Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 132 (2019) 270-276 EISSN 2392-2192

SHORT COMMUNICATION

On triple sequence spaces of χ^3 of rough λ −statistical convergence in probability defined by Musielak-Orlicz function of p − metric space

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ABSTRACT

In this paper we have introduced and studied some new classes of triple sequence spaces of χ^3 via rough λ- statistical convergence in probability using by Musielak-Orlicz functions. Furthermore we have discussed some inclusion properties of among these triple sequence spaces.

***Keywords*:** rough $\lambda -$ statistical convergence, triple sequences, $\chi -$ sequence

2010 Mathematics Subject Classification: 40A35, 40G15, 60B10

1. INTRODUCTION

The idea of rough convergence was introduced by Phu [11], who also introduced the concepts of rough limit points and roughness degree. The idea of rough convergence occurs

(Received 17 July 2019; Accepted 27 July 2019; Date of Publication 28 July 2019)

very naturally in numerical analysis and has interesting applications. Aytar [1] extended the idea of rough convergence into rough statistical convergence using the notion of natural density just as usual convergence was extended to statistical convergence. Pal et al. [10] extended the notion of rough convergence using the concept of ideals which automatically extends the earlier notions of rough convergence and rough statistical convergence.

A triple sequence (real or complex) can be defined as a function $x: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by Sahiner et al. [12,13], Esi et al. [2-4], Dutta et al. [5], Subramanian et al. [14], Debnath et al. [6], Esi and Nagarajan [16], Esi et al.[17-18] and many others.

A triple sequence $\mathrm{x} = (\mathrm{x}_{\mathrm{mnk}})$ is said to be triple analytic if

$$sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3. A triple sequence $x = (x_{mnk})$ is called triple gai sequence if

$$\left((m+n+k)!\,|x_{mnk}|\right)^{\frac{1}{m+n+k}} \to 0 \text{ as } m,n,k \to \infty.$$

The space of all triple gai sequences are usually denoted by χ^3.

2. DEFINITIONS AND PRELIMINARIES

2. 1. Definition

An Orlicz function ([see [7]) is a function $M: [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x) + M(y)$, then this function is called modulus function.

Lindenstrauss and Tzafriri ([8]) used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = sup\{|v|u - (f_{mnk})(u): u \geq 0\}, m,n,k = 1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f, [see [9]] the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f = \left\{x \in w^3 : I_f(|x_{mnk}|)^{1/m+n+k} \to 0 \quad as \quad m,n,k \to \infty\right\},$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk}(|x_{mnk}|)^{1/m+n+k}, x = (x_{mnk}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk} \left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk} \right)$$

is an extended real number.

2. 2. Definition

Let X, Y be a real vector space of dimension w, where $n \leq m$. A real valued function $d_p(x_1, \ldots, x_n) = \| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p$ on X satisfying the following four conditions:

(i) $\| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p = 0$ if and only if $d_1(x_1, 0), \ldots, d_n(x_n, 0)$ are linearly dependent,

(ii) $\| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p$ is invariant under permutation,

(iii) $\| (\alpha d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p = |\alpha| \, \| (d_1(x_1, 0), \ldots, d_n(x_n, 0)) \|_p, \alpha \in \mathbb{R}$

(iv) $d_p((x_1, y_1), (x_2, y_2) \cdots (x_n, y_n)) = (d_X(x_1, x_2, \cdots x_n)^p + d_Y(y_1, y_2, \cdots y_n)^p)^{1/p} for 1 \leq p < \infty$; (or)

(v) $d((x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)) := \sup\{d_X(x_1, x_2, \cdots x_n), d_Y(y_1, y_2, \cdots y_n)\}$,

for $x_1, x_2, \cdots x_n \in X, y_1, y_2, \cdots y_n \in Y$ is called the p product metric of the Cartesian product of n metric spaces (see [15]) .

2. 3. Definition

Let $\eta = (\lambda_{abc})$ be a non-decreasing sequence of positive real numbers tending to infinity and $\lambda_{111} = 1$ and $\lambda_{a+b+c+3} \leq \lambda_{a+b+c+3} + 1$, for all $a, b, c \in \mathbb{N}$. The collection of all such triple sequences λ is denoted by $\mathfrak{I}$.

