Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 136 (2019) 226-240 EISSN 2392-2192

# An Integrated Inventory Model for Deteriorating Items under Cash Discount and Permissible Delay in Payments

**Yogita Sanas[1,*], Sunil Kawale[2], Lahji Abdul Aziz[3]**

[1]Department of Mathematics and Statistics, L. D. Sonawane College, Kalyan 421301, Maharashtra, India

[2]Department of Statistics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad 431001, Maharashtra, India

[3]Department of Mathematics, College of Science, University of Bahrain, Bahrain

*E-mail address: yogita1510@gmail.com

**ABSTRACT**

Numerous studies have been undertaken to explain inventory models with different features. While findings from earlier studies have been conflicting, recent industrial-level studies indicate that multi features inventory models have a positive impact on business scenario. We propose an inventory model with integration of many real features like constant deterioration under cash discount scheme and permissible delay in payments. That is, we want to investigate the buyer’s optimal replenishment policy with quadratic demand under trade credit and cash discount to maximise joint total profit per unit time. Furthermore, numerical example and sensitivity analysis are presented to illustrate the results of the proposed model and to draw managerial insights.

***Keywords*:** Integrated inventory model, Cash discount, permissible delay in payments, quadratic demand, deterioration

( Received 27 September 2019; Accepted 14 October 2019; Date of Publication 15 October 2019 )

## 1. INTRODUCTION

Trade credit is an invaluable promotional tool for the suppliers to increase profit through stimulating more sales and a unique opportunity for the retailers to reduce demand uncertainty and its associated risks. It is a regular component of market transactions and constitutes a major source of short-term financing. In this practice, the supplier is willing to offer the retailer a certain credit period without interest during the permissible delay period to promote market competition. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. This type of model was first discussed by Haley and Higgins (1973). Goyal (1985) explored a single-item EOQ model under permissible delay in payments. Deterioration is defined as the decay, spoilage, evaporation which loses the utility of a production from the original one. Fruits and vegetables, pharmaceutical drugs, electronic items, blood components, radioactive chemicals are some of the examples of deteriorating items. Jaggi and Aggarwal (1994) presented the economic ordering policies of deteriorating items in the presence of trade credit using a discounted cash-flows (DCF) approach. Hwang and Shinn (1997) developed the joint price and lot size determination problem for an exponentially deteriorating product when the supplier offers a certain fixed credit period. Jamal et al. (1997) developed a model for an optimal ordering policy for deteriorating items with allowable shortage and permissible delay in payment. Further Chu et al. (1998), Sarkar et al. (2000), Liao et al. (2000), Chung et al. (2001), Chang et al. (2003), Chung and Liao (2004), Teng et al. (2005), Chung and Liao (2006), Chung (2006), Chung and Huang (2007) and Teng et al. (2009), Huang and Liao (2008), Thangam and Uthayakumar (2010), Shah (2010), Dye and Ouyang (2011), Roy and Samanta (2011), Teng et al. (2011), Mahata (2012), Liao et al. (2012), Thangam (2012), Guchhait et al. (2013), Chung et al. (2014), Swami et al. (2015), Mahata (2015) developed inventory models for deteriorating items under trade credit considering different features.

In most business transactions, the supplier will offer the credit terms mixing cash discount to the retailer to avoid the possibility of resulting in bad debt. The retailer can obtain the cash discount when the payment is paid within cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. One can refer Chang (2002), Huang and Chung (2003), Ouyang et al. (2002), Huang (2005), Huang and Hsu (2007), Ho *et al.* (2008), Jain et al. (2008), Shah and Shukla (2011), Shah et al. (2013), Kumar et al. (2011) for trade credit and cash discount inventory models.

There are many models derived either from the supplier's or the retailer's end. However, the two players in supply chain may have their own goals. Lee et al. (1997) pointed out that the absence of coordinated inventory management throughout the supply chain results in excessive inventory investment, revenue reduction and delays in response to customer requirements. Therefore, determining the optimal integrated policies is more reasonable than considering the buyer's or the supplier's individual profit/cost. Goyal (1976), Banerjee (1986), Goyal (1988), Bhatnagar et al. (1993), Goyal (1995), Viswanathan (1998), Hill (1997, 1999), Kim and Ha (2003), Kelle et al. (2003), Li and Liu (2006) developed optimal joint inventory policies for supplier and retailer.

However, these articles did not incorporate the effect of trade credit on the integrated optimal decision. Abad and Jaggi (2003) developed a vendor – buyer integrated model assuming lot – for – lot production under a permissible delay in payments. Later, Shah (2009) extended Abad and Jaggi's (2003) model for deteriorating items. Shah et al. (2013) developed an integrated inventory model with trapezoidal price-sensitive demand with two-part trade credit. Tayal et al. (2016) developed an integrated production-distribution model for deteriorating items in a two echelon supply chain with allowable shortages and investment in preservation technology.

