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A study on energy of an intuitionistic fuzzy graph

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ABSTRACT

Let $\hat{G} = (V, E, \varsigma, \eta)$ be a simple intuitionistic fuzzy graph. In this article, the notion of energy of the fuzzy graph is expanded to the energy of an intuitionistic fuzzy graph. The adjacency matrix of an intuitionistic fuzzy graph has been determined and evaluated the energy of an intuitionistic fuzzy graph in terms of its adjacency matrix with suitable illustrative examples. We investigated some lower and upper bounds on the energy of an intuitionistic fuzzy graph.

***Keywords*:** Fuzzy graph, Intuitionistic fuzzy graph, Intuitionistic fuzzy set, Energy

AMS Classification: 05C72, 05C50

1. INTRODUCTION

First, Euler established the concept of graph theory, in 1736. The solution of the Konigsberg bridge problem given by Euler is appraised to be the first theorem in the history of graph theory. To know more about graph theory we can refer [13]. The graph theory is used in many real-life problems and different zones such as Computer science, Chemistry, Biology, Statistical physics, and Optimization. In computer science: to represent a network of communication, data organization, computational devices, and flow computation. We used the Huckel molecular orbital theory in chemistry. In classical graph theory, when the structure is

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larger and complicated it is tough to extricate the constrain information about the structure. For such a case we used a fuzzy graph to analyze the structure.

In 1965, Zadeh [14] established the theory of fuzzy sets by deals with uncertainties in real-life problems. Atanassov [1] presented the concept of an intuitionistic fuzzy set is from an expanded form of a fuzzy set by introducing a new element in 1986. In 1975, Rosenfeld [12] introduced the idea of a fuzzy graph. Operations on fuzzy graphs are defined and studied by J. N. Mordeson and Peng. Atanassov K.T [2] introduced the notion of the Intuitionistic fuzzy graph in 1999. Parvathi et.al [10] have been explored the intuitionistic fuzzy graph and operations on the intuitionistic fuzzy graph [9]. We can know more and distinguished about types of graphs based on their energy in [4]. A concept of energy is associated with the spectrum of a graph and it is motivated by the energy in chemistry. The energy approximate the total π - electron energy of molecule Huckel theory is the sum of the energies of all electrons in a molecule it is called total π - electron energy E_π.

The concept of 'graph energy' as the sum of the absolute values of the eigenvalues of the adjacency matrix of the graph was initiated by I. Gutman [7] in 1978. In 2011, Shao and Gung defined energy for weighted graphs [11]. The notion of energy of the fuzzy graph expanded from the energy of the graph [3]. Specific bounds on energies are analyzed in [6, 8]. Cvetkovic and I. Gutman have been explored the application of graph spectra in detail [5]. This paper is arranged as follows: In section 2, we present some basic definitions related to energy and intuitionistic fuzzy graph with illustrative examples. In section 3, we discussed results related to the energy of an intuitionistic fuzzy graph and some upper and lower bounds for energy of an intuitionistic fuzzy graph. In section 4, we give a conclusion.

2. PRELIMINARIES

In this section, we discussed some basic definitions related to the fuzzy graph theory.

Definition 2.1: Let G = (V, E) be a simple graph with vertex set V, Edge set E. The components of sets V and E are called vertices and edges respectively. If a set of elements in V are connected together, where all edges are directed then G is called a directed graph (or) digraph.

Definition 2.2: Let G = (V, E) be a simple graph with n vertices and m edges, where V = $\{v_1, v_2, v_3, \ldots v_n\}$. This can be represented by a $n \times n$ matrix giving the adjacency between all vertices. That matrix is called the adjacency matrix $A = \lfloor a_{ij} \rfloor$, where $a_{ij} = \begin{cases} 1, if\,(v_i, v_j) \in E, \\ 0, otherwise \end{cases}$.