The generalized de la Vall`ee-Poussin means are defined by $t_{abc}(x) = \lambda_{abc}^{-1} \sum_{m,n,k \in I_{abc}} x_{mnk}$, where $I_{abc} = [abc - \lambda_{abc} + 1, abc]$. A sequence $x = (x_{mnk})$ is said to $(V, \lambda) -$ summable to a number L if $t_{abc}(x) \rightarrow L$, as $a, b, c \rightarrow \infty$.

2. 4. Definition

A triple sequence spaces of (X_{mnk}) is said to strong (V, λ) summable (or shortly : $[V, \lambda] -$ convergent to $\bar{0}$ if $lim_{a,b,c \rightarrow \infty} \frac{1}{\lambda_{abc}} \sum_{m \in I_a} \sum_{n \in I_b} \sum_{k \in I_c} |X_{mnk}, \bar{0}| = 0$. In this case write $X_{mnk} \rightarrow^{[V,\lambda]} \bar{0}$.

2. 5. Definition

A triple sequence spaces of (X_{mnk}) is said to be $\lambda -$ statistically convergent (or shortly: $S_\lambda -$ convergent) to $\bar{0}$ if for any $\varepsilon > 0$,

$lim_{a,b,c\to\infty}\frac{1}{\lambda_{abc}}|\{(m,n,k)\in I_{abc}: |X_{mnk},\bar{0}|\geq\varepsilon\}|=0$. In this case we write $S_\lambda - limX_{mnk} = \bar{0}$ or by $X_{mnk}\to^{S_\lambda}\bar{0}$. Now we introduce the following main definition:

2. 6. Definition

Let α be non negative real number. A triple sequence spaces of (X_{mnk}) of random variables is said to be rough $[V,\lambda]-$ summable in probability to $X: W\times W\times W\to\mathbb{R}\times\mathbb{R}\times\mathbb{R}$ with respect to the roughness of degree α (or shortly: $\alpha-[V,\lambda]-$ summable in probability to $\bar{0}$) if for any $\varepsilon>0$, $lim_{abc\to\infty}\frac{1}{\lambda_{abc}}\sum_{m\in I_a}\sum_{n\in I_b}\sum_{k\in I_c}P(|X_{mnk},\bar{0}|\geq\alpha+\varepsilon)=0$. In this case we write $X_{mnk}\to_\alpha^{[V,\lambda]^P}\bar{0}$. The class of all rough $[V,\lambda]-$ summable triple sequence spaces of random variables in probability will be denoted simply by $\alpha[V,\lambda]^P$.

2. 7. Definition

Let α be non negative real number. A triple sequence spaces of (X_{mnk}) of random variables is said to be rough $\lambda-$ statistically convergent in probability to $X: W\times W\times W\to\mathbb{R}\times\mathbb{R}\times\mathbb{R}$ with respect to the roughness of degree α (or shortly: $\alpha-\lambda-$ statistically convergent in probability to $\bar{0}$) if for any $\varepsilon,\delta>0$, $lim_{a,b,c\to\infty}\frac{1}{\lambda_{abc}}|\{(m,n,k)\in I_{abc}: P(|X_{mnk},\bar{0}|\geq\alpha+\varepsilon)\geq\delta\}|=0$. In this case we write $X_{mnk}\to_\alpha^{S_\lambda^P}\bar{0}$. The class of all $\alpha-\lambda-$ statistically convergent triple sequence spaces of random variables in probability will be denoted simply by αS_λ^P.

2. 8. Note

Let f be an Musielak-Orlicz function and triple sequence spaces of

$$\left\|\chi_f^3,\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p=\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right],$$

where $\mu_{mnk}(X)=\left(\left((m+n+k)!\,X_{mnk}\right)^{1/m+n+k},\bar{0}\right)$.

3. MAIN RESULTS

3. 1. Theorem

Let a triple sequence spaces of (X_{mnk}) of random variables are equivalent:

(i) $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p$ is $\alpha-[V,\lambda]-$ summable in probability to $\bar{0}$.

(ii) $\left\|\chi_f^3,\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p$ is $\alpha-\lambda-$ statistically convergent in probability to $\bar{0}$.

Proof: Similar to the proof of Theorem (3.1) in (see [17]).

