, whereas the velocity reduces as we move towards the ends of the channel walls. This is because when we increase the magnetic field (M) the magnetic and electric forces are dominant over the effect of viscous forces owing to a decrease in the velocity. Figure (4) depicts that the velocity reduces with an enhancement in Deborah number (Γ). This is because Deborah number is the ratio of relaxation time to characteristic time scale. It is commonly used to characterize how "fluid" a material is. The smaller the Deborah number, the more the fluid material appears. Thus increasing Deborah number reduces the fluid viscosity and thereby reduces the velocity of the fluid. Figure (5) shows the plot of pressure rise per wavelength (Δp_λ) verses dimensionless volume flow rate (Θ). It indicates that, for $\Gamma = 0$ the pressure rise is linear where as non-linearity approaches as we move towards the non-zero values of Γ. Figure (6) represents the plot of pressure rise per wavelength (Δp_λ) verses dimensionless Volume flow rate (Θ) for different values of Magnetic field (M). It is found that the Volume flow rate (Θ) reduces with enhancement of (M) in the free pumping region $(\Delta p_\lambda = 0)$ and pumping region $(\Delta p_\lambda > 0)$, while in the co-pumping region $(\Delta p_\lambda < 0)$ the flow rate enhances with enhancement in (M). Figures (7) and (8) are plotted to study the frictional force $(F_{\lambda 1})$ profile at lower wall $y = h_1$. Figure (7) depicts that $F_{\lambda 1}$ resists the flow along the channel wall at a critical value (Θ) Further the frictional force $(F_{\lambda 1})$ at the upper wall behaves complementary to pressure rise (Δp_λ). Figure (9) is plotted to analyse the amplitude of the shear stress distribution (S_{xy}) on the lower wall of the channel for various values of Deborah number (Γ) and phase difference (ϕ). It is found that the amplitude of the shear stress distribution (S_{xy}) enhances with an increment in Γ, whereas an opposite trend is shown by phase difference (ϕ) as depicted in Figure (10).

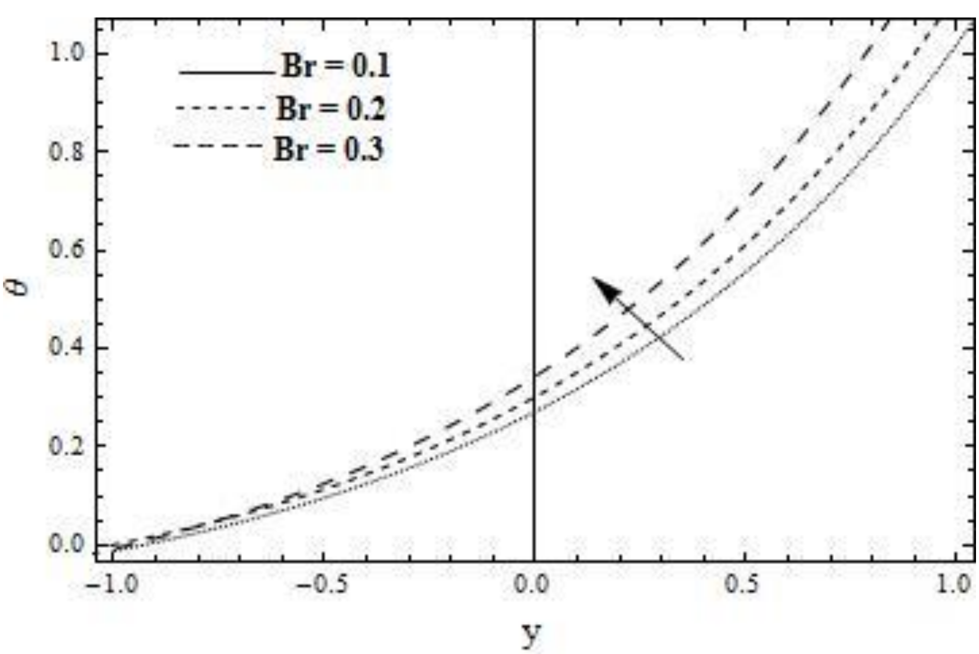

Figure 2. Temperature plot for various values of Brinkmann number (Br) for a = 0.1, b = 0.2, d = 1, $\phi = 0$

