Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 136 (2019) 52-65 EISSN 2392-2192

μ-lacunary $\chi^3_{A_{uvw}}$-convergence defined by Musielak Orlicz function

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ABSTRACT

We study some connections between μ-lacunary strong $\chi^3_{A_{uvw}}$ - convergence with respect to a mnk sequence of Musielak Orlicz function and μ-lacunary $\chi^3_{A_{uvw}}$ - statistical convergence, where A is a sequence of four dimensional matrices $A(uvw) = \left(a^{m_1 \ldots m_r n_1 \ldots n_s}_{k_1 \ldots k_r \ell_1 \ldots \ell_s}(uvw)\right)$ of complex numbers.

***Keywords*:** Analytic sequence, x^2 space, difference sequence space, Musielak-modulus function, *p*-metric space, *mn*-sequences

2010 Mathematics Subject Classification: 40A05, 40C05, 40D05

1. INTRODUCTION

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}), where $m, n, k \in \mathbb{N}$, the set of positive integers. Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

(Received 10 September 2019; Accepted 27 September 2019; Date of Publication 28 September 2019)

We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double series is found in Apostol [1] and double sequence spaces is found in Hardy [9], Deepmala et al. [10, 11] and many others. Later on investigated by some initial work on triple sequence spaces is found in Esi [2], Esi et al. [3-8], Şahiner et al. [12], Subramanian et al. [13], Prakash et al. [14] and many others.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ give one space is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq} \ (m,n,k = 1,2,3,\dots).$$

A sequence $x = (x_{mnk})$ is said to be triple analytic if

$$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0 \text{ as } m,n,k \to \infty.$$

A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m+n+k)!\,|x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m,n,k \to \infty$. The triple gai sequences will be denoted by χ^3.

2. DEFINITIONS AND PRELIMINARIES

A triple sequence $x = (x_{mnk})$ has limit 0 (denoted by $P - \lim x = 0$) (i.e) $\left((m+n+k)!\,|x_{mnk}|\right)^{1/m+n+k} \to 0$ as $m,n,k \to \infty$. We shall write more briefly as $P - convergent\ to\ 0$.

Definition 2.1 An Orlicz function (see [15]) is a function $\mathrm{M}: [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $\mathrm{M}(0) = 0$, $\mathrm{M}(\mathrm{x}) > 0$, for $\mathrm{x} > 0$ and $\mathrm{M}(\mathrm{x}) \to \infty$ as $\mathrm{x} \to \infty$. If convexity of Orlicz function M is replaced by $\mathrm{M}(\mathrm{x}+\mathrm{y}) \le \mathrm{M}(\mathrm{x}) + \mathrm{M}(\mathrm{y})$, then this function is called modulus function.

Lindenstrauss and Tzafriri (see [16]) used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = \sup\{|v|u - (f_{mnk})(u): u \geq 0\}, m, n, k = 1,2, \ldots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f, (see [17]) the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f = \left\{x \in w^3: I_f(|x_{mnk}|)^{1/m+n+k} \to 0 \quad \text{as} \quad m, n, k \to \infty\right\},$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk}(|x_{mnk}|)^{1/m+n+k}, \qquad x = (x_{mnk}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk}\left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk}\right)$$

is an extended real number.

Definition 2.2 Let $\mathrm{mnk}(\geq 3)$ be an integer. A function $\mathrm{x}: (\mathrm{M} \times \mathrm{N} \times \mathrm{K}) \times (\mathrm{M} \times \mathrm{N} \times \mathrm{K}) \times \cdots \times (\mathrm{M} \times \mathrm{N} \times \mathrm{K}) \times (\mathrm{M} \times \mathrm{N} \times \mathrm{K})$ $[\mathrm{m} \times \mathrm{n} \times \mathrm{k} - \text{factors}] \to \mathbb{R}(\mathbb{C})$ is called a real or complex mnk-sequence, where $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denote the sets of natural numbers and complex numbers respectively. Let $\mathrm{m}_1, \mathrm{m}_2, \ldots \mathrm{m}_r, \mathrm{n}_1, \mathrm{n}_2, \ldots, \mathrm{n}_s, \mathrm{k}_1, \mathrm{k}_2, \ldots, \mathrm{k}_t \in \mathbb{N}$ and X be a real vector space of dimension w, where $\mathrm{m}_1, \mathrm{m}_2, \ldots \mathrm{m}_r, \mathrm{n}_1, \mathrm{n}_2, \ldots, \mathrm{n}_s, \mathrm{k}_1, \mathrm{k}_2, \ldots, \mathrm{k}_t \leq \mathrm{w}$. A real valued function

$$d_p(x_{11}, \ldots, x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}) \\ = \| (d_1(x_{11}, 0), \ldots, d_n(x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}, 0)) \|_p$$

on X satisfying the following four conditions:

