Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 130 (2019) 25-41 EISSN 2392-2192

An EOQ inventory model for Gompertz deteriorating items with quadratic demand and constant holding cost in a fuzzy environment

K. Senbagam[1] and **M. Kokilamani**[2]

Department of Mathematics, P.S.G College of Arts and Science,
Coimbatore, Tamil Nadu, India

[1,2]E-mail address: visnusen@gmail.com , gokila88@gmail.com

ABSTRACT

In this paper we discussed an economic order quantity inventory model for deteriorating items with time-dependent demand rate. Here the demand is considered as a quadratic function of time and deteriorating items follow a Gompertz distribution deterioration. Stock out is not permitted and holding cost is constant. The deterioration cost, holding cost and the parameters α, β are assumed as a triangular fuzzy numbers. Our goal is to maximize the total profit of inventory optimization by using Pascal triangular method. A few numerical examples illustrate the implementation of the proposed model. Sensitivity analysis is the study of how to divide and allocate the uncertainty in the output of a mathematical model or system to different sources of input uncertainty.

***Keywords*:** Inventory system, Deteriorating items, Gompertz distribution, Quadratic function, Triangular fuzzy number, Pascal triangular method

AMS Subject Classification: 90B05

1. LITERATURE REVIEW

An important aspect of any successful business is inventory management. It is the process of monitoring and controlling the flow of inventory units that a business uses for sale or distribution in the production or manufacture of goods. Inventories usually consist of

(Received 8 May 2019; Accepted 24 May 2019; Date of Publication 25 May 2019)

a combination of goods, raw materials and finished products, and efficient management of these items is critical to ensuring optimum stock levels and maximizing the company's earning potential. It can be used to predict inventory and price levels, as well as the expected demand for the product. Dutta and Pavankumar (2013) developed a fuzzy inventory model for deteriorating items with fully backordered model by using signed distance method. Maragatham and Lakshmidevi (2014) expanded a fuzzy inventory model for deteriorating items with price dependent demand. He used the signed distance method for defuzzification process. Sensitivity analysis is presented for both crisp and fuzzy model. An inventory model for Gompertz deteriorating items with time-varying holding cost and price dependent demand was considered by Nurul Azeez Khan, Verma and Vijay kumar (2017). Pandit Jagatananda Mishra, Trailokyanath Singh and Hadi bandhu pattanayak (2016) presented an optimal policy of an inventory model for deteriorating items with generalized demand rate and deterioration rate. Shortages are allowed and partially back-ordered. The salvage value is included into deteriorated units. Sahidul Islam and Abhishek Kanti Biswas (2017) introduced deterministic economic order quantity inventory model with exponential demand rate, weibull distribution for deterioration and time dependent holding cost in crisp and fuzzy environment. Shukla, Tripathi, Suhilkumar Yadav and Vivek Shukla (2015) developed inventory model for deteriorating items with quadratic time dependent demand rate and composed shortages. Mathematical models are derived under two deterrent circumstances i.e., case I: The permissible delay period is less than or equal to time to finish positive inventory and case II: The permissible delay period is greater than time to finish positive inventory. Sujatha and Parvathi (2015) studied an EOQ inventory model for weibull deteriorating items with linear demand and partial back-ordering in fuzzy environment. Sujata Saha and Tripti Chakrabarti (2017) generated a supply chain production inventory model for deteriorating items under fuzzy environment. He considered demand as linear price dependent. The signed distance method and graded mean integration method have been used for defuzzification. Sujata Saha (2017) presented a fuzzy continuous review inventory model for deteriorating items in a supply chain management system with price dependent demand. Signed distance method is used to defuzzify the cost function. Sushil kumar and Rajput (2015) expanded fuzzy inventory model for deteriorating items with time dependent demand and partial back-ordering. The back-ordering rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. In this paper we have considered demand as a quadratic function of time. Deterioration is permitted on inventory which follows Gompertz distributions. Stock out is not permitted. The deterioration cost, holding cost and the parameters α, β are assumed as a triangular fuzzy numbers. The model has been studied to maximize the total profit of inventory optimization by using Pascal triangular method. Numerical example and sensitivity analysis is performed to show the effect of changes in the parameters of the optimum solution.

2. PRELIMINARIES

When we consider the fuzzy inventory model, the following definitions are required.

(i) A fuzzy set $\tilde{A}$ on the given universal set X is denoted and defined by

$$\tilde{A} = \left\{ \left(x, \lambda_{\tilde{A}}(x) \right) : x \in X \right\}$$

where $\lambda_{\bar{A}} : X \to [0,1]$ is called the membership function, and $\lambda_{\bar{A}}(x) =$ degree of x in $\lambda_{\bar{A}}$.

(ii) A triangular fuzzy number is specified by the triplet (a_1, a_2, a_3) where $a_1 < a_2 < a_3$ and defined by its continuous membership function $\lambda_{\bar{A}} : X \to [0,1]$ as follows

$$\lambda_{\bar{A}}(x) = \begin{Bmatrix} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_2 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{Bmatrix}$$

(iii) If $A = (a_1, a_2, a_3)$ is a triangular fuzzy number then the Pascal triangular method of $\tilde{A}$ is defined as $P(\tilde{A}, 0) = \frac{1}{4}(a_1 + 2a_2 + a_3)$

(iv) A random variable X is said to have a Gompertz distribution if its probability density function is $f_G(x) = \beta e^{\alpha x} e^{\frac{-\beta}{\alpha}(e^{\alpha x} - 1)}$; $x \ge 0,\ \alpha, \beta > 0$.

(v) Hazard rate function of Gompertz distribution is $h_G(x) = \beta e^{\alpha x}$.

3. PREMISE

To elaborate on the proposing pattern, we allow the following premises:

1) The time between the initiation and completion of a production process is zero.
2) Time horizon is finite.
3) Stock out is not permitted.
4) The demand rate is a quadratic function of time, i.e., $D(t) = a + bt + ct^2$, where $a,\ b, c > 0$ and a is the initial rate of demand, b is the rate that decreases or increases the demand rate and c is the rate at which the demand rate changing.
5) The deterioration rate follows a Gompertz distribution function is given by $\theta(t) = \beta e^{\alpha t}$, where $0 < \beta < 1, \quad \alpha > 0, \quad t > 0$.
6) Replenishment rate is infinite.

