Solving Hexagonal Intuitionistic Fuzzy Fractional Transportation Problem Using Ranking and Russell’s Method

A. Anju
Department of Mathematics, PSG College of Arts and Science, Coimbatore - 641014, Tamil Nadu, India
E-mail address: anjurojin2017@gmail.com

ABSTRACT
This paper presents a solution for fractional transportation problem in an intuitionistic fuzzy environment in which cost are represented by hexagonal intuitionistic fuzzy numbers. Fuzzy fractional transportation is a special kind of optimization problem which is associated with our day to day activities. An optimal solution is found out to show the effectiveness of this method. In this, the problem is solved using ranking and Russell’s method for hexagonal intuitionistic fuzzy numbers. An illustrative example is provided to demonstrate the feasibility of this method.

Keyword: Intuitionistic Fuzzy, Fractional Transportation Problem, Ranking Method, Russell’s Method

Mathematics Subject Classification (2010): 90C39, 03E72, 90C32

1. INTRODUCTION

Transportation problem is a special kind of linear programming problem where the objective is to minimize the cost of distributing a commodity from one source to another destination. Since it is having a special structure the usual simplex method is not suitable for solving transportation problems. Therefore it requires special method of solution. The problem of finding the minimum-cost distribution of a given commodity from a group of supply centers
(sources) $i = 1, 2, \ldots, m$ to a group of receiving centers (destinations) $j = 1, 2, \ldots, n$. Each source has a certain supply ($s_i$) and each destination has a certain demand ($d_j$). In this the cost of shipping from a source to destination is directly proportional to the number of units shipped.

The fractional transportation problem was originally proposed by Swarup [21] in 1966 and it has an important role in logistics and supply chain management for reducing cost and improving service. The fractional transportation problem plays a magnificent role in logistics, transportation and supply management for reducing cost and improving service. In fractional transportation problem the main objective is to optimize the ratios of two cost functions or damage functions or demand functions. As the ratio of two functions is considered, the fractional programming models become more suitable for real world problems. Keeping in view that the complexities associated with real life transportation problem like vagueness and uncertainty involved with the parameters. Therefore it can be very useful if we implement these fuzzy techniques.

The idea of intuitionistic fuzzy sets are introduced by Atanassov in 1986 [1-3]. It deals with vagueness or uncertainty. The main benefit of intuitionistic fuzzy sets is that it include both the degree of membership and non-membership of each element in the set. Recently intuitionistic fuzzy sets plays a vital role in decision making in fuzzy environment. In many cases the decision maker has no precise knowledge about the coefficient belonging to the transportation problem. An intuitionistic fuzzy set is a powerful tool to deal with such vagueness.

In this paper the author introduced fractional transportation problem with hexagonal intuitionistic fuzzy numbers in a more simplified and precise way. This paper is motivated by [4]. Many researchers have keen interest in intuitionistic fuzzy theory, which is applied in the field of decision making. Here the author found out the optimal solution of a fractional transportation problem with intuitionistic fuzzy numbers using the proposed method. In this paper for the first time the author found out the hexagonal intuitionistic fuzzy fractional transportation problem using ranking and Russell’s method.

This paper is organized as follows: Section 2 discusses the review of literature of the proposed problem. Section 3 gives preliminary background of the paper. Section 4 analysed problem formulation of fractional transportation problem, intuitionistic fuzzy fractional transportation problem, Ranking method and Russell’s method. Illustrative numerical example is given in Section 5. Section 6 gives the conclusion.

2. LITERATURE REVIEW

A lot of researchers have been studied the intuitionistic fuzzy number and ranking methods which is one way or the other relates to this paper.


3. PRELIMINARIES

In this section, some necessary background and definitions related to the fuzzy set theory are explained.

**Definition 3.1** [27, 28]

Let $X$ be a nonempty set. An intuitionistic fuzzy set $\tilde{H}$ of $X$ is defined as

$$\tilde{H} = \left\{ \left( x, \mu_{\tilde{H}}(x), \nu_{\tilde{H}}(x) \right) : x \in X \right\}$$

where the function $\mu_{\tilde{H}}: X \to [0,1]$ and $\nu_{\tilde{H}}: X \to [0,1]$ define the degree of membership and the degree of non-membership functions $x \in X$ and $0 \leq \mu_{\tilde{H}}(x), \nu_{\tilde{H}}(x) \leq 1, \forall x \in X$.