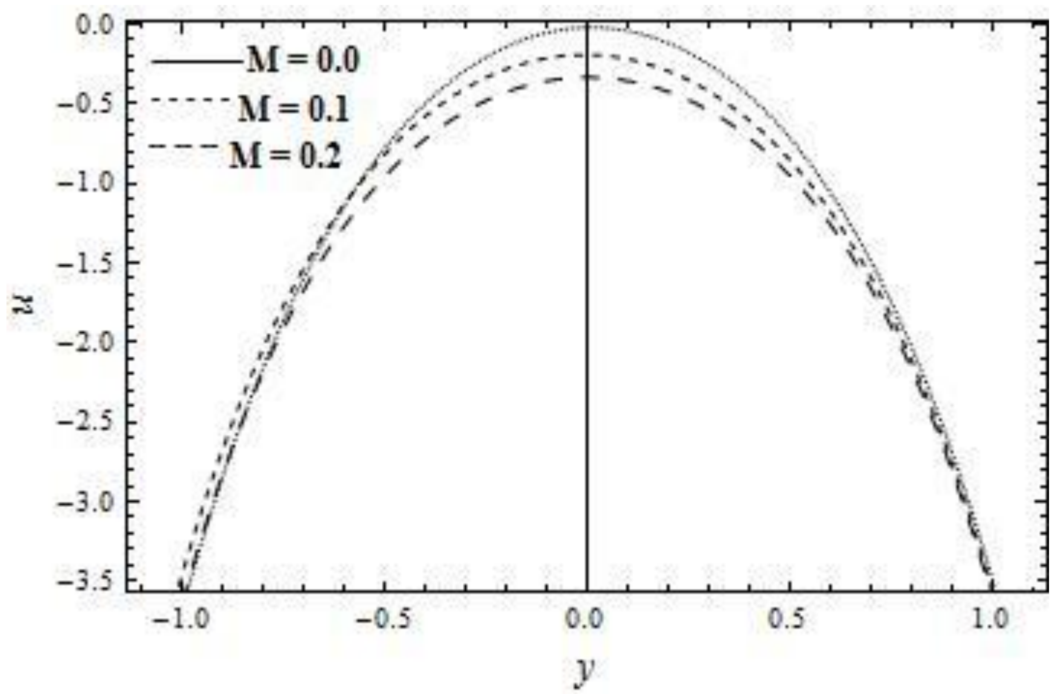

Figure 3. Velocity plot for various values of Magnetic field (M) for a = 0.1, b = 0.2, d = 1, $\phi = 0$