In a simple graph, the adjacency matrix is (0, 1) matrix with zeros on its diagonal. Then A is symmetric and so the spectrum of A is real since there are no loops. The spectrum of a matrix is defined as a set of its eigenvalues. The spectrum of A is called a spectrum of G. Similarly, eigenvalues of A are called eigenvalues of G.

Definition 2.3: Energy of a simple graph G = (V, E) with an adjacency matrix. A is defined as the sum of the absolute values of eigenvalues of A. It is denoted by E (G).

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

where λ_i is an eigenvalue of A, i = {1, 2, 3…n.}

Definition 2.4: Let X be a nonempty set. A fuzzy subset A of X is defined as $A = \{x, \mu_A(x) / x \in X\}$, which is characterized by a membership function $\mu_A(x) : X \to [0,1]$, for all $x \in X$.

Definition 2.5: A fuzzy graph $G = (V, \sigma, \mu)$ is a pair of functions (σ, μ) where σ is a subset of V and μ is fuzzy relation on σ, for all $u, v \in V$ we have $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

Definition 2.6: An intuitionistic fuzzy set A in a set X, where X is a nonempty set. A is defined as an element of the form $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ where $\mu_A : X \to [0,1]$ is a degree of membership and $\nu_A : X \to [0,1]$ is a degree of non-membership of elements of $x \in X$ and satisfying the following condition $0 \le \mu_A(x) + \nu_A(x) \le 1$, for all $x \in X$.

3. THE ENERGY OF AN INTUITIONISTIC FUZZY GRAPH

Definition 3.1: Let Ĝ = (V, E, ς, η) be an intuitionistic fuzzy graph. The adjacency matrix of an intuitionistic fuzzy graph $\hat{G}$ is defined as $A(I\hat{G}) = [a_{ij}]$ where $[a_{ij}] = [(\zeta_{ij}), (\eta_{ij})]$. Then note that ζ_{ij} denotes the membership values of $\hat{G}$ between u_i and u_j, η_{ij} is denote the non-membership values of $\hat{G}$ between u_i' and u_j'. They can be written as two matrices of one containing membership values and another one non-membership values.

Example 3.2: Adjacency matrix of intuitionistic fuzzy paw graph Ĝ (Fig. 1.) is,

$$A(\zeta_{ij}(\hat{G})) = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.2 & 0.2 \\ 0 & 0.2 & 0 & 0.2 \\ 0 & 0.2 & 0.2 & 0 \end{bmatrix} \quad \text{and} \quad A(\eta_{ij}(\hat{G})) = \begin{bmatrix} 0 & 0.3 & 0 & 0 \\ 0.3 & 0 & 0.8 & 0.3 \\ 0 & 0.8 & 0 & 0.8 \\ 0 & 0.3 & 0.8 & 0 \end{bmatrix}$$

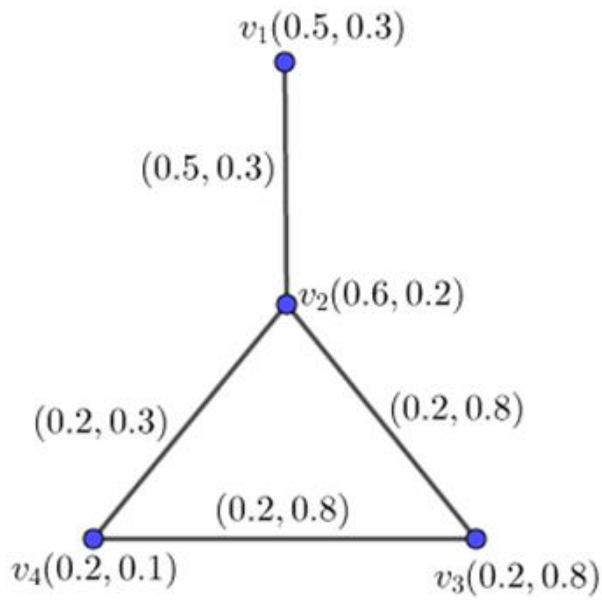

Figure 1. Intuitionistic fuzzy paw graph

Definition 3.3: The characteristic polynomial of an intuitionistic fuzzy graph Ĝ

$$\psi_n(\hat{G}, \mu) = \det\left[\mu I - A(\zeta_{ij}(\hat{G}))\right] \quad \text{and} \quad \psi_n(\hat{G}, \gamma) = \det\left[\gamma I - A(\eta_{ij}(\hat{G}))\right]$$

