An EOQ inventory model for Gompertz deteriorating items with quadratic demand and constant holding cost in a fuzzy environment

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ABSTRACT

In this paper we discussed an economic order quantity inventory model for deteriorating items with time-dependent demand rate. Here the demand is considered as a quadratic function of time and deteriorating items follow a Gompertz distribution deterioration. Stock out is not permitted and holding cost is constant. The deterioration cost, holding cost and the parameters $\alpha, \beta$ are assumed as a triangular fuzzy numbers. Our goal is to maximize the total profit of inventory optimization by using Pascal triangular method. A few numerical examples illustrate the implementation of the proposed model. Sensitivity analysis is the study of how to divide and allocate the uncertainty in the output of a mathematical model or system to different sources of input uncertainty.

Keywords: Inventory system, Deteriorating items, Gompertz distribution, Quadratic function, Triangular fuzzy number, Pascal triangular method

AMS Subject Classification: 90B05

1. LITERATURE REVIEW

An important aspect of any successful business is inventory management. It is the process of monitoring and controlling the flow of inventory units that a business uses for sale or distribution in the production or manufacture of goods. Inventories usually consist of
a combination of goods, raw materials and finished products, and efficient management of these items is critical to ensuring optimum stock levels and maximizing the company’s earning potential. It can be used to predict inventory and price levels, as well as the expected demand for the product. Dutta and Pavankumar (2013) developed a fuzzy inventory model for deteriorating items with fully backordered model by using signed distance method. Maragatham and Lakshmidevi (2014) expanded a fuzzy inventory model for deteriorating items with price dependent demand. He used the signed distance method for defuzzification process. Sensitivity analysis is presented for both crisp and fuzzy model. An inventory model for Gompertz deteriorating items with time-varying holding cost and price dependent demand was considered by Nurul Azeez Khan, Verma and Vijay kumar (2017). Pandit Jagatananda Mishra, Trailokyanath Singh and Hadi bandhu pattanayak (2016) presented an optimal policy of an inventory model for deteriorating items with generalized demand rate and deterioration rate. Shortages are allowed and partially back-ordered. The salvage value is included into deteriorated units. Sahidul Islam and Abhishek Kanti Biswas (2017) introduced deterministic economic order quantity inventory model with exponential demand rate, weibull distribution for deterioration and time dependent holding cost in crisp and fuzzy environment. Shukla, Tripathi, Suhilkumar Yadav and Vivek Shukla (2015) developed inventory model for deteriorating items with quadratic time dependent demand rate and composed shortages. Mathematical models are derived under two deterrent circumstances i.e., case I: The permissible delay period is less than or equal to time to finish positive inventory and case II: The permissible delay period is greater than time to finish positive inventory. Sujatha and Parvathi (2015) studied an EOQ inventory model for weibull deteriorating items with linear demand and partial back-ordering in fuzzy environment. Sujata Saha and Tripti Chakrabarti (2017) generated a supply chain production inventory model for deteriorating items under fuzzy environment. He considered demand as linear price dependent. The signed distance method and graded mean integration method have been used for defuzzification. Sujata Saha (2017) presented a fuzzy continuous review inventory model in a supply chain management system with price dependent demand. Signed distance method is used to defuzzify the cost function. Sushil kumar and Rajput (2015) expanded fuzzy inventory model for deteriorating items with time dependent demand and partial back-ordering. The back-ordering rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. In this paper we have considered demand as a quadratic function of time. Deterioration is permitted on inventory which follows Gompertz distributions. Stock out is not permitted. The deterioration cost, holding cost and the parameters $\alpha, \beta$ are assumed as a triangular fuzzy numbers. The model has been studied to maximize the total profit of inventory optimization by using Pascal triangular method. Numerical example and sensitivity analysis is performed to show the effect of changes in the parameters of the optimum solution.

2. PRELIMINARIES

When we consider the fuzzy inventory model, the following definitions are required.

(i) A fuzzy set $\tilde{A}$ on the given universal set $X$ is denoted and defined by

$$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)): x \in X\}$$
where $\lambda_{\tilde{A}} : X \rightarrow [0,1]$ is called the membership function, and $\lambda_{\tilde{A}}(x) = \text{degree of } x \text{ in } \lambda_{\tilde{A}}$.

