Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 132 (2019) 220-232 EISSN 2392-2192

A New Approach for Solving Intuitionistic Dual Fuzzy Nonlinear Fractional Transportation Problem

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ABSTRACT

In this paper a new approach is explained for solving intuitionistic dual fuzzy fractional nonlinear equations. Here we have suggested a numerical method for solving a dual fuzzy nonlinear fractional equations instead of standard analytical techniques which are not suitable everywhere. Initially we wrote a dual fuzzy non-linear fractional equations in parametric form and then solve it by iterative method. An illustrative example is given to show the efficiency of our approach.

***Keywords*:** Dual, Intuitionistic Fuzzy, Nonlinear Equations, Newton Raphson Method

Mathematics Subject Classification: 65H11, 03E72, 90C30

1. INTRODUCTION

The notion of fuzzy set provides a convenient point of departure for the construction of a conceptual frame-work which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Fuzziness can be found in many areas of our day to day life, such as in engineering, medicine, meteorology, manufacturing and others. It is particularly frequent, however in all areas in which human judgement, evaluation and decisions are important. In today's highly

(Received 05 July 2019; Accepted 22 July 2019; Date of Publication 23 July 2019)

competitive market, many organizations trying to find better ways to create and deliver value to customers become stronger, this becomes more challenging. To meet this challenge, transportation models provide a powerful framework. The transportation problems are basically concerned with the optimal (much better) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations).

The main objective of transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales. It plays an important role in logistics and supply management for reducing cost and improving service.

The intuitionistic fuzzy set theory is an extension of the fuzzy set theory by Atanassov [1-3]. It is a special type of fuzzy linear fractional programming problem. The information available in real life is insufficient and uncertain and so the parameters of the problem are considered as fuzzy numbers. Since fuzziness in decision making problems are very limited in scope, the Intuitionistic Fuzzy Set (IFS) theory seems to be applicable to address this issue of uncertainty. Intuitionistic fuzzy sets have been found to be highly useful to deal with vagueness and it is applied in many fields such as decision making, medical diagnosis. The main advantage of intuitionistic fuzzy sets is that in this both the degree of membership and non-membership of each elements are included in the set.

Nonlinear equations have applications in various areas of science such as operational research, physics, chemistry, statistics, engineering, and social sciences. Equations of this type are necessary to solve for the involved parameters. It is simple and straight forward when the variables involving the equations are crisp number. However, in this case the variables associated with the measurement are uncertain. Therefore, these variables will either be an interval or a fuzzy number.

This paper is organized as follows: Section 2 discusses the review of literature of the proposed problem. Section 3 gives preliminary background of the paper. Section 4 analyzed problem formulation of fractional transportation problem, intuitionistic fuzzy fractional transportation problem and Newton Raphson's method. Approach for Solving Dual Fuzzy Non-linear Fractional Equations are discussed in section 5. In Section 6, illustrative example are solved.

2. LITERATURE REVIEW

A lot of researchers have been studied the intuitionistic fuzzy number and nonlinear equations which is one way or the other relates to this paper. Abbasbandy and Asady [4] explained Newton's method for solving fuzzy nonlinear equations. Tavassoli Kajani, Asady and Hadi Venchehm [9] discussed an iterative method for solving dual fuzzy nonlinear equations. Dennis [6] proposed numerical methods for unconstrained optimization and nonlinear equations.

Kelley [8] explained iterative methods for linear and nonlinear equations. Wang et al., [20] discussed iteration algorithms for solving fuzzy linear equations. Nurhakimah and Abdullah [12] explained about an interval type-2 dual fuzzy polynomial equations and ranking method of fuzzy numbers. Chakraborty et al., [5] found out duality in fuzzy multi-objective linear programming with multi constraint. Maryam Mosleh, [10] found out the solution of dual fuzzy polynomial equations by modified Adomian decomposition method. N. Mahdavi-Amiri and S. H. Nasseri [11] well described duality results and a dual simplex for linear programming problems with trapezoidal fuzzy variables. A. Noor'ani, J. Kavikumar, M. Mustafa and S. Nor [13] solved dual fuzzy Polynomial equation by ranking method. W. Rodder and H.J. Zimmermann [14] explained duality in fuzzy linear programming. Poonam Shugani, S. H. Abbas and Vijay Gupta [15] found out the computation of fuzzy transportation problem with dual simplex method.

