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Reduction of a Pythagorean Fuzzy Matrix to Fuzzy Matrix

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ABSTRACT

We have read some of the results of $\vee_L$ and $\wedge_L$ from the Lukasiewicz type over Pythagorean fuzzy matrices. We presented four reduction operations about the Pythagorean fuzzy matrices. We associate the reduction operations to relate $\vee_L$ and $\wedge_L$ activities.

***Keywords*:** Intuitionistic fuzzy matrix, Pythagorean fuzzy set, Pythagorean fuzzy matrix, Disjunction, Conjunction

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1. INTRODUCTION

The concept of intuitionistic fuzzy matrix (IFM) was introduced by Pal [3] and simultaneously by Im et al. [2] to generalize the concept of Thomason's [8] fuzzy matrix. Each element in an IFM is expressed by an ordered pair $\langle a_{ij}, a'_{ij} \rangle$ with $a_{ij}, a'_{ij} \in [0,1]$. The sum $a_{ij} + a'_{ij}$ of each ordered pair is less than or equal to 1. Since the appearance of IFM in 2001, many

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researchers [4, 5, 9, 10] have importantly contributed for the development of IFM theory and its applications.

Pythagorean fuzzy set (PFS) was introduced by Yager [12]. Silambarasan and Sriram [7] introduced Pythagorean fuzzy matrix (PFM) and studied its algebraic operations. Atanassov and Tcvetkov [1] introduced the operations $\vee_L$ and $\wedge_L$ from Lukasiewicz's type over IFSs.

In [11], we studied the algebraic properties of two operations $\vee_L$ and $\wedge_L$ from Lukasiewicz type over PFMs. Also, using the relations between $\vee_L$ and $\wedge_L$ using modal operations. In [11], we proved the set of all IFMs is a commutative monoid under these operations. Muthuraji and Sriram [6] they are studied reduction of an IFM to FM with some algebraic properties.

This paper as follows. In section 2, we will briefly review PFMs and their opertions. In section 3, some properties of the operations $\vee_L$ and $\wedge_L$ from Lukasiewicz's type over PFMs are studied. In section 4, we introduce four operations called reduction operations on PFM which reduce an PFM to a FM. Also we relate $\vee_L$ and $\wedge_L$ operations with reduction operations.

2. PRELIMINARIES

In this section, we will summarize the PFMs and their operations.

Definition 2.1 ([7])

An PFM is a matrix of pairs $A = \left(\langle a_{ij}, a'_{ij}\rangle\right)$ of a positive real numbers $a_{ij}, a'_{ij} \in [0,1]$ satisfing the condition $a_{ij}^2 + {a'_{ij}}^2 \leq 1$ for all i, j.

Definition 2.2 ([11])

For any two PFMs $A, B \in \mathcal{F}_{mn}$, we have

(i) $A \geq B$ iff $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$,

(ii) $A^C = \left(\langle a'_{ij}. a_{ij}\rangle\right)$,

(iii) $A \vee B = \left(\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})\rangle\right)$,

(iv) $A \wedge B = \left(\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})\rangle\right)$,

(v) $AX_1B = \left(\langle \max(a_{ij}, b_{ij}), a'_{ij}b'_{ij}\rangle\right)$,

(vi) $AX_2B = \left(\langle a_{ij}b_{ij}, \max(a'_{ij}, b'_{ij})\rangle\right)$,

(vii) $A \oplus_P B = \left(\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2},\ a'_{ij}b'_{ij}\rangle\right)$,

$(viii)$ $A \odot_P B = \left(\langle a_{ij}b_{ij}, \sqrt{{a'_{ij}}^2 + {b'_{ij}}^2 - {a'_{ij}}^2 {b'_{ij}}^2}\rangle\right)$,

(ix) $A \otimes_{1P} B = \left(\langle \sqrt{\max\left(a_{ij}^2, b_{ij}^2\right)}\ , a'_{ij}b'_{ij}\rangle\right)$,

$(x)\ A \otimes_{2P} B = \left(\langle a_{ij} b_{ij}, \sqrt{\max({a'_{ij}}^2, {b'_{ij}}^2)}\rangle\right)$,

(xi) An PFM $O = \langle 0,1\rangle$ for all entries is known as the Zero matrix,

An PFM $J = \langle 1,0\rangle$ for all entries is known as the Universal matrix.