4. ENTRIES

- A – Ordering cost per order
- C_1 – Deterioration cost per unit time

- $h-$ Carrying cost per unit time
- $T-$ The fixed length of each cycle
- $Q-$ The maximum inventory level for each ordering cycle
- $t-$ Time of deterioration, $t>0$
- $TP-$ The total profit cost per unit time
- $I(t)-$ The level of inventory at any time t, $(0\leq t\leq T)$
- $\alpha-$ Scale parameter
- $\beta-$ Shape parameter
- $S-$ Selling price
- $\tilde{C}_1-$ fuzzy deterioration cost per unit time
- $\tilde{\alpha}-$ Scale parameter in fuzzy sense
- $\tilde{\beta}-$ Shape parameter in fuzzy sense
- $\tilde{h}-$ fuzzy carrying cost per unit time
- $T\tilde{P}-$ The total fuzzy profit cost per unit time
- TP_{dp} – is the defuzzify value of $T\tilde{P}$ by applying Pascal triangular method

5. MATHEMATICAL FORMULATION

In this model, a quadratic type of demand is studied with Gompertz distribution deterioration rate. During stock period $(0,\ T)$ is governed by the differential equations,

$$\frac{dI(t)}{dt}+\beta e^{\alpha t}I(t)=-(a+bt+ct^2),\qquad 0\leq t\leq T \tag{1}$$

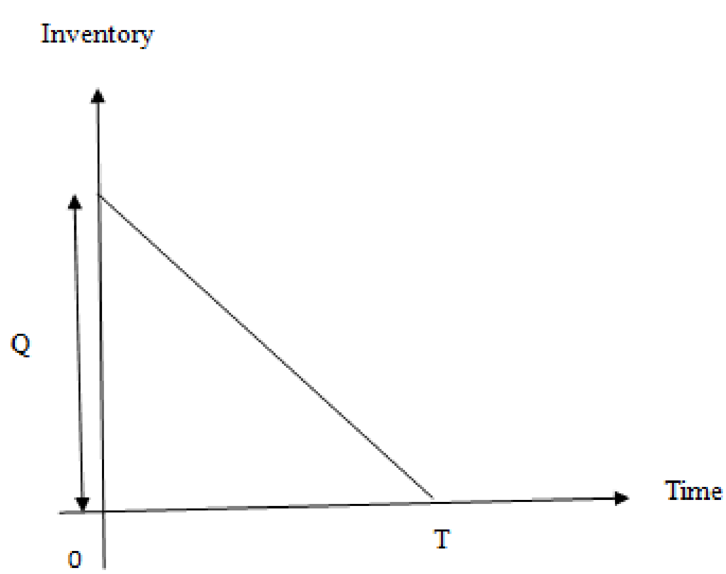

Figure 1. Diagram demonstration of the inventory level

The solution of equation (1) with boundary condition $I(T)=0$ at $t=T$ is given by,

$$I(t)=e^{-\frac{\beta}{\alpha}e^{\alpha t}}\left\{\begin{array}{l} a(T-t)+\frac{b}{2}\left(T^2-t^2\right)+\frac{c}{3}\left(T^3-t^3\right)+\frac{a\beta}{\alpha^2}\left(e^{\alpha T}-e^{\alpha t}\right)+\frac{b\beta}{\alpha^2}\left(T e^{\alpha T}-t e^{\alpha t}\right) \\ -\frac{b\beta}{\alpha^3}\left(e^{\alpha T}-e^{\alpha t}\right)+\frac{c\beta}{\alpha^2}\left(T^2 e^{\alpha T}-t^2 e^{\alpha t}\right)-\frac{2c\beta}{\alpha^3}\left(T e^{\alpha T}-t e^{\alpha t}\right) \\ +\frac{2c\beta}{\alpha^4}\left(e^{\alpha T}-e^{\alpha t}\right) \end{array}\right\}, \quad 0\le t\le T \qquad (2)$$

(by omitting the excessive order terms of β as $0<\beta<1$)

The maximum positive inventory level for each cycle can be obtained from $I(0)=Q$ in equation (2) is given by,

$$\begin{aligned} Q=I(0)=aT&+\frac{b}{2}T^2+\frac{c}{3}T^3+\frac{a\beta}{\alpha^2}e^{\alpha T}-\frac{a\beta}{\alpha^2}+\frac{b\beta}{\alpha^2}T e^{\alpha T}-\frac{b\beta}{\alpha^3}e^{\alpha T}+\frac{b\beta}{\alpha^3}+\frac{c\beta}{\alpha^2}T^2 e^{\alpha T}-\frac{2c\beta}{\alpha^3}T e^{\alpha T} \\ &+\frac{2c\beta}{\alpha^4}e^{\alpha T}-\frac{2c\beta}{\alpha^4}-\frac{a\beta}{\alpha}T-\frac{b\beta}{2\alpha}T^2-\frac{c\beta}{3\alpha}T^3-\frac{a\beta^2}{\alpha^3}e^{\alpha T}+\frac{a\beta^2}{\alpha^3}-\frac{b\beta^2}{\alpha^3}T e^{\alpha T}+\frac{b\beta^2}{\alpha^4}e^{\alpha T} \qquad (3) \\ &-\frac{b\beta^2}{\alpha^4}-\frac{c\beta^2}{\alpha^3}T^2 e^{\alpha T}+\frac{2c\beta^2}{\alpha^4}T e^{\alpha T}-\frac{2c\beta^2}{\alpha^5}e^{\alpha T}+\frac{2c\beta^2}{\alpha^5} \end{aligned}$$

Total revenue inventory cost per unit time consists of the following costs:

1. Ordering cost = A
2. Carrying cost per unit time is given by,

$$HC=h\int_0^T I(t)dt$$

$$=h\left\{\begin{array}{l} \frac{a}{2}T^2+\frac{b}{3}T^3+\frac{c}{4}T^4+\frac{a\beta}{\alpha^2}T e^{\alpha T}-\frac{2a\beta}{\alpha^3}e^{\alpha T}+\frac{2a\beta}{\alpha^3}+\frac{b\beta}{\alpha^2}T^2 e^{\alpha T}-\frac{3b\beta}{\alpha^3}T e^{\alpha T} \\ +\frac{3b\beta}{\alpha^4}e^{\alpha T}-\frac{3b\beta}{\alpha^4}+\frac{c\beta}{\alpha^2}T^3 e^{\alpha T}-\frac{4c\beta}{\alpha^3}T^2 e^{\alpha T}+\frac{8c\beta}{\alpha^4}T e^{\alpha T}-\frac{8c\beta}{\alpha^5}e^{\alpha T}+\frac{8c\beta}{\alpha^5} \\ +\frac{a\beta}{\alpha^2}T+\frac{b\beta}{2\alpha^2}T^2+\frac{c\beta}{3\alpha^2}T^3-\frac{a\beta^2}{2\alpha^4}e^{2\alpha T}+\frac{a\beta^2}{\alpha^4}e^{\alpha T}-\frac{a\beta^2}{2\alpha^4}-\frac{b\beta^2}{2\alpha^4}T e^{2\alpha T}+\frac{b\beta^2}{\alpha^4}T e^{\alpha T} \\ +\frac{b\beta^2}{4\alpha^5}e^{2\alpha T}+\frac{3b\beta^2}{4\alpha^5}-\frac{b\beta^2}{\alpha^5}e^{\alpha T}-\frac{c\beta^2}{2\alpha^4}T^2 e^{2\alpha T}+\frac{c\beta^2}{\alpha^4}T^2 e^{\alpha T}+\frac{c\beta^2}{2\alpha^5}T e^{2\alpha T}-\frac{c\beta^2}{4\alpha^6}e^{2\alpha T} \\ -\frac{7c\beta^2}{4\alpha^6}-\frac{2c\beta^2}{\alpha^5}T e^{\alpha T}+\frac{2c\beta^2}{\alpha^6}e^{\alpha T} \end{array}\right\} \qquad (4)$$