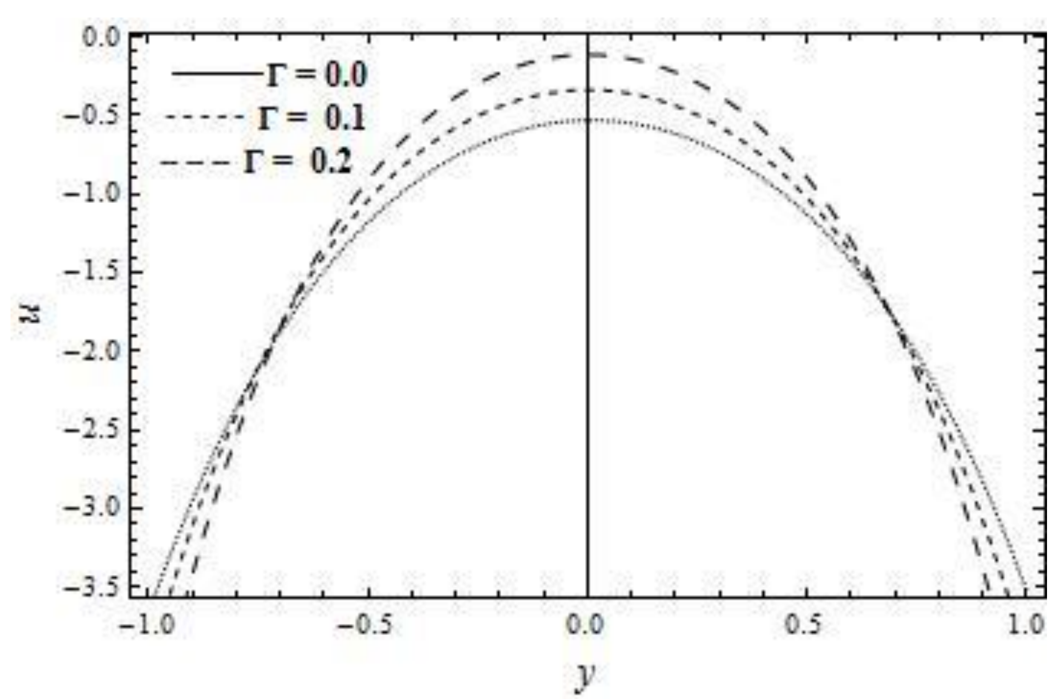

Figure 4. Velocity plot for various values of Deborah number (Γ) for a = 0.1, b = 0.2, d = 1, $\phi = 0$

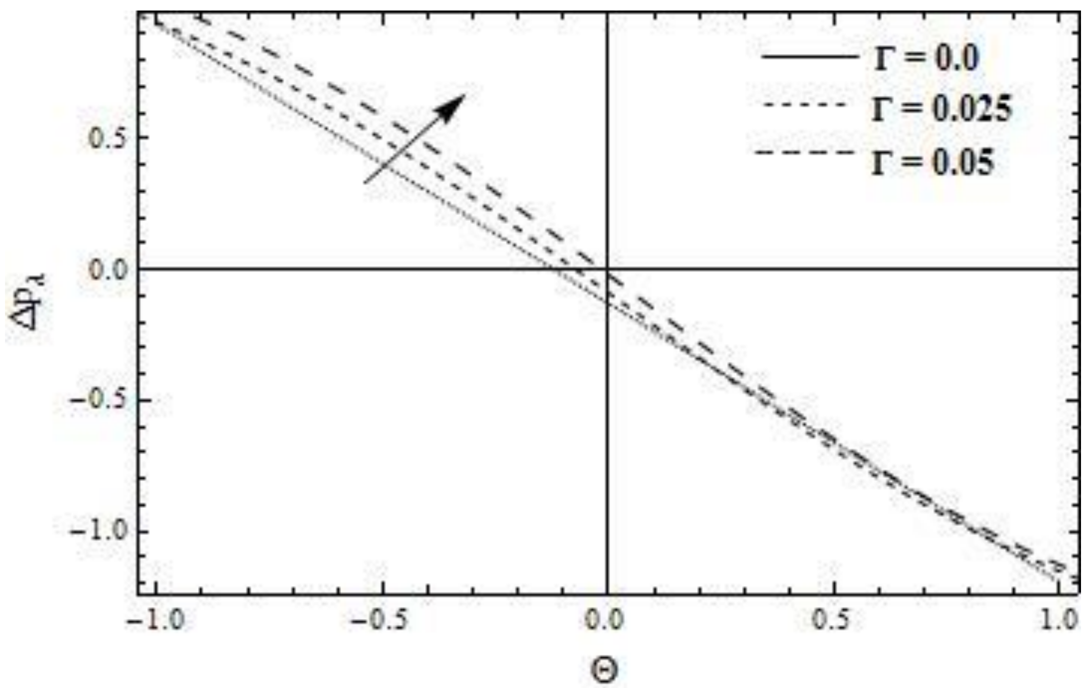

Figure 5. Pressure rise per wavelength plot for various values of Deborah number (Γ) where a = 0.1, b = 0.2, d = 1, $\phi = 0$