(i) $\| (d_1(x_{11}, 0), \ldots, d_n(x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}, 0)) \|_p = 0$ if and and only if $d_1(x_{11}, 0), \ldots, d_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s}(x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}, 0)$ are linearly dependent,

(ii) $\| (d_1(x_{11}, 0), \ldots, d_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}(x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}, 0)) \|_p$ is invariant under permutation,

(iii) For $\alpha \in \mathbb{R}$,

$$\| (\alpha d_1(x_{11}, 0), \ldots, d_{m_1,m_2,\ldots m_p,n_1,n_2,\ldots,n_q,k_1,k_2,\ldots,k_t}(x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}, 0)) \|_p \\ = |\alpha| \, \| (d_1(x_{11}, 0), \ldots, d_n(x_{m_1,m_2,\ldots m_p,n_1,n_2,\ldots,n_q,k_1,k_2,\ldots,k_t}, 0)) \|_p$$

(iv) For $1 \leq p < \infty$,

$$d_p\big((x_{11}, y_{11}), (x_{12}, y_{12}) \ldots (x_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t}, y_{m_1,m_2,\ldots m_r,n_1,n_2,\ldots,n_s,k_1,k_2,\ldots,k_t})\big)$$

$$= \Big(d_X(x_{11}, x_{12}, \dots x_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t})^p +$$
$$d_Y(y_{11}, y_{12}, \dots y_{m_1,m_2,\dots m_p,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t})^p\Big)^{1/p}$$

(or)

(v) $$d\big((x_{11}, y_{11}), (x_{12}, y_{12}), \dots (x_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s}, y_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t})\big)$$
$$:= \sup\{d_X(x_{11}, x_{12}, \dots x_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t}),$$
$$d_Y(y_{11}, y_{12}, \dots y_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t})\},$$

for $x_{11}, x_{12}, \dots x_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t} \in X$, $y_{11}, y_{12}, \dots y_{m_1,m_2,\dots m_r,n_1,n_2,\dots,n_s,k_1,k_2,\dots,k_t} \in Y$ is called the p-product metric of the Cartesian product of $m_1, m_2, \dots m_r, n_1, n_2, \dots, n_s, k_1, k_2, \dots, k_t$ metric spaces is the p-norm of the $m \times n \times k$-vector of the norms of the $m_1, m_2, \dots m_r, n_1, n_2, \dots, n_s, k_1, k_2, \dots, k_t$ subspaces.

Definition 2.3 The triple sequence $\theta_{i,\ell,j} = \{(m_i, n_\ell, k_j)\}$ is called triple lacunary if there exist three increasing sequences of integers such that

$$m_0 = 0, h_i = m_i - m_{r-1} \to \infty \text{ as } i \to \infty$$

and

$$n_0 = 0, \overline{h_\ell} = n_\ell - n_{\ell-1} \to \infty \text{ as } \ell \to \infty.$$
$$k_0 = 0, \overline{h_j} = k_j - k_{j-1} \to \infty \text{ as } j \to \infty.$$

Let $m_{i,\ell,j} = m_i n_\ell k_j$, $h_{i,\ell,j} = h_i \overline{h_\ell h_j}$, and $\theta_{i,\ell,j}$ is determine by

$$I_{i,\ell,j} = \{(m, n, k): m_{i-1} < m < m_i \quad \text{and} \quad n_{\ell-1} < n \le n_\ell \quad \text{and} \quad k_{j-1} < k \le k_j\},$$
$$q_k = \frac{m_k}{m_{k-1}}, \overline{q_\ell} = \frac{n_\ell}{n_{\ell-1}}, \overline{q_j} = \frac{k_j}{k_{j-1}}.$$

Let $F = (f_{mnk})$ be a mnk-sequence of Musielak Orlicz functions such that $\lim_{u \to 0^+} \sup_{mnk} f_{mnk}(u) = 0$. Throughout this paper $\chi^3_{A_{uvw}}$-convergence of p-metric of mnk-sequence of Musielak Orlicz function determinated by F will be denoted by $f_{mnk} \in F$ for every $m, n, k \in \mathbb{N}$.

The purpose of this paper is to introduce and study a concept of triple lacunary strong $\chi^3_{A_{uvw}}$-convergence of p-metric with respect to a mnk-sequence of Musielak Orlicz function.