3. The number of deteriorating units (NDU) is given by,

$$DC = Q - \int_0^T D(t)dt, \quad \text{where } D(t) = a + bt + ct^2 \tag{5}$$

4. Deteriorating cost per unit time is given by,

$$DC = C_1 \left[Q - \int_0^T D(t)dt \right]$$

$$= C_1 \left\{ \begin{array}{l} \frac{a\beta}{\alpha^2} e^{\alpha T} - \frac{a\beta}{\alpha^2} + \frac{b\beta}{\alpha^2} T e^{\alpha T} - \frac{b\beta}{\alpha^3} e^{\alpha T} + \frac{b\beta}{\alpha^3} + \frac{c\beta}{\alpha^2} T^2 e^{\alpha T} - \frac{2c\beta}{\alpha^3} T e^{\alpha T} \\ + \frac{2c\beta}{\alpha^4} e^{\alpha T} - \frac{2c\beta}{\alpha^4} - \frac{a\beta}{\alpha} T - \frac{b\beta}{2\alpha} T^2 - \frac{c\beta}{3\alpha} T^3 - \frac{a\beta^2}{\alpha^3} e^{\alpha T} + \frac{a\beta^2}{\alpha^3} \\ - \frac{b\beta^2}{\alpha^3} T e^{\alpha T} + \frac{b\beta^2}{\alpha^4} e^{\alpha T} - \frac{b\beta^2}{\alpha^4} - \frac{c\beta^2}{\alpha^3} T^2 e^{\alpha T} + \frac{2c\beta^2}{\alpha^4} T e^{\alpha T} \\ - \frac{2c\beta^2}{\alpha^5} e^{\alpha T} + \frac{2c\beta^2}{\alpha^5} \end{array} \right\} \tag{6}$$

5. Sales revenue = $S \int_0^T (a + bt + ct^2)\, dt$

$$= S\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} \right) \tag{7}$$

The total profit per unit time is,

$$TP = \frac{1}{T}\left\{ S\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} \right) \right\} - \frac{1}{T}\left[Ordering \;\; Cost + Carrying \;\; Cost + Deterioration \;\; Cost \right]$$

$$= S\left(a+\frac{bT}{2}+\frac{cT^2}{3}\right)-\frac{1}{T}\left\{\begin{array}{l} A+h\left\{\begin{array}{l} \frac{a}{2}T^2+\frac{b}{3}T^3+\frac{c}{4}T^4+\frac{a\beta}{\alpha^2}T\,e^{\alpha T}-\frac{2a\beta}{\alpha^3}e^{\alpha T}+\frac{2a\beta}{\alpha^3} \\ +\frac{b\beta}{\alpha^2}T^2e^{\alpha T}-\frac{3b\beta}{\alpha^3}T\,e^{\alpha T}+\frac{3b\beta}{\alpha^4}e^{\alpha T}-\frac{3b\beta}{\alpha^4}+\frac{c\beta}{\alpha^2}T^3e^{\alpha T} \\ -\frac{4c\beta}{\alpha^3}T^2e^{\alpha T}+\frac{8c\beta}{\alpha^4}T\,e^{\alpha T}-\frac{8c\beta}{\alpha^5}e^{\alpha T}+\frac{8c\beta}{\alpha^5}+\frac{a\beta}{\alpha^2}T \\ +\frac{b\beta}{2\alpha^2}T^2+\frac{c\beta}{3\alpha^2}T^3-\frac{a\beta^2}{2\alpha^4}e^{2\alpha T}+\frac{a\beta^2}{\alpha^4}e^{\alpha T}-\frac{a\beta^2}{2\alpha^4} \\ -\frac{b\beta^2}{2\alpha^4}T\,e^{2\alpha T}+\frac{b\beta^2}{\alpha^4}T\,e^{\alpha T}+\frac{b\beta^2}{4\alpha^5}e^{2\alpha T}+\frac{3b\beta^2}{4\alpha^5}-\frac{b\beta^2}{\alpha^5}e^{\alpha T} \\ -\frac{c\beta^2}{2\alpha^4}T^2e^{2\alpha T}+\frac{c\beta^2}{\alpha^4}T^2e^{\alpha T}+\frac{c\beta^2}{2\alpha^5}T\,e^{2\alpha T}-\frac{c\beta^2}{4\alpha^6}e^{2\alpha T} \\ -\frac{7c\beta^2}{4\alpha^6}-\frac{2c\beta^2}{\alpha^5}T\,e^{\alpha T}+\frac{2c\beta^2}{\alpha^6}e^{\alpha T} \end{array}\right\} \\ +C_1\left\{\begin{array}{l} \frac{a\beta}{\alpha^2}e^{\alpha T}-\frac{a\beta}{\alpha^2}+\frac{b\beta}{\alpha^2}T\,e^{\alpha T}-\frac{b\beta}{\alpha^3}e^{\alpha T}+\frac{b\beta}{\alpha^3}+\frac{c\beta}{\alpha^2}T^2e^{\alpha T} \\ -\frac{2c\beta}{\alpha^3}T\,e^{\alpha T}+\frac{2c\beta}{\alpha^4}e^{\alpha T}-\frac{2c\beta}{\alpha^4}-\frac{a\beta}{\alpha}T-\frac{b\beta}{2\alpha}T^2-\frac{c\beta}{3\alpha}T^3 \\ -\frac{a\beta^2}{\alpha^3}e^{\alpha T}+\frac{a\beta^2}{\alpha^3}-\frac{b\beta^2}{\alpha^3}T\,e^{\alpha T}+\frac{b\beta^2}{\alpha^4}e^{\alpha T}-\frac{b\beta^2}{\alpha^4} \\ -\frac{c\beta^2}{\alpha^3}T^2e^{\alpha T}+\frac{2c\beta^2}{\alpha^4}T\,e^{\alpha T}-\frac{2c\beta^2}{\alpha^5}e^{\alpha T}+\frac{2c\beta^2}{\alpha^5} \end{array}\right\} \end{array}\right\} \quad (8)$$