Definition 2.3 ([11])

Using intuitionistic fuzzy form of Lukasiewicz implication, we will introduced a disjunction

$$A \vee_L B = \left(\langle \sqrt{\min\left(1, a_{ij}^2 + b_{ij}^2\right)}, \sqrt{\max(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\ \rangle\right).$$

We will call the new disjunction Lukasiewicz Pythagorean fuzzy disjunction.
Also, we can constract,

$$A \wedge_L B = \left(\langle \sqrt{\max\left(0, a_{ij}^2 + b_{ij}^2 - 1\right)}, \sqrt{\min(1,\ {a'_{ij}}^2 + {b'_{ij}}^2)}\ \rangle\right).$$

We will call the new conjunction Lukasiewicz Pythagorean fuzzy conjunction.

3. SOME RESULTS OF PFMS

In this section, in [11], we introduced the operations $\vee_L$ and $\wedge_L$ from Lukasiewicz type over Pythagorean fuzzy matrices. Some results of these operations are studied with other predefined operations.

Property 3.1:

For any two PFMs $O, J \in \mathcal{F}_{mn}$, we have

$(i)\ O \vee_L J = J; O \wedge_L J = O,$

$(ii)\ O \vee_L O = O; O \wedge_L O = O,$

$(iii)\ J \vee_L O = J;\ J \wedge_L O = O,$

$(iv)\ J \vee_L J = J;\ J \wedge_L J = J.$

Proof.

$(i)\ O \vee_L J = \left(\langle \sqrt{\min(1, 0+1)}\ , \sqrt{\max(0, 1+0-1)}\ \rangle\right),$

$= (\langle 1,0\ \rangle)$

$= J.$

Hence, $O \vee_L J = J.$

The proof (ii), (iii) and (iv) are similar to that of (i).

Property 3.2:

For any three PFMs $A, B, C \in \mathcal{F}_{mn}$, we have

(i) If $A \leq B$, then $A \vee_L C \leq B \vee_L C$ and $A \wedge_L C \leq B \wedge_L C$.

Proof.

(i) Let as consider

$$A \vee_L C = \left(\langle \sqrt{\min\left(1, a_{ij}^2 + c_{ij}^2\right)}\ , \sqrt{\max(0,\ {a'_{ij}}^2 + {c'_{ij}}^2 - 1)}\ \rangle \right),$$

$$B \vee_L C = \left(\langle \sqrt{\min\left(1, b_{ij}^2 + c_{ij}^2\right)}\ , \sqrt{\max(0,\ {b'_{ij}}^2 + {c'_{ij}}^2 - 1)}\ \rangle \right).$$

Since, $a_{ij}^2 \leq b_{ij}^2$ and $a'_{ij} \geq b'_{ij}$, we have

$a_{ij}^2 + c_{ij}^2 \leq b_{ij}^2 + c_{ij}^2$ and $a'_{ij} + c'_{ij} \geq b'_{ij} + c'_{ij}$.

$\Rightarrow \min\left(1, a_{ij}^2 + c_{ij}^2\right) \leq \min\left(1, b_{ij}^2 + c_{ij}^2\right)$. …(3.2.1)

Clearly, $(\ {a'_{ij}}^2 + {c'_{ij}}^2 - 1) \geq\ ({b'_{ij}}^2 + {c'_{ij}}^2 - 1)$

$\Rightarrow \max(0,\ {a'_{ij}}^2 + {c'_{ij}}^2 - 1) \geq \max(0,\ {b'_{ij}}^2 + {c'_{ij}}^2 - 1)$ …(3.2.2)

From (3.2.1) and (3.2.2) we get

$A \vee_L C \leq B \vee_L C$.

Similarly, we can prove $A \wedge_L C \leq B \wedge_L C$.

Hence, the operations $\vee_L$ and $\wedge_L$ are monotonically increasing operations.