(ii) A triangular fuzzy number is specified by the triplet $(a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined by its continuous membership function $\lambda_{\tilde{A}} : X \rightarrow [0,1]$ as follows

$$\lambda_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_2-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

(iii) If $A = (a_1, a_2, a_3)$ is a triangular fuzzy number then the Pascal triangular method of $\tilde{A}$ is defined as $P(\tilde{A},0) = \frac{1}{4}(a_1 + 2a_2 + a_3)$

(iv) A random variable $X$ is said to have a Gompertz distribution if its probability density function is $f_G(x) = \beta e^{ax} e^{-\beta(e^{ax}-1)}$; $x \geq 0$, $\alpha, \beta > 0$.

(v) Hazard rate function of Gompertz distribution is $h_G(x) = \beta e^{ax}$.

3. PREMISE

To elaborate on the proposing pattern, we allow the following premises:

1) The time between the initiation and completion of a production process is zero.
2) Time horizon is finite.
3) Stock out is not permitted.
4) The demand rate is a quadratic function of time, i.e., $D(t) = a + bt + ct^2$, where $a, b, c > 0$ and $a$ is the initial rate of demand, $b$ is the rate that decreases or increases the demand rate and $c$ is the rate at which the demand rate changing.
5) The deterioration rate follows a Gompertz distribution function is given by $\theta(t) = \beta e^{\alpha t}$, where $0 < \beta < 1$, $\alpha > 0$, $t > 0$.
6) Replenishment rate is infinite.

4. ENTRIES

- $A$ – Ordering cost per order
- $C_1$ – Deterioration cost per unit time
• $h$ – Carrying cost per unit time
• $T$ – The fixed length of each cycle
• $Q$ – The maximum inventory level for each ordering cycle
• $t$ – Time of deterioration, $t > 0$
• $TP$ – The total profit cost per unit time
• $I(t)$ – The level of inventory at any time $t$, $(0 \leq t \leq T)$
• $\alpha$ – Scale parameter
• $\beta$ – Shape parameter
• $S$ – Selling price
• $\tilde{C}_1$ – fuzzy deterioration cost per unit time
• $\tilde{\alpha}$ – Scale parameter in fuzzy sense
• $\tilde{\beta}$ – Shape parameter in fuzzy sense
• $\tilde{h}$ – fuzzy carrying cost per unit time
• $T\tilde{P}$ – The total fuzzy profit cost per unit time
• $TP_{dp}$ – is the defuzzify value of $T\tilde{P}$ by applying Pascal triangular method

5. MATHEMATICAL FORMULATION

In this model, a quadratic type of demand is studied with Gompertz distribution deterioration rate. During stock period $(0, T)$ is governed by the differential equations,

$$\frac{dI(t)}{dt} + \beta e^{\alpha t} I(t) = -(a + bt + ct^2), \quad 0 \leq t \leq T \quad (1)$$

Figure 1. Diagram demonstration of the inventory level
The solution of equation (1) with boundary condition $I(T) = 0$ at $t = T$ is given by,

$$I(t) = e^{-\frac{b}{\alpha}t} \begin{cases} a(T-t) + \frac{b}{2}(T^2-t^2) + \frac{c}{3}(T^3-t^3) + \frac{a\beta}{\alpha^2}(e^{\alpha T} - e^{\alpha t}) + \frac{b\beta}{\alpha^2}(T e^{\alpha T} - t e^{\alpha t}) \\ - \frac{b\beta}{\alpha^3}(e^{\alpha T} - e^{\alpha t}) + \frac{c\beta}{\alpha^2}(T^2 e^{\alpha T} - T e^{\alpha t}) - \frac{2c\beta}{\alpha^3}(T e^{\alpha T} - t e^{\alpha t}) \\ + \frac{2c\beta}{\alpha^4}(e^{\alpha T} - e^{\alpha t}) \end{cases}$$

$0 \leq t \leq T$ (2)

(by omitting the excessive order terms of $\beta$ as $0 < \beta < 1$)