**Definition 3.2**

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its range between 0 and 1. This range is called the membership function.

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line $R$ such that:

(i) There exist at least one $x \in R$ with $\mu_{\tilde{A}}(x) = 1$. 

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(ii) $\mu^A_\lambda(x)$ is piecewise continuous.

**Definition 3.3**

A subset of intuitionistic fuzzy set $\tilde{H} = \left\{ x, \mu^{\tilde{H}}_\lambda(x), \nu^{\tilde{H}}_\lambda(x) : x \in X \right\}$ of the real line $\mathbb{R}$ is called an intuitionistic fuzzy number if the following conditions hold

(i) their exists $a \in \mathbb{R}, \mu^{\tilde{H}}_\lambda(a) = 1$ and $\nu^{\tilde{H}}_\lambda(a) = 0$

(ii) $\mu^{\tilde{H}}_\lambda(x) : \mathbb{R} \to [0,1]$ is continuous and for every $0 \leq \mu^{\tilde{H}}_\lambda(x), \nu^{\tilde{H}}_\lambda(x) \leq 1$ holds.

The membership and non-membership function of $\tilde{H}$ is defined as follows:

$$
\mu^{\tilde{H}}_\lambda(x) = \begin{cases}
  p_1(x), & x \in [a - \gamma_1, a) \\
  1, & x = a \\
  q_1(x), & x \in (a, a + \delta_1] \\
  0, & \text{otherwise}.
\end{cases}
$$

$$
\nu^{\tilde{H}}_\lambda(x) = \begin{cases}
  1, & x \in (-\infty, a - \gamma_2) \\
  p_2(x), & x \in (a - \gamma_2, a) \\
  0, & x = a, x \in [a + \delta_2, \infty)] \\
  q_2(x), & x \in (a, a + \delta_2].
\end{cases}
$$

where $p_i(x)$ and $q_i(x); i = 1, 2$ which are strictly increasing and decreasing functions in $[a - \gamma_1, a]$ and $(a, a + \delta_1]$ respectively. $\gamma_i$ and $\delta_i$ are the left and right spreads of $\mu^{\tilde{H}}_\lambda(x)$ and $\nu^{\tilde{H}}_\lambda(x)$.

**Definition 3.4 Hexagonal fuzzy number [27]**

An hexagonal fuzzy number is given by 6-tuples $R^{\tilde{H}}_H$ is denoted as $\tilde{R}^{\tilde{H}}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5$ and $a_6$ are real numbers and $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6$. Its membership function is given as
\[ \alpha_{\mu'}(x) = \begin{cases} \frac{1}{2} \left( \frac{x-a_1}{a_2-a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left( \frac{x-a_2}{a_3-a_2} \right), & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left( \frac{x-a_4}{a_5-a_4} \right), & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left( \frac{a_6-x}{a_6-a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{otherwise} \end{cases} \]

4. PROBLEM FORMULATION

4.1. Transportation Problem (FTP)

The FTP is the problem of minimizing \(q\) interval valued objective functions with interval cost. When the objective functions coefficients \(\frac{C^q_{ij}}{D^q_{ij}}\), \(A_i\) is the source parameters, \(B_j\) is the destination parameter and \(C^q_{ij}\), \(D^q_{ij}\) are the conveyance parameters, which are in the form of interval, where \(A_i = [s_{L_i}, s_{R_i}]\), \(i = 1, 2, \ldots, m\) and \(B_j = [t_{L_i}, t_{R_i}]\), \(j = 1, 2, \ldots, n\), are interval values of source and destination. The formulation for interval fuzzy problem is