From Property (3.2), we get

$O = O \vee_L O \leq A \vee_L B \leq A \vee_L J = J$,

$O = A \wedge_L J \leq A \wedge_L B \leq J \vee_L J = J$.

Property 3.3:

For any two PFMs $A, B \in \mathcal{F}_{mn}$, we have

(i) $A \wedge_L B \leq A \wedge B \leq A \vee B \leq A \vee_L B$.

Proof.

$$A \wedge_L B = \left(\langle \sqrt{\max\left(0, a_{ij}^2 + b_{ij}^2 - 1\right)}\ , \sqrt{\min(1,\ {a'_{ij}}^2 + {b'_{ij}}^2)}\ \rangle \right).$$

Then $a_{ij}^2 + b_{ij}^2 - 1 = a_{ij}^2 + (b_{ij}^2 - 1) \leq a_{ij}^2$ and $a_{ij}^2 + b_{ij}^2 - 1 = (a_{ij}^2 - 1) + b_{ij}^2 \leq b_{ij}^2$.

Thus $(a_{ij}^2 + b_{ij}^2 - 1) \le (a_{ij} \wedge b_{ij})$.

So $\max\left(0, a_{ij}^2 + b_{ij}^2 - 1\right) \le (a_{ij} \wedge b_{ij})$. …(3.3.1)

Similarly, $\min(1,\ {a'_{ij}}^2 + {b'_{ij}}^2) \ge (a'_{ij} \vee b'_{ij})$. …(3.3.2)

From (3.3.1) and (3.3.2) we get,

$A \wedge_L B \le A \wedge B$.

Since, $a_{ij}^2 \le \left(a_{ij}^2 + b_{ij}^2\right), b_{ij}^2 \le \left(a_{ij}^2 + b_{ij}^2\right), (a_{ij} \vee b_{ij}) \le \left(a_{ij}^2 + b_{ij}^2\right)$,

$$(a_{ij} \vee b_{ij}) \le \min\left(1, a_{ij}^2 + b_{ij}^2\right).$$

Also ${a'_{ij}}^2 + {b'_{ij}}^2 - 1 = {a'_{ij}}^2 + ({b'_{ij}}^2 - 1) \le {a'_{ij}}^2$ and

$${a'_{ij}}^2 + {b'_{ij}}^2 - 1 = ({a'_{ij}}^2 - 1) + {b'_{ij}}^2 \le {b'_{ij}}^2.$$

Thus ${a'_{ij}}^2 + {b'_{ij}}^2 - 1 \le (a'_{ij} \wedge b'_{ij})$. …(3.3.3)

So, $(a'_{ij} \wedge b'_{ij}) \ge \max\left(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1\right)$. …(3.3.4)

From (3.3.3) and (3.3.4) we get,

$A \vee B \le A \vee_L B$.

Since, $A \wedge B \le A \vee B$ is straightforward,

We conclude that,

$A \wedge_L B \le A \wedge B \le A \vee B \le A \vee_L B$.

Property 3.4:

For any PFM $A \in \mathcal{F}_{mn}$, we have

(i) $A \vee_L A \ge A$,

(ii) $A \wedge_L A \le A$.

Proof.

(i) Let $(A \vee_L A)$ as follows

$$A \vee_L A = \left(\langle \sqrt{\min\left(1, a_{ij}^2 + a_{ij}^2\right)}\ , \sqrt{\max\left(0,\ {a'_{ij}}^2 + {a'_{ij}}^2 - 1\right)}\ \rangle\right)$$

$$= \left(\langle \sqrt{\min\left(1, 2a_{ij}^2\right)}\ , \sqrt{\max\left(0,\ 2{a'_{ij}}^2 - 1\right)}\ \rangle\right)$$

It is enough to prove that

$\min\left(1, 2a_{ij}^2\right) > a_{ij}^2$ and $\max\left(0,\ 2{a'_{ij}}^2 - 1\right) < {a'_{ij}}^2$

Since $2a_{ij}^2 > a_{ij}^2$ and $\min\left(1, 2a_{ij}^2\right) > a_{ij}^2$. …(3.4.1)

Since $2{a'_{ij}}^2 - 1 = {a'_{ij}}^2 + ({a'_{ij}}^2 - 1) \leq {a'_{ij}}^2$,

$$\max\left(0,\ 2{a'_{ij}}^2 - 1\right) < {a'_{ij}}^2. \qquad \ldots(3.4.2)$$

From (3.4.1) and (3.4.2) we get,

$A \vee_L A \geq A$.

