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Induced weak cycle number of path and its derived graphs

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ABSTRACT

Let G = (V, E) be a simple connected graph. The induced weak cycle partition of G is defined as the partition of V(G) into subsets such that each subset induces a cycle or K_2 or K_1. The induced weak cycle number of G, denoted by $\rho_{wc}(G)$, is the minimum cardinality taken over all induced weak cycle partitions. In this paper, the concept of induced weak cycle number is introduced and induced weak cycle number of path and some of its derived graphs are studied.

***Keywords*:** induced weak cycle partition and induced weak cycle number, path, derived graphs

AMS Classification: 05C15

1. INTRODUCTION

Graphs considered in this paper are finite, undirected and simple. The notion and facts of graphs that are used but not described here can be found in [1]. Let G be a graph with vertex set V(G) and edge set E(G).

The concept of an induced-path number was introduced by Chartrand et al. in [3]. The f-induced path-number, denoted by, $\rho_f(G)$ of a graph G is the minimum cardinality of an induced path-partition of G. Induced- path numbers of complete bipartite graphs, complete binary trees,

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2-dimensional meshes, butterflies and general trees were studied in [3]. Broere et al. determined the exact values for complete multipartite graphs in [2].

In [5], Zhiquan Hu et al. studied the partition of a graph in to cycles and vertices. Suppose H_1, H_2, H_3, …, H_k are disjoint sub graphs of G such that $V(G) = \bigcup_{i=1}^{k} V(H_i)$ and for all i, $1 \leq i \leq k$, H_i is a cycle or K_2 or K_1.Then we call $\mathcal{H} = \{H_1, H_2, H_3, \ldots, H_k\}$ a k-weak cycle partition, abbreviated as k-WCP, of G.

Motivated by the above definitions and results, in this paper, the authors introduce the concept of Induced weak cycle number of a graph G is and induced weak cycle number of path and some of its derived graphs are studied. Throughout this paper, 'disjoint' means 'vertex disjoint' since only partitions of vertex set is dealt with.

2. RESULT: INDUCED WEAK CYCLE NUMBER

Definition 2.1: Let G = (V, E) be a simple connected graph. The induced weak cycle partition of G is defined as the partition of V(G) into subsets such that each subset induces a cycle or K_2 or K_1. The induced weak cycle number of G, denoted by $\rho_{wc(G)}$, is the minimum cardinality taken over all induced weak cycle partitions.

Definition 2.2: Number of induced weak cycle number of a graph G, denoted by $\#,\rho_{wc}(G)$, is the number of distinct induced weak cycle partitions of the graph G.

Definition 2.3: A path is a walk in which all the vertices as well as the edges are distinct. The path on n vertices is denoted by P_n .

Theorem 2.4: $\rho_{wc}(P_n) = \frac{n+1}{2}$ if n is odd

$= \frac{n}{2}$ if n is even and

$\#\, \rho_{wc}(P_n) = 2$ if n is odd

$= 1$ if n is even

Proof: Let $\{v_1, v_2, \ldots., v_n\}$ be the vertex set and $\{(v_i v_{i+1}) : 1 \leq i \leq n\}$ be the edge set of the path P_n . Then $|V(P_n)|$ = n and $|E(P_n)|$ = n+1.

Case (i) Let n be odd

Obviously $\{(v_i v_{i+1}), v_n \,; i = 1, 3, 5, \ldots n-2\}$ and $\{ v_{1\,,} (v_i v_{i+1}), \,; i = 2, 4, 6, \ldots n-1 \}$ are two distinct partitions of the vertex set into $(\frac{n-1}{2})\, K_2\,'s$ and a K_1 with minimum cardinality. Therefore $\rho_{wc}(P_n) = (\frac{n-1}{2}) + 1 = (\frac{n+1}{2})$ and $\#\, \rho_{wc}(P_n) = 2$.

Case (ii) Let n be even

Obviously $\{(v_i v_{i+1}), v_n \,; i = 1, 3, 5, \ldots n-1\}$ is the unique partition of the vertex set into $(\frac{n}{2})$ $K_2\,'s$ with minimum cardinality.

Therefore $\rho_{wc}(P_n) = (\frac{n}{2})$ and $\# \rho_{wc}(P_n) = 1$.