\[
\text{Minimize } Z^q(x) \approx \underbrace{\sum_{i=1}^{m} \sum_{j=1}^{n} [C^{q}_{L_{ij}}, C^{q}_{R_{ij}}] x_{ij} + \alpha}_{\text{subject to}} \underbrace{\sum_{i=1}^{m} \sum_{j=1}^{n} [C^{q}_{L_{ij}}, C^{q}_{R_{ij}}] x_{ij} + \beta}_{\text{balanced condition is a necessary and sufficient condition for the existence of a feasible solution}}
\]

\[
\sum_{j=1}^{n} x_{ij} - A_i - [s_{L_i}, s_{R_i}], i = 1, 2, \ldots, m
\]

\[
\sum_{j=1}^{n} x_{ij} - B_j - [t_{L_i}, t_{R_i}], j = 1, 2, \ldots, n
\]

\[
x_{ij} \geq 0, \quad \forall \ i, j
\]

The formulation for interval fuzzy problem is

\[
\text{Minimize } Z^q(x) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} [C^{q}_{L_{ij}}, C^{q}_{R_{ij}}] x_{ij} + \alpha
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} [C^{q}_{L_{ij}}, C^{q}_{R_{ij}}] x_{ij} + \beta
\]

subject to

\[
\sum_{j=1}^{n} x_{ij} - A_i - [s_{L_i}, s_{R_i}], i = 1, 2, \ldots, m
\]

\[
\sum_{j=1}^{n} x_{ij} - B_j - [t_{L_i}, t_{R_i}], j = 1, 2, \ldots, n
\]

\[
x_{ij} \geq 0, \quad \forall \ i, j
\]

balanced condition is a necessary and sufficient condition for the existence of a feasible solution

\[
[C^{q}_{L_{ij}}, C^{q}_{R_{ij}}] \quad \text{and} \quad [D^{q}_{L_{ij}}, D^{q}_{R_{ij}}](q = 1, 2, \ldots, Q), P^{q}_{ij} = [P^{q}_{L_{ij}}, P^{q}_{R_{ij}}] = \frac{C^{q}_{ij}}{D^{q}_{ij}} = \frac{C^{q}_{L_{ij}}, C^{q}_{R_{ij}}}{D^{q}_{L_{ij}}, D^{q}_{R_{ij}}} \quad \text{is an interval representing the uncertain cost for the transportation problem. By the above definition the equivalent multi-objective deterministic transportation problem as}
\]
4. 2. Intuitionistic Fuzzy Fractional Transportation Problem

The proposed method is a simple method to find the optimal solution of an intuitionistic fuzzy fractional transportation problem having supply and demand which are real numbers and transportation cost \( \frac{C_{ij}}{D_{ij}} \) \((i=1,2,\ldots,m);(j=1,2,\ldots,n) \) from \( i^{th} \) source to \( j^{th} \) destination, taken as intuitionistic fuzzy fractional transportation problem.

4. 3. Hexagonal Intuitionistic Fuzzy Number [27]

An hexagonal intuitionistic fuzzy number is defined as

\[
\tilde{R}_H = (a_1, a_2, a_3, a_4, a_5, a_6) (a_1', a_2', a_3', a_4', a_5', a_6')
\]

where \( a_1, a_2, a_3, a_4, a_5, a_6, a_1', a_2', a_3', a_4', a_5', a_6' \) are real numbers and \( a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_3' \leq a_4 \leq a_4' \leq a_5' \leq a_5 \leq a_6' \leq a_6 \) its membership and non-membership functions are given as,

\[
\alpha_{H'}(x) = \begin{cases} \frac{1}{2} \left( \frac{x - a_1}{a_2 - a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left( \frac{x - a_2}{a_3 - a_2} \right), & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left( \frac{x - a_4}{a_5 - a_4} \right), & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left( \frac{a_6 - x}{a_6 - a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{otherwise} \end{cases}
\]

and
\[
\beta_H^I(x) = \begin{cases} 
1 - \frac{1}{2} \left( \frac{x - a_1}{a_2 - a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\
\frac{1}{2} \left( \frac{a_3 - x}{a_3 - a_2} \right), & \text{for } a_2 \leq x \leq a_3 \\
0, & \text{for } a_3 \leq x \leq a_4 \\
\frac{1}{2} \left( \frac{x - a_4}{a_5 - a_4} \right), & \text{for } a_4 \leq x \leq a_5 \\
\frac{1}{2 + 2} \left( \frac{x - a_5}{a_6 - a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\
1, & \text{otherwise}
\end{cases}
\]

