An Integrated Inventory Model for Deteriorating Items under Cash Discount and Permissible Delay in Payments

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**ABSTRACT**

Numerous studies have been undertaken to explain inventory models with different features. While findings from earlier studies have been conflicting, recent industrial-level studies indicate that multi features inventory models have a positive impact on business scenario. We propose an inventory model with integration of many real features like constant deterioration under cash discount scheme and permissible delay in payments. That is, we want to investigate the buyer’s optimal replenishment policy with quadratic demand under trade credit and cash discount to maximise joint total profit per unit time. Furthermore, numerical example and sensitivity analysis are presented to illustrate the results of the proposed model and to draw managerial insights.

**Keywords:** Integrated inventory model, Cash discount, permissible delay in payments, quadratic demand, deterioration
1. INTRODUCTION

Trade credit is an invaluable promotional tool for the suppliers to increase profit through stimulating more sales and a unique opportunity for the retailers to reduce demand uncertainty and its associated risks. It is a regular component of market transactions and constitutes a major source of short-term financing. In this practice, the supplier is willing to offer the retailer a certain credit period without interest during the permissible delay period to promote market competition. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. This type of model was first discussed by Haley and Higgins (1973). Goyal (1985) explored a single-item EOQ model under permissible delay in payments. Deterioration is defined as the decay, spoilage, evaporation which loses the utility of a production from the original one. Fruits and vegetables, pharmaceutical drugs, electronic items, blood components, radioactive chemicals are some of the examples of deteriorating items. Jaggi and Aggarwal (1994) presented the economic ordering policies of deteriorating items in the presence of trade credit using a discounted cash-flows (DCF) approach. Hwang and Shinn (1997) developed the joint price and lot size determination problem for an exponentially deteriorating product when the supplier offers a certain fixed credit period. Jamal et al. (1997) developed a model for an optimal ordering policy for deteriorating items with allowable shortage and permissible delay in payment. Further Chu et al. (1998), Sarkar et al. (2000), Liao et al. (2000), Chung et al. (2001), Chang et al. (2003), Chung and Liao (2004), Teng et al. (2005), Chung and Liao (2006), Chung (2006), Chung and Huang (2007) and Teng et al. (2009), Huang and Liao (2008), Thangam and Uthayakumar (2010), Shah (2010), Dye and Ouyang (2011), Roy and Samanta (2011), Teng et al. (2011), Mahata (2012), Liao et al. (2012), Thangam (2012), Guchhait et al. (2013), Chung et al. (2014), Swami et al. (2015), Mahata (2015) developed inventory models for deteriorating items under trade credit considering different features.

In most business transactions, the supplier will offer the credit terms mixing cash discount to the retailer to avoid the possibility of resulting in bad debt. The retailer can obtain the cash discount when the payment is paid within cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. One can refer Chang (2002), Huang et al. (2003), Ouyang et al. (2002), Huang (2003), Huang and Hsu (2007), Ho et al. (2008), Jain et al. (2008), Shah and Shukla (2011), Shah et al. (2013), Kumar et al. (2011) for trade credit and cash discount inventory models.

There are many models derived either from the supplier’s or the retailer’s end. However, the two players in supply chain may have their own goals. Lee et al. (1997) pointed out that the absence of coordinated inventory management throughout the supply chain results in excessive inventory investment, revenue reduction and delays in response to customer requirements. Therefore, determining the optimal integrated policies is more reasonable than considering the buyer’s or the supplier’s individual profit/cost. Goyal (1976), Banerjee (1986), Goyal (1988), Bhatnagar et al. (1993), Goyal (1995), Viswanathan (1998), Hill (1997, 1999), Kim and Ha (2003), Kelle et al. (2003), Li and Liu (2006) developed optimal joint inventory policies for supplier and retailer.

However, these articles did not incorporate the effect of trade credit on the integrated optimal decision. Abad and Jaggi (2003) developed a vendor – buyer integrated model...