Our goal is to maximize the total profit per unit time. Necessary conditions for total variable inventory cost to be maximized are $(i)\ \frac{\partial TP}{\partial T}=0$ and $(ii)\ \frac{\partial^2 TP}{\partial^2 T}<0.$

$$\frac{\partial TP}{\partial T}=-\frac{1}{T}\left\{\begin{array}{l} h\left\{\begin{array}{l} aT+bT^{2}+cT^{3}+\frac{a\beta}{\alpha}T\,e^{\alpha T}-\frac{a\beta}{\alpha^{2}}e^{\alpha T}+\frac{b\beta}{\alpha}T^{2}\,e^{\alpha T} \\ -\frac{b\beta}{\alpha^{2}}T\,e^{\alpha T}+\frac{c\beta}{\alpha}T^{3}e^{\alpha T}-\frac{c\beta}{\alpha^{2}}T^{2}\,e^{\alpha T}+\frac{a\beta}{\alpha^{2}}+\frac{b\beta}{\alpha^{2}}T \\ +\frac{c\beta}{\alpha^{2}}T^{2}-\frac{a\beta^{2}}{\alpha^{3}}e^{2\alpha T}+\frac{a\beta}{\alpha^{3}}e^{\alpha T}-\frac{b\beta^{2}}{\alpha^{3}}Te^{2\alpha T}+\frac{b\beta^{2}}{\alpha^{3}}T\,e^{\alpha T} \\ -\frac{c\beta^{2}}{\alpha^{3}}T^{2}\,e^{2\alpha T}+\frac{c\beta^{2}}{\alpha^{3}}T^{2}\,e^{\alpha T} \end{array}\right\} \\ +C_{1}\left\{\begin{array}{l} \frac{a\beta}{\alpha}e^{\alpha T}+\frac{b\beta}{\alpha}T\,e^{\alpha T}+\frac{c\beta}{\alpha}T^{2}\,e^{\alpha T}-\frac{a\beta}{\alpha}-\frac{b\beta}{\alpha}T-\frac{c\beta}{\alpha}T^{2} \\ -\frac{a\beta^{2}}{\alpha^{2}}e^{\alpha T}-\frac{b\beta^{2}}{\alpha^{2}}T\,e^{\alpha T}-\frac{c\beta^{2}}{\alpha^{2}}T^{2}\,e^{\alpha T} \end{array}\right\} \end{array}\right\} \quad (9)$$

$$\frac{\partial^{2} TP}{\partial T^{2}}=-\frac{1}{T}\left\{\begin{array}{l} h\left\{\begin{array}{l} a+2bT+3cT^{2}+a\beta T\,e^{\alpha T}+b\beta T^{2}\,e^{\alpha T}+\frac{b\beta}{\alpha}T\,e^{\alpha T}-\frac{b\beta}{\alpha^{2}}e^{\alpha T} \\ +c\beta T^{3}\,e^{\alpha T}+\frac{2c\beta}{\alpha}T^{2}\,e^{\alpha T}-\frac{2c\beta}{\alpha^{2}}T\,e^{\alpha T}+\frac{b\beta}{\alpha^{2}}+\frac{2c\beta}{\alpha^{2}}T-\frac{2a\beta^{2}}{\alpha^{2}}e^{2\alpha T} \\ +\frac{a\beta^{2}}{\alpha^{2}}e^{\alpha T}-\frac{2b\beta^{2}}{\alpha^{2}}T\,e^{2\alpha T}-\frac{b\beta^{2}}{\alpha^{3}}e^{2\alpha T}+\frac{b\beta^{2}}{\alpha^{2}}T\,e^{\alpha T} \\ +\frac{b\beta^{2}}{\alpha^{3}}e^{\alpha T}-\frac{2c\beta^{2}}{\alpha^{2}}T^{2}e^{2\alpha T}-\frac{2c\beta^{2}}{\alpha^{3}}T\,e^{2\alpha T}+\frac{c\beta^{2}}{\alpha^{2}}T^{2}\,e^{\alpha T}+\frac{2c\beta^{2}}{\alpha^{3}}T\,e^{\alpha T} \end{array}\right\} \\ +C_{1}\left\{\begin{array}{l} a\beta\,e^{\alpha T}+b\beta Te^{\alpha T}+\frac{b\beta}{\alpha}e^{\alpha T}+c\beta\,T^{2}\,e^{\alpha T}+\frac{2c\beta}{\alpha}T\,e^{\alpha T}-\frac{b\beta}{\alpha} \\ -\frac{2c\beta}{\alpha}T-\frac{a\beta^{2}}{\alpha}e^{\alpha T}-\frac{b\beta^{2}}{\alpha}T\,e^{\alpha T}-\frac{b\beta^{2}}{\alpha^{2}}e^{\alpha T}-\frac{c\beta^{2}}{\alpha}T^{2}\,e^{\alpha T}-\frac{2c\beta^{2}}{\alpha^{2}}Te^{\alpha T} \end{array}\right\} \end{array}\right\}<0 \quad (10)$$

The optimal value of T can be obtained by using condition *(i)*. Condition *(ii)* is also satisfied for the value of T obtained from condition *(i)*. The value of T is used to find the optimal values of Q and TP. Since equation *(i)* is non-linear it is solved by using appropriate software.

6. FUZZY MODEL

Let us consider the inventory model in fuzzy environment due to deterioration cost, holding cost and the parameters α, β.

Le $\tilde{C}_1=(C_{11}, C_{12}, C_{13})$, $\tilde{h}=(h_1, h_2, h_3)$, $\tilde{\alpha}=(\alpha_1, \alpha_2, \alpha_3)$ and $\tilde{\beta}=(\beta_1, \beta_2, \beta_3)$ are triangular fuzzy numbers then the total cost per unit time in fuzzy sense is