3. 2. Theorem

If $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p \rightarrow_{\alpha}^{S^P} \bar{0}$ and

$\left\|\chi_f^3(Y_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p \rightarrow_{\beta}^{S^P} \bar{0}$ then

$P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right| \geq \alpha+\varepsilon\right)=0.$

Proof: Similar to the proof of Theorem (3.1) in (see [16]).

3. 3. Theorem

If $\lambda \in \Im$ is such that $\frac{\lambda_{abc}}{abc}=1$ then $\alpha S_\lambda^P \subset \alpha S^P$.

Proof: Let $0<\eta<1$ be given. Since $lim_{a,b,c\rightarrow\infty}\frac{\lambda_{abc}}{abc}=1$, we can choose $(u,v,w)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that $\left|\frac{\lambda_{abc}}{abc}-1\right|<\frac{\eta}{2}$ for all $a>u,b>v,c>w$. Now, for $\varepsilon,\delta>0$ $\frac{1}{abc}\left|\left\{m\leq a,n\leq b,k\leq c: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| = \frac{1}{abc}\left|\left\{m\leq a-\lambda_{abc},n\leq b-\lambda_{abc},k\leq c-\lambda_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| = \frac{1}{abc}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| \leq \frac{(abc)-\lambda_{abc}}{(abc)}+\frac{1}{abc}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| \leq 1-\left(1-\frac{\eta}{2}\right)+\frac{1}{abc}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| = \frac{\eta}{2}+\frac{\lambda_{abc}}{abc}\frac{1}{\lambda_{abc}}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| < \frac{\eta}{2}+\frac{1}{\lambda_{abc}}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right|$ holds for all $a>u,b>v,c>w$.

3. 4. Theorem

$\alpha S^P \subset \alpha S_\lambda^P$ if and only if $lim_{a,b,c\rightarrow\infty}\frac{\lambda_{abc}}{(abc)}>0$.

Proof: Let $lim_{abc\rightarrow\infty}\frac{\lambda_{abc}}{(abc)}>0$. Then for $\varepsilon,\delta>0$, we have

$\frac{1}{abc}\left|\left\{m\leq a,n\leq b,k\leq c: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\right\}\right| \geq \frac{1}{abc}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\right.\right.\right.$

$\alpha+\varepsilon\Big)\geq\delta\Big\}\Big|=\frac{\lambda_{abc}}{abc}\frac{1}{\lambda_{abc}}\Big|\Big\{(m,n,k)\in$
$I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq\alpha+\varepsilon\right)\geq\delta\Big\}\Big|.$

Taking limit $a,b,c\to\infty$ we get $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\to_\alpha^{S^P}\bar{0}\Rightarrow$ $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\to_\alpha^{S_\lambda^P}\bar{0}.$

Conversely, let $lim_{a,b,c\to\infty}\frac{\lambda_{abc}}{(abc)}=0$ then we can choose a subsequence $(a_u,b_v,c_w)_{u,v,w\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ such that $\frac{\lambda_{a_u,b_v,c_w}}{a_ub_vc_w}<\frac{1}{uvw}$ for all $u,v,w\in\mathbb{N}$. Define a triple sequence spaces of (X_{mnk}) of random variables whose probability density function is

$$\mu_{abc}(X)=\begin{cases}1, & if\quad 0<X<1\\ 0, & otherwise\end{cases},where(u,v,w)\in I_{abc}forsomeu,v,w\in\mathbb{N}$$

Let $0<\varepsilon,\delta<1$. Then

$$P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq 1+\varepsilon\right)=\begin{cases}1, & if\quad (u,v,w)\in I_{abc}forsomeu,v,w\in\mathbb{N}\\ \left(1-\frac{\varepsilon}{2}\right)^n, & otherwise\end{cases}.$$

We have

$$\frac{1}{\lambda_{abc}}\left|\left\{(m,n,k)\in I_{abc}: P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right]\right|\geq 1+\varepsilon\right)\geq\delta\right\}\right|=\begin{cases}1, & if\quad (u,v,w)\in I_{abc}forsomeu,v,w\in\mathbb{N}\\ 0, & otherwise\end{cases}.$$

Hence $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\notin_\alpha^{S_\lambda^P}$.

4. CONCLUSIONS AND FUTURE WORK

We introduced triple sequence spaces of rough $\lambda-$ statistical convergence in probability with respect sequence of Musielak-Orlicz function. For the reference sections, consider the following introduction described the main results are motivating the research.

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