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Labeling of 2-regular graphs by even edge magic

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ABSTRACT

In this paper we introduced the new notion of an even edge magic total labeling of some 2-regular graphs. An edge magic total labeling of a graph $G(p,q)$ is said to be an even edge magic total labeling if $g(E)=\{2,4,...,2q\}$ with the condition that for each edg $xy \in E, g(x)+g(xy)+g(y)=k_g$, where k_g is said to be magic constant. We determined cycles of odd length, disjoint union of p cycles of q length for p and q are odd, disjoint union , disjoint union $C_3 \cup C_{4r+2} (r \geq 1)$, disjoint union $C_4 \cup C_{4r-1} (r > 1)$ and disjoint union $C_3 \cup C_{4r} (r > 1)$ $C_4 \cup C_{4r-3} (r > 1)$ are even edge magic total labeling.

***Keywords*:** Labeling, Magic Labeling, edge magic total labeling, even edge magic total labeling

2010 Mathematics Subject Classification: 05C78

1. INTRODUCTION

Graph theory is used in different fields like chemistry,social sciences, computer sciences and operations research. Graph Labeling is one of the most growing areas in graph theory. In graph theory, the labeling of graphs noticed to be a theoretical topic. It is used in countless

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applications like coding theory, X-Ray crystallography and astronomy etc. Designof graph labeling is helpful to network security, network addressing and social networkin communication network. An edge-magic total labeling of a graph is a motivating research area. In this paper, we mean only finite, simple and undirected $2-$regular graphs. A graph G has vertex set $V(G)$, edge set $E(G)$ and the number of vertices equals the number of edges. A graph has labeling if $V(G)$ or $E(G)$ maps onto integers. Magic labeling is a one-to-one map that takes $V(G)\cup E(G)$ onto the integers from 1 with constant sum-property. Origin of graph labeling was introduced by Rosa. In 1996, edge-magic labeling was rediscovered by Ringel and Llado. In 2001, Wallis introduced edge magic total labeling. Enomota, Llado Nakamigawa and Ringel introduced super edge-magic total labeling. Wallis called these labeling as strongly edge-magic total labeling.

2. LITERATURE REVIEW

In 1970, Kotzig and Rosa [5] defined a magic valuation of a graph $G(V,E)$ as a bijection f from $V\cup E$ to $\{1,2,3,\ldots,|V\cup E|\}$ such that for all edge $xy, f(x)+f(y)+f(xy)$ is a magic constant. In 1988, Godbold and Slater [3] made the following conjecture. If n is odd, $n\neq 5, C_n$ has an edge-magic labeling with valence k, when $\frac{(5n+3)}{2}\leq k\leq\frac{(7n+3)}{2}$.If n is even, C_n has an edge-magic with valence k, when $\frac{5n}{2}+2\leq k\leq\frac{7n}{2}+1$.

Omer Berkman et al., [17] proved that all cycles $n\geq 3, C_n$ are edge-magic. They also proved that all cycles of odd length are edge-magic. Y. Rodity and T. Bachar [19] noted that the cycles $C_n, 3\leq n\leq 7$ and odd cycles are edge magic. They also showed that some examples of even edge-magic cycles. K. Manicham and M. Marudai [7] explained that the corona admits an edge-magic labeling where the set of vertex labels is $\{1,2,3,\ldots,|V|\}$.

A. Llado and J. Moragas [6] determined that the wheel graph W_n for $n\geq 5$ odd , is C_4-super magic; The graph $G\times K_2$ is C_4-super magic if G is a C_4-free super magic graph of odd size; For any two integers $k \geq 2$ and $r \geq 3$, the windmill $W(r,k)$ is C_r – super magic and $W_n(r,1)$ is C_{2r+1} – super magic. Toru Kojima [24] proved the following: Let G be a C_4 – free super edge-magic (p,q) -graph with the minimum degree at least one and $m \geq 2$. If q odd $m = 2$ or $|p-q| \geq 2$, then $P_m \times G$ is C_4 – super magic; If p is odd and $m = 2$ or $|p-q| = 2$ and $m \leq 5$, then $P_m \times G$ is C_4 – super magic.