In this paper we develop an inventory model for deteriorating items with constant deterioration under cash discount and permissible delay in payments for quadratic demand. This paper is organized as follows. In section 2 assumptions and in section 3 notations are presented. In section 4 the mathematical model is formulated where joint total profit per unit time is maximised to create win-win strategy between players in the supply chain. In section 5 numerical example is cited and sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

## 2. NOTATIONS

The following notations are used in the proposed article:

$S_v$: Vendor's set up cost per set up.

$S_b$: Buyer's ordering cost per order.

$C_v$: Production cost per unit.

$C_b$: Buyer's purchase cost per unit.

$C_c$: The unit retail price to customers where $C_c > C_b > C_v$.

$I_v$: Vendor's inventory holding cost rate per unit per annum, excluding interest charges.

$I_b$: Buyer's inventory holding cost rate per unit per annum, excluding interest charges.

$I_{v0}$: Vendor's opportunity cost/\$/unit time.

$I_{b0}$: Buyer's opportunity cost/\$/unit time.

$I_{be}$: Buyer's interest earned/\$/unit time.

$\varrho$: Capacity utilisation which is ratio of demand to the production rate, $\varrho$ <1 and known constant.

$M_1$: Period of cash discount

$M_2$: Allowable credit period for the buyer offered by the vendor. ( $M_2 > M_1$)

Q: Buyer's order quantity.

T: cycle time (decision variable).

n: Number of shipments from vendor to the buyer.

ɵ: constant rate of deterioration.

ʎ: cash discount rate

fvc: vendor's cash flexibility rate

TVP: Vendor's total profit per unit time.

TBP: Buyer's total profit per unit time.

п: TVP + TBP Joint total profit per unit time.

## 3. ASSUMPTIONS

In addition, the following assumptions are made in derivation of the model:

- The supply chain under consideration comprise of single vendor and single buyer for a single product.
- Shortages are not allowed.
- The demand rate considered is time dependent, increasing demand rate. The constant part of quadratic demand pattern changes with each cycle.
- Replenishment rate is instantaneous for retailer
- The units in inventory are subject to deteriorate at a constant rate of ɵ, $0 < ɵ < 1$. The deteriorated units can neither be repaired nor replaced during the cycle time.
- Finite production rate.
- Vendor produces the nQ items and then fulfils the buyer's demand, so at the beginning of production item, there is small possibility of deterioration in general. Moreover vendor is a big merchant who can have capacity to prevent deterioration. So, in this model, deterioration cost is considered for buyer only at the rate ɵ is assumed to be constant.
- The vendor offers a discount ʎ ($0 < ʎ < 1$) in the purchase price if the buyer pays by time M1; otherwise full account is to be settled within allowable credit period M2, where $M2 > M1 \geq 0$. The offer of discount in unit purchase price from the vendor will increase cash in-flow, thereby reducing the risk of cash flow shortage.
- By offering a trade credit to the buyer, the vendor receives cash at a later date and hence incurs an opportunity cost during the delivery and payment of the product. On the buyer's end, the buyer can generate revenue by selling the items and earning interest by depositing it in an interest bearing account during this permissible delay period. At the end of this period, the vendor charges to the buyer on the unsold stock.
- During the time [M1, M2], a cash flexibility rate fvc is used to quantize the favor of early cash income for the vendor.

## 4. MATHEMATICAL FORMULATION

Let *I(t)* be the inventory level at any time *t*, ($0 \leq t \leq T$ ). Depletion due to deterioration and demand will occur simultaneously. The differential equation describing the instantaneous state of *I(t)* over (0,*T*) is given by:

$$\frac{dI(t)}{dt} + ɵ\, I(t) = -(a + bt + ct^2) \qquad 0 \leq t \leq T \qquad (1)$$

Solution to the equation (1) (using the boundary condition *I(t)* = 0 at *t* = *T*) is given by

$$I(t) = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(e^{\theta(T-t)} - 1\right) + \left(\frac{b}{\theta} - \frac{2c}{\theta}\right)\left(Te^{\theta(T-t)} - t\right) + \frac{c}{\theta}\left(T^2 e^{\theta(T-t)} - t^2\right)$$

$$0 \le t \le T \qquad (2)$$

Also at t = 0, I(t) = Q

$$Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T} \qquad 0 \le t \le T \qquad (3)$$

**4. 1. Vendor's total profit per unit time**

For each unit of item, the vendor charges $((1 - k_j \Lambda)C_b)$ if the buyer pays at time $M_j$, j = 1,2, $k_1$ = 1 and $k_2$ = 0.