J. L. Verdeagay [16] derived a dual approach to solve the fuzzy linear programming problems. M. Zangiabadi and H. R. Maleki [21] Fuzzy Goal programming Technique to solve multi-objective transportation problems with some non-linear membership functions. M. Venkatachalapathy and A. Edward Samuel [17] explained a new dual based approach for the unbalanced fuzzy transportation problem.

3. PRELIMINARIES

In this section, some necessary background and definitions related to the fuzzy set theory are explained.

Definition 3.1 [7, 20, 22]

Let X be a nonempty set. An intuitionistic fuzzy set $\tilde{H}^I$ of X is defined as $\tilde{H}^I = \left\{ \left\langle x, \mu_{\tilde{H}^I}(x), \nu_{\tilde{H}^I}(x) \right\rangle : x \in X \right\}$ where the function $\mu_{\tilde{H}^I}(x) : X \to [0,1]$ and $\nu_{\tilde{H}^I}(x) : X \to [0,1]$ define the degree of membership and the degree of non-membership functions $x \in X$ and $0 \le \mu_{\tilde{H}^I}(x), \nu_{\tilde{H}^I}(x) \le 1, \forall x \in X$.

Definition 3.2

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its range between 0 and 1. This range is called the membership function.

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that:

(i) There exist at least one $x \in R$ with $\mu_{\tilde{A}}(x) = 1.$

(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 3.3

A triangular fuzzy number $\tilde{A}$ is denoted by 3-tuples (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and $a_1 \le a_2 \le a_3$ with membership function defined as,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & for\ \ a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & for\ \ a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

Definition 3.4

A subset of intuitionistic fuzzy set $\tilde{H}^I = \left\{ \left\langle x, \mu_{\tilde{H}^I}(x), \nu_{\tilde{H}^I}(x) \right\rangle : x \in X \right\}$ of the real line $\Re$ is called an intuitionistic fuzzy number if the following conditions hold

(i) their exists $a \in \Re$, $\mu_{\tilde{H}^I}(a) = 1$ and $\nu_{\tilde{H}^I}(a) = 0$

(ii) $\mu_{\tilde{H}^I}(x) : \Re \to [0,1]$ is continuous and for every $0 \le \mu_{\tilde{H}^I}(x), \nu_{\tilde{H}^I}(x) \le 1$ holds.

The membership and non-membership function of $\tilde{H}^I$ is defined as follows:

$$\mu_{\tilde{H}^I}(x) = \begin{cases} p_1(x), & x \in [a-\gamma_1, a) \\ 1, & x = a \\ q_1(x), & x \in (a, a+\delta_1] \\ 0, & otherwise. \end{cases}$$

$$\nu_{\tilde{H}^I}(x) = \begin{cases} 1, & x \in (-\infty, a-\gamma_2) \\ p_2(x), & x \in (a-\gamma_2, a) \\ 0, & x = a, x \in [a+\delta_2, \infty)] \\ q_2(x), & x \in (a, a+\delta_2\,]. \end{cases}$$

where $p_i(x)$ and $q_i(x); i = 1,2$ which are strictly increasing and decreasing functions in $[a-\gamma_i, a)$ and $(a, a+\delta_i]$ respectively. γ_i and δ_i are the left and right spreads of $\mu_{\tilde{H}^I}(x)$ and $\nu_{\tilde{H}^I}(x)$.

4. PROBLEM FORMULATION

4. 1. Fractional Transportation Problem (FTP)

The FTP is the problem of minimizing q interval valued objective functions with interval cost. When the objective functions coefficients $\frac{C^q{}_{ij}}{D^q{}_{ij}}, A_i$ is the source parameters, B_j is the destination parameter and C_q, D_q are the conveyance parameters, which are in the form of interval, where $A_i = [s_{Li}, s_{Ri}], i = 1,2,\ldots,m$ and $B_j = [t_{Li}, t_{Ri}], j = 1,2,\ldots,n,$ are interval values of source and destination. The formulation for interval fuzzy problem is