(ii) Similarly, we can prove $A \wedge_L A \leq A$.

Property 3.5:

For any two PFMs $A, B \in \mathcal{F}_{mn}$, we have

$(i)\ A \wedge (A \vee_L B) = A$,

$(ii)\ A \vee (A \wedge_L B) = A$.

Proof.

$(i)\ A \wedge (A \vee_L B)$

$$= \left(\langle a_{ij}, a'_{ij}\rangle\right) \wedge \left(\langle \sqrt{\min\left(1, a_{ij}^2 + b_{ij}^2\right)}, \sqrt{\max(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\ \rangle\right)$$

$$= \left(\langle \sqrt{\min(a_{ij}^2, \min(1, a_{ij}^2 + b_{ij}^2))}, \sqrt{\max(\ {a'_{ij}}^2, \max(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1))}\rangle\right)$$

$$= \left(\langle a_{ij}, a'_{ij}\rangle\right)$$

$$= A.$$

The proof (ii) is similar to prove that of (i).

Property 3.6:

For any three PFMs $A, B, C \in \mathcal{F}_{mn}$, we have

$(i)\ A \vee_L (B \wedge C) = (A \vee_L B) \wedge (A \vee_L C)$,

$(ii)\ A \vee_L (B \vee C) = (A \vee_L B) \vee (A \vee_L C)$,

$(iii)\ A \wedge_L (B \wedge C) = (A \wedge_L B) \wedge (A \wedge_L C)$,

$(iv)\ A \wedge_L (B \vee C) = (A \wedge_L B) \vee (A \wedge_L C)$.

Proof.

$(i)\ A \vee_L (B \wedge C)$

$$= \left(\langle a_{ij}, a'_{ij}\rangle\right) \vee_L \left(\langle \min\left(b_{ij}, c_{ij}\right), \max(b'_{ij}, c'_{ij})\rangle\right) =$$

$$\left(\langle \sqrt{\min(1, a_{ij}^2 + \min(b_{ij}^2, c_{ij}^2))}, \sqrt{\max(0,\ {a'_{ij}}^2 + \max(\ {b'_{ij}}^2, {c'_{ij}}^2) - 1)}\rangle\right) =$$

$$\left(\langle \sqrt{\min(1, a_{ij}^2 + b_{ij}^2, a_{ij}^2 + c_{ij}^2)}, \sqrt{\max(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1,\ {a'_{ij}}^2 + {c'_{ij}}^2 - 1)}\rangle\right)$$

$$= \left(\langle \sqrt{\min\left(1, a_{ij}^2 + b_{ij}^2\right)}, \sqrt{\max\left(0,\ {a'_{ij}}^2 +\ {b'_{ij}}^2 - 1\right)} \rangle \right) \vee$$

$$\left(\langle \sqrt{\min(1, a_{ij}^2 + c_{ij}^2)}, \sqrt{\max(0,\ {a'_{ij}}^2 + {c'_{ij}}^2 - 1)} \rangle \right)$$

$= (A \vee_L B) \wedge (A \vee_L C)$.

Hence, $A \vee_L (B \wedge C) = (A \vee_L B) \wedge (A \vee_L C)$.

The proof (ii),(iii) and (iv) are similar to that of (i).

Property 3.7:

For any three PFMs $A, B, C \in \mathcal{F}_{mn}$, we have

$(i)\ (A \wedge B) \vee_L C = (A \vee_L C) \wedge (B \vee_L C)$,

$(ii)\ (A \vee B) \vee_L C = (A \vee_L C) \vee (B \vee_L C)$,

$(iii)\ (A \wedge B) \wedge_L C = (A \wedge_L C) \wedge (B \wedge_L C)$,

$(iv)\ (A \vee B) \wedge_L C = (A \wedge_L C) \vee (B \wedge_L C)$.