Example 3.4: The characteristic polynomial of the graph Ĝ in (Fig. 1),

$$\mu^4 - 0.37\mu^2 - 0.16\mu + 0.01 \quad \text{and} \quad \mu^4 - 0.342\mu^2 - 0.384\mu + 0.576$$

Definition 3.5: Let Let Ĝ = (V, E, ς, η) be an intuitionistic fuzzy graph and $A(I\hat{G})$ be its adjacency matrix and it is defined as $A(I\hat{G}) = [(\zeta_{ij}), (\eta_{ij})]$. The eigenvalues of A are called the eigenvalues of Ĝ. The spectrum of A is called the spectrum of Ĝ It is denoted by spec (Ĝ).

Example 3.6: The spectrum of Ĝ for the graph (in Fig 1.),

Spec [$\zeta_{ij}(\hat{G})$] ={0.6065, -0.3158+0.3697i, -0.3198-0.3697i, 0.331, 0}

Spec [$\eta_{ij}(\hat{G})$] ={-0.4284,-0.0341+0.3463i,-0.0341-0.3463i, 0.4661, 0.2904}

Definition 3.7: The energy of an intuitionistic fuzzy graph Ĝ = (*V*, *E*, ς, η) is defined as

$$\left[\sum_{\mu_i \in X} |\mu_i|, \sum_{\gamma_i \in Y} |\gamma_i| \right]$$

where, $\sum_{\mu_i \in X} |\mu_i|$ is the energy of membership matrix denoted by $E[\zeta_{ij}(\hat{G})]$ and $\sum_{\gamma_i \in Y} |\gamma_i|$ is the energy of non-membership matrix denoted by $E[\eta_{ij}(\hat{G})]$.

Example 3.8: The energy of an intuitionistic fuzzy paw graph in (Fig 1.)

$$E[\zeta_{ij}(\hat{G})] = 0.4986 \quad \text{and} \quad E[\eta_{ij}(\hat{G})] = 0.$$

Result: The energy of a non – trivial graph is always greater than 1 **[4]**. But that is not true for an intuitionistic fuzzy graph Ĝ as given in the following illustration.