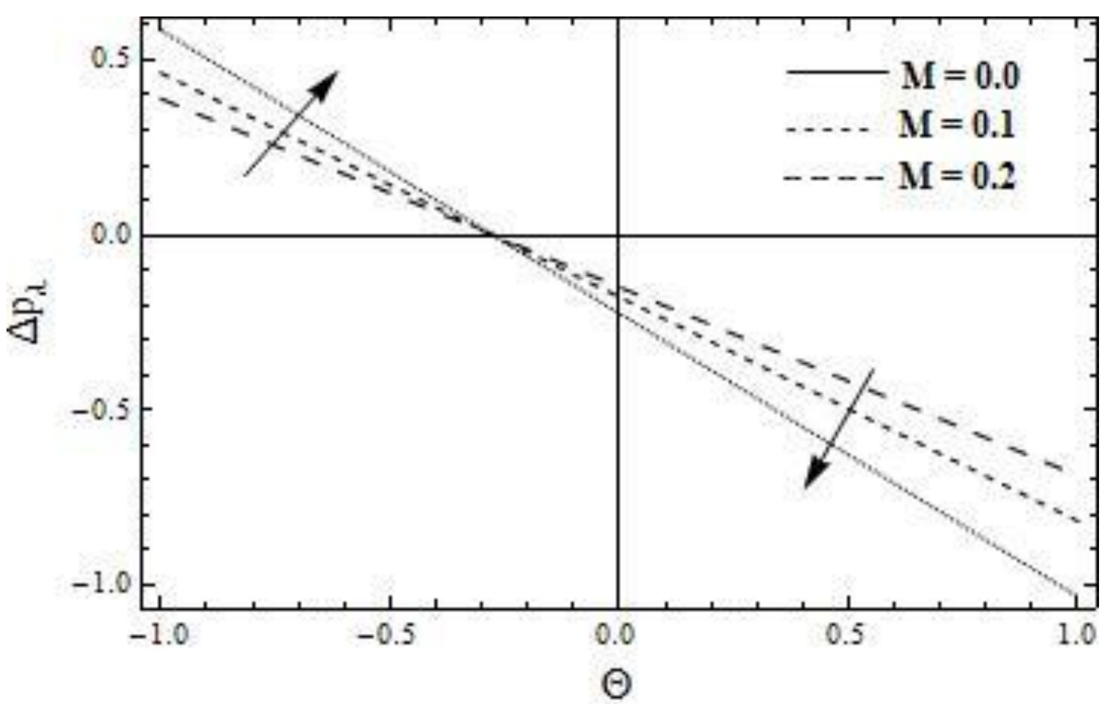

Figure 6. Pressure rise per wavelength plot for various values of Magnetic field (M) where a = 0.1, b = 0.2, d = 1, $\phi = 0$

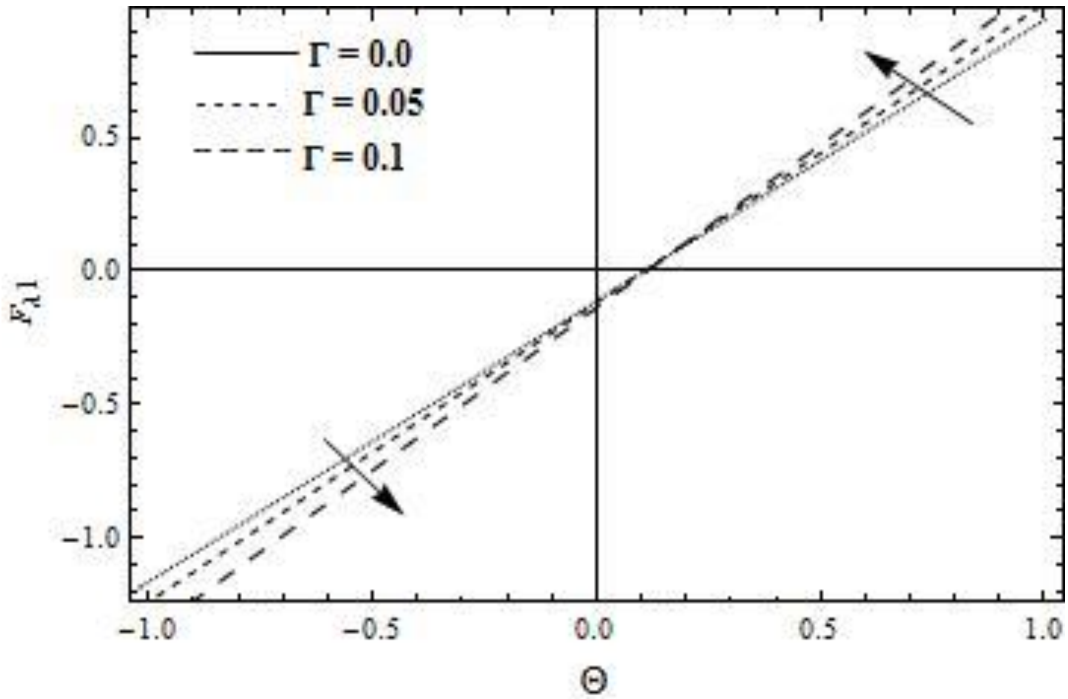

Figure 7. Frictional force plot for various values of Deborah number (Γ) for a = 0.1, b = 0.2, d = 1, $\phi = 0$