$$\left\{\begin{array}{l} Minimize\ Z^q(x) \approx \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n}[C^q{}_{Lij}, C^q{}_{Rij}]x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n}[C^q{}_{Lij}, C^q{}_{Rij}]x_{ij} + \beta} \quad where\ q = 1,2,\ldots,Q \\ subject\ to \\ \quad \sum_{j=1}^{n} x_{ij} - A_i - [s_{Li}, s_{Ri}],\ i = 1,2,\ldots,m \\ \quad \sum_{j=1}^{n} x_{ij} - B_j - [t_{Li}, t_{Ri}],\ j = 1,2,\ldots,n \\ \quad x_{ij} \geq 0,\ \forall\ i, j \end{array}\right\}$$

the balanced condition is a necessary and sufficient condition for the existence of a feasible solution $[C^q{}_{Lij}, C^q{}_{Rij}]$ and $\left[D^q{}_{Lij}, D^q{}_{Rij}\right], (q = 1,2,\ldots,Q), P^q{}_{ij} = [P^q{}_{Lij}, P^q{}_{Rij}] = \frac{C^q{}_{ij}}{D^q{}_{ij}} = \frac{C^q{}_{Lij}, C^q{}_{Rij}}{D^q{}_{Lij}, D^q{}_{Rij}}$ is an interval representing the uncertain cost for the transportation problem. By the above definition the equivalent multi-objective deterministic transportation problem as

$$\left\{\begin{array}{l} Minimize\ Z^q{}_R(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} P^q{}_{Cij}\, x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} P^q{}_{Wij}\, x_{ij} \\ Minimize\ Z^q{}_C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} P^q{}_{Cij}\, x_{ij} \\ Subject\ to\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \leq s_{Ri}, \quad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \leq s_{Li}, \\ \qquad\qquad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \leq t_{Rj}, \quad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \leq t_j, \\ \qquad\qquad\qquad x_{ij} \geq 0, \quad \forall\ i, j. \end{array}\right\}$$

4. 2. Intuitionistic Fuzzy Fractional Transportation Problem

The proposed method is a simple method to find the optimal solution of an intuitionistic fuzzy fractional transportation problem having supply and demand which are real numbers and transportation cost $\frac{C_{ij}}{D_{ij}}$ (i=1,2,...,m);(j=1,2,...,n) from ith source to jth destination, taken as intuitionistic fuzzy fractional transportation problem.

4. 3. Newton-Raphson Method

Newton's method (or the Newton-Raphson) method is one of the most powerful and well-Known numerical methods for solving a root finding problem. It is defined as follows.

Let $x_1, x_2,...$ is calculated from the general formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f^{'}(x_n)},$$

where $n \geq 0$ and is an integer.

This method is simple to implement and has small number of iterations.

5. APPROACH FOR SOLVING DUAL FUZZY NON-LINEAR FRACTIONAL EQUATIONS

Let $a + b = 0$, where $b \in D$[19], here there exist no inverse of any given fuzzy number say $a \in D$. It is correct that $a + (-a) \neq 0$. Now we consider the dual fuzzy non-linear fractional equation as $\frac{P(a)}{Q(a)} = \frac{R(a)+c}{S(a)+d}$, where all parameters are fuzzy numbers. This paper is motivated by [18].

The main idea of this paper to obtain a solution for the above dual fuzzy non-linear fractional equations, whose parametric version is given as

$$\underline{P}(\underline{a},\bar{a},t) = \underline{R}(\underline{a},\bar{a},t) + \underline{c}(t) \tag{1}$$

$$Q(\underline{a},\bar{a},t) = \underline{S}(\underline{a},\bar{a},t) + \underline{c}(t)$$

$$\bar{P}(\underline{a},\bar{a},t) = \bar{R}(\underline{a},\bar{a},t) + \bar{c}(t) \tag{2}$$

$$\bar{Q}(\underline{a},\bar{a},t) = \bar{S}(\underline{a},\bar{a},t) + \bar{c}(t)$$