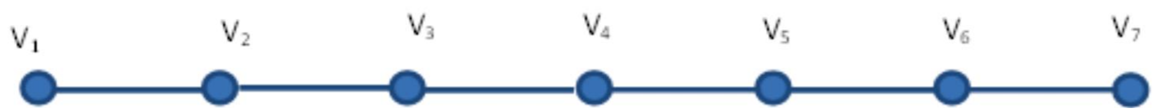

Fig. 2.1

Definition 2.5:: The Line graph L(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are adjacent in G.

Theorem 2.6: $\rho_{wc} L[(P_n)] = \frac{n-1}{2}$ if n is odd

$= \frac{n}{2}$ if n is even

$\# \rho_{wc}(P_n) = 1$ if n is odd

$= 2$ if n is even

Proof: Let $\{e_1, e_2, \ldots., e_{n-1}\}$ be the vertex set and $\{(e_i e_{i+1}) : 1 \le i \le n-2\}$ be the edge set of the graph $L(P_n)$. Then $|V[L(P_n)]|$ = n-1 and $|E[L(P_n)]|$ = n-2.

Case (i) Let n be odd

Obviously $\{(e_i e_{i+1}); i = 1, 3, 5, \ldots n-2\}$ is the unique partition of the vertex set into $(\frac{n-1}{2})$ K_2's with minimum cardinality.

Therefore $\rho_{wc}[L(P_n)] = (\frac{n-1}{2})$ and $\# \rho_{wc}(P_n) = 1$.

Case (ii) Let n be even

Obviously $\{(ee_{i+1}), e_{n-1} ; i = 1, 3, 5, \ldots n-3\}$ and $\{ e_1, (e_i e_{i+1}), ; i = 2, 4, 6, \ldots n-2 \}$ are two distinct partitions of the vertex set into $(\frac{n-2}{2})$ K_2's and a K_1 with minimum cardinality.

Therefore $\rho_{wc}[L(P_n)] = (\frac{n-2}{2}) + 1 = (\frac{n}{2})$ and $\# \rho_{wc}(P_n) = 2$.

Fig. 2.2

Definition 2.7: The subdivision graph S(G) of a graph G is obtained from G by inserting a new vertex into every edge of G.

Theorem 2.8: $\rho_{wc}[S(P_n)]$ = 1for all n ≥ 2 and # ($\rho_{wc}[S(P_n)]$) = 2.

Proof: Let $\{v_1, u_1, v_2, u_2, \ldots., u_{n-1}, v_n\}$ be the vertex set and $\{(v_i u_i): 1 \le i \le n-1)\}$U $\{(u_{i-1}v_i): 2 \le i \le n\ )\ \}$ be the edge set of ($S(P_n)$). Then │V ($S(P_n)$) │ = 2n-1 and │E ($S(P_n)$) │= 2n-2. obviously {($(v_i u_i)v_n : i = 1, 2, \ldots, n-1$) } and { $(u_1(u_{i-1}v_i) : i = 2, 3, \ldots, n$) } are two distinct partitions of the vertex set into n-1 and a with minimum cardinality.

Therefore ($\rho_{wc}[S(P_n)]$) = n-1+1 = n. Hence $\rho_{wc}[S(P_n)]$ = 1for all n ≥ 2 and # ($\rho_{wc}[S(P_n)]$) = 2.

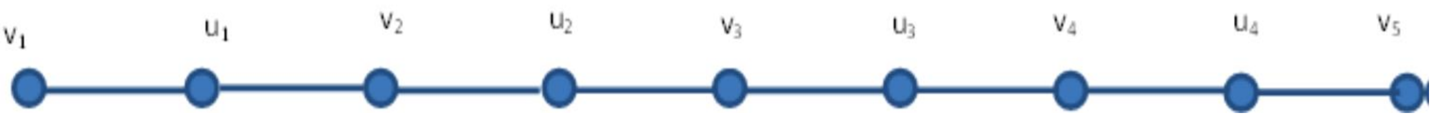

Fig. 2.3

Definition 2.9: The Paraline graph PL(G) is a line graph of subdivision graph of G.

Theorem 2.10: $\rho_{wc}[PL(P_n)] = n - 1$ for all n ≥ 2 and # ($\rho_{wc}[PL(P_n)]$) = 1.