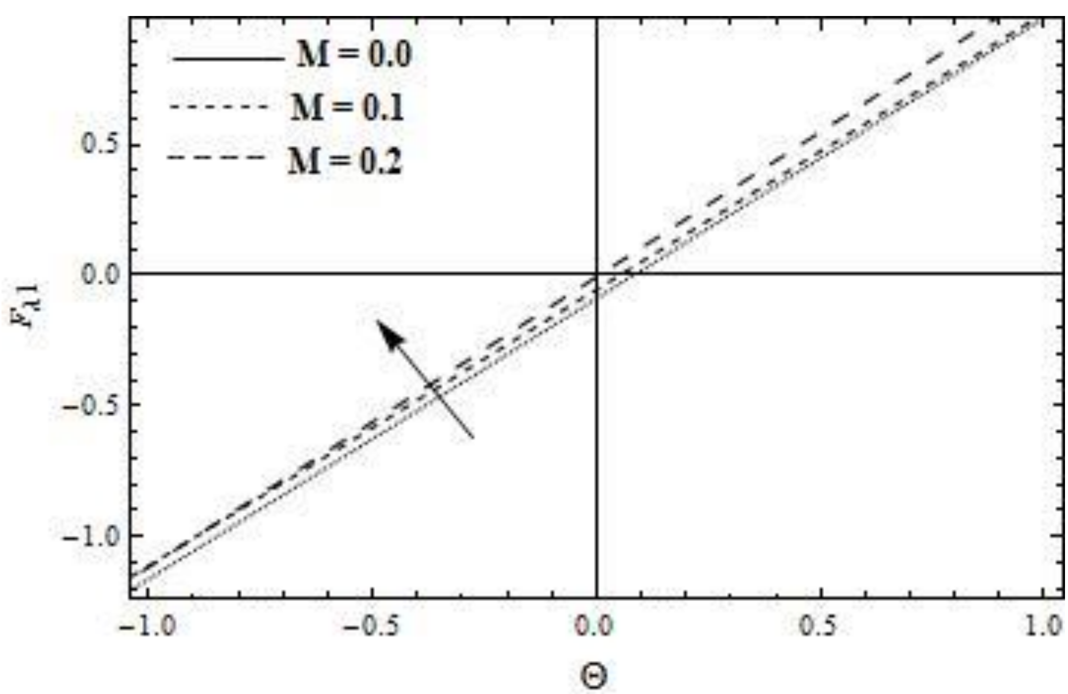

Figure 8. Frictional force plot for various values of Magnetic field (M) for a = 0.1, b = 0.2, d = 1, $\phi = 0$