Proof.

$(i)\ (A \wedge B) \vee_L C$

$$= \left(\langle \min\left(a_{ij}, b_{ij}\right), \max\left(a'_{ij}, b'_{ij}\right)\rangle\right) \vee_L \left(\langle c_{ij}, c'_{ij}\rangle\right)$$

$$= \left(\langle \sqrt{\min(1, \min\left(a_{ij}^2, b_{ij}^2\right) + c_{ij}^2)}, \sqrt{\max(0, \max(\ {a'_{ij}}^2, {b'_{ij}}^2) + {c'_{ij}}^2 - 1)} \rangle \right)$$

$$= \left(\langle \sqrt{\min(1, a_{ij}^2 + c_{ij}^2, b_{ij}^2 + c_{ij}^2)}, \sqrt{\max(0,\ {a'_{ij}}^2 +\ {c'_{ij}}^2 - 1,\ {b'_{ij}}^2 +\ {c'_{ij}}^2 - 1)} \rangle \right)$$

$$= \left(\langle \sqrt{\min\left(1, a_{ij}^2 + c_{ij}^2\right)}, \sqrt{\max\left(0,\ {a'_{ij}}^2 +\ {c'_{ij}}^2 - 1\right)} \rangle \right) \vee$$

$$\left(\langle \sqrt{\min(1, b_{ij}^2 + c_{ij}^2)}, \sqrt{\max(0,\ {b'_{ij}}^2 + {c'_{ij}}^2 - 1)} \rangle \right)$$

$= (A \vee_L C) \wedge (B \vee_L C)$.

Hence, $(A \wedge B) \vee_L C\ = (A \vee_L C) \wedge (B \vee_L C)$.

The proof (ii), (iii) and (iv) are similar to that of (i).

4. PROPERTIES OF REDUCTION OPERATIONS IN PFMs

In this section, we introduce four operations called reduction operations on PFM which reduce an PFM to a FM. Also we relate $\vee_L$ and $\wedge_L$ operations with reduction operations.

Definition 4.1.

For any PFM $A \in \mathcal{F}_{mn}$ and $r \in [0,1]$, we define

$(i)\ r_1(A) = r\sqrt{a_{ij}{}^2} + (1-r)\sqrt{(1-a'_{ij}{}^2)}$,

$(ii)\ r_2(A) = 1 - r_1(A)$,

$(iii)\ r_3(A) = (1-r)\sqrt{a_{ij}{}^2} + r\sqrt{(1-a'_{ij}{}^2)}$,

$(iv)\ r_4(A) = 1 - r_3(A)$.

Property 4.2:

For any PFM $A \in \mathcal{F}_{mn}$, we have

(i) When $r = 0.5$, $r_1(A) = r_3(A)$ and $r_2(A) = r_4(A)$,

$(ii)\ r_1(1,0) = r_3(1,0) = r_2(0,1) = r_4(0,1) = 1$ and

$r_2(1,0) = r_4(1,0) = r_1(0,1) = r_2(0,1) = 0$,

$(iii)\ r_1(A^C) = r_4(A)$ and $r_2(A^C) = r_3(A)$.

Proof.

(i) When $r = 0.5$, then $1 - r = 0.5$.

By Definition (4.1),

$r_1(A) = r_3(A)$ and $r_2(A) = r_4(A)$.

(ii) It is strightforward from Definition (4.1).

(iii) $r_1(A^C) = r_4(A)$ and $r_2(A^C) = r_3(A)$.

$$
\begin{aligned}
r_1(A^C) &= r\sqrt{a'_{ij}{}^2} + (1-r)\sqrt{(1-a_{ij}{}^2)}, \\
&= r\sqrt{a'_{ij}{}^2} + \sqrt{(1-a_{ij}{}^2)} - r\sqrt{(1-a_{ij}{}^2)} \\
&= 1 - \left(\sqrt{a_{ij}{}^2} - r\sqrt{a_{ij}{}^2} + r - r\sqrt{a'_{ij}{}^2}\right) \\
&= 1 - \left((1-r)\sqrt{a_{ij}{}^2} + r\sqrt{(1-a'_{ij}{}^2)}\right), \\
&= 1 - r_3(A) \\
&= r_4(A).
\end{aligned}
$$

Similarly, we can prove $r_2(A^C) = r_3(A)$.