The maximum positive inventory level for each cycle can be obtained from $I(0) = Q$ in equation (2) is given by,

$$Q = I(0) = aT + \frac{b}{2}T^2 + \frac{c}{3}T^3 + \frac{a\beta}{\alpha^2}T e^{\alpha T} - \frac{a\beta}{\alpha^2}T e^{\alpha T} + \frac{b\beta}{\alpha^2}T^2 e^{\alpha T} - \frac{b\beta}{\alpha^3}T e^{\alpha T} + \frac{b\beta}{\alpha^2}T^2 e^{\alpha T} - \frac{2c\beta}{\alpha^3}T e^{\alpha T}$$

$$+ \frac{2c\beta}{\alpha^4}T^2 e^{\alpha T} - \frac{2c\beta}{\alpha^4}T e^{\alpha T} - \frac{b\beta}{\alpha^4}T^2 e^{\alpha T} - \frac{b\beta}{\alpha^4}T e^{\alpha T} + \frac{b\beta}{\alpha^4}T^2 e^{\alpha T} + \frac{b\beta}{\alpha^4}T e^{\alpha T}$$

$$- \frac{2c\beta}{\alpha^4}T^2 e^{\alpha T} + \frac{2c\beta}{\alpha^4}T^2 e^{\alpha T} - \frac{2c\beta}{\alpha^5}T^2 e^{\alpha T} + \frac{2c\beta}{\alpha^5}T^2 e^{\alpha T}$$

(3)

Total revenue inventory cost per unit time consists of the following costs:

1. Ordering cost = $A$

2. Carrying cost per unit time is given by,

$$HC = \frac{h}{T} \int_{0}^{T} I(t) dt$$

$$= \frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{c}{4}T^4 + \frac{a\beta}{\alpha^2}T e^{\alpha T} - \frac{2a\beta}{\alpha^3}T e^{\alpha T} + \frac{2a\beta}{\alpha^2}T^2 e^{\alpha T} - \frac{3b\beta}{\alpha^3}T e^{\alpha T}$$

$$+ \frac{3b\beta}{\alpha^4}T e^{\alpha T} - \frac{3b\beta}{\alpha^4}T^2 e^{\alpha T} - \frac{4c\beta}{\alpha^3}T^2 e^{\alpha T} - \frac{8c\beta}{\alpha^4}T e^{\alpha T} - \frac{8c\beta}{\alpha^5}T e^{\alpha T} + \frac{8c\beta}{\alpha^5}T e^{\alpha T}$$

$$+ \frac{b\beta}{\alpha^2}T + \frac{b\beta}{\alpha^2}T^2 + \frac{b\beta}{\alpha^2}T^3 + \frac{b\beta}{\alpha^2}T^4 + \frac{b\beta}{\alpha^3}T^2 e^{\alpha T} + \frac{b\beta}{\alpha^3}T^3 e^{\alpha T} + \frac{b\beta}{\alpha^4}T^2 e^{\alpha T} + \frac{b\beta}{\alpha^4}T e^{\alpha T}$$

$$+ \frac{b\beta}{\alpha^4}T^2 e^{\alpha T} + \frac{b\beta}{\alpha^4}T e^{\alpha T} - \frac{b\beta}{\alpha^4}T^2 e^{\alpha T} + \frac{b\beta}{\alpha^4}T e^{\alpha T} + \frac{b\beta}{\alpha^4}T^2 e^{\alpha T} + \frac{b\beta}{\alpha^4}T e^{\alpha T}$$

$$+ \frac{7c\beta}{\alpha^5}T e^{\alpha T} + \frac{2c\beta}{\alpha^6}T e^{\alpha T}$$

(4)
3. The number of deteriorating units (NDU) is given by,

\[ DC = Q - \int_0^T D(t) dt, \text{ where } D(t) = a + bt + ct^2 \]  

(5)

4. Deteriorating cost per unit time is given by,

\[
DC = C_1 \left[ Q - \int_0^T D(t) dt \right]
\]

\[
= C_1 \left\{ \frac{a^2}{\alpha^2} e^{aT} - \frac{a^2}{\alpha^2} + \frac{b\beta}{\alpha^2} T e^{aT} - \frac{b\beta}{\alpha^3} e^{aT} + \frac{b\beta}{\alpha^2} T^2 e^{aT} - \frac{c\beta}{\alpha^3} T e^{aT} \right. \\
+ \frac{2c\beta}{\alpha^4} e^{aT} - \frac{2c\beta}{\alpha^4} - \frac{a\beta}{\alpha} T - \frac{b\beta}{2\alpha} T^2 - \frac{c\beta}{3\alpha} T^3 - \frac{a\beta^2}{\alpha^3} e^{aT} + \frac{a\beta^2}{\alpha^3} \\
- \frac{b\beta^2}{\alpha^3} T e^{aT} + \frac{b\beta^2}{\alpha^4} e^{aT} - \frac{b\beta^2}{\alpha^4} - \frac{c\beta^2}{\alpha^3} T^2 e^{aT} + \frac{2c\beta^2}{\alpha^4} T e^{aT} \\
\left. - \frac{2c\beta^2}{\alpha^5} e^{aT} + \frac{2c\beta^2}{\alpha^5} \right\} \]  