C. T. Nagaraj et al. [13-15] introduced even vertex-magic total labeling. A vertex-magic total labeling is called an even vertex-magic total labeling if $f : V \to \{2,4,...,2n\}$. They explained that the cycle C_n is an even vertex-magic if n is odd; rC_s is an even vertex magic iff r and s are odd. The fan graph F_n is an even vertex magic if $n = 2$; for $p \geq 3$, sun graph S_p has an even vertex magic. They also find some families of graphs that admit an even vertex-magic total labeling.

D. Mc Quillan [10, 11] proved that G has vertex-magic total labeling with a magic constant h then the graph G_J has vertex-magic total labeling with magic constants of $nh - 3m$ and $6|V|m + h$ for any subset J of I; If the cycle C_k has VMTL with magic constant g then C_{nk} has a VMTL with magic constant $ng - 3m$ and C_{nk} has a VMTL with the magic constant $6km + g$. D. Mc Quillan and J. Mc Quillan [12] explained that the four graphs $G \cup 2mC_3, G \cup (2m+2)C_3, G \cup mC_8$ and $G \cup (m+1)C_8$ has the strong vertex-magic total labeling.

A vertex-magic total labeling of a graph G is V – super vertex-magic total if $f(V(G)) = \{1,2,..., p\}$ and E – super vertex-magic total if $f(E(G)) = \{1,2,..., q\}$. G. Marimuthu and G. Kumar [8] proved that necessary and sufficient condition for the existence of V – super vertex-magic labeling and they find V – super vertex-magic total labeling of some families of graphs.

Jermy Holden et al., [4] showed that for each integer $t \geq 2$, disjoint union $C_5 \cup (2t)C_3$ has strong vertex and edge magic total labeling. Also for each integer $t \geq 3$, the disjoint union

$C_4 \cup (2t-1)C_3$ and for each integer $t \geq 1$, the disjoint union $C_7 \cup (2t)C_3$ has strong vertex magic total labeling.

V. Swaminathan and P. Jayanthi [23] determined that $P_m + \overline{K}_n$ admits a super edge-magic labeling iff $m \leq 2, n \geq 1$ and $sm(P_m + \overline{k}_n) = 3n + 6$; $K_{1,n} + \overline{K}_1$ admits a super edge-magic labeling and $sm(K_{1,n} + \overline{K}_1) = 3n + 6$. Ngurah et al. [16] determined that $T(m,n,k)$ is an edge-magic graph with the magic constant c lies in the interval $\frac{1}{2t}(5t^2 + 3t + 6) \leq c \leq \frac{1}{2t}(5t^2 + 11t - 6)$. They also showed that for all $m,n \geq 1, T(m,n,n+3)$ and $T(m,n,n+4)$ are super edge magic graph.

H. Enomota et al., [1] proved the following: A cycle C_n is super edge magic iff n is odd; $K_{m,n}$ is super edge magic if $m = 1$ or $n = 1$; A wheel graph W_n of order n is not super edge-magic. Moreover, W_n is not edge-magic if $n \equiv 0 (\text{mod}) 4$. They also proved that every tree is edge magic and super edge-magic. A. N. M. Salman et al. [20] proved that S_n^m is super edge-magic when $m = 1$ or 2 where ($S_n^m, n \geq 3$ is a graph obtaining by intersecting m vertices in every edge of the star S_n .)

More results about graph labeling in [2] detailed by J. A. Gallian. It contains more than 2000 papers. A. S. Sitohang et al. [21] defined that the book graph B_n has an edge magic with the magic constant $k = 4n + 12$ and B_n has an super edge magic total labeling for $n = 1$ and 2. Also proved that B_3 is not super edge magic total labeling.