Proof: Let $\{v_1, u_1, v_2, u_2, \ldots., u_{n-1}, v_n\}$ be the vertex set and $\{(v_i u_i): 1 \le i \le n-1)\}$U $\{(u_{i-1}v_i): 2 \le i \le n\ )\ \}$ be the edge set of ($S(P_n)$)

Let $\{e_1, e_2, \ldots., e_{n-1}\}$ is the vertex set of PL(P_n) Then, | V(PL(P_n))| = 2n-2 and | E(PL(P_n))| = 2n-3.

Obviously, $\{(e_i e_{i+1}): 1 \le i \le 2n-3\ \}$ is the unique partition of the vertex set into n-1 with minimum cardinality.

Therefore, $\rho_{wc}[PL(P_n)]$ = n - 1for all n ≥ 2 and # ($\rho_{wc}[PL(P_n)]$) = 1.

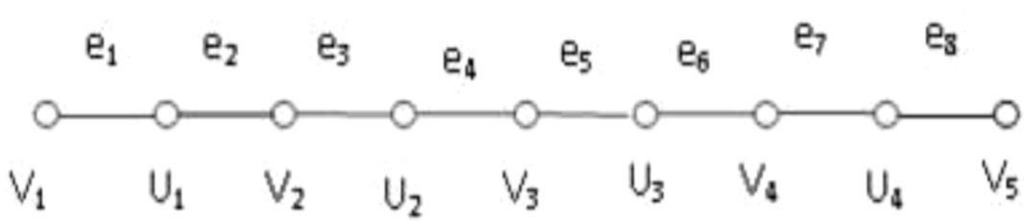

Fig. 2.4

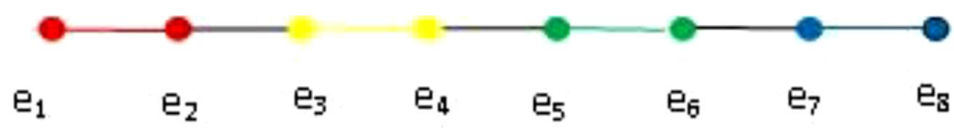

Fig. 2.5

Definition 2.11: The Jump Graph J(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are non-adjacent in G.

Theorem 2.12: $\rho_{wc}[J(P_n)] = \begin{cases} 2 \; if \; n = 3,4,5 \\ 1 \; if \; n = 2 \, , n \geq 6 \end{cases}$ # $(\rho_{wc}[J(P_n)])$ = 1 for all n

Proof: Let $\{e_1, e_2, \dots, e_{n-1}\}$ be the vertex set. Then, | V(J(P_n))| = n – 1.
| E(J(P_n)) | = $\frac{(n-3)(n-2)}{2}$.

Case (i) : n=3 , 4, 5.

Subcase (i): n = 3

Now, | V(J(P_n))| = 3 – 1 = 2.

Obviously, $\{e_1, e_2\}$ is an unique distinct partition of the vertex set K1's with minimum cardinality.

Therefore, $\rho_{wc}[J(P_n)]$ = 2 and # $(\rho_{wc}[J(P_n)])$ = 1.

Subcase (ii): n = 4

Now, | V(J(P_n))| = 4 – 1 = 3.

Obviously, { (e_1, e_3) , e_2} is the unique distinct partition of the vertex set as one K1 and one K2 with minimum cardinality.

Therefore, $\rho_{wc}[J(P_n)]$ = 2 and # $(\rho_{wc}[J(P_n)])$ = 1.

Subcase (iii): n = 5

Now, | V(J(P_n))| = 5 – 1 = 4.

Obviously, { (e_1, e_3) (e_2, e_4)} is the only distinct partition of the vertex set as two K2's with the minimum cardinality.

Therefore, $\rho_{wc}[J(P_n)]$ = 2 and # $(\rho_{wc}[J(P_n)])$ = 1.

Case (ii): n=2 and n ≥ 6.

Subcase (i): n = 2.

Now, | V(J(P_n))| = 2 – 1 = 1.

Obviously, {e_1 } is the only vertex and further J(P_n) is a trivial graph.

Therefore, $\rho_{wc}[J(P_n)]$ = 1 and # ($\rho_{wc}[J(P_n)]$) = 1.

Subcase (ii):

Obviously, $\{e_1, e_{n-3}, e_{n-1} \ldots., e_3\}$ is the unique induced cycle of maximum length in $J(P_n)$.