In this paper we develop an inventory model for deteriorating items with constant deterioration under cash discount and permissible delay in payments for quadratic demand. This paper is organized as follows. In section 2 assumptions and in section 3 notations are presented. In section 4 the mathematical model is formulated where joint total profit per unit time is maximised to create win-win strategy between players in the supply chain. In section 5 numerical example is cited and sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

2. NOTATIONS

The following notations are used in the proposed article:

\( S_v \): Vendor’s set up cost per set up.
\( S_b \): Buyer’s ordering cost per order.
\( C_v \): Production cost per unit.
\( C_b \): Buyer’s purchase cost per unit.
\( C_c \): The unit retail price to customers where \( C_c > C_b > C_v \).
\( I_v \): Vendor’s inventory holding cost rate per unit per annum, excluding interest charges.
\( I_b \): Buyer’s inventory holding cost rate per unit per annum, excluding interest charges.
\( I_v0 \): Vendor’s opportunity cost/$/unit time.
\( I_b0 \): Buyer’s opportunity cost/$/unit time.
\( I_{be} \): Buyer’s interest earned/$/unit time.
\( \varrho \): Capacity utilisation which is ratio of demand to the production rate, \( \varrho < 1 \) and known constant.
\( M_1 \): Period of cash discount
\( M_2 \): Allowable credit period for the buyer offered by the vendor. ( \( M_2 > M_1 \))
\( Q \): Buyer’s order quantity.
\( T \): cycle time (decision variable).
\( n \): Number of shipments from vendor to the buyer.
\( \theta \): Constant rate of deterioration.
\( \lambda \): Cash discount rate
\( fvc \): Vendor’s cash flexibility rate
\( TVP \): Vendor’s total profit per unit time.
TBP: Buyer’s total profit per unit time.
π: TVP + TBP Joint total profit per unit time.

3. ASSUMPTIONS

In addition, the following assumptions are made in derivation of the model:

- The supply chain under consideration comprise of single vendor and single buyer for a single product.
- Shortages are not allowed.
- The demand rate considered is time dependent, increasing demand rate. The constant part of quadratic demand pattern changes with each cycle.
- Replenishment rate is instantaneous for retailer.
- The units in inventory are subject to deteriorate at a constant rate of \( \rho \), \( 0 < \rho < 1 \). The deteriorated units can neither be repaired nor replaced during the cycle time.
- Finite production rate.
- Vendor produces the \( nQ \) items and then fulfils the buyer’s demand, so at the beginning of production item, there is small possibility of deterioration in general. Moreover vendor is a big merchant who can have capacity to prevent deterioration. So, in this model, deterioration cost is considered for buyer only at the rate \( \rho \) is assumed to be constant.
- The vendor offers a discount \( \zeta \) \( (0 < \zeta < 1) \) in the purchase price if the buyer pays by time \( M_1 \); otherwise full account is to be settled within allowable credit period \( M_2 \), where \( M_2 > M_1 \geq 0 \). The offer of discount in unit purchase price from the vendor will increase cash in-flow, thereby reducing the risk of cash flow shortage.
- By offering a trade credit to the buyer, the vendor receives cash at a later date and hence incurs an opportunity cost during the delivery and payment of the product. On the buyer’s end, the buyer can generate revenue by selling the items and earning interest by depositing it in an interest bearing account during this permissible delay period. At the end of this period, the vendor charges to the buyer on the unsold stock.
- During the time \( [M_1, M_2] \), a cash flexibility rate \( f_{vc} \) is used to quantize the favor of early cash income for the vendor.