(6)

5. Sales revenue = \[ S \int_0^T (a + bt + ct^2) \, dt \]

\[ = S \left( aT + \frac{bT^2}{2} + \frac{cT^3}{3} \right) \]  

(7)

The total profit per unit time is,

\[ TP = \frac{1}{T} \left\{ S \left( aT + \frac{bT^2}{2} + \frac{cT^3}{3} \right) \right\} - \frac{1}{T} \left[ \text{Ordering Cost} + \text{Carrying Cost} + \text{Deterioration Cost} \right] \]
\[ A + h + \left\{ \begin{align*}
&\frac{a}{2} T^2 + \frac{b}{3} T^3 + \frac{c}{4} T^4 + \frac{a \beta}{\alpha^2} T e^{\alpha^2 T} - \frac{2a \beta}{\alpha^3} e^{\alpha^2 T} + \frac{2a \beta}{\alpha^3} \\
&+ \frac{b \beta}{\alpha^2} T^2 e^{\alpha^2 T} - \frac{3b \beta}{\alpha^3} T e^{\alpha^2 T} + \frac{3b \beta}{\alpha^4} e^{\alpha^2 T} - \frac{3b \beta}{\alpha^4} T^3 e^{\alpha^2 T} \\
&- \frac{4c \beta}{\alpha^3} T^2 e^{\alpha^2 T} + \frac{8c \beta}{\alpha^3} T e^{\alpha^2 T} - \frac{8c \beta}{\alpha^5} e^{\alpha^2 T} + \frac{8c \beta}{\alpha^5} + \frac{a \beta}{\alpha^2} T \\
&\frac{b \beta^2}{2 \alpha^4} T e^{2 \alpha^2 T} + \frac{c \beta^2}{2 \alpha^5} T^2 e^{2 \alpha^2 T} + \frac{2 \beta^2}{2 \alpha^5} T e^{2 \alpha^2 T} - \frac{c \beta^2}{4 \alpha^6} e^{2 \alpha^2 T} \\
&- \frac{7c \beta^2}{4 \alpha^6} - \frac{2c \beta^2}{\alpha^5} T e^{\alpha^2 T} + \frac{2c \beta^2}{\alpha^5} e^{\alpha^2 T} \\
&\frac{a \beta^2}{\alpha^2} e^{\alpha^2 T} - \frac{a \beta}{\alpha^2} + \frac{b \beta}{\alpha^2} T e^{\alpha^2 T} - \frac{b \beta}{\alpha^3} e^{\alpha^2 T} + \frac{b \beta}{\alpha^3} + \frac{c \beta}{\alpha^2} T^2 e^{\alpha^2 T} \\
&- \frac{2c \beta}{\alpha^3} T e^{\alpha^2 T} + \frac{2c \beta}{\alpha^4} e^{\alpha^2 T} - \frac{2c \beta}{\alpha^4} T - \frac{b \beta}{\alpha} T - \frac{b \beta}{3 \alpha} T^3 \\
&- \frac{a \beta}{\alpha^3} e^{\alpha^2 T} + \frac{b \beta}{\alpha^3} - \frac{b \beta}{\alpha^3} T e^{\alpha^2 T} + \frac{b \beta}{\alpha^4} e^{\alpha^2 T} - \frac{b \beta}{\alpha^4} \\
&- \frac{c \beta^2}{\alpha^3} T^2 e^{\alpha^2 T} + \frac{2c \beta^2}{\alpha^3} T e^{\alpha^2 T} - \frac{2c \beta^2}{\alpha^5} e^{\alpha^2 T} + \frac{2c \beta^2}{\alpha^5} \\
&\left(8\right)
\end{align*} \right\} \]

Our goal is to maximize the total profit per unit time. Necessary conditions for total variable inventory cost to be maximized are (i) \( \frac{\partial TP}{\partial T} = 0 \) and (ii) \( \frac{\partial^2 TP}{\partial^2 T} < 0. \)
can be obtained by using condition \( \beta \). Since equation (11) \( (i) \) is used to find the triangular fuzzy numbers then the total cost per unit time in fuzzy sense is satisfied for the value of \( \beta \) satisfies

The optimal value of \( T \) can be obtained by using condition \( (i) \). Condition \( (ii) \) is also satisfied for the value of \( T \) obtained from condition \( (i) \). The value of \( T \) is used to find the optimal values of \( Q \) and \( TP \). Since equation \( (i) \) is non-linear it is solved by using appropriate software.