Therefore, $\rho_{wc}[J(P_n)]$ = 1 for all n ≥ 6 and # ($\rho_{wc}[J(P_n)]$) = 1 for all n ≥ 6.

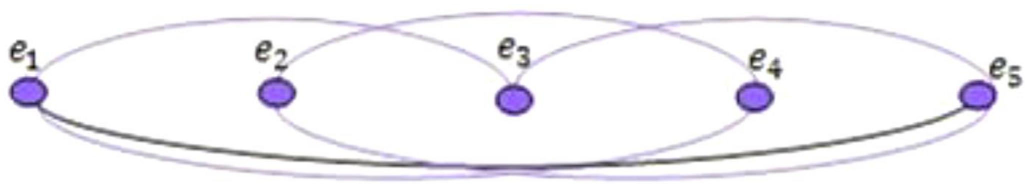

Fig.2.6

Definition 2.13: The Semitotal point graph (G) as the graph whose vertex set is V(G)UE(G), where two vertices are adjacent if and only if

(i) They are adjacent vertices of G
(ii) one is a vertex of G and other is an edge of G incident with it.

Theorem 2.14: $\rho_{wc}[T_2(P_n)]$ = n – 1 for all n ≥ 2 and # ($\rho_{wc}[T_2(P_n)]$) = 1.

Proof: Let $\{v_1, v_2, \ldots., v_n, e_1, e_2, \ldots., e_n\}$ be the vertex set and $\{(v_i v_{i+1}): 1 \leq i \leq n-1\}$U $\{(v_i e_i): 1 \leq i \leq n-1\}$ U $\{(e_i v_{i+1}): 1 \leq i \leq n-1\}$ be the edge set of the Path (P_n) .

Then, | V($T_2(P_n)$)| = 2n – 1. | E($T_2(P_n)$)| = 3n – 3.

Case (i): if n is odd.
$\{(v_i e_i v_{i+1} : i = 1, 3, \ldots, 2n-1)\}$ be the induced cycle of length 3 and $\{(v_n e_{n+1})\}$ be the distinct partition of the vertex set as K_2's. $\{e_i : i = 2, 4, \ldots, 2n-1\}$ be the distinct partition of the vertex set as K_1's.

There are $\frac{n-1}{2}$ C3's and only one K_2 and $\frac{n-3}{2}$ K_1's.

Therefore, $\rho_{wc}[T_2(P_n)] = \frac{n-1}{2} + 1 + \frac{n-3}{2} = \frac{n-1+2+n-3}{2} = \frac{2n-2}{2}$ = n – 1.

Case (ii): if n is even.

$\{(v_i e_i v_{i+1} : i = 1, 3, \ldots, 2n-1)\}$ be the induced cycle of length 3 and $\{e_i : i = 2, 4, \ldots, \frac{n}{2}\}$ be the distinct partition of the vertex set as K_1's.

There are $\frac{n}{2}$ C_3's and $\frac{n-2}{2}$ K_1's.

Therefore, $\rho_{wc}[T_2(P_n)] = \frac{n}{2} + \frac{n-2}{2} = \frac{2n-2}{2}$ = n – 1.

Hence, $\rho_{wc}[T_2(P_n)]$ = n – 1 for all n ≥ 2 and # ($\rho_{wc}[T_2(P_n)]$) = 1.