Slamin et al., [22] proved the following: For $n \equiv 6 (\text{mod } 8)$, every wheel W_n has an edge-magic total labeling with the magic constant $k = 5n + 2$; Fan graph f_n has an edge-magic total labeling with the magic constant $k = 4n + 2$; For $n \geq 2$, the friendship graph T_n is super edge-magic if $3 \leq n \leq 5$ and $n = 7$.

G. Ringel and A. S. Llado [18] proved that a graph with p vertices and q edges is not edge magic total if q is even and $p + q \equiv 2(\text{mod} 4)$ and each vertex has odd degree. They conjecture that trees are edge-magic total. Additional information about vertex and edge magic total labeling of graphs see [9, 25].

The concept of [13-15] motivated us to present this paper. Throughout this paper we defined even edge magic total labeling on cycles and disconnected cycles.

3. PRELIMINARIES

Before proving the result we give some basic definition as follows.

Definition 3.1: [25] A cycle of length n is a closed walk of length n , $n \geq 3$, in which all vertices are different.

Definition 3.2: [25] A graph is called $2-$regular if all its vertices have degree 2. A $2-$regular graph consists of one or more (disconnected) cycles.

Definition 3.3: [17] Let L be some labeling of G. The valence of edge $e = (u, v) \in E$ under the labeling L is $val(e) = l(u) + l(v) + l(e)$.

Definition 3.4: [17] A graph G is called edge-magic if there exist a labelling L, for which all edge valences are equal.

Definition 3.5: [21] A bijective function $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ such that $f(w) + f(wz) + f(z) = k$ for any edge $(w, z) \in E(G)$ is called an edge magic total labeling of G and that graph G is called an edge magic.

4. PROPOSED METHOD

The notion of an even vertex magic total labeling motivated us to define the following new definition of an even edge magic total labeling.

Definition 4.1: An edge magic total labeling g of a graph G is called an even edge magic total labeling if $g(E) = \{2, 4, ..., 2q\}$ and that graph G is called an even edge magic total graph.

Example 4.2:

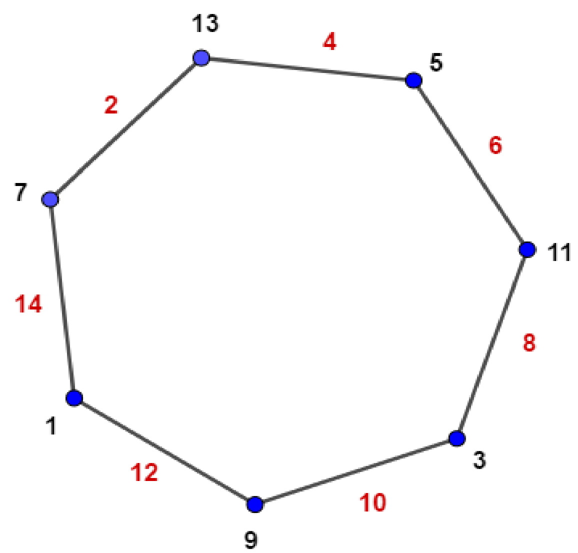

Figure 1. $p=7, q=7, k_g=22$

Using this new definition, we prove some results as follows.

4. 1. Even edge magic total labeling on cycles

Theorem 4.1.1: Let G be a $2-$regular graph with p vertices. If G is an even edge magic total graph then the magic constant $k_g=3p+1$.

Proof: Assign a $2-$regular graph G with p vertices. Since G is a $2-$regular graph, number of edges is equal to p. Let g be an even edge magic total labeling of a graph G with the magic constant k_g. Then $g(E)=\{2,4,...,2p\}$, $k_g=g(x)+g(xy)+g(y), \forall xy \in E$

$$pk_g=(1+3+...+(2p-1))+(2+4+...+2p)+(1+3+...+(2p-1))$$

$$k_g=3p+1.$$

Theorem 4.1.2: A cycle C_p is an even edge magic total graph if and only if p is odd.

Proof: Suppose C_p is an even edge magic total graph, by theorem 4.1, $k_g=3p+1$.