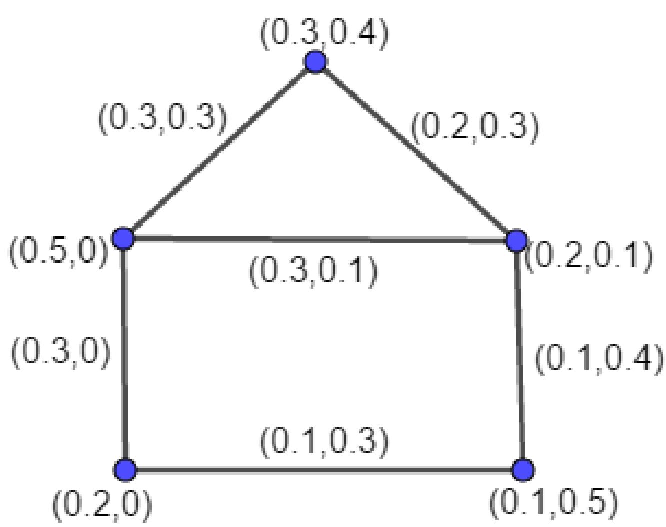

Figure 2. An intuitionistic fuzzy graph

Adjacency matrices of the above intuitionistic fuzzy graph,

$$A(\zeta_{ij}(\hat{G})) = \begin{bmatrix} 0 & 0.2 & 0 & 0 & 0.3 \\ 0.2 & 0 & 0.1 & 0 & 0.2 \\ 0 & 0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0 & 0.2 \\ 0.3 & 0.3 & 0 & 0.3 & 0 \end{bmatrix} \quad \text{and} \quad A(\eta_{ij}(\hat{G})) = \begin{bmatrix} 0 & 0.3 & 0 & 0 & 0.3 \\ 0.3 & 0 & 0.4 & 0 & 0.1 \\ 0 & 0.4 & 0 & 0.3 & 0 \\ 0 & 0 & 0.3 & 0 & 0 \\ 0.3 & 0.1 & 0 & 0.3 & 0 \end{bmatrix}$$

$\text{Spec}[\zeta_{ij}(\hat{G})]$ = 0.6065, -0.3158+0.3697i,-0.3198-0.3697i, 0.331, 0

$\text{Spec}[\eta_{ij}(\hat{G})]$ = -0.4284, -0.0341+0.3463i, -0.0341-0.3463i, 0.4661, 0.2904.

The energy of an above intuitionistic fuzzy graph

$E[\zeta_{ij}(\hat{G})] = 0.2979 < 1$ and $E[\eta_{ij}(\hat{G})] = 0.2599 < 1$.

4. BOUNDS FOR THE ENERGY OF AN INTUITIONISTIC FUZZY GRAPH

Theorem 4.1: Let Let $\hat{G} = (V, E, \varsigma, \eta)$ be intuitionistic fuzzy undirected, loopless and connected graph with order p and size q and $A(IG) = [(\zeta_{ij}), (\eta_{ij})]$ be IF adjacency matrix of G then,

$$(i)\sqrt{2\sum_{1\le i\le j\le p}\zeta_{ij}^2 + p(p-1)|A|^{\frac{2}{p}}} \le E[\zeta_{ij}(\hat{G})] \le \sqrt{2p\sum_{1\le i\le j\le p}\zeta_{ij}^2}.$$

where $|A|$ – determinant of $A[\zeta_{ij}(\hat{G})]$ and

$$(ii)\sqrt{2\sum_{1\le i\le j\le p}\eta_{ij}^2 + p(p-1)|B|^{\frac{2}{p}}} \le E[\eta_{ij}(\hat{G})] \le \sqrt{2p\sum_{1\le i\le j\le p}\eta_{ij}^2}.$$

where $|B|$ - determinant of $A[\eta_{ij}(\hat{G})]$.

Proof: Upper bound

Apply Cauchy Schwarz inequality to the n numbers (1, 1, 1,….) and $|\mu_1|, |\mu_2|, |\mu_3|, \ldots |\mu_p|$.

$$\sum_{i=1}^{p}|\mu_i| \le \sqrt{p}\sqrt{\sum_{i=1}^{p}|\mu_i|^2} \tag{1}$$

$$(\sum_{i=1}^{p}|\mu_i|^2) \le \sum_{i=1}^{p}|\mu_i|^2 + 2\sum_{1\le i<j\le p}(\mu_i\mu_j) \tag{2}$$

By comparing the coefficients of μ^{n-2}, in the characteristic polynomial

$$\prod_{i=1}^{p}(\mu-\mu_i) = |A-\lambda I|$$

We get,

$$\sum_{1\le i<j\le p}\mu_i\mu_j = -\sum \zeta_{ij}^2 \tag{3}$$

Substitute Equation (3) in (2), we get

$$\sum_{i=1}^{p}|\mu_i|^2 = 2\sum_{1\le i<j\le p}\zeta_{ij}^2 \tag{4}$$

Substitute Equation (4) in (1),

$$\sum_{i=1}^{p}|\mu_i|^2 \le \sqrt{p}\sqrt{2\sum_{1\le i<j\le p}\zeta_{ij}^2} = \sqrt{2p\sum_{1\le i<j\le p}\zeta_{ij}^2}$$