**1)** Sales revenue: the total sales revenue per unit time is $\left((1 - k_j \Lambda)C_b - C_V\right)\frac{Q}{T}$

$$= \frac{((1-k_j\Lambda)C_b - C_V)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right. \qquad (4)$$

**2)** Set-up cost: nQ units are manufactured in one production run by the vendor. Therefore the setup cost per unit time is $\frac{S_v}{nT}$

**3)** Holding cost: using method given by Joglekar (1988), vendor's average inventory per unit time is

$$\frac{C_v(I_v + I_{v0})}{T}\left[(n-1)(1-\varrho) + \varrho\right]\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^3}\right)\left(1 + \theta T - e^{\theta T}\right) + \left(-\frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(T - Te^{\theta T} + \frac{\theta T^2}{2}\right)\right.$$
$$\left. - \frac{c}{\theta^2}\left(T^2 - T^2 e^{\theta T} + \theta\frac{T^3}{3}\right)\right\} \qquad (5)$$

**4)** Opportunity cost: opportunity cost per unit time because of offering permissible delay period is $\frac{(1-k_j\Lambda)C_b I_{v0} MQ}{T}$

$$= \frac{(1 - k_j\Lambda)C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right\} \qquad (6)$$

However, if the buyer pays at $M_1$ –time, during $M_2$ - $M_1$ the vendor can use the revenue $((1 - \Lambda)C_b)$ to avoid a cash flow crisis. The advantage gain per unit time from early payment at a cash flexibility rate fvc is

$$\frac{k_j(1 - \Lambda)C_b fvc\,(M2 - M1)Q}{T}$$
$$= \frac{k_j(1 - \Lambda)C_b fvc\,(M2 - M1)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right\} \qquad (7)$$

Hence the total profit per unit time for vendor is = Sales revenue – Set up cost – Holding cost – Opportunity cost + Advantage gain

$$\mathrm{TVP_j} = \frac{((1-k_j\Lambda)C_b - C_V)}{T}\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\} - \frac{S_v}{nT} - \frac{C_v(I_v+I_{v0})}{T}[(n-1)(1-\varrho)+\varrho]\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)(1+\theta T-e^{\theta T}) + (-\frac{b}{\theta^2}+\frac{2c}{\theta^2})(T - Te^{\theta T}+\frac{\theta T^2}{2}) - \frac{c}{\theta^2}(T^2 - T^2 e^{\theta T}+\theta\frac{T^3}{3})\} - \frac{(1-k_j\Lambda)C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\} + \frac{k_j(1-\Lambda)C_b fvc\,(M2-M1)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\} \tag{8}$$

$j = 1,2$; $k_1 = 1$, $k_2 = 0$

**4. 2. Net profit function for the buyer consists of following elements**

**1)** Sales revenue: The total sales revenue per unit time is $\frac{(C_c-(1-k_j\Lambda)C_b)Q}{T}$

$$= \frac{(C_c-(1-k_j\Lambda)C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\} \tag{9}$$

**2)** Ordering cost: Ordering cost per unit time is $\frac{S_b}{T}$

**3)** Holding cost: The buyer's holding cost (excluding interest charges) per unit time is

$$\frac{(1-k_j\Lambda)C_b I_b}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)(1+\theta T-e^{\theta T}) + (-\frac{b}{\theta^2}+\frac{2c}{\theta^2})(T - Te^{\theta T}+\frac{\theta T^2}{2}) - \frac{c}{\theta^2}(T^2 - T^2e^{\theta T}+\theta\frac{T^3}{3})\} \tag{10}$$

**4)** Deteriorating cost: Deteriorating cost per unit time is $\frac{(1-k_j\Lambda)C_b}{T}[Q-\int_0^T(a+bt+ct^2)dt]$

$$= \frac{(1-k_j\Lambda)C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T} - aT - \frac{bT^2}{2} - \frac{cT^3}{3}\right\} \tag{11}$$

Based on the length of the payment time, two cases arise namely $\mathrm{M_j} < T$ and $\mathrm{M_j} \geq T$; $j = 1,2$

Case 1. When $\mathrm{M_j} < T$ ; $j = 1,2$.

**5)** Interest earned per unit time during the period $[0, \mathrm{M_j}]$ is $\frac{I_{be}C_c}{T}\int_0^{\mathrm{M_j}}(a+bt+ct^2)tdt$

$$= \frac{I_{be}C_c}{T}\left[\frac{a\mathrm{M_j}^2}{2}+\frac{b\mathrm{M_j}^3}{3}+\frac{c\mathrm{M_j}^4}{4}\right] \tag{12}$$

**6)** Interest payable per unit time during time span $[\mathrm{M_j}, \mathrm{T}]$ is $\frac{(1-k_j\Lambda)C_b I_{b0}}{T}\int_{\mathrm{M_j}}^{T} I(t)dt$

$$= \frac{(1-k_j\lambda)C_b I_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)\left(1+\theta\left(T-M_j\right)-e^{\theta\left(T-M_j\right)}\right)+(-\frac{b}{\theta^2}+\frac{2c}{\theta^2})(T-\ Te^{\theta\left(T-M_j\right)}+\theta(\frac{T^2}{2}-\frac{{M_j}^2}{2}))\text{-}\frac{c}{\theta^2}(T^2-T^2e^{\theta\left(T-M_j\right)}+\theta(\frac{T^3}{3}-\frac{{M_j}^3}{3}))\} \quad (13)$$

Therefore profit of the buyer in this case can be expressed as:

$TBP_{j1}$ = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned – Interest paid.

