Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 134(2) (2019) 326-334 EISSN 2392-2192

SHORT COMMUNICATION

Modeling the Impact of Agriculture, Export Earnings and Inflation on Gross Domestic Product Using the Generalized Least Square (GLS) Approach

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ABSTRACT

The paper explored the impact of Agriculture, export earnings and inflation on gross domestic product (GDP). Time series data were obtained from the central bank of Nigeria statistical bulletin from 1981 to 2018. Each series consist of 38 observations. Evidence from our study showed that the predictor variables (Agriculture, export earnings and inflation) were significantly joint predictors of Gross Domestic Product. The predictor variables jointly explained 68.958% of GDP. Result of the analysis also revealed that both agriculture and export earnings have a positive impact on gross domestic product reaffirming the importance of the sectors to economic growth while inflation has a negative impact on gross domestic product. With evidence that agriculture has the potential to cause economic growth, spur industrialization as well as to enhance the living condition of the nation's majority, there should be increased investment in the development of the sector. This study also revealed that inflation is detrimental to sustainable economic growth in Nigeria. The result has important policy implications for both domestic policy makers and development partners. It also implies that controlling inflation is a necessary condition for promoting economic growth. Thus, policy makers should focus on maintaining inflation at a low rate probably single digit.

***Keywords*:** Agriculture, Inflation, Gross Domestic Product (GDP), Export Earnings

(Received 12 August 2019; Accepted 31 August 2019; Date of Publication 02 September 2019)

1. INTRODUCTION

Classical regression model seeks to determine the relationship between the dependent variable and the independent variables. This regression model could be simple (consisting of one dependent and one independent variable) or multiple (consisting of one dependent and two or more independent variables). However, in the linear regression model, certain assumptions are made on how a dataset will be produced by an underlying data-generating process [1]. Ordinary Least Squares (OLS) is the most common estimation method for linear models—and that's true for a good reason. As long as your model satisfies the OLS assumptions for linear regression, you can rest easy knowing that you're getting the best possible estimates. According to [2] some of the assumptions are: (1) No heteroscedasticity, here the variance of the errors should be consistent for all observations. In other words, the variance does not change for each observation or for a range of observations. This preferred condition is known as homoscedasticity. If the variance changes, we refer to that as heteroscedasticity. (2) The regression model is linear in the coefficients and the error term. This assumption addresses the functional form of the model. In statistics, a regression model is linear when all terms in the model are either the constant or a parameter multiplied by an independent variable.(3) Observations of the error term are uncorrelated with each other. One observation of the error term should not predict the next observation. For instance, if the error for one observation is positive and that systematically increases the probability that the following error is positive, that is a positive correlation. If the subsequent error is more likely to have the opposite sign, that is a negative correlation. This problem is known both as serial correlation and autocorrelation. (4) The error term is normally distributed. This assumption ensures that the error term is normally distributed.

When a linear regression model satisfies the OLS assumptions, the procedure generates unbiased coefficient estimates that tend to be relatively close to the true population values (minimum variance). However, in problems concerning time series data, it is often the case that there are disturbances, in fact, correlate. The most serious implication of auto correlated disturbances is not the resulting inefficiency of OLS, but the misleading inference when standard tests are used. The auto correlated nature of disturbances is accounted for in the generalized least squares (GLS). The superiority of GLS over OLS is due to the fact that GLS has a smaller variance. According to the Generalized Gauss Markov Theorem, the GLS estimator provides the Best Linear Unbiased Estimator (BLUE) of β [3-7].

In an attempt to overcome the weaknesses of ordinary least squares estimation method in the presence of autocorrelation, this study seeks to apply the generalized least squares estimation method since the ordinary least squares estimation method does not make use of the information of the unexplained variance as captured by the error terms in the dependent variable, whereas the generalized least squares (GLS) takes such information, the unexplained variance into account explicitly.