$$E[\zeta_{ij}(\hat{G})] \le \sqrt{2p\sum_{1\le i<j\le p}\zeta_{ij}^2}$$

Lower bound

$$E[\zeta_{ij}(\hat{G})] = (\sum_{i=1}^{p}|\mu_i|^2) = \sum_{i=1}^{p}|\mu_i|^2 + 2\sum_{1\le i<j\le p}|\mu_i\mu_j|$$

$$= 2\sum_{1\le i<j\le p}\zeta_{ij}^2 + \frac{2p(p-1)}{2}$$

$$AM\{|\mu_i\mu_j|\} \ge GM\{|\mu_i\mu_j|\}$$

$$E[\zeta_{ij}(\hat{G})] \ge \sqrt{2\sum\zeta_{ij}^2 + p(p-1)GM\{|\mu_i\mu_j|\}}$$

$$GM\{|\mu_i\mu_j|\} = \left(\prod_{1\le i<j\le n}|\mu_i\mu_j|\right)^{\frac{2}{p(p-1)}}$$

$$= |A|^{\frac{2}{p}}$$

Hence,

$$\sqrt{2\sum_{1\le i<j\le p}\zeta_{ij}^2 + p(p-1)|A|^{\frac{2}{p}}} \le E[\zeta_{ij}(\hat{G})] \le \sqrt{2p\sum_{1\le i<j\le p}\zeta_{ij}^2}$$

In a similar way, we can prove that

$$\sqrt{2\sum_{1\le i<j\le p}\eta_{ij}^2 + p(p-1)|B|^{\frac{2}{p}}} \le E[\eta_{ij}(\hat{G})] \le \sqrt{2p\sum_{1\le i<j\le p}\eta_{ij}^2}$$

Now, we have another solution giving an upper bound for the energy of an intuitionistic fuzzy graph which has fewer numbers of nodes.

Theorem 4.2: Let $G = (V, \hat{E}, \zeta, \eta)$ be an intuitionistic fuzzy graph with order p and size q, $\zeta_B^* = \{e_1, e_2, e_3, ..e_p\}$ and $\eta_B^* = \{e_1', e_2', e_3', ..e_p'\}$. Let A be the adjacency matrix of Ĝ If $\xi_i = \zeta_B(e_i)$ and $p \le 2\sum_{i=1}^{q}\xi_i^2$ then

$$E[\zeta_{ij}(\hat{G})] \le \frac{2\sum_{i=1}^{q}\xi_i^2}{p} + \sqrt{(p-1)\left\{2\sum_{i=1}^{q}\xi_i^2 - \left(\frac{2\sum_{i=1}^{q}\xi_i^2}{p}\right)^2\right\}} \quad (5)$$

and for non-membership value, if $\chi_i = \eta_B(e_i)$ and $p \le 2\sum_{i=1}^{q}\chi_i^2$ then,

$$E[\eta_{ij}(\hat{G})] \le \frac{2\sum_{i=1}^{q}\chi_i^2}{p} + \sqrt{(p-1)\left\{2\sum_{i=1}^{q}\chi_i^2 - \left(\frac{2\sum_{i=1}^{q}\chi_i^2}{p}\right)^2\right\}} \quad (6)$$

Proof: If $A = \lfloor a_{ij} \rfloor_{m\times n}$ it is a symmetric matrix with zero diagonal. Then,

$$\mu_{\max} \le \frac{2\sum_{1\le i\le j\le p} a_{ij}}{p}$$

where $\mu_{\max}$ is the maximum eigenvalues of A. If A is the adjacency matrix of Ĝ, then

$$\mu_1 \ge \frac{2\sum_{i=1}^{q}\xi_i}{p}$$

where $\mu_1 \geq \mu_2 \geq \mu_3 \ldots \geq \mu_p$.

$$\sum_{i=1}^{p} |\mu_i|^2 = 2\sum_{i=1}^{q} \xi_i^2$$

$$\sum_{i=2}^{p} |\mu_i|^2 = 2\sum_{i=1}^{q} \xi_i^2 - \mu_1^2 \tag{7}$$

By Cauchy Schwarz inequality,

$$E[\zeta_{ij}(\hat{G})] - \mu_1 = \sum_{i=1}^{p} |\mu_i| \leq \sqrt{(p-1)\sum_{i=2}^{p} |\mu_i|^2} \tag{8}$$