$$TBP_{j1} = \frac{(C_c-(1-k_j\lambda)C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\}-\frac{S_b}{T}-\frac{(1-k_j\lambda)C_b I_b}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)\left(1+\theta T-e^{\theta T}\right)+(-\frac{b}{\theta^2}+\frac{2c}{\theta^2})(T-\ Te^{\theta T}+\frac{\theta T^2}{2})-\frac{c}{\theta^2}(T^2-T^2e^{\theta T}+\theta\frac{T^3}{3})\}-\frac{(1-k_j\lambda)C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}-aT-\frac{bT^2}{2}-\frac{cT^3}{3}\right\}+\frac{I_{be}C_c}{T}\left[\frac{a{M_j}^2}{2}+\frac{b{M_j}^3}{3}+\frac{c{M_j}^4}{4}\right]-\frac{(1-k_j\lambda)C_b I_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)\left(1+\theta\left(T-M_j\right)-e^{\theta\left(T-M_j\right)}\right)+(-\frac{b}{\theta^2}+\frac{2c}{\theta^2})(T-\ Te^{\theta\left(T-M_j\right)}+\theta(\frac{T^2}{2}-\frac{{M_j}^2}{2}))\text{-}\frac{c}{\theta^2}(T^2-T^2e^{\theta\left(T-M_j\right)}+\theta(\frac{T^3}{3}-\frac{{M_j}^3}{3}))\} \quad (14)$$

j = 1,2

Case2. When $M_j \geq T$; j = 1,2.

The first 4 components of the profit function remain same. The sixth cost component does not exist for $M_j \geq T$. The interest earned per unit time during time span [0, $M_j$] is

$$\frac{I_{be}C_c}{T}\{\int_0^T (a+bt+ct^2)tdt+Q(M_j-T)\}$$
$$=\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}\right]+\frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\}(M_j-T) \quad (15)$$

In this case profit for the buyer is given by

$TBP_{j2}$ = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned.

$$TBP_{j2} = \frac{(C_c-(1-k_j\lambda)C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\}-\frac{S_b}{T}-\frac{(1-k_j\lambda)C_b I_b}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)\left(1+\theta T-e^{\theta T}\right)+(-\frac{b}{\theta^2}+\frac{2c}{\theta^2})(T-\ Te^{\theta T}+\frac{\theta T^2}{2})-\frac{c}{\theta^2}(T^2-T^2e^{\theta T}+\theta\frac{T^3}{3})\}-\frac{(1-k_j\lambda)C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}-aT-\frac{bT^2}{2}-\frac{cT^3}{3}\right\}+\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}\right]+\frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\}(M_j\text{-}T) \quad (16)$$

j = 1,2

### 4. 3. Joint total profit per unit time

In integrated system, the vendor and the buyer to take joint decision which maximizes the profit of the supply chain, the joint total profit per unit time for integrated system is

$$\pi_j = \begin{cases} \pi_{j1} = TVP_j + TBP_{j1} & M_j < T \\ \pi_{j2} = TVP_j + TBP_{j2} & M_j \geq T; j = 1,2. \end{cases}$$

Considering $e^{\theta T} = 1 + \theta T + \frac{\theta^2 T^2}{2}$

$$TVP_j = \Big((1-k_j\Lambda)C_b - C_v - (1-k_j\Lambda)C_b I_{v0} M_j + k_j(1-\Lambda)C_b fvc\,(M2 - M1)\Big)\Big(a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\Big) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho)+\varrho]\Big(\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{2cT^2}{3\theta} + cT^3\Big) \tag{17}$$

$$TBP_{j1} = \big(C_c - (1-k_j\Lambda)C_b\big)\Big\{a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\Big\} - \frac{S_b}{T} - (1-k_j\Lambda)C_b I_b\Big\{\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - \frac{cT^3}{3\theta}\Big\} - (1-k_j\Lambda)C_b\Big\{\frac{a\theta T}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2 - \frac{cT^2}{3}\Big\} + \frac{I_{be}C_c}{T}\Big[\frac{aM_j^2}{2} + \frac{bM_j^3}{3} + \frac{cM_j^4}{4}\Big] - \frac{(1-k_j\Lambda)C_b I_{b0}}{T}\Big\{\frac{a(T-M_j)^2}{2} - \frac{b(T-M_j)^2}{2\theta} + \frac{c(T-M_j)^2}{\theta} + \frac{bT(T-M_j)}{\theta} + \frac{bT(T-M_j)^2}{2} - \frac{b}{\theta}\Big(\frac{T^2}{2} - \frac{M_j^2}{2}\Big) - \frac{2cT(T-M_j)}{\theta} + \frac{c(T^2-M_j^2)}{\theta} - cT(T-M_j)^2 + \frac{cT^2(T-M_j)}{\theta} + \frac{cT^2(T-M_j)^2}{2} - \frac{c}{\theta}\Big(\frac{T^3}{3} - \frac{M_j^3}{3}\Big)\Big\} \tag{18}$$