This study was motivated by the fact that some previous studies have failed to use GLS to explore the additional information embedded in the error terms of Ordinary Least Squares (OLS) estimated regression model involving Gross Domestic Product, agriculture production, export earnings and inflation in Nigeria. For example, [8] investigate the contribution of agriculture to economic growth from 1960 to 2011; their result revealed that agriculture sector has contributed positively and consistently to economic growth in Nigeria, reaffirming the sector's importance in the economy.

2. MATERIALS AND METHODS

2. 1. Method of Ordinary Least Square for Simple Linear Regression

The least squares estimation procedure uses the criterion that the solution must give the smallest possible sum of squared deviations of the observed Y_t from the estimates of their true means provided by the solution [1]. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be numerical estimates of β_0 and β_1 respectively and let

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 X_t \tag{1}$$

Be the estimate of Y_t for eachX_t, t = 1,…,n (see 9, 10, 11)

The least square principle choses $\hat{\beta}_0$ and $\hat{\beta}_1$ that minimises the sum of squares residuals, SSE.

$$\text{SSE} = \sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2 = \sum_{t=1}^{n} \varepsilon_t^2 \tag{2}$$

where $\varepsilon_t = (Y_t - \hat{Y}_t)$ is the observed residual for the observation

Also we can express ε_i in terms of Y_t, X_t, and $\beta_o$and β_1. Hence, we have

$$\varepsilon_t = Y_t - \beta_0 - \beta_1 X_t \tag{3}$$

Equation (3) becomes

$$SSE = \sum_{t=1}^{n}(Y_t - \beta_0 - \beta_1 X_t)^2 \tag{4}$$

The partial derivative of SSE with respect to the regression constant $\hat{\beta}_0$ that is

$$\frac{\sigma SSE}{\sigma \beta_0} = \frac{\sigma}{\sigma \beta_0}\left[\sum_{t=1}^{n}(Y_t - \beta_0 - \beta_1 X_t)^2\right] \tag{5}$$

With some subsequent rearrangement, the estimate of β_0 is obtained as

$$\hat{\beta}_0 = \left[\frac{\sum_{t=1}^{n} Y_t}{n}\right] - \beta_1\left[\frac{\sum_{t=1}^{n} X_t}{n}\right] \tag{6}$$

The partial derivative of SSE with respect to the regression coefficient β_1. That is

$$\frac{\sigma SSE}{\sigma \beta_1} = \frac{\sigma}{\sigma \beta_1}\left[\sum_{t=1}^{n}(Y_t - \beta_0 - \beta_1 X_t)^2\right] \tag{7}$$

Rearranging equation (7), we obtained the estimate of β_1.

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{n} Y_t X_t \frac{\sum_{t=1}^{n} Y_t \sum_{t=1}^{n} X_t}{n}}{\sum_{t=1}^{n} X_t^2 - \frac{(\sum_{t=1}^{n} X_t)^2}{n}} \tag{8}$$

2. 2. Method of Generalized Least Squares

Most at times we have situations where the following OLS assumptions are not satisfied

- Errors are uncorrelated
- The error variance are homoscedastic
- Errors are normally distributed

When these assumptions are not satisfied, the OLS estimation will be efficient and the OLS standard errors be biased. But $\hat{\beta}_{OLS}$ will continue to be unbiased and also consistent.

In situations where the homoscedasticity assumption is met we have Var (ϵ) = σ^2I. when the homoscedasticity assumption is violated then we have unequal variance for the error terms and it is denoted by Var (ϵ) = $\sigma^2\sum$, where $\sum$ is any matrix, not necessary diagonal.

The basic ideal behind GLS is to transform the observation matrix (Y, X) such that the variance of the error terms in the transformed model is I or σ^2I.