We now introduce the generalizations of triple lacunary strongly $\chi^3_{A_{uvw}}$-convergence of p-metric with respect a mnk-sequence of Musielak Orlicz function and investigate some inclusion relations.

Let A denote a sequence of the matrices $A^{uvw} = \left(a_{k_1\ldots k_r \ell_1 \ldots \ell_s}^{m_1\ldots m_r n_1 \ldots n_s, k_1, k_2, \ldots, k_t}(uvw)\right)$ of complex numbers. We write for any sequence $x = (x_{mnk})$,

$$y_{ij}(uv) = A_{ij}^{uvw}(x) = \sum_{m_1\ldots m_r}^{\infty} \sum_{n_1\ldots n_s}^{\infty} \sum_{k_1,k_2,\ldots,k_t}^{\infty} \left(a_{k_1\ldots k_r \ell_1\ldots \ell_s}^{m_1\ldots m_r n_1\ldots n_s}(uvw)\right) \cdot$$

$$\left((m_1 \ldots m_r + n_1 \ldots n_s + k_1, k_2, \ldots, k_t)! \left|x_{m_1\ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}\right|\right)^{1/m_1\ldots m_r + n_1\ldots n_s + k_1, k_2, \ldots, k_t}$$

if it exits for each ijq and uvw. We $A^{uvw}(x) = \left(A_{ijq}^{uvw}(x)\right)_{ijq}$, $Ax = \left(A^{uvw}(x)\right)_{uvw}$.

Definition 2.4 Let μ be a valued measure on $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and $F = \left(f_{m_1\ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}^{ijq}\right)$ be a mnk-sequence of Musielak Orlicz function , A denote the sequence of four dimensional infinte matrices of complex numbers and X be locally convex Hausdorff topological linear space whose topology is determined by a set of continuous semi norms η and $\left(X, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right)$ be a p-metric space, $q = (q_{ijq})$ be triple analytic sequence of strictly positive real numbers.

By $w^3(p - X)$ we denote the space of all sequences defined over $\left(X, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right)^{\mu}$.

In the present paper we define the following sequence spaces:

$$\left[\chi_{AfN_\theta^\alpha}^{3q\eta}, \left\|\left(d(x_{111}), d(x_{122}), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}\right)\right)\right\|_p\right]^{\mu}$$

$$= \mu \lim_{rst} \Big[f_{ijq}(\|N_\theta^\alpha(x),, (d(x_{111}, 0), d(x_{122}, 0), \ldots,$$

$$d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\Big)\Big\|_p\Big)\Big]^{q_{ijq}} \geq \epsilon = 0,$$

where

$$N_\theta^\alpha(x) = \frac{1}{h_{rst}^\alpha} \sum_{i \in I_{rst}} \sum_{j \in I_{rst}} \sum_{q \in I_{rst}} \eta A_{ij}^{uvw} \Big(\big((m_1 \ldots m_r + n_1 \ldots n_s + k_1, k_2, \ldots, k_t)!$$

$$\left|x_{m_1\ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}\right|\big)^{1/m_1\ldots m_r + n_1 \ldots n_s + k_1, k_2, \ldots, k_t}\Big),$$

uniformly in u, v, w

$$\left[\Lambda_{AfN_\theta^\alpha}^{3q\eta}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$$

$$= \mu \sup_{rst} [f_{uvw}(\|N_\theta^\alpha(x), (d(x_{111}, 0), d(x_{122}, 0), \ldots,$$

$$d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\Big\|_p\Big)\Big]^{q_{ijq}} \geq k = 0,$$

where $e = \begin{pmatrix} 1 & 1 & \cdots 1 \\ 1 & 1 & \cdots 1 \\ . & & \\ . & & \\ . & & \\ 1 & 1 & \cdots 1 \end{pmatrix}$.

The main aim of this paper is to introduce the idea of summability of triple lacunary sequence spaces in p-metric spaces using a three valued measure. We also make an effort to study μ-of lacunary triple sequences with respect to a sequence of Musielak Orlicz function in p-metric spaces and three valued measure μ. We also plan to study some topological properties and inclusion relation between these spaces.

3. MAIN RESULTS

Proposition 3.1 Let μ be a three valued measure,

$$\left[\chi^{3q\eta}_{AfN^\alpha_\theta}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^\mu$$

and

$$\left[\Lambda^{3q\eta}_{AfN^\alpha_\theta}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^\mu$$

are linear spaces.