Substitute equation (7) in (8),

$$E[\zeta_{ij}(\hat{G})] - \mu_1 \leq \sqrt{(p-1)\left(2\sum_{i=1}^{q} \xi_i^2 - \mu_1^2\right)}$$

$$E[\zeta_{ij}(\hat{G})] \leq \mu_1 + \sqrt{(p-1)\left(2\sum_{i=1}^{q} \xi_i^2 - \mu_1^2\right)}$$

Let the function,

$$G(x) = x + \sqrt{(n-1)\left(2\sum_{i=1}^{q} \xi_i^2 - x^2\right)}$$

be a non-increasing (decreasing) in the interval

$$\left[\frac{2\sum_{i=1}^{q} \xi_i^2}{p}, \sqrt{2\sum_{i=1}^{q} \xi_i^2}\right].$$

Since,

$$p \leq 2\sum_{i=1}^{q} \xi_i^2$$

$$1 \leq \left(\frac{2\sum_{i=1}^{q} \xi_i^2}{p} \right)$$

Hence,

$$\sqrt{\frac{2\sum_{i=1}^{q} \xi_i^2}{p}} \leq \frac{2\sum_{i=1}^{q} \xi_i}{p}$$

$$\frac{2\sum_{i=1}^{q} \xi_i}{p} \leq \mu_1 \leq \sqrt{2\sum_{i=1}^{q} \xi_i^2} \tag{9}$$

$$E[\zeta_{ij}(\hat{G})] \leq \mu_1 + \sqrt{(n-1)\left(2\sum_{i=1}^{q} \xi_i^2 - \mu_1^2\right)} \tag{10}$$

Substitute equation (9) in equation (10) then we get equation (5). Similarly, we can prove this theorem for non- membership value $E[\eta_{ij}(\hat{G})]$.

Theorem 4.3: Let Ĝ be an intuitionistic fuzzy graph (IFG) with order p, $\zeta_B^* = \{e_1, e_2, ..e_p\}$ and $\eta_B^* = \{e_1^{'}, e_2^{'}, ...e_p^{'}\}$. Then, $E[\zeta_{ij}(\hat{G})] \leq 2\sum_{i=1}^{q} \zeta_B(e_i)$ and $E[\eta_{ij}(\hat{G})] \leq 2\sum_{i=1}^{q} \eta_B(e_i^{'})$.

By **Theorem 4.3,** upper bounds for the energy of an IFG can be derived as membership values of its nodes and similarly derived as non-membership values of its nodes.

Proposition 4.4: Let Ĝ be an IFG with p nodes and q links $\zeta_B^* = \{e_1, e_2, ..e_p\}$ and $\eta_B^* = \{e_1^{'}, e_2^{'}, ..e_p^{'}\}$. Then $E[\zeta_{ij}(\hat{G})] \leq (p-1)\sum_{i=1}^{p} \zeta_A(u_i)$ and $E[\eta_{ij}(\hat{G})] \leq (p-1)\sum_{i=1}^{p} \eta_A(u_i^{'})$

Proof: From theorem 4.3, we have

$$E[\zeta_{ij}(\hat{G})] \leq 2\sum_{i=1}^{q} \zeta_B(e_i)$$

$$= 2 \sum_{i=1}^{\frac{p(p-1)}{2}} \zeta_B(e_i)$$

where $\zeta_B(e_i) = 0 \quad \forall i > m.$

By the definition of an Intuitionistic fuzzy graph,

$$\zeta_B(e_i) = \min\{\zeta_A(u_i), \eta_A(u_j)\} \qquad forsome\ u_i, u_j \in V.$$