Suppose p is even, $k_g = 3p+1$ is odd.

For any even edge magic total labeling g, $k_g = g(x)+g(xy)+g(y) \forall e = xy \in E$

In particular, $g(v_j)+g(v_j v_{j+1})+g(v_{j+1}) = k_g$.

Since $g(v_j), g(v_{j+1})$ are odd numbers and $g(v_j v_{j+1})$ is an even number. Therefore k_g is even, which is a contraction.

Hence p is odd.

Conversely, suppose p is odd number.

Let $V(C_p) = \{v_1, v_2, \ldots, v_p\}$ and $E(C_p) = \{e_1, e_2, \ldots, e_{p-1}\} \cup \{e_p = v_p v_1\}$

Define $g(V \cup E) = \{1, 2, \ldots, 2p\}$ by

$$g(e_p) = 2$$

$$g(e_j) = 2j+2,\ 1 \le j \le p-1 \text{ and}$$

$$g(v_j) = \begin{cases} 2p-j & ; \ j \text{ is odd} \\ p-j & ; \ j \text{ is even} \end{cases}$$

Hence the cycle C_p is an even edge magic total graph with magic constant $k_g = 3p+1$.

4. 2. Even edge magic total labeling on disconnected cycles

In this part, we introduced an even edge magic total labeling for disconnected cycles pC_q i.e., the disjoint union of p cycles of q length, where p and q odd.

Theorem 4.2.1: The graph pC_q is an even edge magic total graph if p and q are odd.

Proof: Assume that pC_q is an even edge magic total graph.

$$\Rightarrow n[V(pC_q)] = pq \ \& \ n[E(pC_q)] = pq.$$

By theorem 4.1.1, $k_g = 3pq+1$

For any even edge magic total labeling g, $k_g = g(x)+g(xy)+g(y) \forall e = xy \in E$.

Since any edge of pC_q is incident to only two vertices, $g(x)+g(e)+g(y) = k_g$ where $e = xy$, x and y are end points of e.

Here $g(x), g(y)$ are odd numbers and $g(e)$ is even number.

Therefore $k_g = 3pq+1$ is even number if and only if p and q are odd.

Conversely, let p and q are odd integers.

Assume that the graph pC_q has the vertex set $V = V_1 \cup V_2 \cup \ldots \cup V_p$ where $V_j = \{v_j^1, v_j^2, \ldots, v_j^q\}$ and the edge set $E = E_1 \cup E_2 \cup \ldots \cup E_p$ where $E_j = \{e_j^1, e_j^2, \ldots, e_j^q\}$ and $e_j^k = v_j^k v_j^{k+1}$ for $1 \le k \le q-1, e_j^q = v_j^q v_j^1$.

Define $g(V \cup E) = \{1,2,3,\ldots,2pq\}$

$$g(e_j^q) = 2j \ , j = 1,2,\ldots,p$$

$$g(e_j^1) = \begin{cases} 4p-4j+2; & 1 \le j \le \frac{p-1}{2} \\ 6p-4j+2; & \frac{p+1}{2} \le j \le p \end{cases}$$

For $k = 2,3,4,\ldots,(q-1)$

$$g(e_j^k) = \begin{cases} (2k+1)p+2j+1, & 1 \le j \le \frac{p-1}{2} \\ (2k-1)p+2j+1, & \frac{p+1}{2} \le j \le p \end{cases}$$

$$g(v_j^1) = \begin{cases} 2pq+2j-p; & 1 \le j \le \frac{p-1}{2} \\ 2pq-3p+2j; & \frac{p+1}{2} \le j \le p \end{cases}$$

$k = 2,4,6,\ldots,(q-1)$

$$g(v_j^k) = pq+2j-1-(k+1)p, \quad 1 \le j \le p$$

$k = 3,5,7,\ldots,q$

$$g(v_j^k) = \begin{cases} 2pq-4j+1-(k-1)p, & 1 \le j \le \frac{p-1}{2} \\ 2pq-4j+1-(k-3)p, & \frac{p+1}{2} \le j \le p \end{cases}$$

It is clear that g is an even edge magic total labeling of pC_q with the magic constant $k_g = 3pq+1$.