Proof. It is routine verification. Therefore the proof is omitted.

The inclusion relation between

$$\left[\boldsymbol{\chi^{3q\eta}_{AfN^\alpha_\theta}}, \left\|\left(\mathbf{d}(\mathbf{x}_{111}, \mathbf{0}), \mathbf{d}(\mathbf{x}_{122}, \mathbf{0}), \ldots, \mathbf{d}\left(\mathbf{x}_{\mathbf{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}}}, \mathbf{0}\right)\right)\right\|_\mathbf{p}\right]^{\boldsymbol{\mu}}$$

and

$$\left[\boldsymbol{\Lambda^{3q\eta}_{AfN^\alpha_\theta}}, \left\|\left(\mathbf{d}(\mathbf{x}_{111}, \mathbf{0}), \mathbf{d}(\mathbf{x}_{122}, \mathbf{0}), \ldots, \mathbf{d}\left(\mathbf{x}_{\mathbf{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}}}, \mathbf{0}\right)\right)\right\|_\mathbf{p}\right]^{\boldsymbol{\mu}}$$

Theorem 3.1 Let μ be a three valued measure and A be a mnk-sequence the four dimensional infinite matrices $A^{uv} = \left(a^{m_1 \ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}_{k_1 \ldots k_r \ell_1 \ldots \ell_s}(uvw)\right)$ of complex numbers and $F = \left(f^{ijq}_{mnk}\right)$ be a mn-sequence of Musielak Orlicz function. If $x = (x_{mnk})$ triple lacunary strong A_{uvw}-convergent of orer α to zero then $x = (x_{mnk})$ triple lacunary strong A_{uvw}-convergent of order α to zero with respect to mnk-sequence of Musielak Orlicz function, (i.e)

$$\left[\chi^{3q\eta}_{AN^{\alpha}_{\theta}}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$$

$$\subset \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}.$$

Proof. Let $F = \left(f^{ijq}_{mnk}\right)$ be a mnk-sequence of Musielak Orlicz function and put $\sup f^{ijq}_{mnk}(1) = T$. Let

$$x = (x_{mnk}) \in \left[\chi^{2q\eta}_{AN^{\alpha}_{\theta}}, \|(d(x_{11},0), d(x_{12},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\Big\|_p\right]^{\mu}$$

and $\epsilon > 0$. We choose $0 < \delta < 1$ such that $f^{ijq}_{mnk}(u) < \epsilon$ for every u with $0 \le u \le \delta$ $(i, j, q \in \mathbb{N})$. We can write

$$\left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$$

$$= \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu} +$$

$$\left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$$

where the first part is over $\le \delta$ and second part is over $> \delta$. By definition of Musielak Orlicz function of f^{ijq}_{mnk} for every ijq, we have

$$\left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$$

$$\le \epsilon^{H_2} + (3T\delta^{-1})^{H_2} \cdot \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \|(d(x_{111},0), d(x_{122},0), \ldots,\right.$$

$$\left. d\left(x_{m_1,m_2,\ldots m_{r-1} n_1,n_2,\ldots n_{s-1} k_1,k_2,\ldots,k_{t-1}}, 0\right)\right)\Big\|_p\right]^{\mu}.$$

Therefore

$$x = (x_{mnk}) \in \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \|(d(x_{111},0), d(x_{122},0), \ldots,\right.$$

$$d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\Big)\Big\|_p\Big]^{\mu}.$$

Theorem 3.2 Let μ be a three valued measure and A be a mnk-sequence of the four dimensional infinite matrices $A^{uvw} = \left(a_{k_1\ldots k_r\ell_1\ldots\ell_s}^{m_1\ldots m_r n_1\ldots n_s}(uvw)\right)$ of complex numbers, $q = (q_{ijq})$ be a mnk-sequence of positive real numbers with $0 < \inf q_{ijq} = H_1 \le \sup q_{ijq} = H_2 > \infty$ and $F = \left(f_{mnk}^{ijq}\right)$ be a mnk-sequence of Musielak Orlicz function. If $\text{limm}_{u,v,w\to\infty}\inf_{ijq}\frac{f_{ijq}(uvw)}{uvw} > 0$, then

$$\left[\chi_{AfN_\theta^\alpha}^{3q\eta},\left\|\left(d(x_{111},0),d(x_{122},0),\ldots,d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$
$$=\left[\chi_{AN_\theta^\alpha}^{3q\eta},\left\|\left(d(x_{111},0),d(x_{122},0),\cdots,d\left(x_{m_1,m_2,\cdots m_{r-1}n_1,n_2,\cdots n_{s-1}k_1,k_2,\cdots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}.$$