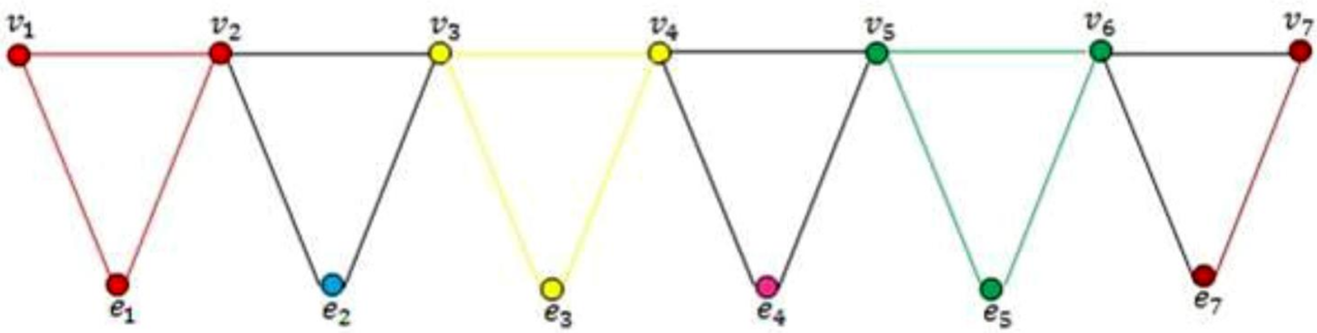

Fig. 2.7

Definition 2.15: The semitotal –line graph T_1(G) as the graph whose vertex set is V(G) U E(G) where two vertices are adjacent if and only if

(i)They are adjacent edges of G.

(ii) one is a vertex of G and other is an edge of G incident with it.

Theorem 2.16: $\rho_{wc}[T_1(P_n)]$ = n for all n ≥ 2 and # ($\rho_{wc}[T_1(P_n)]$) = 1.

Proof: Let $\{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n\}$ be the vertex set and $\{(e_i e_{i+1}): 1 \le i \le n-1\}$U $\{(v_i e_i): 1 \le i \le n-1\}$ U $\{(e_i v_{i+1}): 1 \le i \le n-1\}$ be the edge set of the Path $T_1(P_n)$.

Then, | V($T_1(P_n)$)| = 2n – 1. | E($T_1(P_n)$)| = 3n – 4.

Case (i): if n is odd.

$\{(v_{i+1} e_i e_{i+1} : i = 1, 3, \ldots, 2n-1)\}$ be the induced cycle of length 3 and $\{v_i : i = 1, 3, \ldots, n\}$ be the distinct partition of the vertex set as K_1's.

There are $\frac{n-1}{2}$ C_3's and $\frac{n+1}{2}$ K_1's.

Therefore, $\rho_{wc}[T_1(P_n)] = \frac{n-1}{2} + \frac{n+1}{2} = \frac{2n}{2}$ = n.

Case (ii): if n is even.

$\{(e_{i+1}e_iv_{i+1} : i = 1, 3, \ldots, n-3)\}$ be the induced cycle of length 3 and $\{v_i : i = 2, 4, \ldots, n-1\}$ be the distinct partition of the vertex set as K_1's and $\{(v_ne_{n-1})\}$ be the distinct partition of the vertex set as K_2's.

There are $\frac{n-2}{2}$ C_3's and only one K_2 and $\frac{n}{2}$ K_1's.

Therefore, $\rho_{wc}[T_2(P_n)] = \frac{n}{2} + 1 + \frac{n-2}{2} = \frac{2n+2-2}{2} = \frac{2n}{2} = n$.

Hence, $\rho_{wc}[T_1(P_n)] = n$ for all $n \geq 2$ and $\#(\rho_{wc}[T_2(P_n)]) = 1$.

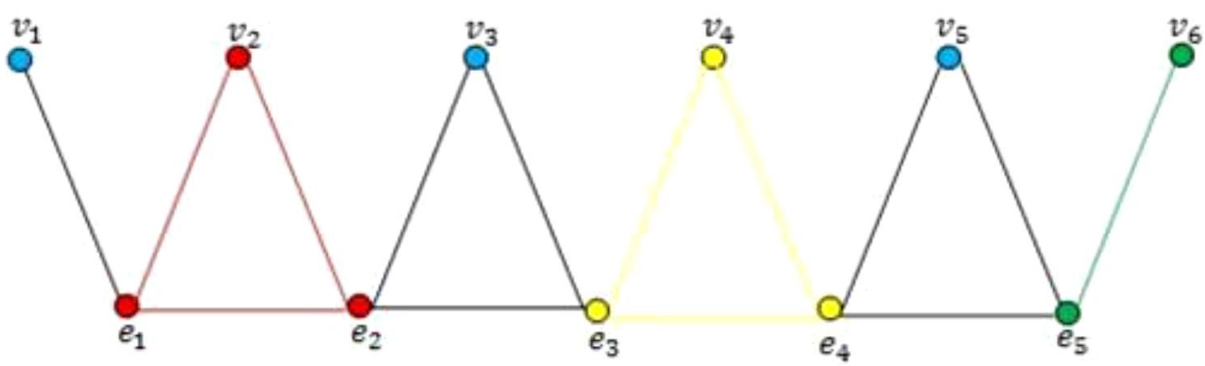

Fig. 2.8

Definition 2.17: The Total graph T(G) as the graph whose vertex set is V(G)UE(G) where two vertices are adjacent if and only if

(i)They are adjacent vertices of G

(ii)They are adjacent edges of G

(iii) one is a vertex of G and other is an edge of G incident with it.