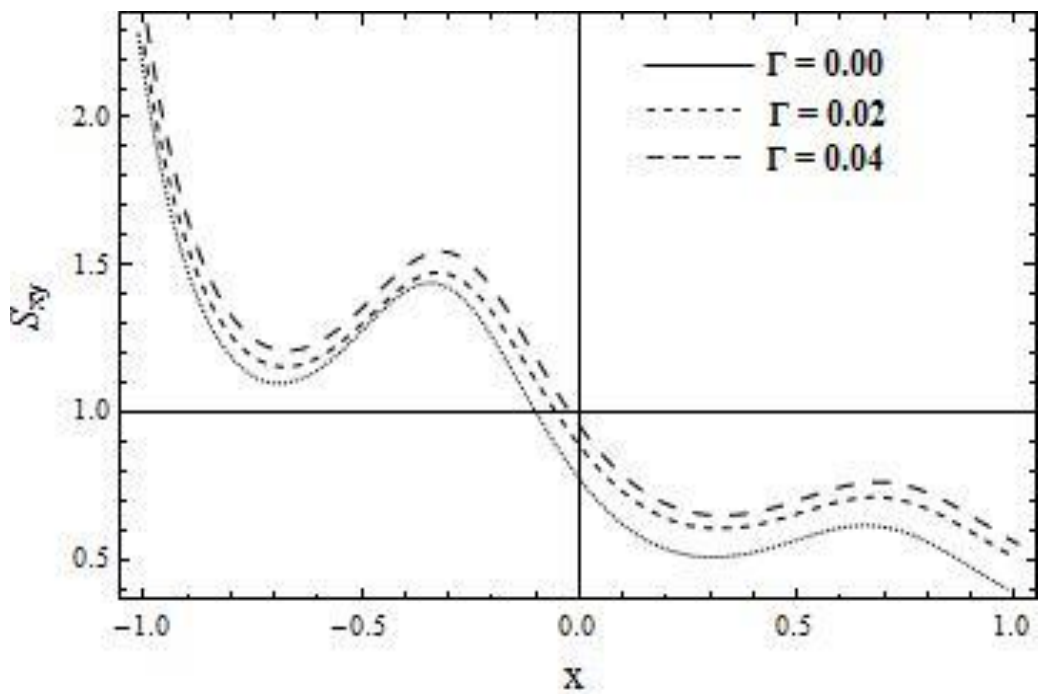

Figure 9. Stress tensor plot for various values of Deborah number (Γ) for a = 0.1, b = 0.2, d = 1, $\phi = 0$, M =0.5

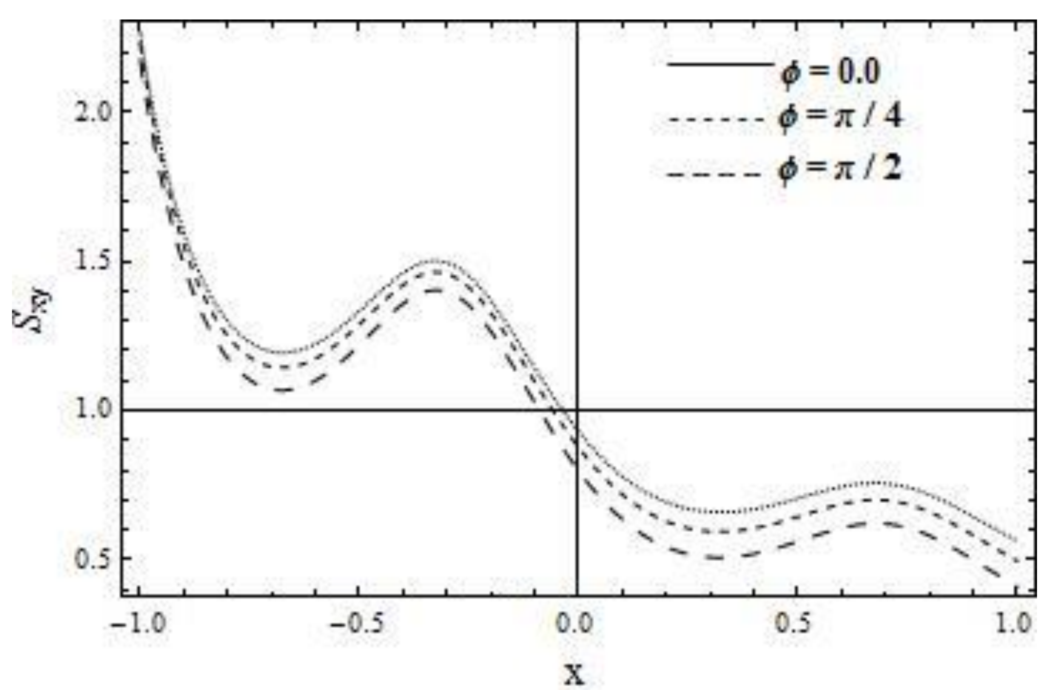

Figure 10. Stress tensor plot for various values of phase difference ϕ for a = 0.1, b = 0.2, d = 1, M =0.5

5. CONCLUDING REMARKS

The research problem considered here is to analyse the impacts of Joule heating and magnetic field on peristaltic flow of a third grade fluid. It is found that the results obtained are applicable to various fields of engineering, biomedical and industry.

The following are the key points obtained from this study;

- It is found that the Brinkmann number (Br) enhances the temperature (θ) values.
- Both Magnetic field (M) and Deborah number (Γ) show opposite trend for velocity profile.
- It is found that the pressure rise per wavelength (Δp_{λ}) enhances with an enhancement in the Deborah number (Γ) and shows opposite behaviour with an enhancement in Magnetic field (M).
- The frictional force $(F_{\lambda 1})$ shows complementary behaviour to the pressure rise (Δp_{λ}).

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