6. FUZZY MODEL

Let us consider the inventory model in fuzzy environment due to deterioration cost, holding cost and the parameters \( \alpha, \beta \).

Le \( \tilde{C}_1 = (C_{11}, C_{12}, C_{13}), \tilde{h} = (h_1, h_2, h_3), \tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3) \) and \( \tilde{\beta} = (\beta_1, \beta_2, \beta_3) \) are triangular fuzzy numbers then the total cost per unit time in fuzzy sense is
Next defuzzify the total cost $TP$ by using Pascal triangular method, we have

$$TP_{dp} = \frac{1}{4} \left( TP_{dp_1} + 2TP_{dp_2} + TP_{dp_3} \right)$$
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\[
T \tilde{p}_{\phi_2} = S \left( A + \frac{bT}{2} + \frac{cT^2}{3} \right) - \frac{1}{T} \left\{ \begin{array}{c}
\frac{a}{2} T^2 + \frac{b}{3} T^3 + \frac{c}{4} T^4 + a \tilde{p}_2 \frac{T e^{\tilde{a}_1 T}}{\tilde{a}_2^2} - \frac{2a \tilde{p}_2}{\tilde{a}_2^3} e^{\tilde{a}_1 T} + \frac{2a \tilde{p}_2}{\tilde{a}_2^3} \\
+ \frac{c \tilde{p}_2}{\tilde{a}_2^4} e^{\tilde{a}_1 T} - b \tilde{p}_2 \frac{T e^{\tilde{a}_1 T}}{\tilde{a}_2^3} + \frac{3b \tilde{p}_2}{\tilde{a}_2^4} e^{\tilde{a}_1 T} - \frac{3b \tilde{p}_2}{\tilde{a}_2^4} T e^{\tilde{a}_1 T} + c \tilde{p}_2 \frac{T^3 e^{\tilde{a}_1 T}}{\tilde{a}_2^5} \\
\frac{4c \tilde{p}_2}{\tilde{a}_2^5} e^{\tilde{a}_1 T} + \frac{8c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} - \frac{8c \tilde{p}_2}{\tilde{a}_2^6} e^{\tilde{a}_1 T} + \frac{8c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + a \tilde{p}_2 \\
+ \frac{2a \tilde{p}_2}{3 \tilde{a}_2^2} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{4 \tilde{a}_2^4} T e^{\tilde{a}_1 T} - \frac{a \tilde{p}_2}{\tilde{a}_2^4} T e^{\tilde{a}_1 T} + \frac{a \tilde{p}_2}{\tilde{a}_2^4} e^{\tilde{a}_1 T} - \frac{a \tilde{p}_2}{\tilde{a}_2^4} \\
\frac{2a \tilde{p}_2}{3 \tilde{a}_2^2} T^4 e^{\tilde{a}_1 T} + \frac{b \tilde{p}_2}{2 \tilde{a}_2^3} T e^{\tilde{a}_1 T} + \frac{b \tilde{p}_2}{2 \tilde{a}_2^3} T e^{\tilde{a}_1 T} - \frac{b \tilde{p}_2}{2 \tilde{a}_2^3} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{4 \tilde{a}_2^4} T e^{\tilde{a}_1 T} - \frac{c \tilde{p}_2}{4 \tilde{a}_2^4} T e^{\tilde{a}_1 T} \\
\frac{3c \tilde{p}_2}{\tilde{a}_2^5} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{2 \tilde{a}_2^5} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{2 \tilde{a}_2^5} T e^{\tilde{a}_1 T} - \frac{c \tilde{p}_2}{2 \tilde{a}_2^5} T e^{\tilde{a}_1 T} + \frac{7c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + \frac{2c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + \frac{2c \tilde{p}_2}{\tilde{a}_2^6} e^{\tilde{a}_1 T} \\
\frac{2a \tilde{p}_2}{\tilde{a}_2^4} T e^{\tilde{a}_1 T} + \frac{b \tilde{p}_2}{\tilde{a}_2^5} T e^{\tilde{a}_1 T} + \frac{b \tilde{p}_2}{\tilde{a}_2^5} T e^{\tilde{a}_1 T} - \frac{b \tilde{p}_2}{\tilde{a}_2^5} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{\tilde{a}_2^6} e^{\tilde{a}_1 T} \\
\frac{a \tilde{p}_2}{\tilde{a}_2^3} e^{\tilde{a}_1 T} + \frac{b \tilde{p}_2}{\tilde{a}_2^4} T e^{\tilde{a}_1 T} - \frac{b \tilde{p}_2}{\tilde{a}_2^4} e^{\tilde{a}_1 T} + \frac{b \tilde{p}_2}{\tilde{a}_2^4} T e^{\tilde{a}_1 T} - \frac{b \tilde{p}_2}{\tilde{a}_2^4} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{\tilde{a}_2^5} T e^{\tilde{a}_1 T} + \frac{c \tilde{p}_2}{\tilde{a}_2^5} e^{\tilde{a}_1 T} \\
\frac{2c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + \frac{2c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} - \frac{2c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + \frac{2c \tilde{p}_2}{\tilde{a}_2^6} e^{\tilde{a}_1 T} + \frac{2c \tilde{p}_2}{\tilde{a}_2^6} T e^{\tilde{a}_1 T} + \frac{2c \tilde{p}_2}{\tilde{a}_2^6} e^{\tilde{a}_1 T}
\end{array} \right\}
\right.