Now assume that $a = (\underline{\gamma}, \bar{\gamma})$ is the solution to the above fuzzy non-linear fractional equation, then

$$\frac{\underline{P}(\underline{\gamma},\overline{\gamma},t)}{\underline{Q}(\underline{\gamma},\overline{\gamma},t)}=\frac{\underline{R}(\underline{\gamma},\overline{\gamma},t)+\underline{c}(t)}{\underline{S}(\underline{\gamma},\overline{\gamma},t)+\underline{d}(t)}\qquad,\forall t\in[0,1] \tag{3}$$

$$\frac{\overline{P}(\underline{\gamma},\overline{\gamma},t)}{\overline{Q}(\underline{\gamma},\overline{\gamma},t)}=\frac{\overline{R}(\underline{\gamma},\overline{\gamma},t)+\overline{c}(t)}{\overline{S}(\underline{\gamma},\overline{\gamma},t)+\overline{d}(t)}\qquad,\forall t\in[0,1] \tag{4}$$

From (3) and (4)

$$\frac{\underline{P}(\underline{\gamma},\overline{\gamma},t)-\underline{R}(\underline{\gamma},\overline{\gamma},t)}{\underline{Q}(\underline{\gamma},\overline{\gamma},t)-\underline{S}(\underline{\gamma},\overline{\gamma},t)}=\frac{\underline{c}(t)}{\underline{d}(t)}\qquad,\forall t\in[0,1] \tag{5}$$

$$\frac{\overline{P}(\underline{\gamma},\overline{\gamma},t)-\overline{R}(\underline{\gamma},\overline{\gamma},t)}{\overline{Q}(\underline{\gamma},\overline{\gamma},t)-\overline{S}(\underline{\gamma},\overline{\gamma},t)}=\frac{\overline{c}(t)}{\overline{d}(t)}\qquad,\forall t\in[0,1] \tag{6}$$

Now, let $\underline{G}(\underline{\gamma},\overline{\gamma},t)=\underline{c}(t)$ and $\overline{G}(\underline{\gamma},\overline{\gamma},t)=\overline{c}(t)$

$$\underline{H}(\underline{\gamma},\overline{\gamma},t)=\underline{d}(t) \text{ and } \overline{H}(\underline{\gamma},\overline{\gamma},t)=\overline{d}(t)$$

The equation (5) and (6) becomes

$$\frac{\underline{P}(\underline{\gamma},\overline{\gamma},t)-\underline{R}(\underline{\gamma},\overline{\gamma},t)}{\underline{Q}(\underline{\gamma},\overline{\gamma},t)-\underline{S}(\underline{\gamma},\overline{\gamma},t)}=\frac{\underline{G}(\underline{\gamma},\overline{\gamma},t)}{\underline{H}(\underline{\gamma},\overline{\gamma},t)}\quad,\forall t\in[0,1]$$

and

$$\frac{\bar{P}(\underline{\gamma},\bar{\gamma},t) - \bar{R}(\underline{\gamma},\bar{\gamma},t)}{\bar{Q}(\underline{\gamma},\bar{\gamma},t) - \bar{S}(\underline{\gamma},\bar{\gamma},t)} = \frac{\bar{G}(\underline{\gamma},\bar{\gamma},t)}{\bar{H}(\underline{\gamma},\bar{\gamma},t)}, \forall t \in [0,1].$$

If $a_k = (\underline{a}_k, \bar{a}_k)$ is an approximate solution, then there exist $x(t)$ and y(t) and $,\forall t \in [0,1]$

$$\underline{\gamma}(t) = \underline{a}_k(t) + g(t)$$

$$\bar{\gamma}(t) = \bar{a}_k(t) + g(t), \qquad \forall\ k = 0, 1, 2, \ldots$$

Consider the Taylor's expansion of the functions $\underline{G}, \bar{G}, \underline{H},$ *and* $\bar{H}$ about $(\underline{a}_0, \bar{a}_0)$ and by eliminating highest terms, we have

$$\frac{\underline{G}(\underline{a}_0,\bar{a}_0,t) + h\underline{G}_{\underline{a}}(\underline{a}_0,\bar{a}_0,t) + g\underline{G}_{\bar{a}}(\underline{a}_0,\bar{a}_0,t)}{\underline{H}(\underline{a}_0,\bar{a}_0,t) + h\underline{H}_{\underline{a}}(\underline{a}_0,\bar{a}_0,t) + g\underline{H}_{\bar{a}}(\underline{a}_0,\bar{a}_0,t)} = \frac{\underline{c}(t)}{\underline{d}(t)} \tag{7}$$

$$\frac{\underline{G}(\underline{a}_0,\bar{a}_0,t) + h\underline{G}_{\underline{a}}(\underline{a}_0,\bar{a}_0,t) + g\bar{G}_{a}(\underline{a}_0,\bar{a}_0,t)}{\bar{H}(\underline{a}_0,\bar{a}_0,t) + h\bar{H}_{\underline{a}}(\underline{a}_0,\bar{a}_0,t) + g\bar{H}_{a}(\underline{a}_0,\bar{a}_0,t)} = \frac{\bar{c}(t)}{\bar{d}(t)} \tag{8}$$

After some simplifications, using Newton Raphson Method to find the solution.