As an alternative to OLS is the GLS

$$\hat{\beta}_{GLS} = (X'\textstyle\sum^{-1}X)^{-1}(X'\sum^{-1}Y) \tag{9}$$

where $\hat{\beta}_{GLS}$ is the estimator of regression parameters

Suppose that $\sum$ has the eigen values, $\lambda_1, \lambda_1, \ldots, \lambda_T$, then by Cholesky decomposition we obtain.

$$\textstyle\sum = S\Lambda S' \tag{10}$$

where, Λ is a diagonal matrix with the diagonal elements $\lambda_1,\ \lambda_1, \ldots, \lambda_T$ and S is an orthogonal matrix. Thus,

$$\textstyle\sum^{-1} = S^{-1}\Lambda^{-1}S'^{-1} \tag{11}$$

$$\textstyle\sum^{-1} = S^{-1}\Lambda^{-1/2}\Lambda'^{-1/2}S'^{-1} \tag{12}$$

$$\textstyle\sum^{-1} = \rho\rho' \tag{13}$$

where, $\rho = S^{-1}\Lambda^{-1/2}$ and $\Lambda^{-1/2}$ is a diagonal matrix with the diagonal elements, $\sqrt{\lambda_1}, \ldots \sqrt{\lambda_T}$

It is then straight forward to prove that $\rho\sum\rho' = I_r$. Now multiply both sides of the linear model $Y = X\beta + \varepsilon$ by ρ

$$\rho Y = \rho X\beta + \rho\varepsilon \tag{14}$$

$$Y^0 = X^0\beta + \varepsilon^0 \tag{15}$$

where, $Y^0 = \rho Y$, $X^0 = \rho X$ and $\varepsilon^0 = \rho\varepsilon$. In this transformed model, we have E[ε^0] = 0 and

$$\text{Var}(\varepsilon^0) = \text{E}(\varepsilon^0\varepsilon^{0\prime}) = \sigma^2\rho\textstyle\sum\rho' = \sigma^2 I \tag{16}$$

The OLS estimate of β are given by

$$\hat{\beta}_{GLS} = (X_0'X_0)^{-1}X_oY_0 \tag{17}$$

$$\hat{\beta}_{GLS} = (X'\rho\rho'X)^{-1}X'\rho\rho'Y \tag{18}$$

$$\hat{\beta}_{GLS} = (X'\textstyle\sum^{-1}X)^{-1}(X'\textstyle\sum^{-1}Y) \tag{19}$$

Equation (19) is defined as GLS estimator.

The variance of GLS estimator is given in equation (20) below

$$\text{Var}\left(\hat{\beta}_{GLS}\right) = \sigma^2(X'\textstyle\sum^{-1}X)^{-1} \tag{20}$$

2. 3. Detecting Autocorrelation in the Error Term

We apply the ACF and PACF of the residual and the Breusch– Godfrey (BG) test to detect the presence of autocorrelation in the residuals of a fitted regression model. Consider equation (21) below

$$Y = X\beta + \varepsilon_t \tag{21}$$

Assume that the error term ε_t follows a pth order autoregressive process

$$\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + \cdots + \rho_p\varepsilon_{t-p} + V_t \tag{22}$$

where, V_t ~ N (0, σ^2)

The null hypothesis H_0 to tested is that

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_p = 0$$

Then, the test statistics is (n-p)$R^2 \sim X_p^2$.

The decision rule is that if the calculated value of the BG test statistic exceeds the critical x^2 value 5% level of significance (also, if the p-value corresponding to the BG test statistic is less than 0.05 level of significance), the hypothesis of no autocorrelation can be rejected; otherwise not rejected (see [1, 4, 12, 13]).

3. RESULTS AND DISCUSSION

In this study we consider Gross Domestic Product as dependent variable while Agricultural sector, Export earnings and inflation as independent variables. The data were obtained from the central bank of Nigeria statistical bulletin form 1981 to 2018. Each series consist of 38 observations. Gretl was used for statistical analysis.