Proof. If $\lim_{u,v,w\to\infty}\inf f_{ijq}\frac{f_{ijq}(uvw)}{uvw} > 0$, then there exists a number $\beta > 0$ such that $f_{ijq}(uvw) \ge \beta u$ for all $u \ge 0$ and $i,j,q \in \mathbb{N}$. Let

$$x = \left(x_{m_1}\ldots m_r n_1\ldots n_s k_1,k_2,\ldots,k_t\right) \in \Big[\chi_{AfN_\theta^\alpha}^{3q\eta},\|(d(x_{111},0),d(x_{122},0),\ldots,$$
$$d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\Big)\Big\|_p\Big]^{\mu}.$$

Clearly

$$\left[\chi_{AfN_\theta^\alpha}^{3q\eta},\left\|\left(d(x_{111},0),d(x_{122},0),\ldots,d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$
$$\ge\beta\left[\chi_{AN_\theta^\alpha}^{3q\eta},\left\|\left(d(x_{111},0),d(x_{122},0),\ldots,d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}.$$

Therefore

$$x = \left(x_{m_1}\ldots m_r n_1\ldots n_s k_1,k_2,\ldots,k_t\right) \in \Big[\chi_{AN_\theta^\alpha}^{3q\eta},\|(d(x_{111},0),d(x_{122},0),\ldots,$$
$$d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\Big)\Big\|_p\Big]^{\mu}.$$

By using Theorem 3.1, the proof is complete.
We now give an example to show that

$$\left[\chi_{AfN_\theta^\alpha}^{3q\eta},\left\|\left(d(x_{111},0),d(x_{122},0),\ldots,d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$
$$\neq\left[\chi_{AN_\theta^\alpha}^{3q\eta},\left\|\left(d(x_{111},0),d(x_{122},0),\ldots,d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$

in the case when $\beta = 0$. Consider $A = I$, unit matrix,

$$\eta(x) = \big((m_1 \cdots m_r + n_1 \cdots n_s + k_1, k_2, \ldots, k_t)! \\ |x_{m_1 \ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}|\big)^{1/m_1 \cdots m_r + n_1 \cdots n_s + k_1, k_2, \ldots, k_t}, \; q_{ijq} = 1$$

for every $i, j, q \in \mathbb{N}$ and

$$f_{mnk}^{ijq}(x) = \frac{|x_{m_1 \ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}|^{1/((m_1 \ldots m_r + n_1 \ldots n_s + k_1, k_2, \ldots, k_t)(i+1)(j+1)(q+1))}}{\big((m_1 \ldots m_r + n_1 \ldots n_s + k_1, k_2, \ldots, k_t)!\big)^{1/m_1 \ldots m_r + n_1 \ldots n_s + k_1, k_2, \ldots, k_t}}$$
$$(i, j, q \geq 1, x > 0)$$

in the case $\beta > 0$. Now we define $x_{ijq} = h_{rst}^{\alpha}$ if $i, j, q = m_r n_s k_t$ for some $r, s, t \geq 1$ and $x_{ijq} = 0$ otherwise. Then we have,

$$\left[\chi_{AfN_\theta^\alpha}^{3q\eta}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right]^\mu \to 1$$
$$\text{as } r, s, t \to \infty$$

and so

$$x = \left(x_{m_1 \ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}\right) \notin \left[\chi_{AN_\theta^\alpha}^{3q\eta}, \|(d(x_{111}, 0), d(x_{122}, 0), \ldots, \right.$$
$$\left. d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\Big\|_p\Big]^\mu.$$

In this section we introduce natural relationship between μ be a three valued measure of triple lacunary A^{uvw}- statistical convergence of order α and μ be a three valued measure of triple lacunary strong A^{uvw}-convergence of order α with respect to mnk-sequence of Musielak Orlicz function.

Definition 3.1 Let μ be a three valued measure and θ be a triple lacunary mnk-sequence. Then a mnk-sequence $x = \left(x_{m_1 \ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}\right)$ is said to be μ-lacunary statistically convergent of order α to a number zero if for every $\epsilon > 0$, $\mu(\lim m_{rst \to \infty} h_{rst}^{-\alpha} |K_\theta(\epsilon)|) = 0$, where $|K_\theta(\epsilon)|$ denotes the number of elements in

$$K_\theta(\epsilon) = \mu\Big\{(i, j, q) \in I_{rst} : \big((m_1 \cdots m_r + n_1 \cdots n_s + k_1, k_2, \ldots, k_t)! \cdot \\ |x_{m_1 \ldots m_r n_1 \ldots n_s k_1, k_2, \ldots, k_t}|\big)^{1/m_1 \cdots m_r + n_1 \cdots n_s + k_1, k_2, \cdots, k_t} \geq \epsilon = 0\Big\}.$$

The set of all triple lacunary statistical convergent of order α of $mnk -$ sequences is denoted by $(S_\theta^\alpha)^\mu$.