$$TBP_{j2} = \Big(C_c - (1-k_j\Lambda)C_b + I_{be}C_c(M_j - T)\Big)\Big\{a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\Big\} - \frac{S_b}{T} - (1-k_j\Lambda)C_b I_b\Big\{\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - \frac{cT^3}{3\theta}\Big\} - (1-k_j\Lambda)C_b\Big\{\frac{a\theta T}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2 - \frac{cT^2}{3}\Big\} + \frac{I_{be}C_c}{T}\Big[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4}\Big] \tag{19}$$

$$\pi_{j1} = \Big((1-k_j\Lambda)C_b - C_v - (1-k_j\Lambda)C_b I_{v0} M_j + k_j(1-\Lambda)C_b fvc\,(M2 - M1)\Big)\Big(a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\Big) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho)+\varrho]\Big(\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{2cT^2}{3\theta} + cT^3\Big) + \big(C_c - (1-k_j\Lambda)C_b\big)\Big\{a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\Big\} - \frac{S_b}{T} - (1-k_j\Lambda)C_b I_b\Big\{\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - \frac{cT^3}{3\theta}\Big\} - (1-k_j\Lambda)C_b\Big\{\frac{a\theta T}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2 - \frac{cT^2}{3}\Big\} + \frac{I_{be}C_c}{T}\Big[\frac{aM_j^2}{2} + \frac{bM_j^3}{3} + \frac{cM_j^4}{4}\Big] - \frac{(1-k_j\Lambda)C_b I_{b0}}{T}\Big\{\frac{a(T-M_j)^2}{2} - \frac{b(T-M_j)^2}{2\theta} + \frac{c(T-M_j)^2}{\theta} + \frac{bT(T-M_j)}{\theta} + \frac{bT(T-M_j)^2}{2} - \frac{b}{\theta}\Big(\frac{T^2}{2} - \frac{M_j^2}{2}\Big) - \frac{2cT(T-M_j)}{\theta} + \frac{c(T^2-M_j^2)}{\theta} - cT(T-M_j)^2 + \frac{cT^2(T-M_j)}{\theta} + \frac{cT^2(T-M_j)^2}{2} - \frac{c}{\theta}\Big(\frac{T^3}{3} - \frac{M_j^3}{3}\Big)\Big\} \tag{20}$$

$$\text{п}_{j2} = \Big((1-k_j \Lambda)C_b - C_v - (1-k_j \Lambda)C_b I_{v0} M_j + k_j(1-\Lambda)C_b fvc\,(M2 - M1)\Big)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\right) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho)+\varrho]\left(\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{2cT^2}{3\theta} + cT^3\right) + \Big(C_c - (1-k_j \Lambda)C_b + I_{be}C_c(\text{M}_\text{j} - T)\Big)\left\{a + \frac{a\theta T}{2} + \frac{bT}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2\right\} - \frac{S_b}{T} - (1-k_j \Lambda)C_b I_b\left\{\frac{bT^2}{2} - cT^2 + \frac{aT}{2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - \frac{cT^3}{3\theta}\right\} - (1-k_j \Lambda)C_b\left\{\frac{a\theta T}{2} - cT + \frac{b\theta T^2}{2} + \frac{cT}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3}{2} - c\theta T^2 - \frac{cT^2}{3}\right\} + \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4}\right] \tag{21}$$

The optimum value of cycle time can be obtained by setting $\frac{d\text{п}_j}{dT} = 0$ for fixed n. The necessary condition for maximising total profit is $\frac{d^2\text{п}_j}{dT^2} < 0$.

## 5. NUMERICAL EXAMPLES

To illustrate the above developed model, an inventory system with the following data is considered a = 1000, b = 50 , $\theta$ = 0.1, $\varrho$ = 0.7, $C_v$ = \$5/unit, $C_b$ = \$35/ unit, $C_c$ = \$ 55 / unit, $S_v$ = \$1500/setup, $S_b$ = \$100/order, $I_v$ = 1%/unit/annum, $I_b$=1%/unit/annum, $I_{v0}$ = 2% /unit/annum, $I_{b0}$ = 5%/unit/annum, $I_{be}$ = 8%/unit/annum, $M_1$ = 10days, M2 = 30 days, $\Lambda$ = 2% and fvc = 0.17/\$/annum

Using computational procedure optimum cycle time T* for above data is 20 days for n = 5. The buyer's order quantity Q* are 14,88,400 units/order. Vendor's total profit TVP is \$4,37,360 and buyer's total profit TBP is \$ 15,81,400. The maximum total joint profit of the integrated system п is \$20,18,800.

### 5. 1. Sensitivity analysis

Here we study the effects of changes in the system parameters a,b, c,$\theta$ and $\varrho$ on the optimal length of order cycle $T^*$, the optimal order quantity per cycle $Q^*$, vendor's profit TVP, buyer's profit TBP, total profit of the collaborative vendor-buyer inventory system п. The results are shown in Table 1.