3. 1. Test for Relationship among the Dependent and Independent Variables

The aim of the study is to address the weakness of the ordinary least square by applying the generalised least square method. We begin by applying OLS to modelled the relationship between the dependent and the independent variables. The estimates are presented in the table below.

Table 1. Estimates of OLS model.

Variable	Coefficient	S.e	t-ratio	p-value
Constant	-9.48305e+011	2.89867e+011	-3272	0.7456
Agriculture	0.00675734	0.0431624	0.1573	0.00365
Export	12.3562	6.69348	1.546	0.00457
Inflation	-3.2149e+09	5.50102e+09	-0.5844	0.00267
R-square F-statistic P-value	0.68958 3.55439 0.024547			

The result from the OLS model in table 1 showed that the predictor variables (Agriculture, export earnings and inflation) were significantly joint predictors of Gross Domestic Product. The predictor variables jointly explained 68.958% of GDP, while the remaining 31.042% could be due to other factors that affect GDP.

The model is specify in equation (23) below

$$GDP = -9.48305e^{10} + 0.00675734\text{Agric} + 12.3562\text{Exports} - 3.2149e^{9}infl \qquad (23)$$

3. 2. Model Diagnosis

To diagnose the fitted regression model in equation (23), we plot the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the residual of equation (23). If the lags of the ACF and PACF of the residuals of the fitted model are zero, the there is no serial correlation and hence no additional information in the residual and as such the model can be used for inference. Conversely, if the coefficient of the terms of both ACF and PACF are significant, then there is additional information in the residual series [1]. Assessing the correlogram in Figure 1, we observed that lags 1 and lag 5 of the ACF and lags 1, 2 and 4 of the PACF are significant. This indicates that there is additional information in the residual series and such information can be modelled by GLS.

Having confirmed the presence of autocorrelation in the residuals of the model, we move to apply the GLS model to capture the additional information in the auto correlated errors.

Table 2. Output of GLS model

Variable	Coefficient	S.e	t-ratio	p-value
Constant	-1.08e+11	2.94e+11	1.64e-05	<0.0001
Agriculture	0.004543	0.002490	0.18824	0.0000
Export	12.03782	6.053944	2.0410	0.0000
Inflation	-2.72e+09	5.43e+09	-0.500993	<0.0001

From Table 2, we observed that the independent variables are (agriculture, export earnings and inflation) are significant with their corresponding p-value less than 5% significance level. The coefficient of agriculture is 0.004543, it has a positive relationship with GDP ($t = 0.18824$, $P<.005$) showing that a unit increase in agriculture will increase GDP by 0.004543. The coefficient of export earnings is 12.03782, it has a positive relationship with GDP ($t = 2.0410$, $P<.005$) showing that a unit increase in industry will increase GDP by 12.03782. Also, the coefficient of inflation is -2.72e+09, it has a negative relationship with GDP ($t = -0.500993$, $P< 0.05$) showing that a unit increase in inflation will decrease GDP by -2.72e+09. Comparing the estimates of OLS model and GLS model showed that the OLS and the GLS produce almost the same values for the coefficients. The major difference is in the standard error and calculations based on the estimated variance of the coefficient probability distribution. The standard error are smaller when accounting for autocorrelation; that is in GLS regression, the standard error, t-statistic and p-value are reasonably different from those of OLS [1]. The implication is that GLS regression gives better estimates than OLS regression.