Theorem 2.18: $\rho_{wc}[T(P_n)] = 1$ for all $n \geq 2$ and $\#(\rho_{wc}[T(P_n)]) = 1$.

Proof: Let $\{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n\}$ be the vertex set and $\{(v_iv_{i+1}): 1 \leq i \leq n-1\}$U $\{(v_ie_i): 1 \leq i \leq n-1\}$ U $\{(e_ie_{i+1}): 1 \leq i \leq n-2\}$ be the edge set of the Path $T(P_n)$. Then, $|V(T(P_n))| = 2n - 1$ and $|E(T(P_n))| = 4n - 5$.

Now, $\{(v_1e_1e_2, \ldots, e_n, v_n, v_{n-1}, \ldots, v_1)\}$ be the unique induced cycle of maximum length in T(P_n).

Hence, $\rho_{wc}[T(P_n)] = 1$ for all $n \geq 2$ and $\#(\rho_{wc}[T(P_n)]) = 1$.

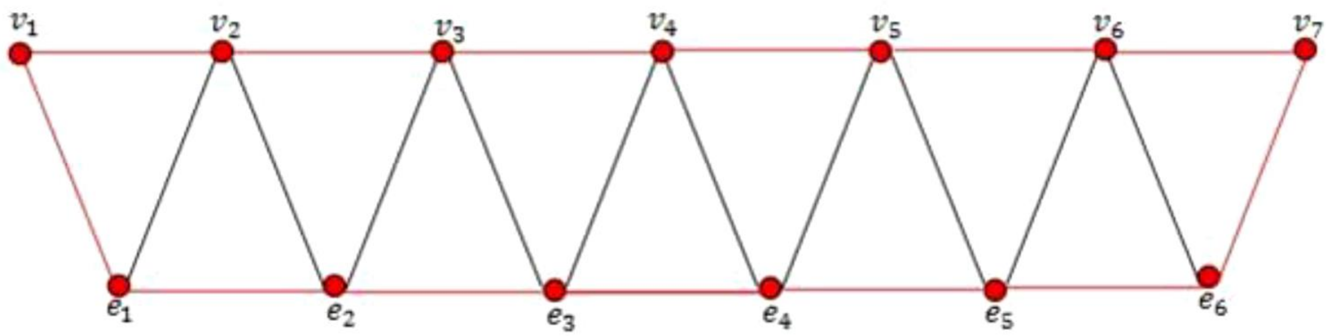

Fig. 2.9

Definition 2.19: The quasi-total graph P(G) as the graph whose vertex set is V(G)UE(G) where two vertices are adjacent if and only if

(i)They are non-adjacent vertices of G

(ii) they are adjacent edges of G

(iii) one is a vertex of G and other is an edge of G incident with it.

Theorem 2.20: $\rho_{wc}[P(P_n)]$ = 1 for all n ≥ 2 and # ($\rho_{wc}[P(P_n)]$) = 1.

Proof: Let $\{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$ be the vertex set and $\{(v_i v_{j+2}): 1 \leq i \leq n-2, j = i, i+1, i+2, \dots, n-2\}$ U $\{(v_i e_i): 1 \leq i \leq n-1\}$ U $\{(e_i e_{i+1}): 1 \leq i \leq n-2\}$ U $\{(e_i v_{i+1}): 1 \leq i \leq n-1\}$ be the edge set of the Path P(P_n). Then, | V($P(P_n)$)| = 2n – 1 and | E($P(P_n)$)| = 3n – 4 + $\sum_{i=1}^{n-2}(n-1-i)$.

Now, $\{(v_1, e_1, v_2, e_{n-1}, e_{n-2}, \dots, v_1)\}$ be the unique induced cycle of maximum length in P(P_n).

Hence, $\rho_{wc}[P(P_n)]$ = 1 for all n ≥ 2 and # ($\rho_{wc}[P(P_n)]$) = 1.