Let μ be a three valued measure and $A^{uvw} = \left(a_{k_1\cdots k_r\ell_1\cdots\ell_s}^{m_1\cdots m_r n_1\cdots n_s k_1,k_2,\ldots,k_t}(uvw)\right)$ be an four dimensional infinite matrix of complex numbers. Then a mnk-sequence $x = \left(x_{m_1\cdots m_r n_1\cdots n_s k_1,k_2,\ldots,k_t}\right)$ is said to be μ-triple lacunary A-statistically convergent of order α to a number zero if for every $\epsilon > 0$, $\mu(\lim_{rst\to\infty} h_{rst}^{-\alpha}|KA_\theta(\epsilon)|) = 0$, where $|KA_\theta(\epsilon)|$ denotes the number of elements in

$$KA_\theta(\epsilon) = \mu\Big\{(i,j,q) \in I_{rst}: \big((m_1\cdots m_r + n_1\cdots n_s + k_1,k_2,\ldots,k_t)! \cdot \left|x_{m_1\cdots m_r n_1\cdots n_s k_1,k_2,\ldots,k_t}\right|\big)^{1/m_1\cdots m_r+n_1\cdots n_s+k_1,k_2,\ldots,k_t} \geq \epsilon = 0\Big\}.$$

The set of all triple lacunary A-statistical convergent of order α of mnk-sequences is denoted by $\left(S_\theta^\alpha(A)\right)^\mu$.

Definition 3.2 Let μ be a three valued measure and A be a mnk-sequence of the four dimensional infinite matrices $A^{uvw} = \left(a_{k_1\cdots k_r\ell_1\cdots\ell_s}^{m_1\cdots m_r n_1\cdots n_s k_1,k_2,\ldots,k_t}(uvw)\right)$ of complex numbers and let $q = (q_{ijq})$ be a mnk-sequence of positive real numbers with $0 < \inf q_{ijq} = H_1 \leq \sup q_{ijq} = H_2 < \infty$. Then a mnk-sequence $x = \left(x_{m_1\cdots m_r n_1\cdots n_s k_1,k_2,\ldots,k_t}\right)$ is said to be μ-lacunary A^{uvw}-statistically convergent of order α to a number zero if for every $\epsilon > 0$, $\mu\left(\lim_{rst\to\infty} h_{rst}^{-\alpha}\left|KA_{\theta\eta}(\epsilon)\right|\right) = 0$, where $\left|KA_{\theta\eta}(\epsilon)\right|$ denotes the number of elements in

$$KA_{\theta\eta}(\epsilon) = \mu\Big\{(i,j,q) \in I_{rst}: \big((m_1\cdots m_r + n_1\cdots n_s + k_1,k_2,\ldots,k_t)! \cdot \left|x_{m_1\cdots m_r n_1\cdots n_s k_1,k_2,\ldots,k_t}\right|\big)^{1/m_1\cdots m_r+n_1\cdots n_s+k_1,k_2,\ldots,k_t} \geq \epsilon = 0\Big\}.$$

The set of all μ-lacunary A_η-statistical convergent of order α of mnk-sequences is denoted by $\left(S_\theta^\alpha(A,\eta)\right)^\mu$.

The following theorems give the relations between μ-lacunary A^{uvw}-statistical convergence of order α and μ-lacunary strong A^{uvw}-convergence of order α with respect to a mnk-sequence of Musielak Orlicz function.

Theorem 3.3 Let μ be a three valued measure and $F = (f_{ijq})$ be a mnk-sequence of Musielak Orlicz function. Then

$$\left[\chi_{AfN_\theta^\alpha}^{3q\eta}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\ldots m_{r-1}n_1,n_2,\ldots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^\mu$$

$$\subseteq \left[\chi_{AS_\theta^\alpha}^{3\eta}, \left\|\left(d(x_{111},0), d(x_{122},0), \ldots, d\left(x_{m_1,m_2,\cdots m_{r-1}n_1,n_2,\cdots n_{s-1}k_1,k_2,\ldots,k_{t-1}},0\right)\right)\right\|_p\right]^\mu$$

if and only if $\mu\left(\lim_{ijq\to\infty} f_{ijq}(u)\right) > 0$, $(u > 0)$.