5. 1. Algorithm Used

1) Firstly the author tried to solve the problem using intuitionistic method. Then transform the intuitionistic fuzzy fractional number into its simplified form.
2) Transform the dual fuzzy nonlinear fractional equations into its parametric form.
3) Find the initial guess a_0 by solving the parametric equations for $t=0$ and $t=1$.
4) Using Newton Raphson method solve the problem, after some iterations we will get the result.

6. NUMERICAL EXAMPLE

In this section numerical examples were illustrated to show the performance of Newton-Raphson method for solving intuitionistic dual fuzzy nonlinear fractional equations.

Example: Consider

$$\frac{(10,11,12)(8,11,14)a^3+(13,15,16)(11,15,18)a^2+(8,12,14)(6,12,16)a}{(5,6,8)(3,6,10)a^3+(9,10,12)(7,10,14)a^2+(11,13,15)(9,13,17)a}=\frac{(8,10,13)(6,10,15)a+(6,10,12)(4,10,14)}{(13,14,15)(11,14,17)a+(3,4,8)(1,4,10)}$$

$$\frac{(9,11,13)a^3+(12,15,17)a^2+(7,12,15)a}{(4,6,9)a^3+(8,10,13)a^2+(10,13,11)a}=\frac{(7,10,14)a+(5,10,13)}{(12,14,16)a+(2,4,9)}$$

Here numerator and denominator values are calculated separately, without loss of generality, let a be positive, and the parametric form of the numerator is given as,

$$\frac{(-2+t)\underline{a}^3(t)+(-3+t)\underline{a}^2(t)+(-5+t)\underline{a}(t)}{(-2+t)\underline{a}^3(t)+(-2+t)\underline{a}^2(t)+(-3+t)\underline{a}(t)}=\frac{(-3+t)\underline{a}(t)+(-5+t)}{(-2+t)\underline{a}(t)+(-2+t)}$$

After cross multiplication and little simplifications we arrive at the equation

$$(t-2)\underline{a}^3(t)+(3t-6)\underline{a}^2(t)+(9-6t+t^2)\underline{a}\ (t)+(t-5)\ .4$$

Now to obtain the initial values let $t=0$, and $t=1$ in parametric form, therefore we get

$$-2\underline{a}^3(0)-6\underline{a}^2(0)+9\underline{a}\,(0)-5=0$$

$$-\underline{a}^3(1)-3\underline{a}^2(1)+3\underline{a}\ (1)-4=0$$

Again, the parametric form of the denominator is given as

$$\frac{(20+t)\bar{a}^3(t)+(27+t)\bar{a}^2(t)+(19+t)\bar{a}(t)}{(10+t)\bar{a}^3(t)+(18+t)\bar{a}^2(t)+(23+t)\bar{a}(t)}=\frac{(17+t)\bar{a}(t)+(15+t)}{(26+t)\bar{a}(t)+(6+t)}$$

After cross multiplication and little simplifications we arrive at the equation

$$(350+19t)\underline{a}^3(t)+(366+19t)\underline{a}^2(t)+(-5+5t)\underline{a}\ (t)+(9t-231)$$