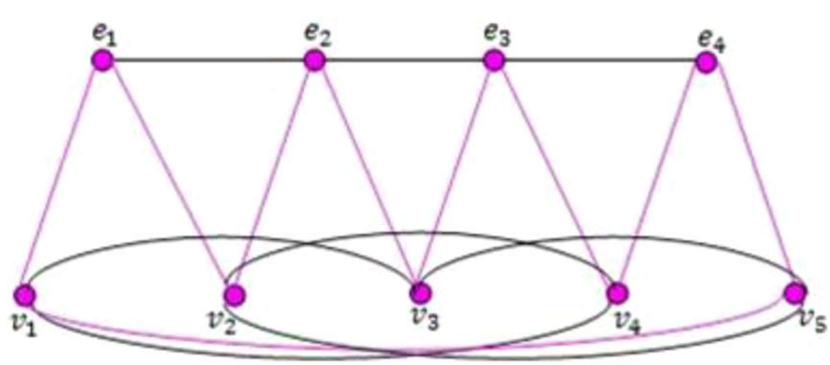

Fig. 2.10

Definition 2.21: The quasi vertex-total graph Q(G) as the graph whose vertex set is V(G)UE(G) where two vertices are adjacent if and only if

(i) They are adjacent vertices of G
(ii) They are non-adjacent edges of G
(iii) They are adjacent edges of G

Theorem 2.22: $\rho_{wc}[Q(P_n)]$ = 1 for all n ≥ 2 and # ($\rho_{wc}[Q(P_n)]$) = 1.

Proof: Let $\{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n\}$ be the vertex set and $\{(v_i v_{i+1}): 1 \le i \le n-1\}$ U $\{(e_i e_{i+1}): 1 \le i \le n-2\}$ U $\{(v_i e_i): 1 \le i \le n-1\}$ U $\{(e_i v_{i+1}): 1 \le i \le n-1\}$ U $\{(v_i v_{j+2}): 1 \le i \le n-2, j = i, i+1, i+2, \ldots, n-2\}$ be the edge set of the Path Q(P_n).

Then, | V($Q(P_n)$)| = 2n – 1 and | E($P(P_n)$)| = 4n – 5 + $\sum_{i=1}^{n-2}(n-1-i)$.

Now, $\{(v_1, v_2, \ldots, v_n, e_{n-1}, e_{n-2}, \ldots, v_1)\}$ be the unique induced cycle of maximum length in Q(P_n).

Hence, $\rho_{wc}[Q(P_n)]$ = 1 for all n ≥ 2 and # ($\rho_{wc}[Q(P_n)]$) = 1.

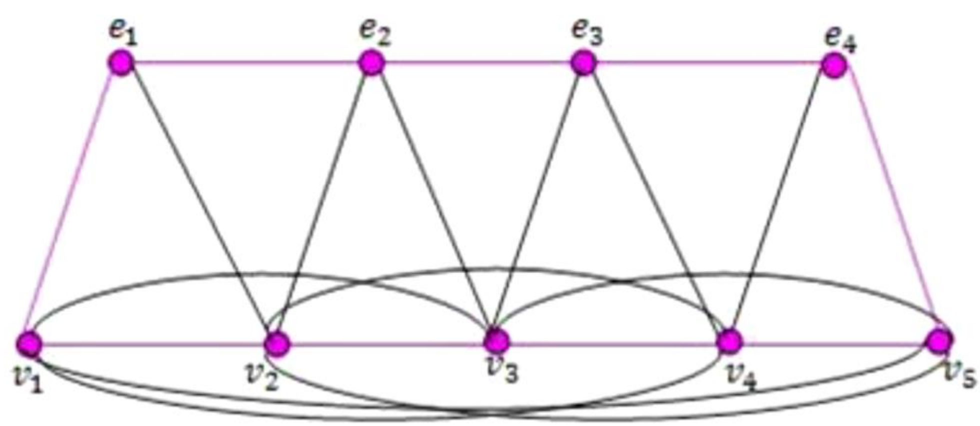

Fig. 2.11

3. CONCLUSION

Thus in this paper the author has studied the induced weak cycle number for path and its derived graphs such as line graph, subdivision graph, paraline graph, total graph etc. Also, the number of induced weak cycles for each graph is studied.

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