Proof. Let $\epsilon > 0$ and $x = \left(x_{m_1 \cdots m_r n_1 \cdots n_s k_1, k_2, \ldots, k_t}\right) \in \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \|(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\Big\|_p\Big]^{\mu}$. If $\mu\left(\lim_{ijq\to\infty} f_{ijq}(u)\right) > 0$, $(u > 0)$, then there exists a number $d > 0$ such that $f_{ijq}(\epsilon) > d$ for $u > \epsilon$ and $i, j, q \in \mathbb{N}$. Let

$$\left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$$
$$\geq h^{-\alpha}_{rst} d^{H_1} K A_{\theta\eta}(\epsilon).$$

It follows that

$$\left[\chi^{3\eta}_{AfS^{\alpha}_{\theta}}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}.$$

Conversely, suppose that $\mu\left(\lim_{ijq\to\infty} f_{ijq}(u)\right) > 0$ does not hold, then there is a number $t > 0$ such that $\mu\left(\lim_{ijq\to\infty} f_{ij}(t)\right) = 0$. We can select a lacunary mn-sequence $\theta = (m_1 \cdots m_r n_1 \cdots n_s k_1, k_2, \ldots, k_t)$ such that $f_{ijq}(t) < 3^{-rst}$ for any $i > m_1 \cdots m_r, j > n_1 \cdots n_s, k > k_1, k_2, \ldots, k_t$. Let $A = I$, unit matrix, define the mnk-sequence x by putting $x_{ijq} = t$ if

$$m_1, m_2, \cdots m_{r-1} n_1, n_2, \cdots n_{s-1} k_1, k_2, \ldots, k_{t-1} < i, j, q < \frac{m_1, m_2, \ldots m_r n_1, n_2, \ldots n_s k_1, k_2, \ldots, k_t + m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}{2}$$

and $x_{ijq} = 0$ if

$$\frac{m_1, m_2, \ldots m_r n_1, n_2, \ldots n_s k_1, k_2, \ldots, k_t + m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}{2} \leq i, j, q \leq m_1, m_2, \cdots m_r n_1, n_2, \cdots n_s k_1, k_2, \ldots, k_t.$$

We have

$$x = \left(x_{m_1 \cdots m_r n_1 \cdots n_s k_1, k_2, \ldots, k_t}\right) \in \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \|(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\Big\|_p\Big]^{\mu}.$$

but $x \notin \left[\chi^{3\eta}_{AS^{\alpha}_{\theta}}, \left\|\left(d(x_{111}, 0), d(x_{122}, 0), \ldots, d\left(x_{m_1, m_2, \ldots m_{r-1} n_1, n_2, \ldots n_{s-1} k_1, k_2, \ldots, k_{t-1}}, 0\right)\right)\right\|_p\right]^{\mu}$.

Theorem 3.4 Let μ be a three valued measure and $F = (f_{ijq})$ be a mnk-sequence of Musielak Orlicz function. Then

$$\left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$

$$\supseteq \left[\chi^{3\eta}_{AS^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$

if and only if $\mu\left(\sup_u \sup_{ijq} f_{ijq}(u)\right) < \infty$.

Proof. Let

$$x \in \left[\chi^{3\eta}_{AS^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}.$$

Suppose that $h(u) = \sup_{ijq} f_{ijq}(u)$ and $h = \mathrm{supp}_u h(u)$. Since $f_{ijq}(u) \leq h$ for all i,j,q and $u > 0$, we have for all u, v, w

$$\left[\chi^{3\eta}_{AS^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$
$$\leq h^{H_2} h^{-\alpha}_{rst} \left|KA_{\theta\eta}(\epsilon)\right| + |h(\epsilon)|^{H_2}.$$

It follows from $\epsilon \to 0$ that

$$x \in \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}.$$

Conversely, suppose that $\mu\left(\sup_u \sup_{ijq} f_{ijq}(u)\right) = \infty$. Then we have $0 < u_{111} < \cdots < u_{r-1s-1t-1} < u_{rst} < \cdots$, such that $f_{m_r n_s k_t}(u_{rst}) \geq h^{\alpha}_{rst}$ for $r, s, t \geq 1$. Let $A = I$, unit matrix, define the mnk-sequence x by putting $x_{ijq} = u_{rst}$ if $i, j, q = m_1 m_2 \cdots m_r n_1 n_2 \cdots n_s$ for some $r, s, t = 1,2, \dots$ and $x_{ijq} = 0$ otherwise. Then we have

$$x \in \left[\chi^{3\eta}_{AS^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}$$

but

$$x \notin \left[\chi^{3q\eta}_{AfN^{\alpha}_{\theta}}, \left\| \left(d(x_{111},0), d(x_{122},0), \dots, d\left(x_{m_1,m_2,\dots m_{r-1}n_1,n_2,\dots n_{s-1}k_1,k_2,\dots,k_{t-1}},0\right)\right)\right\|_p\right]^{\mu}.$$