$$\tilde{TP} = S\left(a + \frac{bT}{2} + \frac{cT^2}{3}\right) - \frac{1}{T}\left\{\begin{array}{l} A + \tilde{h}\left\{\begin{array}{l} \frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{c}{4}T^4 + \frac{a\tilde{\beta}}{\tilde{\alpha}^2}T\,e^{\tilde{\alpha}T} - \frac{2a\tilde{\beta}}{\tilde{\alpha}^3}e^{\tilde{\alpha}T} + \frac{2a\tilde{\beta}}{\tilde{\alpha}^3} \\ + \frac{b\tilde{\beta}}{\tilde{\alpha}^2}T^2 e^{\tilde{\alpha}T} - \frac{3b\tilde{\beta}}{\tilde{\alpha}^3}T\,e^{\tilde{\alpha}T} + \frac{3b\tilde{\beta}}{\tilde{\alpha}^4}e^{\tilde{\alpha}T} - \frac{3b\tilde{\beta}}{\tilde{\alpha}^4} + \frac{c\tilde{\beta}}{\tilde{\alpha}^2}T^3 e^{\tilde{\alpha}T} \\ - \frac{4c\tilde{\beta}}{\tilde{\alpha}^3}T^2 e^{\tilde{\alpha}T} + \frac{8c\tilde{\beta}}{\tilde{\alpha}^4}T\,e^{\tilde{\alpha}T} - \frac{8c\tilde{\beta}}{\tilde{\alpha}^5}e^{\tilde{\alpha}T} + \frac{8c\tilde{\beta}}{\tilde{\alpha}^5} + \frac{a\tilde{\beta}}{\tilde{\alpha}^2}T \\ + \frac{b\tilde{\beta}}{2\tilde{\alpha}^2}T^2 + \frac{c\tilde{\beta}}{3\tilde{\alpha}^2}T^3 - \frac{a\tilde{\beta}^2}{2\tilde{\alpha}^4}e^{2\tilde{\alpha}T} + \frac{a\tilde{\beta}^2}{\tilde{\alpha}^4}e^{\tilde{\alpha}T} - \frac{a\tilde{\beta}^2}{2\tilde{\alpha}^4} \\ - \frac{b\tilde{\beta}^2}{2\tilde{\alpha}^4}T\,e^{2\tilde{\alpha}T} + \frac{b\tilde{\beta}^2}{\tilde{\alpha}^4}T\,e^{\tilde{\alpha}T} + \frac{b\tilde{\beta}^2}{4\tilde{\alpha}^5}e^{2\tilde{\alpha}T} + \frac{3b\tilde{\beta}^2}{4\tilde{\alpha}^5} - \frac{b\tilde{\beta}^2}{\tilde{\alpha}^5}e^{\tilde{\alpha}T} \\ - \frac{c\tilde{\beta}^2}{2\tilde{\alpha}^4}T^2 e^{2\tilde{\alpha}T} + \frac{c\tilde{\beta}^2}{\tilde{\alpha}^4}T^2 e^{\tilde{\alpha}T} + \frac{c\tilde{\beta}^2}{2\tilde{\alpha}^5}T\,e^{2\tilde{\alpha}T} - \frac{c\tilde{\beta}^2}{4\tilde{\alpha}^6}e^{2\tilde{\alpha}T} \\ - \frac{7c\tilde{\beta}^2}{4\tilde{\alpha}^6} - \frac{2c\tilde{\beta}^2}{\tilde{\alpha}^5}T\,e^{\tilde{\alpha}T} + \frac{2c\tilde{\beta}^2}{\tilde{\alpha}^6}e^{\tilde{\alpha}T} \end{array}\right\} \\ + \tilde{C}_1\left\{\begin{array}{l} \frac{a\tilde{\beta}}{\tilde{\alpha}^2}e^{\tilde{\alpha}T} - \frac{a\tilde{\beta}}{\tilde{\alpha}^2} + \frac{b\tilde{\beta}}{\tilde{\alpha}^2}T\,e^{\tilde{\alpha}T} - \frac{b\tilde{\beta}}{\tilde{\alpha}^3}e^{\tilde{\alpha}T} + \frac{b\tilde{\beta}}{\tilde{\alpha}^3} + \frac{c\tilde{\beta}}{\tilde{\alpha}^2}T^2 e^{\tilde{\alpha}T} \\ - \frac{2c\tilde{\beta}}{\tilde{\alpha}^3}T\,e^{\tilde{\alpha}T} + \frac{2c\tilde{\beta}}{\tilde{\alpha}^4}e^{\tilde{\alpha}T} - \frac{2c\tilde{\beta}}{\tilde{\alpha}^4} - \frac{a\tilde{\beta}}{\tilde{\alpha}}T - \frac{b\tilde{\beta}}{2\tilde{\alpha}}T^2 - \frac{c\tilde{\beta}}{3\tilde{\alpha}}T^3 \\ - \frac{a\tilde{\beta}^2}{\tilde{\alpha}^3}e^{\tilde{\alpha}T} + \frac{a\tilde{\beta}^2}{\tilde{\alpha}^3} - \frac{b\tilde{\beta}^2}{\tilde{\alpha}^3}T\,e^{\tilde{\alpha}T} + \frac{b\tilde{\beta}^2}{\tilde{\alpha}^4}e^{\tilde{\alpha}T} - \frac{b\tilde{\beta}^2}{\tilde{\alpha}^4} \\ - \frac{c\tilde{\beta}^2}{\tilde{\alpha}^3}T^2 e^{\tilde{\alpha}T} + \frac{2c\tilde{\beta}^2}{\tilde{\alpha}^4}T\,e^{\tilde{\alpha}T} - \frac{2c\tilde{\beta}^2}{\tilde{\alpha}^5}e^{\tilde{\alpha}T} + \frac{2c\tilde{\beta}^2}{\tilde{\alpha}^5} \end{array}\right\} \end{array}\right\} \qquad (11)$$

Now defuzzify the total cost $\tilde{TP}$ by using Pascal triangular method, we have

$$TP_{dp} = \frac{1}{4}\left\{TP_{dp_1} + 2\,TP_{dp_2} + TP_{dp_3}\right\} \qquad (12)$$

$$
\widetilde{TP} = S\left(a+\frac{bT}{2}+\frac{cT^2}{3}\right)-\frac{1}{T}\left\{
\begin{array}{l}
A+\tilde{h}_1\left\{
\begin{array}{l}
\frac{a}{2}T^2+\frac{b}{3}T^3+\frac{c}{4}T^4+\frac{a\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}T\,e^{\tilde{\alpha}_1 T}-\frac{2a\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}e^{\tilde{\alpha}_1 T}+\frac{2a\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}\\
+\frac{b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}T^2 e^{\tilde{\alpha}_1 T}-\frac{3b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}T\,e^{\tilde{\alpha}_1 T}+\frac{3b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,4}}e^{\tilde{\alpha}_1 T}-\frac{3b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,4}}+\frac{c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}T^3 e^{\tilde{\alpha}_1 T}\\
-\frac{4c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}T^2 e^{\tilde{\alpha}_1 T}+\frac{8c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,4}}T\,e^{\tilde{\alpha}_1 T}-\frac{8c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,5}}e^{\tilde{\alpha}_1 T}+\frac{8c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,5}}+\frac{a\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}T\\
+\frac{b\tilde{\beta}_1}{2\tilde{\alpha}_1^{\,2}}T^2+\frac{c\tilde{\beta}_1}{3\tilde{\alpha}_1^{\,2}}T^3-\frac{a\tilde{\beta}_1^{\,2}}{2\tilde{\alpha}_1^{\,4}}e^{2\tilde{\alpha}_1 T}+\frac{a\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,4}}e^{\tilde{\alpha}_1 T}-\frac{a\tilde{\beta}_1^{\,2}}{2\tilde{\alpha}_1^{\,4}}\\
-\frac{b\tilde{\beta}_1^{\,2}}{2\tilde{\alpha}_1^{\,4}}T\,e^{2\tilde{\alpha}_1 T}+\frac{b\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,4}}T\,e^{\tilde{\alpha}_1 T}+\frac{b\tilde{\beta}_1^{\,2}}{4\tilde{\alpha}_1^{\,5}}e^{2\tilde{\alpha}_1 T}+\frac{3b\tilde{\beta}_1^{\,2}}{4\tilde{\alpha}_1^{\,5}}-\frac{b\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,5}}e^{\tilde{\alpha}_1 T}\\
-\frac{c\tilde{\beta}_1^{\,2}}{2\tilde{\alpha}_1^{\,4}}T^2 e^{2\tilde{\alpha}_1 T}+\frac{c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,4}}T^2 e^{\tilde{\alpha}_1 T}+\frac{c\tilde{\beta}_1^{\,2}}{2\tilde{\alpha}_1^{\,5}}T\,e^{2\tilde{\alpha}_1 T}-\frac{c\tilde{\beta}_1^{\,2}}{4\tilde{\alpha}_1^{\,6}}e^{2\tilde{\alpha}_1 T}\\
-\frac{7c\tilde{\beta}_1^{\,2}}{4\tilde{\alpha}_1^{\,6}}-\frac{2c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,5}}T\,e^{\tilde{\alpha}_1 T}+\frac{2c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,6}}e^{\tilde{\alpha}_1 T}
\end{array}\right\}\\
+\tilde{C}_1\left\{
\begin{array}{l}
\frac{a\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}e^{\tilde{\alpha}_1 T}-\frac{a\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}+\frac{b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}T\,e^{\tilde{\alpha}_1 T}-\frac{b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}e^{\tilde{\alpha}_1 T}+\frac{b\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}+\frac{c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,2}}T^2 e^{\tilde{\alpha}_1 T}\\
-\frac{2c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,3}}T\,e^{\tilde{\alpha}_1 T}+\frac{2c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,4}}e^{\tilde{\alpha}_1 T}-\frac{2c\tilde{\beta}_1}{\tilde{\alpha}_1^{\,4}}-\frac{a\tilde{\beta}_1}{\tilde{\alpha}_1}T-\frac{b\tilde{\beta}_1}{2\tilde{\alpha}_1}T^2-\frac{c\tilde{\beta}_1}{3\tilde{\alpha}_1}T^3\\
-\frac{a\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,3}}e^{\tilde{\alpha}_1 T}+\frac{a\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,3}}-\frac{b\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,3}}T\,e^{\tilde{\alpha}_1 T}+\frac{b\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,4}}e^{\tilde{\alpha}_1 T}-\frac{b\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,4}}\\
-\frac{c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,3}}T^2 e^{\tilde{\alpha}_1 T}+\frac{2c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,4}}T\,e^{\tilde{\alpha}_1 T}-\frac{2c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,5}}e^{\tilde{\alpha}_1 T}+\frac{2c\tilde{\beta}_1^{\,2}}{\tilde{\alpha}_1^{\,5}}
\end{array}\right\}
\end{array}\right\} \qquad (13)
$$