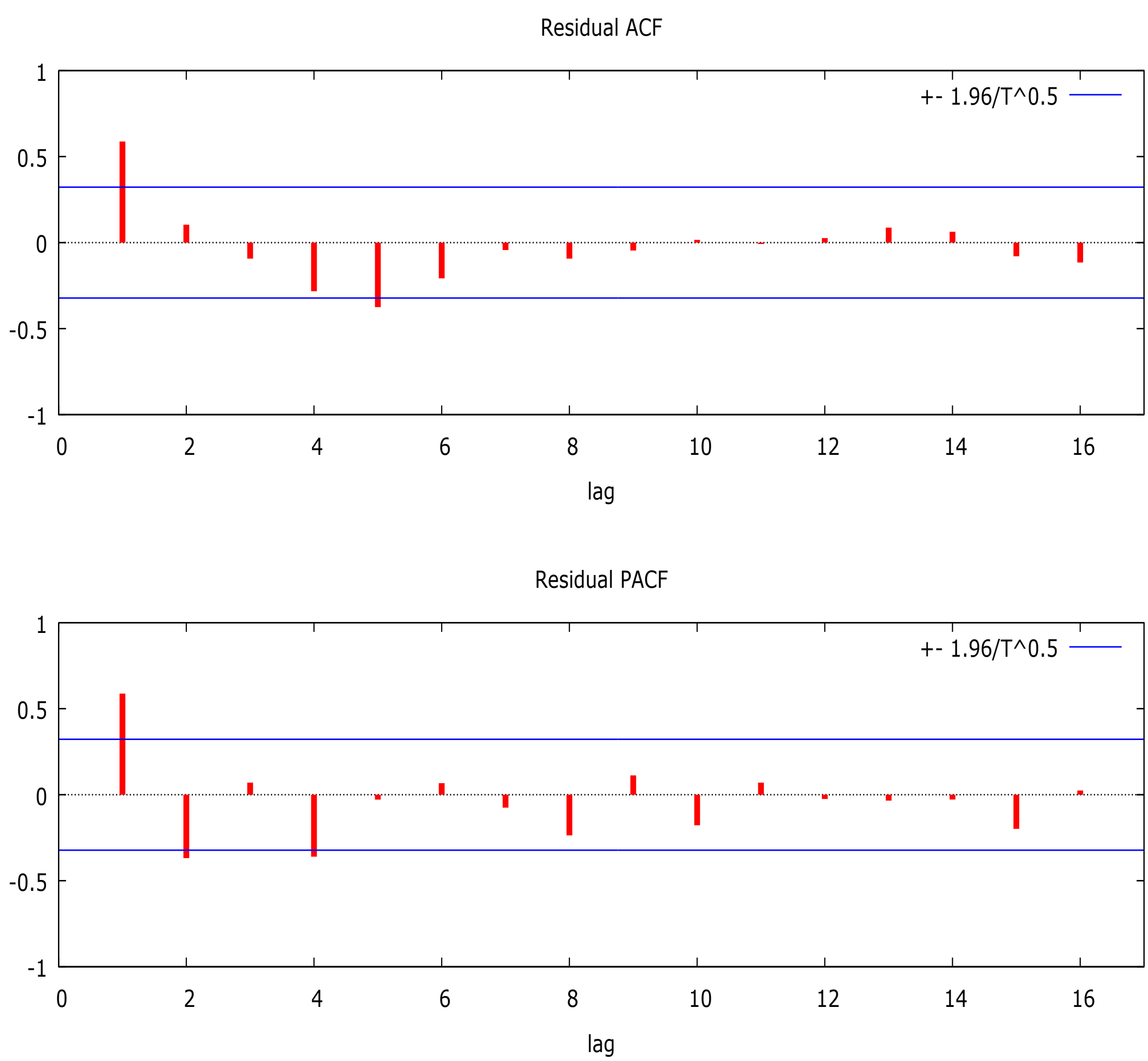

Figure 1. Correlogram of the Residual of Regression model

4. CONCLUSIONS

The study modelled the impact of Agriculture, export earnings and inflation on GDP using the generalized least square approach. The relationship between the dependent variable (GDP) and the independent variables (agriculture, Export earnings and inflation) was determined using the OLS approach. The result of the analysis revealed that the predictor variables (Agriculture, export earnings and inflation) were significantly joint predictors of Gross Domestic Product. The predictor variables jointly explained 68.958% of GDP, while the remaining 31.042% could be as a result of other factors that contribute to gross domestic product. Agriculture and export earnings have a positive impact on GDP while inflation has a negative impact on GDP. Evidence from the correlogram revealed that the errors derived from the regression model were auto correlated, thereby violating some assumptions (uncorrelated errors, homoscedacity, and normally distributed errors) of OLS model. To address the effect of autocorrelation in the OLS model, we apply the GLS approach and the result of the analysis revealed that the estimates from the GLS model were better compared to estimates from OLS model. The result of our work is tandem with studies of [14, 15] that agriculture and export earnings have positive impact on GDP while inflation has negative impact on GDP