4. CONCLUSION

In this paper we have studied study some connections between μ-lacunary strong $\chi^3_{A_{uvw}}$-convergence with respect to a mnk sequence of Musielak Orlicz function and μ-lacunary $\chi^3_{A_{uvw}}$-statistical convergence, where A is a sequence of four dimensional matrices $A(uvw) =$

$\left(a_{k_1 \dots k_r \ell_1 \dots \ell_s}^{m_1 \dots m_r n_1 \dots n_s}(uvw)\right)$ of complex numbers. The results of this of this paper are more general than earlier results.

References

[1] T. Apostol, Mathematical Analysis, Addison-Wesley, London, 1978.

[2] A. Esi, On some triple almost lacunary sequence spaces defined by Orlicz functions, *Research and Reviews: Discrete Mathematical Structures*, 1(2) (2014) 16-25.

[3] A. Esi, N. Subramanian and M.K.Ozdemir, Riesz triple probabilistic of almost lacunary Cesaro C_{111} statistical convergence of Γ^3 defined by a Musielak-Orlicz function. *World Scientofic News* 116 (2019) 115-127.

[4] A. Esi, N.Subramanian and A.Esi, On triple sequence spaces of X^3 of rough λ-statistical convergence in probability defined by Musielak-Orlicz function of p-metric space. *World Scientofic News* 132 (2019) 270-276.

[5] A. Esi and M. Necdet Catalbas, Almost convergence of triple sequences, *Global Journal of Mathematical Analysis,* 2(1) (2014) 6-10.

[6] A. Esi and E. Savas, On lacunary statistically convergent triple sequences in probabilistic normed space. *Appl. Math. and Inf. Sci.* 9(5) (2015) 2529-2534.

[7] A. Esi, N. Subramanian and A. Esi, On triple sequence space of Bernstein operator of rough λ-convergence Pre-Cauchy sequences. *Proyecciones Journal of Mathematics,* 36(4) (2017) 567-587.

[8] A. Esi, N. Subramanian and A. Esi, Triple rough statistical convergence of sequence of Bernstein operators. *Int. J. Adv. Appl. Sci.* 4(2) (2017) 28-34.

[9] G. H. Hardy, On the convergence of certain multiple series. *Proc. Camb. Phil. Soc.* 19 (1917) 86-95.

[10] Deepmala, N. Subramanian and V. N. Mishra, Double almost $(\lambda_m \mu_n)$ in χ^2-Riesz space. *Southeast Asian Bulletin of Mathematics,* 41(3) (2017) 385-395.

[11] Deepmala, L. N. Mishra and N. Subramanian, Characterization of some Lacunary $\chi^2_{A_{uv}}$-convergence of order α with p-metric defined by mn sequence of moduli Musielak. *Appl. Math. Inf. Sci. Lett.* 4(3) (2016) 111-118.

[12] A. Sahiner, M. Gurdal and F. K. Duden, Triple sequences and their statistical convergence. *Selcuk J. Appl. Math.* 8(2) (2007) 49-55.

[13] N. Subramanian and A. Esi, Some New Semi-Normed Triple Sequence Spaces Defined By A Sequence Of Moduli. *Journal of Analysis & Number Theory,* 3(2) (2015) 79-88.

[14] T. V. G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian, Lacunary Triple sequence Γ^3 of Fibonacci numbers over probabilistic p-metric spaces. *International Organization of Scientific Research,* 12(1), Version IV (2016), 10-16.

[15] J. Lindenstrauss and L. Tzafriri, On Orlicz sequence spaces. *Israel J. Math.* 10 (1971) 379-390.

[16] P. K. Kamthan and M. Gupta, Sequence spaces and series, Lecture notes, Pure and Applied Mathematics, 65 Marcel Dekker, Inc. New York, 1981.

[17] J. Musielak, Orlicz Spaces, Lectures Notes in Math., 1034, Springer-Verlag, 1983.