**Table 1.** Effect of change in various parameters of Example1

| Parameter | Value | T | Q | Vendor | Buyer | Joint Profit |
|---|---|---|---|---|---|---|
| a | 50 | 20 | 14,85,200 | 4,36,610 | 15,74,700 | 20,11,300 |
| | 100 | 20 | 14,88,400 | 4,37,360 | 15,81,400 | 20,18,800 |
| | 150 | 20 | 14,91,600 | 4,38,120 | 15,88,100 | 20,26,200 |
| | 200 | 20 | 14,94,800 | 4,38,880 | 15,94,700 | 20,33,600 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| b | 10 | 20 | 14,84,200 | 4,36,300 | 15,72,700 | 20,09,000 |
| | 15 | 20 | 14,88,400 | 4,37,360 | 15,81,400 | 20,18,800 |
| | 20 | 20 | 14,92,600 | 4,38,430 | 15,90,100 | 20,28,500 |
| | 25 | 20 | 14,96,800 | 4,39,490 | 15,98,800 | 20,38,300 |
| | | | | | | |
| c | 3 | 20 | 9,00,630 | 2,64,300 | 9,64,595 | 12,28,892 |
| | 5 | 20 | 14,88,400 | 4,37,360 | 15,81,400 | 20,18,800 |
| | 10 | 20 | 29,57,800 | 8,70,030 | 31,23,400 | 39,93,400 |
| | 15 | 20 | 44,27,200 | 13,02,700 | 46,65,395 | 59,68,100 |
| | | | | | | |
| ɵ | 0.05 | 19 | 18,71,800 | 8,06,210 | 31,95,800 | 40,02,000 |
| | 0.1 | 20 | 14,88,400 | 4,37,360 | 15,81,400 | 20,18,800 |
| | 0.15 | 21 | 23,13,200 | 3,70,330 | 11,79,700 | 15,50,000 |
| | 0.2 | 21 | 37,35,700 | 3,20,550 | 10,34,000 | 13,54,500 |
| | | | | | | |
| ϱ | 0.6 | 20 | 14,88,400 | 4,34,870 | 15,81,400 | 20,16,300 |
| | 0.7 | 20 | 14,88,400 | 4,37,360 | 15,81,400 | 20,18,800 |
| | 0.8 | 20 | 14,88,400 | 4,39,850 | 15,81,400 | 20,21,200 |
| | 0.9 | 20 | 14,88,400 | 4,42,340 | 15,81,400 | 20,23,700 |

Based on the results of Table 1, we can obtain the following managerial insights.

(1) Increase in the value of the parameters a, b and c will result in increase of Q*, vendor's profit, buyer's profit and joint total profit. T* remain same.

(2) Decrease in the value of the parameters a, b and c will result in decrease of Q*, vendor's profit, buyer's profit and joint total profit. T* remain same.

(3) Increase in the value of the parameters ɵ will result in increase of Q* but decrease of vendor's profit, buyer's profit and joint total profit.

(4) Decrease in the value of the parameters ɵ will result in increase of Q*, vendor's profit, buyer's profit and joint total profit but decrease of T*.

(5) Increase in the value of the parameter ϱ will result in increase of vendor's profit and joint total profit but Q*, buyer's profit and T* remain unchanged.

## 6. CONCLUSION

Trade credit with cash discount is very realistic in global competitive market. To stimulate demand of the buyer, vendor always offers trade credit. However, the vendor can also use the cash discount policy to attract buyer to pay the full payment of the amount of purchasing cost to shorten the collection period. In this paper, we formulated an integrated vendor- buyer inventory model for deteriorating items with the assumption that demand is quadratic and vendor offers two payment options: trade credit and cash discount. A mathematical model is developed to find optimal replenishment policies and to maximise joint total profit per unit time of the supply chain system, which helps the inventory manager to take advantage of credit period for repaying the vendor by ordering larger quantity. This result helps the buyer to make a decision between two promotional tools, viz cash discount and trade credit. By using the numerical example, sensitive analysis is performed to study the effects of the changes of the parameter values on the optimal cycle time, optimal order quantity and total relevant profit respectively. In future one can analyze integrated inventory system for weibull distributed deteriorating items.

## References

[1] Abad, P.L., Jaggi, C.K. (2003). A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. *International Journal of Production Economics*, Vol. 83, pp. 115-122.

[2] Banerjee, A. (1986). A joint economic – lot – size model for purchaser and vendor. *Decision Sciences*, Vol. 17, pp. 292-311.

[3] Bhatnagar, R., Chandra, P., Goyal, S.K. (1993). Models for multi – plant coordination. *European Journal of Operational Research*, Vol. 67, pp. 141-160.

[4] Chang, C.T., (2002). Extended economic order quantity model under cash discount and payment delay. *International Journal of Information and Management Sciences*, Vol. 13, pp. 57-69.

[5] Chang, C., Ouyang, L. and Teng, J. (2003). An EOQ model for deteriorating items under supplier credits linked to ordering quantity. *Applied Mathematical Modelling*, Vol. 27, No. 12, pp. 983-996.