4. MATHEMATICAL FORMULATION

Let \( I(t) \) be the inventory level at any time \( t \), \( (0 \leq t \leq T) \). Depletion due to deterioration and demand will occur simultaneously. The differential equation describing the instantaneous state of \( I(t) \) over \( (0,T) \) is given by:

\[
\frac{dI(t)}{dt} + \rho I(t) = -(a + bt + ct^2) \quad 0 \leq t \leq T
\]  

Solution to the equation (1) (using the boundary condition \( I(t) = 0 \) at \( t = T \)) is given by
\[ I(t) = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right) \left(e^{\theta(T-t)} - 1\right) + \left(\frac{b}{\theta} - \frac{2c}{\theta^2}\right) \left(T e^{\theta(T-t)} - t\right) + \frac{c}{\theta} \left(T^2 e^{\theta(T-t)} - t^2\right) \]

\[ 0 \leq t \leq T \] (2)

Also at \( t = 0, \ I(t) = Q \)

\[ Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right) \left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right) e^{\theta T} \quad 0 \leq t \leq T \] (3)

4. 1. Vendor’s total profit per unit time

For each unit of item, the vendor charges \((1 - k_j \Phi) C_b\) if the buyer pays at time \(M_j\), \(j = 1, 2\), \(k_1 = 1\) and \(k_2 = 0\).

1) Sales revenue: the total sales revenue per unit time is \( \left(1 - k_j \Phi\right) C_b - C_V \frac{Q}{T} \)

\[ = \frac{(1-k_j \Phi) C_b - C_V}{T} \left\{ \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right) \left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right) e^{\theta T} \right\} \] (4)

2) Set-up cost: \(nQ\) units are manufactured in one production run by the vendor. Therefore the setup cost per unit time is \( \frac{S_v}{nT} \)

3) Holding cost: using method given by Joglekar (1988), vendor’s average inventory per unit time is

\[ = -\frac{C_v}{\theta^2} \left\{ \left(\frac{-a}{\theta^2} + \frac{b}{\theta^2} - \frac{2c}{\theta^2}\right) \left(1 + \theta T - e^{\theta T}\right) + \left(\frac{-b}{\theta^2} + \frac{2c}{\theta^2}\right) \left(T - T e^{\theta T} + \frac{\theta T^2}{2}\right) \right\} \] (5)

4) Opportunity cost: opportunity cost per unit time because of offering permissible delay period is \( \frac{(1-k_j \Phi) C_b l_{v0} M Q}{T} \)

\[ = \frac{(1-k_j \Phi) C_b l_{v0} M}{T} \left\{ \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right) \left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right) e^{\theta T} \right\} \] (6)

However, if the buyer pays at \(M_1\) –time, during \(M_2 - M_1\) the vendor can use the revenue \((1 - \Phi) C_b\) to avoid a cash flow crisis. The advantage gain per unit time from early payment at a cash flexibility rate \(f_{fc} \) is

\[ k_j (1 - \Phi) C_b f_{fc} (M2 - M1) Q \]

\[ = \frac{k_j (1 - \Phi) C_b f_{fc} (M2 - M1)}{T} \left\{ \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right) \left(e^{\theta T} - 1\right) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right) e^{\theta T} \right\} \] (7)
Hence the total profit per unit time for vendor is 

\[
TVP_j = \frac{(1-k_j)C_b - C_V}{T} \left\{ (\frac{a}{a_0} - \frac{b}{a_0^2} + \frac{2c}{a_0^3}) (e^{\alpha T} - 1) + \left( \frac{bT}{\alpha} + \frac{cT^2}{\alpha^2} - \frac{2cT}{\alpha^3} \right) e^{\alpha T} \right\} - \frac{S_b}{\alpha T} \frac{C_b(\mu \gamma + \lambda \eta)}{T} [(n - 1)(1 - e^{-\alpha T}) + e^{-\alpha T}) (1 + \alpha T - e^{\alpha T}) + (- \frac{b}{\alpha^2} + \frac{2c}{\alpha^3}) (T - T e^{\alpha T} + \frac{\alpha T^2}{2}) - \frac{c}{\alpha^4} (T^2 - T^2 e^{\alpha T} + \theta \frac{T^3}{3})]
\]

\[
= \frac{(1-k_j)C_b}{T} \left\{ (\frac{a}{a_0} - \frac{b}{a_0^2} + \frac{2c}{a_0^3}) (e^{\alpha T} - 1) + \left( \frac{bT}{\alpha} + \frac{cT^2}{\alpha^2} - \frac{2cT}{\alpha^3} \right) e^{\alpha T} \right\}
\]

\[
(8)
\]

