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SHORT COMMUNICATION

Applications of Second Order Derivative of Planck Distribution to Cosmic Microwave Background and Melting Point

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ABSTRACT

The second order derivative of Planck distribution for black body radiation has been applied to conclude that minimum temperature of cosmic microwave background can never be less than 1.6 Kelvin. The same derivative has been applied to derive a melting point of all materials naturally to begin to become nonsolid materials, and to derive a limit of temperature for materials naturally to begin to become radioactive materials which radiate.

Keywords: Cosmic Microwave Background, Second Order Derivative, Planck's Distribution, Wien's Approximation

1. INTRODUCTION

Wien's distribution for intensities of light waves or heat radiation emitted from a black body was refined as Planck's distribution. Planck's distribution was the first significant reliable

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distribution, which was verified experimentally. For some mathematical derivations, giving meaningful physical interpretations is a big problem. Planck mentioned in his Nobel lecture that he had to wait about five years to provide an interpretation to his work. One simpler example is the Einstein's energy mass relation [8].

This relation is simple to state. But, only very few succeeded in giving an unambiguous interpretation, although there are many vague interpretations. The present note provides meaningful interpretations for derivations to be presented. The experimentally verified Wien's displacement law was theoretically derived by equating first order derivative of Planck's distribution to zero [9, 10]. More specifically, if $u(\lambda)$ is the Planck's distribution for intensity of light with wavelengths λ emitted from a black body at temperature T, then the solution λ_{max} for λ of the equation $\frac{du(\lambda)}{d\lambda} = 0$ gives the Wien's displacement law $\lambda_{max}T = b$, where b is the Wien's constant 2.892×10^{-3} in MKS system. Thus λ_{max} is a special point at which there is a maximum intensity in radiation, and it can be obtained as a solution of the equation $\frac{du(\lambda)}{d\lambda} = 0$. The solution of $\frac{d^2u(\lambda)}{d\lambda^2} = 0$ are also special points at which emissions are lost.

This fact is applied to derive the conclusion that minimum temperature of cosmic microwave background cannot be less than 1.6 Kelvin in any year. The same fact is applied to get limits of temperature to keep materials in states like solid, liquid, gaseous, and radioactive states. It was observed in the article [2] that minimum temperature of cosmic wave background is 2.726 Kelvin, which was considered as the best observed value in comparison with previous ones. There are many very recent works [1, 3-7] regarding Planck's distribution, temperature of cosmic microwave background, and melting point of materials [10-13].

Let us use the following constants in MKS system, in addition to the Wien's constant b mentioned above.

Planck's constant $h = 6.626 \times 10^{-34}$.

Boltzmann constant $k = 1.38 \times 10^{-23}$.

Speed of light in vacuum $c = 2.9979 \times 10^{8}$.

Stefan-Boltzmann constant $\sigma = 5.672 \times 10^{-8}$.

For example, let us use the value for $\frac{hc}{k}$ as $14.3942647826087 \times 10^{-3}$ without mentioning units in MKS system. The unit for temperature is Kelvin. Planck's distribution for intensity $u(\lambda)$ corresponding to wavelength λ for a lightwave emitted from a black body with temperature T is considered in the form: $u(\lambda) = \frac{8\pi hc}{\lambda^5}\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1}$. Let us fix these notations and let us have units in MKS system.

2. PRELIMINARY FOR JUSTIFICATION

We shall given an interpretation for the points λ which satisfy $\frac{d^2u(\lambda)}{d\lambda^2} = 0$. For justification of the interpretation, let us do some preliminary work. To approximate $\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1}$ by $e^{\frac{-hc}{\lambda kT}}$, we expect for example that $\frac{hc}{\lambda kT} \geq 2.5$ or $\lambda T \leq 0.005$, approximately.

A compromise can be done to some more extent. Let us write $\lambda T \ll \frac{hc}{k}$ to mention this type of approximation. All parts of this note will not violate the inequality $\lambda T \leq 0.009$, approximately, whenever the notation $\lambda T \ll \frac{hc}{k}$ is used. Let us assume at present that $\lambda T \ll \frac{hc}{k}$. In this case, we have Wien's approximation $u(\lambda) \approx \frac{8\pi hc}{\lambda^5} e^{\frac{-hc}{\lambda kT}}$. One can also use $e^{\frac{-hc}{\lambda kT}} \approx 1$ when $\lambda T \ll \frac{hc}{k}$.