$$
\begin{aligned}
T\tilde{P}_{dp_2} = S\left(a+\frac{bT}{2}+\frac{cT^2}{3}\right)-\frac{1}{T}\Bigg\{ & A+\tilde{h}_2\Bigg\{ \frac{a}{2}T^2+\frac{b}{3}T^3+\frac{c}{4}T^4+\frac{a\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}T\,e^{\tilde{\alpha}_2 T}-\frac{2a\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}}e^{\tilde{\alpha}_2 T}+\frac{2a\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}} \\
& +\frac{b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}T^2\,e^{\tilde{\alpha}_2 T}-\frac{3b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}}T\,e^{\tilde{\alpha}_2 T}+\frac{3b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,4}}e^{\tilde{\alpha}_2 T}-\frac{3b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,4}}+\frac{c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}T^3\,e^{\tilde{\alpha}_2 T} \\
& -\frac{4c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}}T^2\,e^{\tilde{\alpha}_2 T}+\frac{8c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,4}}T\,e^{\tilde{\alpha}_2 T}-\frac{8c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,5}}e^{\tilde{\alpha}_2 T}+\frac{8c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,5}}+\frac{a\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}T \\
& +\frac{b\tilde{\beta}_2}{2\tilde{\alpha}_2^{\,2}}T^2+\frac{c\tilde{\beta}_2}{3\tilde{\alpha}_2^{\,2}}T^3-\frac{a\tilde{\beta}_2^{\,2}}{2\tilde{\alpha}_2^{\,4}}e^{2\tilde{\alpha}_2 T}+\frac{a\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,4}}e^{\tilde{\alpha}_2 T}-\frac{a\tilde{\beta}_2^{\,2}}{2\tilde{\alpha}_2^{\,4}} \\
& -\frac{b\tilde{\beta}_2^{\,2}}{2\tilde{\alpha}_2^{\,4}}T\,e^{2\tilde{\alpha}_2 T}+\frac{b\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,4}}T\,e^{\tilde{\alpha}_2 T}+\frac{b\tilde{\beta}_2^{\,2}}{4\tilde{\alpha}_2^{\,5}}e^{2\tilde{\alpha}_2 T}+\frac{3b\tilde{\beta}_2^{\,2}}{4\tilde{\alpha}_2^{\,5}}-\frac{b\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,5}}e^{\tilde{\alpha}_2 T} \\
& -\frac{c\tilde{\beta}_2^{\,2}}{2\tilde{\alpha}_2^{\,4}}T^2\,e^{2\tilde{\alpha}_2 T}+\frac{c\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,4}}T^2\,e^{\tilde{\alpha}_2 T}+\frac{c\tilde{\beta}_2^{\,2}}{2\tilde{\alpha}_2^{\,5}}T\,e^{2\tilde{\alpha}_2 T}-\frac{c\tilde{\beta}_2^{\,2}}{4\tilde{\alpha}_2^{\,6}}e^{2\tilde{\alpha}_2 T} \\
& -\frac{7c\tilde{\beta}_2^{\,2}}{4\tilde{\alpha}_2^{\,6}}-\frac{2c\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,5}}T\,e^{\tilde{\alpha}_2 T}+\frac{2c\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,6}}e^{\tilde{\alpha}_2 T}\Bigg\} \\
& +\tilde{C}_{12}\Bigg\{ \frac{a\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}e^{\tilde{\alpha}_2 T}-\frac{a\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}+\frac{b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}T\,e^{\tilde{\alpha}_2 T}-\frac{b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}}e^{\tilde{\alpha}_2 T}+\frac{b\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}}+\frac{c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,2}}T^2\,e^{\tilde{\alpha}_2 T} \\
& -\frac{2c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,3}}T\,e^{\tilde{\alpha}_2 T}+\frac{2c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,4}}e^{\tilde{\alpha}_2 T}-\frac{2c\tilde{\beta}_2}{\tilde{\alpha}_2^{\,4}}-\frac{a\tilde{\beta}_2}{\tilde{\alpha}_2}T-\frac{b\tilde{\beta}_2}{2\tilde{\alpha}_2}T^2-\frac{c\tilde{\beta}_2}{3\tilde{\alpha}_2}T^3 \\
& -\frac{a\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,3}}e^{\tilde{\alpha}_2 T}+\frac{a\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,3}}-\frac{b\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,3}}T\,e^{\tilde{\alpha}_2 T}+\frac{b\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,4}}e^{\tilde{\alpha}_2 T}-\frac{b\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,4}} \\
& -\frac{\tilde{c}_2\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,3}}T^2\,e^{\tilde{\alpha}_2 T}+\frac{2\tilde{c}_2\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,4}}T\,e^{\tilde{\alpha}_2 T}-\frac{2\tilde{c}_2\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,5}}e^{\tilde{\alpha}_2 T}+\frac{2\tilde{c}_2\tilde{\beta}_2^{\,2}}{\tilde{\alpha}_2^{\,5}}\Bigg\}\Bigg\}
\end{aligned}
\tag{14}
$$