4. 2. Net profit function for the buyer consists of following elements

1) Sales revenue: The total sales revenue per unit time is 

\[
\left( \frac{C_c - (1-k_j)C_b}{T} \right) Q
\]

\[
= \frac{(1-k_j)C_b}{T} \left\{ (\frac{a}{a_0} - \frac{b}{a_0^2} + \frac{2c}{a_0^3}) (e^{\alpha T} - 1) + \left( \frac{bT}{\alpha} + \frac{cT^2}{\alpha^2} - \frac{2cT}{\alpha^3} \right) e^{\alpha T} \right\}
\]

\[
(9)
\]

2) Ordering cost: Ordering cost per unit time is 

\[
\frac{S_b}{\alpha T}
\]

3) Holding cost: The buyer’s holding cost (excluding interest charges) per unit time is 

\[
\frac{(1-k_j)C_b b}{T} \left\{ (\frac{-a}{a_0^2} + \frac{b}{a_0^3} - \frac{2c}{a_0^4}) (1 + \alpha T - e^{\alpha T}) + (- \frac{b}{a_0^2} + \frac{2c}{a_0^3}) (T - T e^{\alpha T} + \frac{\alpha T^2}{2}) - \frac{c}{a_0^4} (T^2 - T^2 e^{\alpha T} + \theta \frac{T^3}{3}) \right\}
\]

\[
(10)
\]

4) Deteriorating cost: Deteriorating cost per unit time is 

\[
\frac{(1-k_j)C_b}{T} \left\{ (\frac{a}{a_0} - \frac{b}{a_0^2} + \frac{2c}{a_0^3}) (e^{\alpha T} - 1) + \left( \frac{bT}{\alpha} + \frac{cT^2}{\alpha^2} - \frac{2cT}{\alpha^3} \right) e^{\alpha T} - \alpha T - \frac{bT^2}{2} - \frac{cT^3}{3} \right\}
\]

\[
(11)
\]

Based on the length of the payment time, two cases arise namely \(M_j < T\) and \(M_j \geq T\); \(j = 1,2\)

Case 1. When \(M_j < T\); \(j = 1,2\).

5) Interest earned per unit time during the period \([0, M_j]\) is 

\[
\frac{1}{T} \left[ \frac{aM_j^2}{2} + \frac{bM_j^3}{3} + \frac{cM_j^4}{4} \right]
\]

\[
(12)
\]

6) Interest payable per unit time during time span \([M_j, T]\) is 

\[
\frac{(1-k_j)C_b b}{T} \left[ \int_{M_j}^{T} I(t) dt \right]
\]
\[\begin{align*}
&= \frac{(1-k_1\beta)C_p l_b}{T} \left\{ \left( \frac{-a}{a^2} + \frac{b}{a^3} - \frac{2c}{a^2} \right) \left( 1 + e^{\theta(T-M_j)} - e^{\theta(T-M_j)} \right) + \left( - \frac{b}{a^2} + \frac{2c}{a^2} \right) (T - Te^{\theta(T-M_j)} + \theta(T - M_j)) \right\} \\
&\left( \frac{T^2}{2} - \frac{M_j^2}{2} \right) c \frac{e^{\theta(T-M_j)}}{a}
\end{align*}\]