Thus, $u(\lambda) \approx \frac{8\pi hc}{\lambda^5}$ when $\lambda T \ll \frac{hc}{k}$. Since $\int_w^\infty u(\lambda) d\lambda$ tends to zero as w tends to infinity, we can have the following approximations $\int_{p/T}^\infty u(\lambda) d\lambda \approx \int_{p/T}^\infty \frac{8\pi hc}{\lambda^5} d\lambda \approx 32\pi hc \frac{T^4}{p^4} \approx 1997.7594494286 \frac{T^4}{p^4} \times 10^{-26}$, when $p \ll \frac{hc}{k}$. Equate this σT^4 to obtain $p^4 \approx \frac{1997.7594494286}{\sigma} \times 10^{-26} = 35221.428877695 \times 10^{-16}$, subject to a clarification to be explained. Thus $p \approx 1.36994061862192 \times 10^{-3} \ll \frac{hc}{k}$. The clarification is the following.

The Stefan-Boltzmann formula is $E = \sigma T^4$. When it is equated with $\int_{p/T}^\infty u(\lambda) d\lambda$, it is understood that all wavelengths λ of all light waves emitted are greater than or equal to p/T subject to the condition $p = (p/T)T \ll \frac{hc}{k}$. This means that $\lambda \geq 1.36994061862192 \times 10^{-3} \times T^{-1}$, or equivalently, $\lambda T \geq 1.36994061862192 \times 10^{-3}$. This is true for any wavelength λ of a light wave emitted from a black body with temperature T. This conclusion part is to be used as the justification for an interpretation.

3. WIEN'S APPROXIMATION

Let us write $u(\lambda) \approx \frac{8\pi hc}{\lambda^5} e^{\frac{-hc}{\lambda kT}} = C\lambda^{-5} e^{\frac{-a}{\lambda T}}$, where $a = \frac{hc}{k}$ and $C = 8\pi hc$, for $\lambda T \ll \frac{hc}{k}$. Then, $u'(\lambda) \approx C\left(-5\lambda^{-6} e^{\frac{-a}{\lambda T}} + \frac{a}{T}\lambda^{-7} e^{\frac{-a}{\lambda T}}\right)$ and $u''(\lambda) \approx C\left(30\lambda^{-7} e^{\frac{-a}{\lambda T}} - 12\frac{a}{T}\lambda^{-8} e^{\frac{-a}{\lambda T}} + \left(\frac{a}{T}\right)^2 \lambda^{-9} e^{\frac{-a}{\lambda T}}\right)$. If $\frac{d^2u(\lambda)}{d\lambda^2} = 0$, then $30\lambda^2 - 12\left(\frac{a}{T}\right)\lambda + \left(\frac{a}{T}\right)^2 \approx 0$, $\lambda = \left(\frac{12 \pm \sqrt{24}}{60}\right)\frac{a}{T}$, and hence $\lambda = 1.70356615852494 \times 10^{-3} \times T^{-1}$ or $\lambda = 4.05413975451854 \times 10^{-3} \times T^{-1}$. Average of these two values is $2.87885295652174 \times 10^{-3} \times T^{-1} \approx bT^{-1}$, where b is the Wien's constant. Here solutions of $\frac{d^2u(\lambda)}{d\lambda^2} = 0$ are special points at which there are sign changes for $\frac{du(\lambda)}{d\lambda}$. The nature does not allow such changes. The changes which happen are breaks in emission, when $\lambda T \ll \frac{hc}{k}$, and in this case $1.70356615852494 \times 10^{-3} \leq \lambda T \leq 4.05413975451854 \times 10^{-3}$. This conclusion is arrived after having a comparison with the previous conclusion that $\lambda T \geq 1.36994061862192 \times 10^{-3}$.

4. MAJOR CONCLUSIONS

If we assume that $\lambda = 1.063 \times 10^{-3}$ metre is the maximum possible wavelength of a light wave, then $\lambda T \geq 1.7035661582494 \times 10^{-3}$ implies that $T \geq 1.60260221874406$

Kelvin. Thus, if all light emissions are lost, then the temperature of the solid body should be at least 1.602 Kelvin.

This temperature 1.602 Kelvin is comparable with the minimum possible temperature 2.726 Kelvin of cosmic microwave background of our universe. So, minimum temperature of cosmic microwave background of our universe cannot go below 1.6 Kelvin, approximately.

If we assume that minimum wavelength of ultraviolet rays is 10^{-8} metre, then $\lambda T \leq 4.05413975451854 \times 10^{-3}$ implies that $T \leq 405414$ Kelvin. So, a black body with temperature greater than 405414 Kelvin does not emit ultraviolet rays with wavelength 10^{-8} metre. But this is impossible.