$$
\tilde{TP}_{dp_3} = S\left(a + \frac{bT}{2} + \frac{cT^2}{3}\right) - \frac{1}{T}\left\{
\begin{aligned}
& A + \tilde{h}_3 \left\{
\begin{aligned}
& \frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{c}{4}T^4 + \frac{a\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} T e^{\tilde{\alpha}_3 T} - \frac{2a\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} e^{\tilde{\alpha}_3 T} + \frac{2a\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} \\
& + \frac{b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} T^2 e^{\tilde{\alpha}_3 T} - \frac{3b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} T e^{\tilde{\alpha}_3 T} + \frac{3b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,4}} e^{\tilde{\alpha}_3 T} - \frac{3b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,4}} + \frac{c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} T^3 e^{\tilde{\alpha}_3 T} \\
& - \frac{4c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} T^2 e^{\tilde{\alpha}_3 T} + \frac{8c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,4}} T e^{\tilde{\alpha}_3 T} - \frac{8c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,5}} e^{\tilde{\alpha}_3 T} + \frac{8c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,5}} + \frac{a\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} T \\
& + \frac{b\tilde{\beta}_3}{2\tilde{\alpha}_3^{\,2}} T^2 + \frac{c\tilde{\beta}_3}{3\tilde{\alpha}_3^{\,2}} T^3 - \frac{a\tilde{\beta}_3^{\,2}}{2\tilde{\alpha}_3^{\,4}} e^{2\tilde{\alpha}_3 T} + \frac{a\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,4}} e^{\tilde{\alpha}_3 T} - \frac{a\tilde{\beta}_3^{\,2}}{2\tilde{\alpha}_3^{\,4}} \\
& - \frac{b\tilde{\beta}_3^{\,2}}{2\tilde{\alpha}_3^{\,4}} T e^{2\tilde{\alpha}_3 T} + \frac{b\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,4}} T e^{\tilde{\alpha}_3 T} + \frac{b\tilde{\beta}_3^{\,2}}{4\tilde{\alpha}_3^{\,5}} e^{2\tilde{\alpha}_3 T} + \frac{3b\tilde{\beta}_3^{\,2}}{4\tilde{\alpha}_3^{\,5}} - \frac{b\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,5}} e^{\tilde{\alpha}_3 T} \\
& - \frac{c\tilde{\beta}_3^{\,2}}{2\tilde{\alpha}_3^{\,4}} T^2 e^{2\tilde{\alpha}_3 T} + \frac{c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,4}} T^2 e^{\tilde{\alpha}_3 T} + \frac{c\tilde{\beta}_3^{\,2}}{2\tilde{\alpha}_3^{\,5}} T e^{2\tilde{\alpha}_3 T} - \frac{c\tilde{\beta}_3^{\,2}}{4\tilde{\alpha}_3^{\,6}} e^{2\tilde{\alpha}_3 T} \\
& - \frac{7c\tilde{\beta}_3^{\,2}}{4\tilde{\alpha}_3^{\,6}} - \frac{2c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,5}} T e^{\tilde{\alpha}_3 T} + \frac{2c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,6}} e^{\tilde{\alpha}_3 T}
\end{aligned}
\right\} \\
& + \tilde{C}_{13} \left\{
\begin{aligned}
& \frac{a\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} e^{\tilde{\alpha}_3 T} - \frac{a\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} + \frac{b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} T e^{\tilde{\alpha}_3 T} - \frac{b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} e^{\tilde{\alpha}_3 T} + \frac{b\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} + \frac{c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,2}} T^2 e^{\tilde{\alpha}_3 T} \\
& - \frac{2c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,3}} T e^{\tilde{\alpha}_3 T} + \frac{2c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,4}} e^{\tilde{\alpha}_3 T} - \frac{2c\tilde{\beta}_3}{\tilde{\alpha}_3^{\,4}} - \frac{a\tilde{\beta}_3}{\tilde{\alpha}_3} T - \frac{b\tilde{\beta}_3}{2\tilde{\alpha}_3} T^2 - \frac{c\tilde{\beta}_3}{3\tilde{\alpha}_3} T^3 \\
& - \frac{a\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,3}} e^{\tilde{\alpha}_3 T} + \frac{a\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,3}} - \frac{b\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,3}} T e^{\tilde{\alpha}_3 T} + \frac{b\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,4}} e^{\tilde{\alpha}_3 T} - \frac{b\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,4}} \\
& - \frac{c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,3}} T^2 e^{\tilde{\alpha}_3 T} + \frac{2c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,4}} T e^{\tilde{\alpha}_3 T} - \frac{2c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,5}} e^{\tilde{\alpha}_3 T} + \frac{2c\tilde{\beta}_3^{\,2}}{\tilde{\alpha}_3^{\,5}}
\end{aligned}
\right\}
\end{aligned}
\right\} \tag{15}
$$

The necessary condition for TP_{dp} to be maximize is that (i) $\frac{\partial TP_{dp}}{\partial T} = 0$ and (ii) $\frac{\partial^2 TP_{dp}}{\partial^2 T} < 0$. The optimal value of T can be obtained by using condition *(i)*. Condition *(ii)* is also satisfied for the value of T obtained from condition *(i)*. The value of T is used to find the optimal values of Q and TP. Since equation *(i)* is non-linear it is solved by using appropriate software.

7. NUMERICAL ANALYSIS

7. 1. Example

Crisp Model

Assume suitable value for $A = 20,\ a = 60,\ b = 2,\ c = 5,\ C_1 = 2,\ \ h = 0.1,\ \alpha = 1, \beta = 0.2, S = 2$

With appropriated units. The optimal values are T = 0.17890 and TP = 11.13367, Q = 8.11681.

7. 2. Example

Fuzzy Model

Assume suitable value for $A = 20,\ a = 60,\ b = 2,\ c = 5,$ $\tilde{C}_1 = (2, 3, 4),\ \tilde{h} = (0.1, 0.2, 0.3),\ \tilde{\alpha} = (1, 2, 3),\ \tilde{\beta} = (0.2, 0.3, 0.5),\ S = 2$

With appropriated units. The optimal values are T = 0.09121 and $\tilde{TP}_{dp}$ = 4625.13647, Q = 4.65403.

8. SENSITIVITY ANALYSIS FOR FUZZY MODEL AND CRISP MODEL

Table 1. Sensitivity Analysis on parameter $\tilde{C}_1$.