\right.\}

(14)

-35-
\[
T \ddot{P}_{dp} = S \left( a + \frac{bT}{2} + cT^2 \right) - \frac{1}{T} \left\{ \begin{array}{l}
\frac{a}{2} T^2 + \frac{b}{3} T^3 + \frac{c}{4} T^4 + \frac{a \beta_3}{\alpha_3^2} T e^{\alpha_3 T} - \frac{2a \beta_3}{\alpha_3^3} e^{\alpha_3 T} + \frac{2a \beta_3}{\alpha_3^3} e^{\alpha_3 T} \\
+ \frac{b \beta_3}{\alpha_3^2} T^2 e^{\alpha_3 T} - \frac{3b \beta_3}{\alpha_3^3} T e^{\alpha_3 T} + \frac{3b \beta_3}{\alpha_3^3} T e^{\alpha_3 T} - \frac{3b \beta_3}{\alpha_3^4} T^3 e^{\alpha_3 T} + \frac{c \beta_3}{\alpha_3^2} T^3 e^{\alpha_3 T} \\
- \frac{4c \beta_3}{\alpha_3^2} T^2 e^{\alpha_3 T} + \frac{8c \beta_3}{\alpha_3^4} T e^{\alpha_3 T} - \frac{8c \beta_3}{\alpha_3^4} T e^{\alpha_3 T} + \frac{a \beta_3}{\alpha_3^2} T \\
\end{array} \right\} \\
A + \tilde{h}_3 + \tilde{c}_{13} \right\} \\

\}
\]

The necessary condition for \(TP_{dp}\) to be maximize is that \((i)\) \(\frac{\partial TP_{dp}}{\partial T} = 0\) and \((ii)\) \(\frac{\partial^2 TP_{dp}}{\partial^2 T} < 0\). The optimal value of \(T\) can be obtained by using condition \((i)\). Condition \((ii)\) is also satisfied for the value of \(T\) obtained from condition \((i)\). The value of \(T\) is used to find the optimal values of \(Q\) and \(TP\). Since equation \((i)\) is non-linear it is solved by using appropriate software.
7. NUMERICAL ANALYSIS

7.1. Example

Crisp Model

Assume suitable value for \( A = 20, a = 60, b = 2, c = 5, C_1 = 2, h = 0.1, \alpha = 1, \beta = 0.2, S = 2 \)

With appropriated units. The optimal values are \( T = 0.17890 \) and \( TP = 11.13367, Q = 8.11681 \).

7.2. Example

Fuzzy Model

Assume suitable value for \( A = 20, a = 60, b = 2, c = 5, \tilde{C}_1 = (2, 3, 4), \tilde{h} = (0.1, 0.2, 0.3), \tilde{\alpha} = (1, 2, 3), \tilde{\beta} = (0.2, 0.3, 0.5), S = 2 \)

With appropriated units. The optimal values are \( T = 0.09121 \) and \( TP_{dp} = 4625.13647, Q = 4.65403 \).