So, it is expected that some change should happen to the material. It is expected that the material cannot continue to be solid in nature, when the material reaches a temperature 405414 Kelvin. Thus, it is expected that any material in our universe having temperature more than 405414 Kelvin cannot continue to be solid in nature. If we assume that minimum wavelength of X-ray is 10^{-11} metre, then $\lambda T \leq 4.05413975451854 \times 10^{-3}$ implies that $T \leq 40541397546$ Kelvin. It is expected that any material which reaches a temperature greater than 40541397546 Kelvin should begin to become a disintegrating radioactive material, which begins radiation. For a fixed temperature T, the relation $1.70356615852494 \times 10^{-3} \times T^{-1} \leq \lambda \leq 4.05413975451854 \times 10^{-3} \times T^{-1}$ is applicable for all wavelengths λ of all light waves emitted from a black body, if $\lambda T \ll \frac{hc}{k}$. A natural question is the following one. Is it possible to improve significantly these inequalities for many values of λT, when we consider exact Planck's distribution instead of Wien's approximation? The answer is "No". This answer requires the following justification.

5. PLANCK'S DISTRIBUTION

Let $x(\lambda) = \frac{u(\lambda)}{8\pi hc} = \lambda^{-5}\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1} = \lambda^{-5}A$, where $A = \left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1}$. Then $A = \left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1}$. Then $\frac{dA}{d\lambda} = A^2 e^{\frac{hc}{\lambda kT}} \frac{hc}{\lambda^2 kT}$, $\frac{dx(\lambda)}{d\lambda} = -5\lambda^{-6}A + \lambda^{-7}\frac{hc}{kT}e^{\frac{hc}{\lambda kT}}A^2$, and $\frac{d^2x(\lambda)}{d\lambda^2} = 30A\lambda^{-7} - 12\frac{hc}{kT}e^{\frac{hc}{\lambda kT}}A^2\lambda^{-8} + A^3\left(\left(\frac{hc}{kT}\right)^2\left(e^{\frac{hc}{\lambda kT}}\right)\left(1 + e^{\frac{hc}{\lambda kT}}\right)\right)\lambda^{-9}$. Let us consider solutions of $\frac{d^2u(\lambda)}{d\lambda^2} = 0$ for finding two special values of λ discussed previously. For this purpose, we consider the relation $\frac{d^2x(\lambda)}{d\lambda^2} = 0$, or equivalently, we consider the relation $30\lambda^2 - 12A\frac{hc}{kT}e^{\frac{hc}{\lambda kT}}\lambda + A^2\left(\frac{hc}{kT}\right)^2\left(e^{\frac{hc}{\lambda kT}}\right)\left(1 + e^{\frac{hc}{\lambda kT}}\right) = 0$.

For existence of two distinct real solution values of λ corresponding to given T, we must have $144A^2\left(\frac{hc}{kT}\right)^2\left(e^{\frac{2hc}{\lambda kT}}\right) - 120A^2\left(\frac{hc}{kT}\right)^2\left(e^{\frac{hc}{\lambda kT}}\right)\left(1 + e^{\frac{hc}{\lambda kT}}\right) > 0$. Equivalently, after simplification, we must have $\lambda T < \frac{hc/k}{\log 5}$. That is, we must have $\lambda T < 8.9436595667359 \times 10^{-3}$, approximately. This is the main reason for not proceeding further through actual Planck's distribution.

6. IMPLICATIONS

All materials should stop all radiations when temperature is below 1.6 Kelvin. All materials naturally begin to become non solid, when temperature is above 405414 Kelvin. All materials naturally begin to become radioactive materials that begin radiation, when temperature is above 40541400000 Kelvin.

7. CONCLUSIONS AND SCOPE FOR FUTURE WORK

The minimum temperature of cosmic micro wave background of our universe can never be less than 1.6 Kelvin. When $\lambda T \ll \frac{hc}{k}$, for any fixed temperature T of a black body, wavelength λ of light wave satisfies the relation $1.70356615852494 \times 10^{-3} \times T^{-1} \leq \lambda \leq 4.05413975451854 \times 10^{-3} \times T^{-1}$. Similar results should be observed experimentally for other values of λT .

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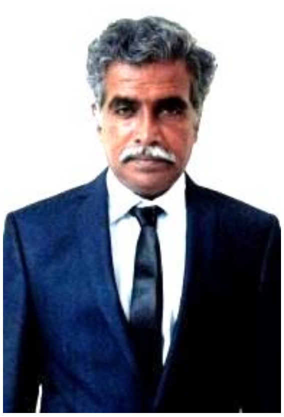
Biography



Dr. C. Ganesa Moorthy is working as a Professor in Department of Mathematics, Alagappa University, and has academic experience about 35 years in teaching and in research. He has published 57 articles in highly reputed journals and published 2 books. He solved a 50 year old open problem for his doctor of philosophy degree, and the solution was published in "Mathematika" in 1992. He is being a renowned theorist in India.

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