$\tilde{C}_1$	T	Q	$\tilde{TP}_{dp}$
(2, 3, 4)	0.19028	8.77322	18.94641
(3, 4, 5)	0.19813	9.22703	24.49464
(4, 5, 6)	0.20291	9.50338	28.16025
(5, 6, 7)	0.20613	9.69049	30.97128
(6, 7, 8)	0.20847	9.82585	33.31656

Table 2. Sensitivity Analysis on parameter C_1

C_1	T	Q	TP
3	0.19171	8.85576	19.89050
4	0.19878	9.26453	24.87095
5	0.20325	9.52331	28.34614
6	0.20634	9.70236	31.07777
7	0.20860	9.83393	33.38342

Table 3. Sensitivity Analysis on parameter $\tilde{h}$

$\tilde{h}$	T	Q	$\tilde{TP}_{dp}$
(0.01,0.02,0.003)	0.21283	10.07907	29.06920
(0.02,0.03,0.04)	0.20797	9.76679	26.85083
(0.03,0.04,0.05)	0.2033	9.52628	24.62396
(0.04,0.05,0.06)	0.19882	9.26705	22.39053
(0.05,0.06,0.07)	0.19452	9.01836	20.14981

Table 4. Sensitivity Analysis on parameter h

h	T	Q	TP
0.01	0.21785	10.37044	31.28039
0.02	0.21278	10.07593	29.07129
0.03	0.20792	9.79396	26.85381
0.04	0.20325	9.52331	24.62439
0.05	0.19878	9.26453	22.39324

Table 5. Sensitivity Analysis on parameter $\tilde{\alpha}$

$\tilde{\alpha}$	T	Q	$\tilde{TP}_{dp}$
(1.1,1.2,1.3)	0.12306	6.11148	-38.28790
(1.2,1.3,1.4)	0.10384	5.32277	-64.63180
(1.3,1.4,1.5)	0.08882	4.66384	-91.48189
(1.4,1.5,1.6)	0.07686	4.11217	-118.03164
(1.5,1.6,1.7)	0.06716	3.64800	-143.22321

Table 6. Sensitivity Analysis on parameter α

α	T	Q	TP
1.1	0.14613	7.00952	-12.65335
1.2	0.12163	6.07297	-38.00392
1.3	0.10283	5.29045	-64.49040
1.4	0.08808	4.63719	-91.54247
1.5	0.07630	4.09052	-118.35223

Table 7. Sensitivity Analysis on parameter $\tilde{\beta}$

$\tilde{\beta}$	T	Q	$\tilde{TP}_{dp}$
(0.2,0.3,0.4)	0.31480	13.60249	55.86797
(0.3,0.4,0.5)	0.4724	19.14375	87.02601
(0.4,0.5,0.6)	0.67126	24.69763	108.39888
(0.5,0.6,0.7)	0.96375	31.26237	127.88494

Table 8. Sensitivity Analysis on parameter β

β	T	Q	TP
0.2	0.17890	8.11681	11.13367
0.3	0.30856	13.56809	61.77936
0.4	0.46319	19.15696	88.77949
0.5	0.65466	24.69298	108.76571
0.6	0.91251	30.24758	127.28462

9. OBSERVATIONS

Based on the results we can make the following conclusions:

1. From Table 1, we can observe that the parameter $\tilde{C}_1$ increases, T and Q increases while $\tilde{TP}_{dp}$ increases.
2. From Table 2, we can observe that the parameter C_1 increases, T and Q increases while TP increases.
3. From Table 3, we can observe that the parameter $\tilde{h}$ increases, T and Q decreases while $\tilde{TP}_{dp}$ decreases.
4. From Table 4, we can observe that the parameter h increases, T and Q decreases while TP decreases.
5. From Table 5, we can observe that the parameter $\tilde{\alpha}$ increases, T and Q decreases while $\tilde{TP}_{dp}$ increases.
6. From Table 6, we can observe that the parameter α increases, T and Q decreases while TP increases.
7. From Table 7, we can observe that the parameter $\tilde{\beta}$ increases, T and Q increases while $\tilde{TP}_{dp}$ increases.
8. From Table 8, we can observe that the parameter β increases, T and Q increases while TP increase.

10. CONCLUSIONS

In the current financial world, a deterministic inventory model for deteriorating items involves Gompertz distribution deterioration rate and quadratic demand of time is improved and clarified in this work. Stock-out is not permitted. The deterioration cost, holding cost and the parameters α, β are assumed as a triangular fuzzy numbers. The aim of this economic order quantity model is to maximize the total profit of inventory optimization by using Pascal triangular method. Here we provide some numerical examples and sensitivity analysis are supported to demonstrate the solving methodology. For instance, this inventory model may be extended incorporating with various considerations like intuitionistic fuzzy sets, Trapezoidal fuzzy sets including shortages, price discount, quantity discount and others.

Acknowledgement

The writer thanks the editor and nameless reviewers for their precious and positive comments, that have brought about a sizable development in the manuscript.

References

[1] D. Dutta and Pavankumar (2013). Fuzzy inventory model for deteriorating items with shortages under fully backlogged condition. *International Journal of Soft Computing and Engineering*, Vol. 3, 393-398.

[2] M. Maragatham and P.K. Lakshmidevi (2014). A Fuzzy inventory model for deteriorating items with price dependent demand. *International Journal of Fuzzy Mathematical Archive,* Vol. 5, No. 1, 39-47

[3] Nurul Azeez Khan, V.S. Verma and Vijay Kumar (2017). An inventory model for Gompertz deteriorating items with time-varying holding cost and price dependent demand. *International Journal of Mathematics Trends and Technology,* Vol. 49, No.3, 183-187

[4] Pandit Jagatananda Mishra, Trailokyanath Singh and Hadi Bandhu Pattanayak (2016). An optimal policy with quadratic demand, three-parameter weibull distribution deterioration rate, Shortages and Salvage value. *American Journal of Computational Mathematics,* 6, 200-211

[5] Sahidul Islam and Abhishek Kanti Biswas (2017). A fuzzy inventory model having exponential demand with weibull distribution for non-instantaneous deterioration, shortages under partially backlogging and time dependent holding cost. *International Journals of Advanced Research in Computer Science and Software Engineering,* Vol. 7, 434-443

[6] H.S. Shukla, R.P. Tripathi, Suhilkumar Yadav and Vivek Shukla (2015). Inventory model for deteriorating items with quadratic time-dependent demand rate and composed shortages. *Journal of Applied Probability and Statistics,* Vol. 10, No. 2, 135-147

[7] J. Sujatha and P. Parvathi (2015). An EOQ model for weibull deteriorating items with linear demand and partial backlogging in fuzzy Environment. *International Journal of Computer Science and Mobile Computing,* Vol. 49, No. 3, 183-187

[8] Sujata Saha and Tripti Chakrabarti (2017). A Fuzzy inventory model for deteriorating items with linear price dependent demand in a supply chain. *International Journal of Fuzzy Mathematical Archive,* Vol. 13, No. 1, 59-67

[9] Sujata Saha (2017). Fuzzy inventory model for deteriorating items in a supply chain system with price dependent demand and without Backorder. *American Journal of Engineering Research,* Vol. 6, 183-187

[10] Sushil Kumar and U.S. Rajput (2015). Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. *Applied Mathematics,* 6, 496-509