8. SENSITIVITY ANALYSIS FOR FUZZY MODEL AND CRISP MODEL

**Table 1.** Sensitivity Analysis on parameter \( \tilde{C}_1 \).

<table>
<thead>
<tr>
<th>( \tilde{C}_1 )</th>
<th>( T )</th>
<th>( Q )</th>
<th>( TP_{dp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 3, 4)</td>
<td>0.19028</td>
<td>8.77322</td>
<td>18.94641</td>
</tr>
<tr>
<td>(3, 4, 5)</td>
<td>0.19813</td>
<td>9.22703</td>
<td>24.49464</td>
</tr>
<tr>
<td>(4, 5, 6)</td>
<td>0.20291</td>
<td>9.50338</td>
<td>28.16025</td>
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<tr>
<td>(5, 6, 7)</td>
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<td>9.69049</td>
<td>30.97128</td>
</tr>
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<td>(6, 7, 8)</td>
<td>0.20847</td>
<td>9.82585</td>
<td>33.31656</td>
</tr>
</tbody>
</table>

**Table 2.** Sensitivity Analysis on parameter \( C_1 \).

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( T )</th>
<th>( Q )</th>
<th>( TP )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.19171</td>
<td>8.85576</td>
<td>19.89050</td>
</tr>
<tr>
<td>4</td>
<td>0.19878</td>
<td>9.26453</td>
<td>24.87095</td>
</tr>
<tr>
<td>5</td>
<td>0.20325</td>
<td>9.52331</td>
<td>28.34614</td>
</tr>
<tr>
<td>6</td>
<td>0.20634</td>
<td>9.70236</td>
<td>31.07777</td>
</tr>
<tr>
<td>7</td>
<td>0.20860</td>
<td>9.83393</td>
<td>33.38342</td>
</tr>
</tbody>
</table>
Table 3. Sensitivity Analysis on parameter $\tilde{h}$

<table>
<thead>
<tr>
<th>$\tilde{h}$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$T\tilde{P}_{dp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0.01,0.02,0.003)$</td>
<td>0.21283</td>
<td>10.07907</td>
<td>29.06920</td>
</tr>
<tr>
<td>$(0.02,0.03,0.04)$</td>
<td>0.20797</td>
<td>9.76679</td>
<td>26.85083</td>
</tr>
<tr>
<td>$(0.03,0.04,0.05)$</td>
<td>0.2033</td>
<td>9.52628</td>
<td>24.62396</td>
</tr>
<tr>
<td>$(0.04,0.05,0.06)$</td>
<td>0.19882</td>
<td>9.26705</td>
<td>22.39053</td>
</tr>
<tr>
<td>$(0.05,0.06,0.07)$</td>
<td>0.19452</td>
<td>9.01836</td>
<td>20.14981</td>
</tr>
</tbody>
</table>

Table 4. Sensitivity Analysis on parameter $h$

<table>
<thead>
<tr>
<th>$h$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$TP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.21785</td>
<td>10.37044</td>
<td>31.28039</td>
</tr>
<tr>
<td>0.02</td>
<td>0.21278</td>
<td>10.07593</td>
<td>29.07129</td>
</tr>
<tr>
<td>0.03</td>
<td>0.20792</td>
<td>9.79396</td>
<td>26.85381</td>
</tr>
<tr>
<td>0.04</td>
<td>0.20325</td>
<td>9.52331</td>
<td>24.62439</td>
</tr>
<tr>
<td>0.05</td>
<td>0.19878</td>
<td>9.26453</td>
<td>22.39324</td>
</tr>
</tbody>
</table>