Property 4.3:

For any two PFMs $A, B \in \mathcal{F}_{mn}$ and $r \in [0,1]$, we have

$(i)\ r_1(A) \leq r_1(B)$,

$(ii)\ r_2(A) \geq r_2(B)$,

$(iii)\ r_3(A) \geq r_3(B)$,

$(iv)\ r_4(A) \leq r_4(B)$.

Proof.

(i) By the Definition (4.1), we have

$$r_1(A) = r\sqrt{a_{ij}{}^2} + (1-r)\sqrt{(1-a'_{ij}{}^2)},$$

$$r_1(B) = r\sqrt{b_{ij}{}^2} + (1-r)\sqrt{(1-b'_{ij}{}^2)},$$

Since $A \leq B$, $a_{ij}{}^2 \leq b_{ij}{}^2$, $ra_{ij}{}^2 \leq rb_{ij}{}^2$ and $a'_{ij}{}^2 \geq a'_{ij}{}^2$.

Thus $(1-r)(1-a_{ij}{}^2) \leq (1-r)(1-b_{ij}{}^2)$.

So $ra_{ij}{}^2 + (1-r)(1-a_{ij}{}^2) \leq rb_{ij}{}^2 + (1-r)(1-b_{ij}{}^2)$.

Hence, $r_1(A) \leq r_1(B)$.

(ii) By Definition (4.1),

$r_2(A) = 1 - r_1(A)$ and from (i), we have $r_1(A) \leq r_1(B)$.

Then $1 - r_1(A) \geq 1 - r_1(B)$.

Thus $r_2(A) \geq r_2(B)$.

(iii) The proof of (iii) similar to (i).

(iv) The proof of (iv) similar to (ii).

Property 4.4:

For any two PFMs $A, B \in \mathcal{F}_{mn}$ and $r \in [0,1]$, we have

$(i)\ r_1(A \vee_L B) = r_1(A) + r_1(B) - r_1(A \wedge_L B)$,

$(ii)\ r_2(A \vee_L B) = r_2(A) + r_2(B) - r_2(A \wedge_L B)$,

$(iii)\ r_3(A \vee_L B) = r_3(A) + r_3(B) - r_3(A \wedge_L B)$,

$(iv)\ r_4(A \vee_L B) = r_4(A) + r_4(B) - r_4(A \wedge_L B)$.

Proof.

(i) By the Definition (4.1), we have

$r_1(A \vee_L B)$

$$= r_1\left(\langle\sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{\max(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\rangle\right)$$

$$= r\left(\sqrt{\min\left(1, a_{ij}^2 + b_{ij}^2\right)}\right) + (1-r)\left(\sqrt{1 - \max(0,\ {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\right).$$

$r_1(A \wedge_L B)$

$$= r\left(\langle\sqrt{\max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{\min(1,\ {a'_{ij}}^2 + {b'_{ij}}^2)}\rangle\right)$$

$$= r\left(\sqrt{\max\left(0, a_{ij}^2 + b_{ij}^2 - 1\right)}\right) + (1-r)\left(\sqrt{1 - \min\left(1,\ {a'_{ij}}^2 + {b'_{ij}}^2\right)}\right).$$

$$r_1(A) + r_1(B) = r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)\left(\sqrt{(1 - {a'_{ij}}^2) + (1 - {b'_{ij}}^2)}\right).$$

1). If $a_{ij}^2 + b_{ij}^2 \geq 1$ and ${a'_{ij}}^2 + {b'_{ij}}^2 - 1 \leq 0$, then

$$r_1(A \vee_L B) = r + (1-r)$$

$$= 1.$$

$$r_1(A \wedge_L B) = r\left(\sqrt{a_{ij}^2 + b_{ij}^2 - 1}\right) + (1-r)\left(\sqrt{1 - {a'_{ij}}^2 - {b'_{ij}}^2}\right).$$

$r_1(A \vee_L B) + r_1(A \wedge_L B)$

$$= r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)\left(\sqrt{(1 - {a'_{ij}}^2) + (1 - {b'_{ij}}^2)}\right)$$

$$= r_1(A) + r_1(B).$$

Hence, $r_1(A \vee_L B) = r_1(A) + r_1(B) - r_1(A \wedge_L B)$.