Now to obtain the initial values let $t=0$, and $t=1$ in parametric form, therefore we get

$$350\bar{a}^3(0)+366\bar{a}^2(0)-5\bar{a}\ (0)-231=0$$

$$369\bar{a}^3(1)+385\bar{a}^2(1)+0\bar{a}\ (1)-222\ \ =0$$

Therefore we can write,

$$-2\underline{a}^4(0)-6\underline{a}^3(0)+9\underline{a}^2(0)-5\underline{a}(0)=0 \tag{9}$$

$$350\bar{a}^4(0)-366\bar{a}^3(0)-5\bar{a}^2(0)-231\bar{a}(0)=0$$

and

$$-\underline{a}^4(1)-3\underline{a}^3(1)+3\underline{a}^2(1)-4\underline{a}(1)=0 \tag{10}$$

$$369\bar{a}^4(1)+386\bar{a}^3(1)+0\bar{a}^2(1)-222\bar{a}(1)=0\,.$$

Thus from (9) and (10) we have $\bar{a}(0)=0.6$, $\underline{a}(0)=0.6, \bar{a}(1)=0.6, \underline{a}(1)=0.7$. From the observation a_0 is very close to the solution. Therefore in order to illustrate the performance of our approach, we consider $a_0=(0.5,0.6,0.7)$.

It has been shown in [5] that the negative root of this dual fuzzy nonlinear system does not exist, for that we are considering positive values. Graphically it can be drawn as in Figure 1.

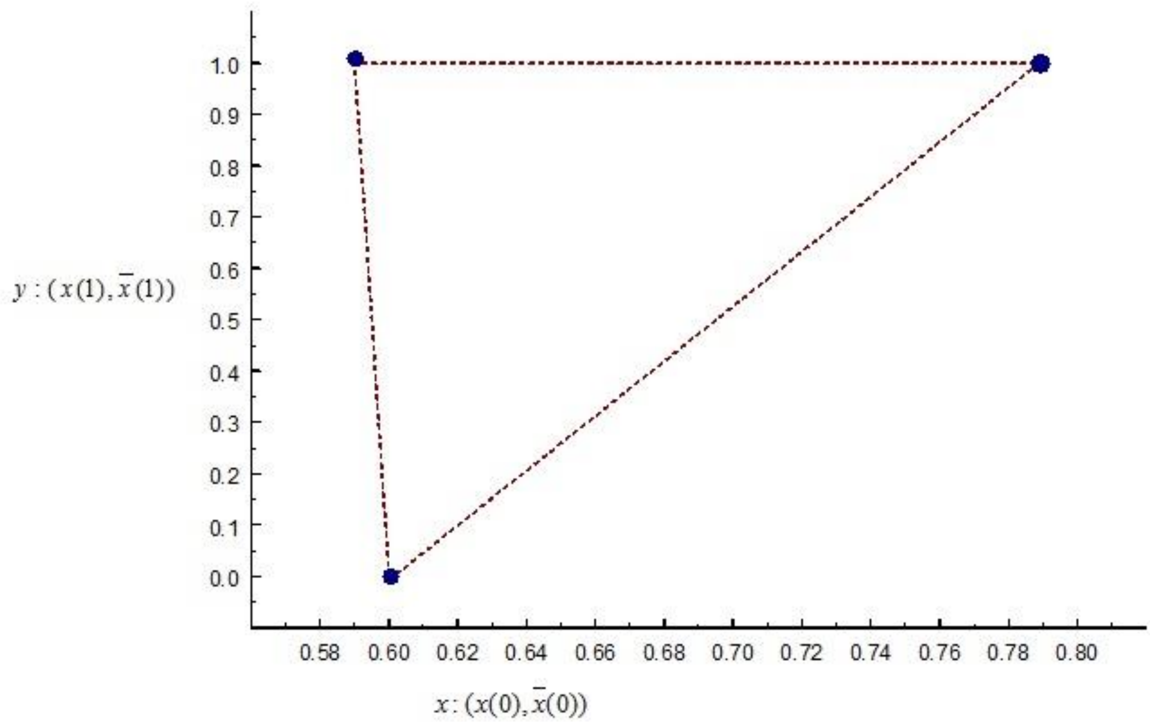

Figure 1. Positive solution of Newton Raphson Method.

7. CONCLUSION

Many research and problems were found in many areas including fuzzy numbers, fuzzy equations and more. However the study of solving intuitionistic dual fuzzy non-linear fractional transportation problem has not been done by many researchers. In this paper for the first time the author found out to solve intuitionistic dual fuzzy non-linear fractional transportation problem. Its efficiency and less computations to solve dual fuzzy problems make it a good alternative. The field of non-linear equations is very wide and has drastic importance, especially in our day to day activities and the solution to such are equally importance as well.

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