Recommendations

- Based on the evidence from this study, we also recommend that the linkages between agriculture and other sectors be strengthened to increase the effect of agriculture growth on growth across the sectors. This can be achieved through increased productivity and the development of agriculture value chain.
- Government needs to take proper steps for improving exports to increase our economic growth.
- This study found out that inflation is detrimental to sustainable economic growth in Nigeria. These results have important policy implications for both domestic policy makers and development partners, implying that controlling inflation is a necessary condition for promoting economic growth. Thus, policy makers should focus on maintaining inflation at a low rate probably single digit.

References

[1] Alphonsus and Moffat (2018). Modelling the Auto correlated Errors in Time Series Regression: A Generalized Least Squares Approach. *Journal of Advances in Mathematics and Computer Science*, 26(4): 1-15.

[2] Safi, Samir and White, Alexander (2006). The Efficiency of OLS In The Presence Of Auto-Correlated Disturbances in Regression Models. *Journal of Modern Applied Statistical Methods* 5(1). DOI: 10.22237/jmasm/1146456540

[3] Koreisha, S. G. and Fang, Y. (2002). Generalized least squares with misspecifiedSerial correlation structures. *Journal of the Royal Statistical Society*, 63, Series B, 515-531.

[4] Koreisha, S. G. and Fang, Y. (2004). Forecasting with serially correlated regression models. *Journal of Statistical Computations and Simulation*, 74, 625-649.

[5] Kramer, W. (1980). Finite sample efficiency of ordinary least squares in the linear regression model with autocorrelated errors. *Journal of the American Statistical Association*, 75, 1005-1009

[6] Ullah, A., Srivastava, V. K., Magee, L., & Srivastava, A. (1983). Estimation of linear regression model with autocorrelated disturbances. *Journal of Time Series Analysis*, 4, 127-135

[7] Choudhury, A., Hubata, R. & Louis, R. (1999). Understanding time-series regression estimators. *The American Statistician*, 53, 342-348.

[8] Adeleke K.M (2014). Impact of Foreign Direct Investment on Nigeria Economic Growth. *International Journal of Academic Research in Business and Social Sciences* 4(8), 234-242

[9] Pankratz A. Forecasting with dynamic regressions models. 3rded. New York, John Wiley and Sons; 1991.

[10] Dalgaard P. Introductory statistics with R. 2nd Ed. Springer. 2008; 228.

[11] Moffat IU, Akpan EA. Modeling and forecasting trend function of a discrete-time stochastic process. *American Journal of Scientific and Industrial Research* 2014; 56: 195-202.

[12] Breusch TS. Testing for autocorrelation in dynamic linear models. *Australian Economic Papers* 1978; 17: 334–355

[13] Akpan EA, Moffat IU, Ekpo NB. Modeling regression with time series errors of gross domestic product on government expenditure. *International Journal of Innovation and Applied Studies* 2016; 18(4): 990-996.

[14] Tolulope and Chinonso E. Contribution of Agriculture to Economic Growth in Nigeria1. The 18th Annual Conference of the African Econometric Society (AES) Accra, Ghana at the session organized by the Association for the Advancement of African Women Economists (AAAWE), 22nd and 23rd July, 2013.

[15] Faraji K and Kenani M (2013). Impact of Inflation on Economic Growth: A Case Study of Tanzania. *Asian Journal of Empirical Research* 3(8): 363-380