Table 5. Sensitivity Analysis on parameter $\tilde{\alpha}$

<table>
<thead>
<tr>
<th>$\tilde{\alpha}$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$T\tilde{P}_{dp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1.1,1.2,1.3)$</td>
<td>0.12306</td>
<td>6.11148</td>
<td>-38.28790</td>
</tr>
<tr>
<td>$(1.2,1.3,1.4)$</td>
<td>0.10384</td>
<td>5.32277</td>
<td>-64.63180</td>
</tr>
<tr>
<td>$(1.3,1.4,1.5)$</td>
<td>0.08882</td>
<td>4.66384</td>
<td>-91.48189</td>
</tr>
<tr>
<td>$(1.4,1.5,1.6)$</td>
<td>0.07686</td>
<td>4.11217</td>
<td>-118.03164</td>
</tr>
<tr>
<td>$(1.5,1.6,1.7)$</td>
<td>0.06716</td>
<td>3.64800</td>
<td>-143.22321</td>
</tr>
</tbody>
</table>
Table 6. Sensitivity Analysis on parameter $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$TP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.14613</td>
<td>7.00952</td>
<td>-12.65335</td>
</tr>
<tr>
<td>1.2</td>
<td>0.12163</td>
<td>6.07297</td>
<td>-38.00392</td>
</tr>
<tr>
<td>1.3</td>
<td>0.10283</td>
<td>5.29045</td>
<td>-64.49040</td>
</tr>
<tr>
<td>1.4</td>
<td>0.08808</td>
<td>4.63719</td>
<td>-91.54247</td>
</tr>
<tr>
<td>1.5</td>
<td>0.07630</td>
<td>4.09052</td>
<td>-118.35223</td>
</tr>
</tbody>
</table>

Table 7. Sensitivity Analysis on parameter $\tilde{\beta}$

<table>
<thead>
<tr>
<th>$\tilde{\beta}$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$TP_{dp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.2,0.3,0.4)</td>
<td>0.31480</td>
<td>13.60249</td>
<td>55.86797</td>
</tr>
<tr>
<td>(0.3,0.4,0.5)</td>
<td>0.4724</td>
<td>19.14375</td>
<td>87.02601</td>
</tr>
<tr>
<td>(0.4,0.5,0.6)</td>
<td>0.67126</td>
<td>24.69763</td>
<td>108.39888</td>
</tr>
<tr>
<td>(0.5,0.6,0.7)</td>
<td>0.96375</td>
<td>31.26237</td>
<td>127.88494</td>
</tr>
</tbody>
</table>

Table 8. Sensitivity Analysis on parameter $\beta$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$TP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.17890</td>
<td>8.11681</td>
<td>11.13367</td>
</tr>
<tr>
<td>0.3</td>
<td>0.30856</td>
<td>13.56809</td>
<td>61.77936</td>
</tr>
<tr>
<td>0.4</td>
<td>0.46319</td>
<td>19.15696</td>
<td>88.77949</td>
</tr>
<tr>
<td>0.5</td>
<td>0.65466</td>
<td>24.69298</td>
<td>108.76571</td>
</tr>
<tr>
<td>0.6</td>
<td>0.91251</td>
<td>30.24758</td>
<td>127.28462</td>
</tr>
</tbody>
</table>

9. OBSERVATIONS

Based on the results we can make the following conclusions:
1. From Table 1, we can observe that the parameter $\tilde{C}_1$ increases, $T$ and $Q$ increases while $TP_{dp}$ increases.

2. From Table 2, we can observe that the parameter $C_1$ increases, $T$ and $Q$ increases while $TP$ increases.

3. From Table 3, we can observe that the parameter $\tilde{h}$ increases, $T$ and $Q$ decreases while $TP_{dp}$ decreases.

4. From Table 4, we can observe that the parameter $h$ increases, $T$ and $Q$ decreases while $TP$ decreases.

5. From Table 5, we can observe that the parameter $\tilde{a}$ increases, $T$ and $Q$ decreases while $TP_{dp}$ increases.

6. From Table 6, we can observe that the parameter $\alpha$ increases, $T$ and $Q$ decreases while $TP$ increases.

7. From Table 7, we can observe that the parameter $\tilde{\beta}$ increases, $T$ and $Q$ increases while $TP_{dp}$ increases.

8. From Table 8, we can observe that the parameter $\beta$ increases, $T$ and $Q$ increases while $TP$ increase.

10. CONCLUSIONS

In the current financial world, a deterministic inventory model for deteriorating items involves Gompertz distribution deterioration rate and quadratic demand of time is improved and clarified in this work. Stock-out is not permitted. The deterioration cost, holding cost and the parameters $\alpha$, $\beta$ are assumed as a triangular fuzzy numbers. The aim of this economic order quantity model is to maximize the total profit of inventory optimization by using Pascal triangular method. Here we provide some numerical examples and sensitivity analysis are supported to demonstrate the solving methodology. For instance, this inventory model may be extended incorporating with various considerations like intuitionistic fuzzy sets, Trapezoidal fuzzy sets including shortages, price discount, quantity discount and others.

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References