Example 4.2.2:

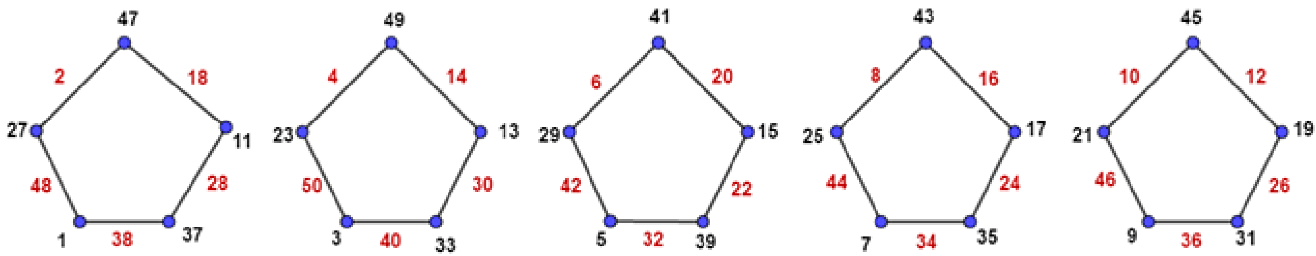

Figure 2. $p=5, q=5, k_g=76$

Theorem 4.2.3: The graph $C_3 \cup C_{4r}, r>1$ is an even edge magic total graph.

Proof: Labeling the vertices of C_3 by $g(v_1), g(v_2), g(v_3) = 2r+5, 2r+1, 2r+3$.

Labeling the edges of C_3 by $g(e_1), g(e_2), g(e_3) = 8r+4, 8r+6, 8r+2$.

Labeling the vertices and edges of C_{4r} by

$$g(v_j)=\begin{cases} j+2 & ;\ j\ odd,\ 1\le j\le 2r-3 \\ j+8 & ;\ j\ odd,\ 2r-1\le j\le 4r-3 \\ 1 & ;\ j=4r-1 \\ 4r+5+j & ;\ j\ even \end{cases}$$

$$g(e_j)=\begin{cases} 8r+2-2j & ;\ j=1,2,3,\dots,2r-3 \\ 8r-4-2j & ;\ 2r-2\le j\le 4r-3 \\ 4r+6 & ;\ j=4r-2 \\ 4r+4 & ;\ j=4r-1 \\ 4r+2 & ;\ j=4r \end{cases}$$

Therefore, the graph $C_3 \cup C_{4r}, r>1$ is an even edge magic total graph with the magic constant, $k_g = 12r+10$.

Example 4.2.4:

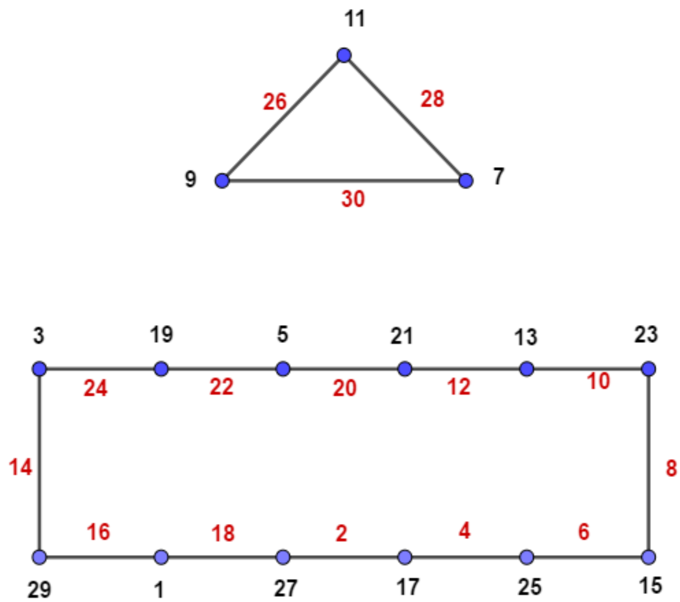

Figure 3. $C_3 \cup C_{12}, k_g = 46$

Theorem 4.2.5: The graph $C_3 \cup C_{4r+2}, r \geq 1$ is an even edge magic total graph.