[6] Chang, H. -J., Dye, C. –Y. and Chuang, B. –R. (2002). An inventory model for deteriorating items under the condition of permissible delay in payments. *Yugoslav Journal of Operations Research,* Vol. 12, No. 1, pp. 73-84.

[7] Chu, P., Chung, K. and Lan, S. (1998). Economic order quantity of deteriorating items under permissible delay in payments. *Computers and Operations Research*, Vol. 25, No. 10, pp. 817-824.

[8] Chung, K. and Liao, J. (2004). Lot-sizing decisions under trade credit depending on the ordering quantity. *Computers and Operations Research*, Vol. 31, No. 6, pp. 909-928.

[9] Chung, K. J., Lin, S. D. and Srivastava, H. M. (2014). The inventory models for deteriorating items in the discounted cash-flows approach under conditional trade credit

and cash discount in a supply chain system. *Applied Mathematics & Information Sciences*, Vol. 8, No. 5, pp. 2103-2111.

[10] Chung, K. J., Chang, S. L. and Yang, W. D. (2001). The optimal cycle time for exponentially deteriorating products under trade credit financing. *Engineering Economics*, Vol. 46, No. 3, pp. 232-242.

[11] Dye, C. and Ouyang, L. (2011). A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing. *Computers and Industrial Engineering*, Vol. 60, No. 1, pp.127-137.

[12] Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, Vol. 36, pp. 335-338

[13] Goyal, S.K. (1976). An integrated inventory model for a single supplier – single customer Problem. *International Journal of Production Research*, Vol. 15, pp. 107-111.

[14] Goyal, S.K. (1988).A joint economic – lot – size model for purchaser and vendor: a comment. *Decision Sciences*, Vol. 19, pp. 236-241.

[15] Goyal, S.K. (1995). A one – vendor multi – buyer integrated inventory model: a comment. *European Journal of Operational Research*, Vol. 82, pp. 209-210.

[16] Guchhait, P., Maiti, M. K. and Maiti, M. (2013). Two storage inventory model of a deteriorating item with variable demand under partial credit period. *Applied Soft Computing Journal*, Vol. 13, No. 1, pp. 428-448.

[17] Haley, C. W., Higgins, R. C. (1973). Inventory policy and trade credit financing. *Management Science*, Vol. 20, No. 4, pp. 464-471.

[18] Hill, R.M. (1997). The single – vendor single – buyer integrated production – inventory model with a generalized policy. *European Journal of operational Research*, Vol. 97, pp. 493-499.

[19] Hill, R.M. (1999). The optimal production and shipment policy for the single – vendor single buyer integrated production – inventory problem. *International Journal of Production Research*, Vol. 37, pp. 2463-2475.

[20] Ho, C.-H., Ouyang, L.-Y. and Su, C.-H. (2008). Optimal pricing, shipment and payment policy for an integrated supplier-buyer inventory model with two-part trade credit. *European Journal of Operational Research*, Vol. 187, No. 2, pp. 496-510.

[21] Huang, Y. F. (2005).A note on EOQ model under cash discount and payment delay. *Information and Management Sciences,* Vol. 16, No. 3, pp. 97-107.

[22] Huang, Y. F. and Hsu, K. H. (2007). An EOQ model with non-instantaneous receipt under Supplier credits. *Journal of the Operations Research Society of Japan*, Vol. 50, No. 1, pp. 1-13.

[23] Huang, Y.-F. and Chung, K. -J. (2003). Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit. *Asia-Pacific Journal of Operational Research*, Vol. 20, No. 2, pp. 177-190.

[24] Huang, K-N. and Liao, J-J. (2008). A simple method to locate the optimal solution for exponentially deteriorating items under trade credit financing. *Computers & Mathematics with Applications*, Vol. 56, No. 4, pp. 965-977.

[25] Hwang, H. and Shinn, S. W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and Operations Research*, Vol. 24, No.6, pp. 539-547.

[26] Jaggi, C. K. and Aggarwal, S. P. (1994). Credit financing in economic ordering policies of deteriorating items,' *International Journal of Production Economics*, Vol. 34, No. 2, pp. 151-155.

[27] Jain, S., Advani, P. and Kumar, M. (2008). An inventory model for weibull distribution deterioration with allowable shortage under cash discount and permissible delay in payments. *ASOR Bulletin*, Vol. 27, No. 4, pp. 2-14.

[28] Jamal, A. M. M., Sarker B. R. and Wang, S. (1997). An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. *Journal of the Operational Research Society*, Vol. 48, No. 8, pp. 826-833.

[29] Joglekar, P. N. (1988). Comments on "A quantity discount pricing model to increase vendor profits". *Management Science*, Vol. 34, No. 11, pp. 1391-1398.

[30] Kelle, P., Al – Khatee, F., Miller, A.P.(2003) 'Partnership and negotiation support by joint optimal ordering / setup policies for JIT,' *International Journal of Production Economics*, Vol. 81-82, pp. 431-441.