2). If $a_{ij}^2 + b_{ij}^2 \leq 1$ and ${a'_{ij}}^2 + {b'_{ij}}^2 - 1 \leq 0$, then

$$r_1(A \vee_L B) = r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)$$

$$r_1(A \wedge_L B) = (1-r)\left(\sqrt{1 - {a'_{ij}}^2 - {b'_{ij}}^2}\right).$$

$r_1(A \vee_L B) + r_1(A \wedge_L B)$

$$= r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)\left(\sqrt{1 - {a'_{ij}}^2 - {b'_{ij}}^2}\right)$$

$$= r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)\left(\sqrt{(1 - {a'_{ij}}^2) + (1 - {b'_{ij}}^2)}\right)$$

$= r_1(A) + r_1(B).$

Hence, $r_1(A \vee_L B) = r_1(A) + r_1(B) - r_1(A \wedge_L B)$.

3). If $a_{ij}^2 + b_{ij}^2 \leq 1$ and ${a'_{ij}}^2 + {b'_{ij}}^2 - 1 \geq 0$, then

$r_1(A \vee_L B) = r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)\left(\sqrt{(1- {a'_{ij}}^2) + (1-{b'_{ij}}^2)}\right).$

$r_1(A \wedge_L B) = 0.$

$r_1(A \vee_L B) + r_1(A \wedge_L B)$

$= r\left(\sqrt{a_{ij}^2 + b_{ij}^2}\right) + (1-r)\left(\sqrt{(1- {a'_{ij}}^2) + (1-{b'_{ij}}^2)}\right)$

$= r_1(A) + r_1(B).$

Hence, $r_1(A \vee_L B) = r_1(A) + r_1(B) - r_1(A \wedge_L B)$.

4). If $a_{ij}^2 + b_{ij}^2 \geq 1$ and ${a'_{ij}}^2 + {b'_{ij}}^2 - 1 \geq 0$, then

A and B are not PFMs.

From all the above cases we conclude that

$r_1(A \vee_L B) = r_1(A) + r_1(B) - r_1(A \wedge_L B).$

(ii) $r_2(A \vee_L B) + r_2(A \wedge_L B)$

$= (1 - r_1)(A \vee_L B) + 1 - r_1(A \wedge_L B)$

$= 1 - \big(r_1(A) + r_1(B) - r_1(A \wedge_L B)\big) + 1 - r_1(A \wedge_L B).$

$= (1 - r_1)(A \vee_L B) + 1 - r_1(A \wedge_L B)$

$= 1 - r_1(A) + 1 - r_1(B)$

$= r_2(A) + r_2(B).$

Hence, $r_2(A \vee_L B) = r_2(A) + r_2(B) - r_2(A \wedge_L B)$.

(iii) Since $r_2(A^C) = r_3(A)$,

$r_3(A \vee_L B) + r_3(A \wedge_L B) = r_2((A \vee_L B)^C) + r_2((A \wedge_L B)^C).$

From De Morgan's law of $\vee_L$ and $\wedge_L$ operations, we can write the above as follows:

$r_3(A \vee_L B) + r_3(A \wedge_L B)$

$= r_2(B^C \vee_L A^C) + r_2(B^C \wedge_L A^C)$

$= r_2(B^C) + r_2(A^C)$

$= r_3(B) + r_3(A).$

Hence, $r_3(A \vee_L B) = r_3(A) + r_3(B) - r_3(A \wedge_L B)$.

(iv) The proof is similar to (ii) or (iii).

5. CONCLUSION

In this work, we have read some of the results of $\vee_L$ and $\wedge_L$ from the Lukasiewicz type over Pythagorean fuzzy matrices. We presented four reduction operations about the Pythagorean fuzzy matrices. We associate the reduction operations to relate $\vee_L$ and $\wedge_L$ activities.

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