4.4. Graphical representation of hexagonal intuitionistic fuzzy number

**Figure 1.** Graphical representation of hexagonal intuitionistic fuzzy number
4. 5. Ranking Method

The ranking of hexagonal intuitionistic fuzzy number

\[ \tilde{R}_{H^l} = (a_1, a_2, a_3, a_4, a_5, a_6)(a_1', a_2', a_3, a_4, a_5', a_6') \]

which maps the set of all fuzzy numbers to set of all real numbers is defined and given as

\[ \tilde{R}_{H^l} = [Mag_{\alpha}(\tilde{R}_{H^l}), Mag_{\beta}(\tilde{R}_{H^l})] \]

where

\[ Mag_{\alpha}(\tilde{R}_{H^l}) = \frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18} \]

and

\[ Mag_{\beta}(\tilde{R}_{H^l}) = \frac{2a_1' + 3a_2' + 4a_3 + 4a_4 + 3a_5' + 2a_6'}{18} \]

4. 6. Russell’s Method [16]

To find the basic feasible solution there are so many methods, one among the method is Russell’s method used for solving transportation problem. This method is better than other methods because it generates near optimal initial basic feasible solution. In this paper the author used this method to solve intuitionistic fuzzy fractional transportation problem.

4. 7. Proposed Algorithm

1) In intuitionistic fuzzy fractional transportation problem, the entries in both numerator and denominator table are reduced into an integer using the Russell’s ranking method numerator and denominator values are calculated separately.

2) In the reduced intuitionistic fuzzy fractional transportation problem, firstly identify the row and column difference considering the least two numbers of the respective row and column.

3) Now select the maximum among the difference, if more than one value, select any one among them and allocate the respective demand or supply to the minimum value of the corresponding column or row.

4) Take the difference of the respective supply or demand of the allocated cell which leads either one to zero, then eliminate the respective row or column. Check whether I both demand and supply is zero then eliminate both row and column.

5) Repeat the above steps until all the rows and columns are eliminated.

6) Lastly total minimum cost is calculated as the sum of the product of the cost and the allocated value for both numerator and denominator.

7) Now find out the intuitionistic fuzzy fractional transportation problem using values in both numerator and denominator tables.
5. NUMERICAL EXAMPLE

Consider a 3X3 hexagonal intuitionistic fuzzy fractional transportation problem given in the following Table 1 for numerator and Table 2 for denominator.

**Table 1.** Numerator values for Intuitionistic Fractional transportation problem

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>[(2,4,5,7,10,11)</td>
<td>[(3,4,5,6,8,10)</td>
<td>[(7,9,11,13,16,20)</td>
<td>15</td>
</tr>
</tbody>
</table>
<pre><code> | (1,3,5,7,12,13)]    | (2,4,5,6,10,13)]   | (5,7,11,13,19,24)]  |        |
</code></pre>
<p>| S2| [(6,8,11,14,19,25)  | [(9,11,13,15,18,20)| [(10,12,14,16,20,24)| 20     |
| (4,7,11,14,21,26)]  | (8,10,13,15,19,22)]| (8,10,14,16,20,25)]  |        |
| S3| [(3,5,6,8,9,10)     | [(4,7,9,12,15,17)  | [(10,12,14,16,20,24)| 35     |
| (2,3,6,8,11,13)]    | (3,6,9,12,16,18)]  | (8,10,14,16,20,25)]  |        |
| Demand| 25                 | 35                  | 10                  |        |</p>