[31] Kim, S.L., Ha, D. (2003). A JIT lot – splitting model for supply chain management: enhancing buyer – supplier linkage. *International Journal of Production Economics*, Vol. 86, pp. 1-10.

[32] Kumar,A., Kaanodiya, K. K. and Pachauri, R.R. (2011). Retailer's optimal ordering policies with cash discount and progressive payment scheme derived algebraically. *International Journal of Industrial Engineering Computations*, Vol. 2, pp. 891-900.

[33] Lee, H.L., Padmanabhan, V., Whang, S. (1997). The bullwhip effect in the supply chains. *Sloan Management Review*, Vol. 38, pp. 93-102.

[34] Li, J., Liu, L. (2006). Supply chain coordination with quantity discount policy. *International Journal of Production Economics*, Vol. 101, pp. 89-98.

[35] Liao, H., Tsai, C. and Su, C. (2000). Inventory model with deteriorating items under inflation when a delay in payment is permissible. *International Journal of Production Economics*, Vol. 63, No.2, pp. 207-214.

[36] Liao, J., Huang, K. and Chung, K. (2012). Lot-sizing decisions for deteriorating items with two warehouses under an order-size dependent trade credit. *International Journal of Production Economics*, Vol. 137, No. 1, pp. 102-115

[37] Mahata, G. C. (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *Expert Systems with Applications*, Vol. 39, No. 3, pp. 3537-3550.

[38] Mahata, G. C. (2015). Retailer's optimal credit period and cycle time in a supply chain for deteriorating items with up-stream and down-stream trade credits. *Journal of Industrial Engineering International*, Vol. 11, pp. 353-366.

[39] Ouyang, L.Y., Chen, M.S., Chuang, K.W. (2002). Economic Order quantity model under cash discount and payment delay. *International Journal of Information and Management Sciences*, Vol. 13, pp. 1-10.

[40] Roy, A. and Samanta, G. P. (2011). Inventory model with two rates of production for deteriorating items with permissible delay in payments. *International Journal of Systems Science*, Vol. 42, No. 8, pp. 1375-1386.

[41] Sarker, B. R., Jamal, A. M. M. and Wang, S. (2000). Supply chain models for perishable products under inflation and permissible delay in payment. *Computers and Operations Research*, Vol.27, No.1, pp. 59-75.

[42] Shah, N. H. and Shukla, K. T. (2011). A collaborative vendor – buyer deteriorating inventory model for optimal pricing, shipment and payment policy with two – part trade credit. *Supply Chain Management*, Dr. Pengzhong Li (Ed.), ISBN: 978-953-307-184-8,

[43] Shah, N.H. (2009). Optimization of pricing and ordering policy for deteriorating inventory when end demand is price and credit period sensitive. *International Journal of Business Performance and Supply Chain Modeling*, Vol. 1, pp. 229-239.

[44] Shah, N.H. (2010). A lot size model for deteriorating items under conditions of a one time only extended credit period. *International Journal of Mathematics and Mathematical Sciences*, Vol. 2010, Article ID 137428.

[45] Shah,N. H., Shah, D. B. and Patel, D. G. (2013. Optimal shipments, ordering and payment policies for integrated supplier-buyer deteriorating inventory system with price-sensitive trapezoidal demand and net credit. *International Journal of Production Technology and Management* Vol. 4, No. 3, pp. 14-31.

[46] Swami, A., Singh, S.R., Pareek, S., Yadav, A. S. (2015). Inventory policies for deteriorating item with stock dependent demand and variable holding costs under permissible delay in payment. *International Journal of Application or Innovation in Engineering & Management* Vol. 4, no. 2, pp. 89-99.

[47] Tayal, S., Singh, S. R. and Sharma, R., (2016). An integrated production inventory model for perishable products with trade credit period and investment in preservation technology. *International Journal of Mathematics in Operation Research* Vol. 8, No. 2, pp. 137-163.

[48] Teng, J. -T., Chen, J. and Goyal, S. K. (2009). A comprehensive note on: an inventory model under two levels of trade credit and limited storage space derived without derivatives. *Applied Mathematical Modelling*, Vol. 33, No.12, pp. 4388-4396.

[49] Teng, J-T., Krommyda, I-P., Skouri, K. and Lou, K-R. (2011). A comprehensive extension of optimal ordering policy for stock dependent demand under progressive payment scheme. *European Journal of Operational Research*, Vol. 215, No. 1, pp. 97-104.

[50] Thangam, A. and Uthayakumar, R. (2010). Optimal pricing and lot sizing policy for a two-warehouse supply chain system with perishable items under partial trade credit financing. *Operational Research*, Vol. 10, No. 2, pp. 133-161.

[51] Thangam, A. (2012). Optimal price discounting and lot-sizing policies for perishable items in a supply chain under advance payment scheme and two-echelon trade credits. *International Journal of Production Economics*, Vol. 139, No. 2, pp. 459-472.

[52] Viswanathan, S. (1998). Optimal strategy for the integrated vendor – buyer inventory model. *European Journal of Operational Research*, Vol. 105, pp. 38-42.