Therefore,

$$E[\zeta_{ij}(\hat{G})] \leq 2 \sum_{i=1}^{\frac{p(p-1)}{2}} \zeta_B(e_i)$$

$$= \sum_{i=1}^{\frac{p(p-1)}{2}} \zeta_B(e_i) + \zeta_B(e_i)$$

$$\leq \sum_{1 \leq i \leq j \leq p} \zeta_A(u_i) + \eta_A(u_i)$$

$$= (p-1) \sum_{i=1}^{p} \zeta_A(u_i)$$

Similarly, we can prove this theorem for non - membership values in the same manner as above

$$E[\eta_{ij}(\hat{G})] \leq (p-1) \sum_{i=1}^{q} \eta_A(u_i')$$

Proposition 4.5: Let Ĝ be an IFG with p nodes, q links, and G^* a cycle and $\zeta_B^* = \{e_1, e_2, e_3, \ldots e_n\}$ and $\eta_B^* = \{e_1', e_2', e_3', .. e_p'\}$. Then

$$E[\zeta_{ij}(\hat{G})] \leq 2 \sum_{i=1}^{p} \zeta_A(u_i)$$

$$E[\eta_{ij}(\hat{G})] \leq 2 \sum_{i=1}^{p} \eta_A(u_i')$$

where $u_i, u_i' \in V$, $i = \{1, 2, .. n\}$.

Proof: From theorem 4.3, we know that

$$E[\zeta_{ij}(\hat{G})] \leq 2\sum_{i=1}^{p} \zeta_B(e_i) \tag{11}$$

$$e_i = \begin{cases} u_i u_{i+1}, i = \{1,2,.n-1\} \\ u_n u_{n+1}, i = n \end{cases}$$

Each edge in $\hat{G}$ can be explicitly mapped to nodes by a map and determined as

$$g(e_i) = (u_i) \qquad i = \{1,2,..n\}.$$

Then Equation (11) can be defined in terms of membership values of nodes of Ĝ

$$E[\zeta_{ij}(\hat{G})] \leq 2\sum_{i=1}^{p} \zeta_A(u_i)$$

Since by the definition of an intuitionistic fuzzy graph,

$$\zeta_B(e_i) \leq \zeta_A(u_i) \ , \ \ i = \{1,2,.n\}$$

The proof of this theorem is the same as for non - membership values of nodes of Ĝ

Theorem 4.6: If Ĝ is an intuitionistic fuzzy graph with p vertices and q edges then

$$\max\left(\frac{4q}{p}, 2\sqrt{q}\right) \leq E\left[\zeta_{ij}(\hat{G})\right] \leq \min\left(2q, \sqrt{2pq}\right)$$

and

$$\max\left(\frac{4q}{p}, 2\sqrt{q}\right) \leq E\left[\eta_{ij}(\hat{G})\right] \leq \min\left(2q, \sqrt{2pq}\right)$$

The (p, q) type approximation given in **Theorem 4.6** is all obtained but not possible for all values of p and q.

Theorem 4.7: For all the intuitionistic fuzzy graph $\hat{G}$, the energy E satisfies $E\left[\zeta_{ij}(\hat{G})\right] \leq \frac{4q}{p}$ and $E\left[\eta_{ij}(\hat{G})\right] \leq \frac{4q}{p}$ for $p < 12$ and $q \leq 36$.

Corollary 4.8: Let Ĝ be an intuitionistic fuzzy graph of order ($p<1$) then $|\mu_1| = |\mu_2| = \ldots.|\mu_p| \geq 0$ and $|\gamma_1| = |\gamma_2| = ..|\gamma_p| \geq 0$ if and only if $\hat{G} \cong \frac{p}{2} K_2$ (p is even).

5. CONCLUSION

An intuitionistic fuzzy graph plays an important role in many research areas and gives more precision, flexibility, and compatibility to the model compared to the classical fuzzy model. In this article, the adjacency matrix and energy of an intuitionistic fuzzy graph are determined with illustrative examples. Then derived some results on bounds for the energy of an intuitionistic fuzzy graph.

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