**Table 2.** Denominator values for Intuitionistic Fractional transportation problem

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>[(3,5,6,8,11,12)</td>
<td>[(7,8,10,12,14,17)</td>
<td>[(8,10,12,14,17,21)</td>
<td>25</td>
</tr>
</tbody>
</table>
<pre><code> | (2,4,6,8,13,14)]    | (6,7,10,12,17,19)]  | (6,8,12,14,20,24)]  |        |
</code></pre>
<p>| S2| [(3,5,7,9,12,15)    | [(9,11,15,17,21,26)| [(7,9,12,15,20,26)  | 30     |
| (2,4,7,9,13,17)]    | (8,10,13,15,19,22)]| (5,7,12,15,22,28)]  |        |
| S3| [(6,8,11,14,19,25)  | [(4,5,6,7,9,11)     | [(7,8,10,12,14,17)  | 35     |
| (4,7,11,14,21,27)]  | (3,5,6,7,11,13)]    | (6,7,10,12,17,19)]  |        |
| Demand| 30                 | 45                  | 15                  |        |</p>

In Table 1 and Table 2 it is clear that it is a balanced transportation problem. By using the proposed algorithm the solution of the problem is given as follows. Now apply Russell’s ranking method to each row and column we have,

Applying ranking method to each [(2,4,5,7,10,11)(1,3,5,7,12,13)]
\[ \text{Mag}_a(R_{H_i}) = \frac{2 \times 2 + 3 \times 4 + 4 \times 5 + 4 \times 7 + 3 \times 10 + 2 \times 11}{18} = 6.44 \]
\[ \text{Mag}_\beta(R_{H_i}) = \frac{2 \times 1 + 3 \times 3 + 4 \times 5 + 4 \times 7 + 3 \times 12 + 2 \times 13}{18} = 6.72 \]

Therefore, \( \frac{\text{Mag}_a(R_{H_i}) + \text{Mag}_\beta(R_{H_i})}{2} = 6.583 = 7 \)

Similarly we get all values using his method and we can reduce the table with these values and are given in Table 3 and Table 4.

**Reduced Table**

**Table 3. Reduced table for Numerator**

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>7</td>
<td>6</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>S2</td>
<td>14</td>
<td>14</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>S3</td>
<td>7</td>
<td>11</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>Demand</td>
<td>25</td>
<td>35</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4. Reduced table for Denominator**

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>25</td>
</tr>
<tr>
<td>S2</td>
<td>8</td>
<td>17</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>S3</td>
<td>14</td>
<td>7</td>
<td>11</td>
<td>35</td>
</tr>
<tr>
<td>Demand</td>
<td>30</td>
<td>45</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

Again by the proposed algorithm, repeating the steps 2, 3 and 4 until all the rows and columns are eliminated the table becomes,
Continuing in the same manner we get the optimal solution for numerator and denominator as given in the following Table 5 and Table 6.

**Table 5.** Optimal solution of numerator

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
<th>Row Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>7</td>
<td>6</td>
<td>15</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>S2</td>
<td>14</td>
<td>14</td>
<td>16</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>S3</td>
<td>7</td>
<td>11</td>
<td>16</td>
<td>35</td>
<td>4</td>
</tr>
<tr>
<td>Demand</td>
<td>25</td>
<td>35</td>
<td>20</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Column Difference</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>Eliminate Row S1</td>
<td></td>
</tr>
</tbody>
</table>

The optimal solution is \( 15 \times 6 + 10 \times 14 + 10 \times 16 + 25 \times 7 + 10 \times 11 = 675 \)

**Table 6.** Optimal solution of Denominator

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>8</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>S2</td>
<td>8</td>
<td>30</td>
<td>17</td>
</tr>
<tr>
<td>S3</td>
<td>14</td>
<td>7</td>
<td>35</td>
</tr>
</tbody>
</table>

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The optimal solution is: $0 \times 8 + 10 \times 11 + 15 \times 14 + 30 \times 8 + 7 \times 35 = 805$

Therefore, the optimal solution of hexagonal intuitionistic fuzzy fractional transportation problem is 0.8385.

6. CONCLUSIONS

Many research and problems were found in many areas including fuzzy numbers, fuzzy equations and more. However, the study of solving hexagonal intuitionistic fuzzy fractional transportation problem has not been done by many researchers, this is for the first time the author found out using ranking method and Russell’s method. The field of intuitionistic transportation problem is very wide and has drastic importance, especially in our day to day activities and the solution to such are equally important as well.

References