Proof: For $r = 1,$ the labeling the vertices and edges of C_3 by $g(v_j) = 3,7,9$ for $j = 1,2,3$ and $g(e_j) = 18,12,16$ for $j = 1,2,3.$

Labeling the vertices and edges of C_6 by $g(v_j) = 1,13,11,15,5,17$ for $j = 1,2,\ldots,6$ and $g(e_j) = 14,4,2,8,6,10$ for $j = 1,2,\ldots,6$ (magic constant, $k_g = 28$).

For $r = 2,$ labeling the vertices and edges of C_3 by $g(v_j) = 5,9,11$ for $j = 1,2,3$ and $g(e_j) = 26,20,24$ for $j = 1,2,3.$

Labeling the vertices and edges of C_{10} by $g(v_j) = 1,17,15,19,3,21,7,23,13,25$ for $j = 1,2,\ldots,10$ and $g(e_j) = 22,8,6,18,16,12,10,4,2,14$ for $j = 1,2,\ldots,10$ (magic constant, $k_g = 40$).

For $r \geq 3,$ labeling the vertices and edges of C_3 by $g(v_1), g(v_2), g(v_3) = 2r+1, 2r+5, 2r+7$ and $g(e_1), g(e_2), g(e_3) = 8r+10, 8r+4, 8r+8.$

Labeling the vertices and edges of C_{4r+2} by

$$g(v_j)=\begin{cases}1 & ;\ j=1\\ 4r+7; & j=3\\ j-2\ ; & j\ \ odd,\ 5\le j\le 2r+1\\ 2r+3\ ; & j\ \ odd,\ j=2r+3\\ j+4\ ; & j\ \ odd,\ 2r+5\le j\le 4r+1\\ j+4r+7\ ; & j\ \ even\end{cases}$$

$$g(e_j)=\begin{cases}8r+6 & ;\ j=1\\ 4r & ;\ j=2\\ 4r-2 & ;\ j=3\\ 8r+10-2j\ ; & 4\le j\le 2r+1\\ 4r+4 & ;\ j=2r+2\\ 4r+2 & ;\ j=2r+3\\ 8r+4-2j\ ; & 2r+4\le j\le 4r+1\\ 4r+6 & ;\ j=4r+2\end{cases}$$

Therefore, the graph $C_3\cup C_{4r+2}, r\ge 1$ is an even edge magic total graph with the magic constant, $k_g=12r+16$.

Example 4.2.6:

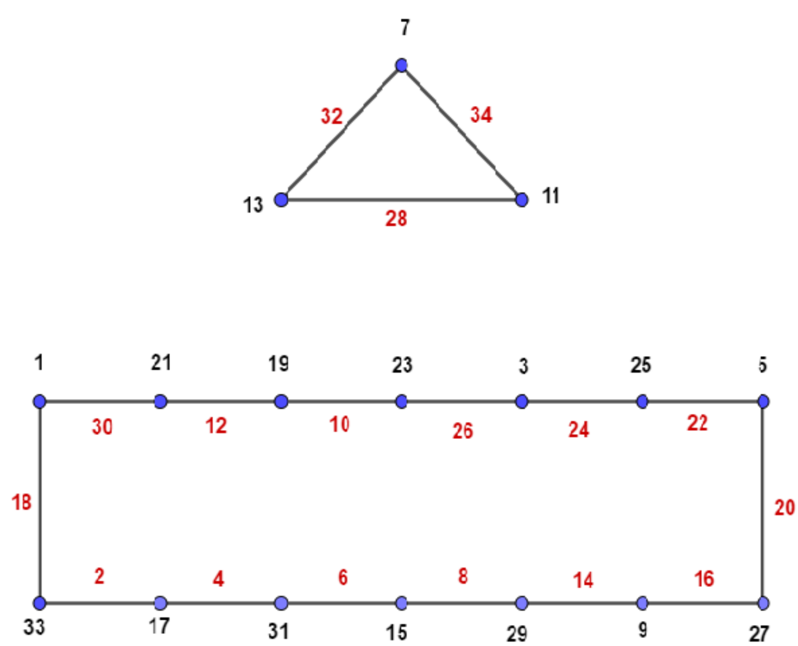

Figure 4. $C_3\cup C_{14}, k_g=52$

Theorem 4.2.7: The graph $C_4 \cup C_{4r-1}, r > 1$ is an even edge magic total graph.

Proof: Labeling the vertices and edges of C_4 by

$g(v_1), g(v_2), g(v_3), g(v_4) = 4r+3, 8r+3, 4r-1, 8r+5$ and $g(e_1), g(e_2), g(e_3), g(e_4) = 4,8,6,2$.

Labeling the vertices and edges of C_{4r-1} by

$$g(v_j) = \begin{cases} j+2 & ;\ j \equiv 1 \bmod 4,\ j < 4r-3 \\ 4r+5 & ;\ j = 2 \\ j+4r+1 & ;\ j \equiv 2 \bmod 4,\ j > 2 \\ j-2 & ;\ j \equiv 3 \bmod 4,\ j < 4r-1 \\ j+4r+5 & ;\ j \equiv 0 \bmod 4 \\ 4r-3 & ;\ j = 4r-3 \\ 4r+1 & ;\ j = 4r-1 \end{cases}$$

$$g(e_j) = \begin{cases} 8r+2 & ;\ j = 1 \\ 8r+4 & ;\ j = 2 \\ 8r-2j+6 & ;\ j \equiv 3 \bmod 4,\ j < 4r-1 \\ 8r-6 & ;\ j = 4, r \neq 2 \\ 12 & ;\ j = 4, r = 2 \\ 14 & ;\ j = 4r-3 \\ 10 & ;\ j = 4r-2 \\ 8r-2j+6 & ;\ j \equiv 1 \bmod 4,\ 1 < j < 4r-3 \\ 8r-2j+10 & ;\ j \equiv 2 \bmod 4,\ 6 \leq j \leq 4r-6 \\ 12 & ;\ j = 4r-4 \\ 8r-2j+2 & ;\ j \equiv 0 \bmod 4,\ 4 < j \leq 4r-8 \\ 8r+6 & ;\ j = 4r-1 \end{cases}$$

Therefore, the graph $C_4 \cup C_{4r-1}, r > 1$ is an even edge magic total graph with the magic constant, $k_g = 12r+10$.

Example 4.2.8:

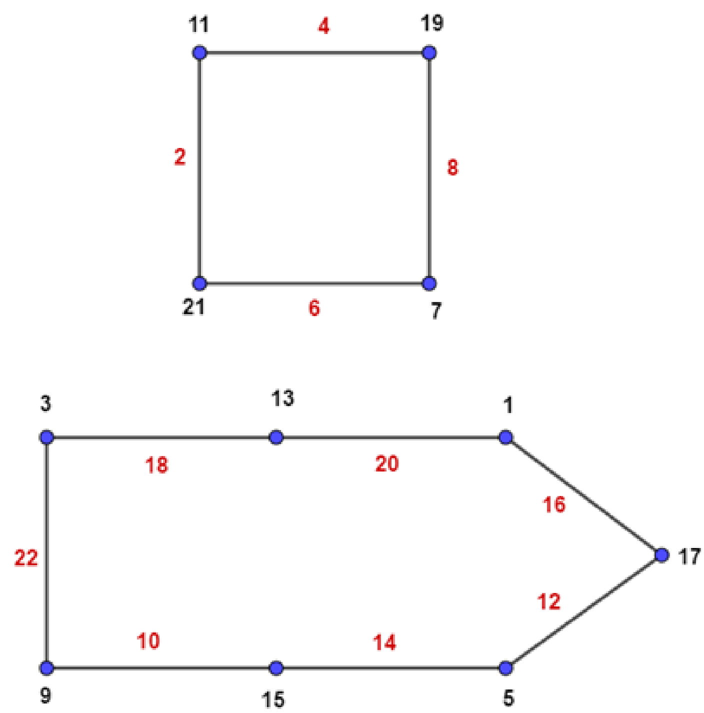

Figure 5. $C_4 \cup C_7, k_g = 34$

Theorem 4.2.9: The graph $C_4 \cup C_{4r-3}, r > 1$ is an even edge magic total graph.

Proof: Labeling the vertices of C_4 by $g(v_1), g(v_2), g(v_3), g(v_4) = 4r+1, 8r-1, 4r-3, 8r+1$.

Labeling the edges of C_4 by $g(e_1), g(e_2), g(e_3), g(e_4) = 4,8,6,2$.

Labeling the vertices and edges of C_{4r-3} by

$$g(v_j) = \begin{cases} j+2 & ;\ j \equiv 1 \bmod 4 \\ j+4r+3 & ;\ j \equiv 2 \bmod 4 \\ j-2 & ;\ j \equiv 3 \bmod 4 \\ j+4r-1 & ;\ j \equiv 0 \bmod 4 \end{cases}$$

$$g(e_j) = \begin{cases} 8r-2j+2 & ;\ j \equiv 0,2 \bmod 4 \\ 8r-2j-2 & ;\ j \equiv 1 \bmod 4, j < 4r-3 \\ 8r-2j+6 & ;\ j \equiv 3 \bmod 4 \\ 8r+2 & ;\ j = 4r-3 \end{cases}$$

Therefore, the graph $C_4 \cup C_{4r-3}, r > 1$ is an even edge magic total graph with the magic constant, $k_g = 12r + 4$.

Example 4.2.10:

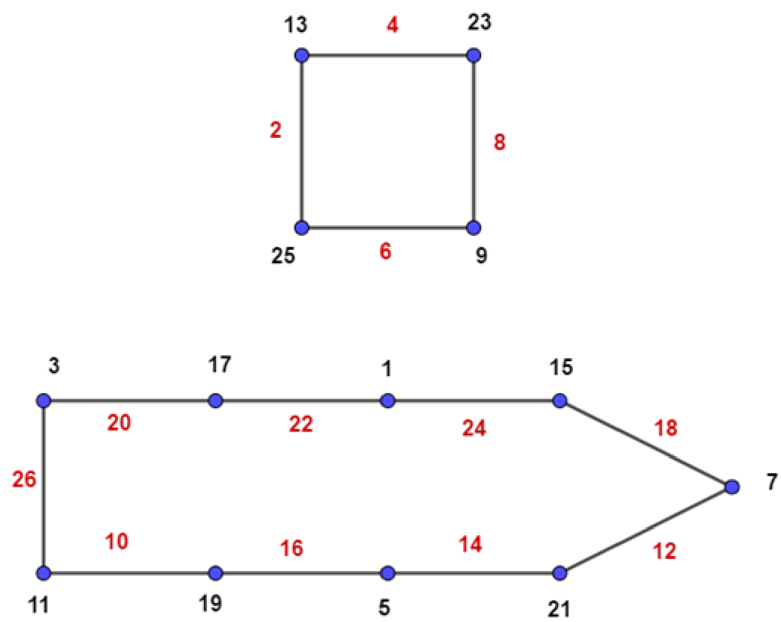

Figure 6. $C_4 \cup C_9, k_g = 40$

5. CONCLUSION

In this paper we discussed about cycles of odd length and disjoint union of some cycles which admits even edge magic total labeling. In future we can prove different types of graphs which satisfy even edge magic total labeling.

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