Therefore profit of the buyer in this case can be expressed as:

\[TBP_{j1} = \text{Sales revenue} - \text{Ordering cost} - \text{Inventory carrying cost} - \text{Deteriorating cost} + \text{Interest earned} - \text{Interest paid.}\]

\[TBP_{j1} = \frac{(C_c - (1-k_1\beta)C_b)}{T} \left\{ \left( \frac{a}{a^2} - \frac{b}{a^3} + \frac{2c}{a^2} \right) (e^{\theta T} - 1) + \left( \frac{bT}{a} + \frac{cT^2}{a^2} - \frac{2cT}{a^2} \right) e^{\theta T} \right\} - \frac{S_b}{T} - \frac{(1-k_1\beta)C_l l_b}{T} \left\{ \left( \frac{-a}{a^2} + \frac{b}{a^3} - \frac{2c}{a^2} \right) (1 + e^{\theta T} - e^{\theta T}) + \left( - \frac{b}{a^2} + \frac{2c}{a^2} \right) (T - Te^{\theta T} + \theta \frac{T^2}{2} - \frac{c}{a^2} (T^2 - T^2 e^{\theta(T-M_j)} + \theta \frac{T^3}{3} - \frac{M_j^3}{3})) \right\} j = 1,2\]

Case 2. When \( M_j \geq T; j = 1,2.\)

The first 4 components of the profit function remain same. The sixth cost component does not exist for \( M_j \geq T.\) The interest earned per unit time during time span \([0, M_j]\) is

\[\frac{I_{beC_c}}{T} \left\{ \int_0^T (a + bt + ct^2) t dt + Q(M_j - T) \right\} \]

\[= \frac{I_{beC_c}}{T} \left\{ \left( \frac{aT^2}{2} + \frac{btT^3}{3} + \frac{ct^4}{4} \right) + \frac{I_{beC_c}}{T} \left\{ \left( \frac{a}{a^2} - \frac{b}{a^3} + \frac{2c}{a^2} \right) (e^{\theta T} - 1) + \left( \frac{bT}{a} + \frac{cT^2}{a^2} - \frac{2cT}{a^2} \right) e^{\theta T} \right\} (M_j - T) \right\} (15)\]

In this case profit for the buyer is given by

\[TBP_{j2} = \text{Sales revenue} - \text{Ordering cost} - \text{Inventory carrying cost} - \text{Deteriorating cost} + \text{Interest earned.}\]

\[TBP_{j2} = \frac{(C_c - (1-k_1\beta)C_b)}{T} \left\{ \left( \frac{a}{a^2} - \frac{b}{a^3} + \frac{2c}{a^2} \right) (e^{\theta T} - 1) + \left( \frac{bT}{a} + \frac{cT^2}{a^2} - \frac{2cT}{a^2} \right) e^{\theta T} \right\} - \frac{S_b}{T} - \frac{(1-k_1\beta)C_l l_b}{T} \left\{ \left( \frac{-a}{a^2} + \frac{b}{a^3} - \frac{2c}{a^2} \right) (1 + e^{\theta T} - e^{\theta T}) + \left( - \frac{b}{a^2} + \frac{2c}{a^2} \right) (T - Te^{\theta T} + \theta \frac{T^2}{2} - \frac{c}{a^2} (T^2 - T^2 e^{\theta(T-M_j)} + \theta \frac{T^3}{3} - \frac{M_j^3}{3})) \right\} + \frac{I_{beC_c}}{T} \left\{ \left( \frac{aT^2}{2} + \frac{btT^3}{3} + \frac{ct^4}{4} \right) + \frac{I_{beC_c}}{T} \left\{ \left( \frac{a}{a^2} - \frac{b}{a^3} + \frac{2c}{a^2} \right) (e^{\theta T} - 1) + \left( \frac{bT}{a} + \frac{cT^2}{a^2} - \frac{2cT}{a^2} \right) e^{\theta T} \right\} (M_j - T) \right\} j = 1,2\]
4.3. Joint total profit per unit time

In integrated system, the vendor and the buyer to take joint decision which maximizes the profit of the supply chain, the joint total profit per unit time for integrated system is

\[
\pi_j = \begin{cases} 
\pi_{j1} = TVP_j + TBP_{j1} & M_j < T \\
\pi_{j2} = TVP_j + TBP_{j2} & M_j \geq T; j = 1, 2.
\end{cases}
\]

Considering \( e^{\theta T} = 1 + \theta T + \frac{\theta^2 T^2}{2} \),

\[
TVP_j = \left( (1 - k_j \lambda) C_b - C_v - (1 - k_j \lambda) C_b I_v \phi M_j + k_j (1 - \lambda) C_b f c (M2 - M1) \right) \left( a + \frac{aoT}{2} + \frac{bt}{2} - \frac{cT+}{\theta^2} + \frac{cT^2}{\theta} + \frac{cT^3 - ceT^2}{2} \right) - \frac{S_v}{nT} - C_v(1 - V) \left[ n - 1 \right] (1 - \eta + \theta) \left( \frac{bt}{2} - \frac{cT^2}{2} + \frac{2ct^2}{3} + cT^3 \right)
\]

\[
TBP_{j1} = \left( C_c - (1 - k_j \lambda) C_b \right) \left\{ \left( a + \frac{aoT}{2} + \frac{bt}{2} - cT+ \frac{ct^2}{\theta^2} + \frac{ct^2}{\theta} + \frac{ct^3 - ceT^2}{2} \right) - \frac{S_b}{nT} - (1 - k_j \lambda) C_b \left( a \frac{aoT}{2} - cT+ \frac{bt}{2} + \frac{ct^2}{\theta^2} + \frac{ct^2}{\theta} + \frac{ct^3}{2} - \frac{ceT^2}{3} \right) \right\}
\]

\[
TBP_{j2} = \left( C_c - (1 - k_j \lambda) C_b + I_{be} \right) \left\{ \left( a + \frac{aoT}{2} + \frac{bt}{2} - cT+ \frac{ct^2}{\theta^2} + \frac{ct^2}{\theta} + \frac{ct^3 - ceT^2}{2} \right) - \frac{S_b}{nT} - (1 - k_j \lambda) C_b \left( \frac{aoT}{2} - cT+ \frac{bt}{2} + \frac{ct^2}{\theta} + \frac{ct^2}{\theta} + \frac{ct^3}{2} - \frac{ceT^2}{3} \right) \right\}
\]
\[\pi_{ij} = ((1 - k_j \lambda)C_b - C_v - (1 - k_j \lambda)C_b I_{v0} M_j + k_j (1 - \lambda)C_b f v c (M2 - M1))(a + \frac{aoT}{2} + \frac{bT}{2} - \frac{cT^2}{2} + \frac{btT^2}{2} + \frac{cT^2}{2} + \frac{ctT^2}{2} - c\epsilon T^2) - \frac{5c}{nT} - C_v (I_v + I_{v0})[(n - 1)(1 - \varphi) + \varphi] \left(\frac{bT^2}{2} - cT^2 + \frac{atT^2}{2} + \frac{ctT^2}{2} + cT^2 + cT^3\right) + \left(C_c - (1 - k_j \lambda)C_b + I_{be} C_c (M_j - T)\right) \left(a + \frac{aoT}{2} + \frac{bT}{2} + \frac{ctT^2}{2} + \frac{ct^2}{2} + \epsilon cT^2\right) - \left(1 - k_j \lambda\right)C_b \left(\frac{aoT}{2} - cT + \frac{btT^2}{2} + \frac{ctT^2}{2} + \frac{ct^2}{2} + \epsilon cT^2 - \frac{ct^3}{3} + \frac{I_{be} c c}{T} \left[\frac{atT^2}{2} + \frac{bt^2}{3} + \frac{ct^4}{4}\right]\right) \right) \]  

(21)

The optimum value of cycle time can be obtained by setting \(\frac{d\pi_{ij}}{dT} = 0\) for fixed \(n\). The necessary condition for maximising total profit is \(\frac{d^2\pi_{ij}}{dT^2} < 0\).

5. NUMERICAL EXAMPLES

To illustrate the above developed model, an inventory system with the following data is considered

- \(a = 1000\), \(b = 50\), \(c = 0.1\), \(\varphi = 0.7\), \(C_v = \$5/\text{unit}\), \(C_b = \$35/\text{unit}\), \(C_c = \$55/\text{unit}\), \(S_v = \$1500/\text{setup}\), \(S_b = \$100/\text{order}\), \(I_v = 1\%/\text{unit/annum}\), \(I_{e0} = 1\%/\text{unit/annum}\), \(I_{e0} = 2\%/\text{unit/annum}\), \(I_{be} = 5\%/\text{unit/annum}\), \(I_{be} = 8\%/\text{unit/annum}\), \(M_1 = 10\text{days}\), \(M_2 = 30\text{days}\), \(\lambda = 2\%\), and \(f v c = 0.17/\text{annum}\).

Using computational procedure optimum cycle time \(T^*\) for above data is 20 days for \(n = 5\). The buyer’s order quantity \(Q^*\) are 14,88,400 units/order. Vendor’s total profit TVP is \$4,37,360 and buyer’s total profit TBP is \$ 15,81,400. The maximum total joint profit of the integrated system \(n = 20,18,800\).

5.1. Sensitivity analysis

Here we study the effects of changes in the system parameters \(a, b, c, \varphi\) and \(\lambda\) on the optimal length of order cycle \(T^*\), the optimal order quantity per cycle \(Q^*\), vendor’s profit TVP, buyer’s profit TBP, total profit of the collaborative vendor-buyer inventory system \(n\). The results are shown in Table 1.

**Table 1. Effect of change in various parameters of Example 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>T</th>
<th>Q</th>
<th>Vendor</th>
<th>Buyer</th>
<th>Joint Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>50</td>
<td>20</td>
<td>14,85,200</td>
<td>4,36,610</td>
<td>15,74,700</td>
<td>20,11,300</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>20</td>
<td>14,88,400</td>
<td>4,37,360</td>
<td>15,81,400</td>
<td>20,18,800</td>
</tr>
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Based on the results of Table 1, we can obtain the following managerial insights.

1. Increase in the value of the parameters \( a, b \) and \( c \) will result in increase of \( Q^* \), vendor’s profit, buyer’s profit and joint total profit. \( T^* \) remain same.
2. Decrease in the value of the parameters \( a, b \) and \( c \) will result in decrease of \( Q^* \), vendor’s profit, buyer’s profit and joint total profit. \( T^* \) remain same.
3. Increase in the value of the parameter \( \varphi \) will result in increase of \( Q^* \) but decrease of vendor’s profit, buyer’s profit and joint total profit.
4. Decrease in the value of the parameter \( \varphi \) will result in increase of \( Q^* \), vendor’s profit, buyer’s profit and joint total profit but decrease of \( T^* \).
5. Increase in the value of the parameter \( \varrho \) will result in increase of vendor’s profit and joint total profit but \( Q^* \), buyer’s profit and \( T^* \) remain unchanged.
6. CONCLUSION

Trade credit with cash discount is very realistic in global competitive market. To stimulate demand of the buyer, vendor always offers trade credit. However, the vendor can also use the cash discount policy to attract buyer to pay the full payment of the amount of purchasing cost to shorten the collection period. In this paper, we formulated an integrated vendor-buyer inventory model for deteriorating items with the assumption that demand is quadratic and vendor offers two payment options: trade credit and cash discount. A mathematical model is developed to find optimal replenishment policies and to maximise joint total profit per unit time of the supply chain system, which helps the inventory manager to take advantage of credit period for repaying the vendor by ordering larger quantity. This result helps the buyer to make a decision between two promotional tools, viz cash discount and trade credit. By using the numerical example, sensitive analysis is performed to study the effects of the changes of the parameter values on the optimal cycle time, optimal order quantity and total relevant profit respectively. In future one can analyze integrated inventory system for weibull distributed deteriorating items.

References


