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Joint Life Term Insurance Reserves Use the Retrospective Method Based on De Moivre Law

Fiyan Handoyo[1,*], Riaman[2], Nurul Gusriani[2], Sudrajat Supian[2], Subiyanto[3]

[1]Master Program of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia

[2]Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Indonesia

[3]Department of Marine Science, Faculty of Fishery and Marine Science, Universitas Padjadjaran, Indonesia

*E-mail address: Handoyo.fiyan@gmail.com

ABSTRACT

Joint Life Insurance futures is life insurance that covers two or more people within n years. The policy holder will get benefits from the insurance company if one of the combined insurance insured dies during the period of protection. It is likely that the insurance company will incur a loss if the claim is greater than predicted. Therefore, it is necessary to calculate premium reserves for insurance companies to predict company losses in the future. The method used to calculate premium reserves is the retrospective method. Premium reserves are calculated based on the 2011 TMI and De Moivre's assumptions. The results of the annual premium calculation based on assumptions are greater than using TMI 2011, because life opportunities based on assumptions are relatively small, while premium reserves are based on smaller assumptions than using 2011 TMI because the size of the reserves depends on the development of premiums.

***Keywords*:** Joint Life Insurance, Premium Reserves, Retrospective Methods, De Moivre Law

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1. INTRODUCTION

Risk is defined as uncertainty where an undesirable situation causes a loss to assets held [1][2]. To avoid unwanted risks, an insurance program is needed to minimize losses [3-6]. Risk aversion, prudence and temperance are the main drivers of an individual's decisions under risk. Indeed, risk aversion increases insurance demand as well as prevention level, and leads to a higher degree of diversification for portfolio choices [7]. In the same vein, prudence is a key determinant of most of the risky decisions, in particular, prevention [8-10].

Life insurance is a payment for a certain amount of money for the death of the insured to the heirs or people who have the right to receive it in accordance with the provisions in the insurance policy, the amount of money paid to the insured is in the form of sum assured [11][12]. Between 2001 and 2010, total insurance premiums in emerging markets expanded by 11%, with growth averaging 1.3% in industrialized markets. This growth in emerging markets was mostly driven by life sector with an average growth rate of 12.6% compared to 0.6% for industrialized markets. However, the growth in penetration of life insurance in emerging markets has been mainly concentrated in emerging Asian markets and in Latin America with China alone accounting for about a third of the total emerging market premium volume [13][14]. In this study will be discussed about term life insurance, which is life insurance that provides protection for a certain period of time. The existence of a combined life insurance product is an innovation carried out by the company to minimize asset losses [15][16], this supports policyholders can insure with members of their families. The policy holder will get benefits if one of the policyholders dies within the period of engagement [17][18].

The difference between the premium and the sum insured will be held by the insurance company as a reserve, which will be used to pay the sum insured in the event of a claim. The possibility of an insurance company experiencing losses due to claims that are greater than the number of claims previously predicted. Different surplus appropriation schemes in participating life insurance are analyzed for the first time from the policyholders' and the insurer's perspectives encompassing mortality and financial risk, hereby also studying the impact on default risk in [19-22]. According to [23], to avoid losses of a company, the insurance reserves are calculated to predict future company losses.

Previous research has discussed approach to claims reserving for life insurance policies with reserve-dependent payments driven by multi-state Markov chains by [24]. [25] Reviewing premium reserves using a prospective method. In both the studies were devoted to one person insured. Therefore, this study will examine insurance products for two insured people. Using the retrospective method to calculate premium reserves, for term life insurance products.

2. LITERATURE REVIEW

2. 1. Joint Life Mortality Table

The combined mortality table is a mortality rate table that has a role in determining premiums [11]. In general (x, y) states policyholders who are x years old, and y years old. The combined function of policyholders aged x years and y years is denoted by l_{xy} so that it is obtained:

$$l_{xy} = l_x l_y \tag{2.1}$$

For t years then the combination is formulated as follows:

$$l_{x+t,y+t} = l_{x+t} l_{x+t} \tag{2.2}$$

The number of people dying in one year is expressed by the following functions:

$$d_{xy} = l_{xy} - l_{x+t,y+t}\,. \tag{2.3}$$

2. 2. Probability of Joint Life

The policyholder aged x years is symbolized as(x), in the life table the number of (x) that lives is l_x , while the number (y) that lives is l_y.

The combination of (x) and (y), the death does not affect each other, the value of the possibility of two- both within 1 year and then remain alive, formulated as follows:

$$p_{xy} = p_x p_y \tag{2.4}$$

Opportunities for policyholders aged x and y to stay alive for the next t year are:

$${}_t p_{xy} = \frac{l_{x+t,y+t}}{l_{xy}} \tag{2.5}$$

The possible value of one between (x) and (y) first dies in the interval $[t, t-1]$ is

$$\begin{aligned} {}_{t|} p_{xy} &= \frac{l_{x+t,y+t} - l_{x+t-1,y+t-1}}{l_{xy}} \\ &= {}_t p_{xy} - {}_{t+1} p_{xy} \end{aligned} \tag{2.6}$$

2. 3. Joint Life Annuity and Joint Life Insurance

Joint life annuity is an annuity contract consisting of two or more insured, where payment is stopped if one of the insured dies. In this research, early life annuity is used, which is an annuity whose payment is made as long as the insured is still alive within the period of payment with payment made at the beginning of the year. Life or death benefits are associated with the insured amounting to 2 people or more, insurance payments with benefits like this are called Joint Insurance. Insurance with 2 or more people at the same time, if each of the insured lives or dies there is no connection, the payment of the sum insured is not included in the joint life insurance. The present value of joint life futures if (x) and (y) are still alive

Last Annuity:

$$a_{xy:\overline{n}|} = \sum_{t=1}^{n} v^t\, {}_t p_{xy} \tag{2.7}$$

First Annuity:

$$\ddot{a}_{xy:\overline{n}|} = \sum_{t=1}^{n-1} v^t \,_t p_{xy} \tag{2.8}$$

A single premium for insurance for which the sum of money R is paid at the end of the insurance period is denoted by $A^{\,1}_{\widehat{xy}:\overline{n}|}$, which is formulated as follows [11],

$$\begin{aligned} A_{x\hat{y}}^{1} &= R\sum_{t=0}^{n-1} v^{t+1} \,_{t|}q_{xy} \\ &= R(v\ddot{a}_{xy:\overline{n}|} - a_{xy:\overline{n}|}) \end{aligned} \tag{2.9}$$

Annual premium for term life insurance $P^{1}_{xy:\overline{n}|}$ which will be paid at the end of the policy year stated by:

$$p^{1}_{xy:\overline{n}|} = \frac{A^{1}_{x\hat{y}:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}} \tag{2.10}$$

2. 4. Retrospective Reserves

Before explaining the retrospective reserve, it will first be explained about reserves. Reserves in life insurance are liabilities (obligations) of insurance companies against policyholders in the form of a number of funds that must be prepared by insurance companies to pay claims that will occur in the future for policies issued by insurance companies [26-29].

In principle, the retrospective premium reserve for a life insurance policy is the actuarially accumulated value of past premiums minus past benefits. Simple cases accepted, however, there is not just one but a number of retrospective reserves, namely one reserve for each state of the system involved. For each state, there is a prospective reserve as well, and the two corresponding reserves need not coincide, though in many situations they do. The difference between the reserves is of special importance when they are computed on a second-order valuation basis intended not to include any safety margins in the model elements while the premiums have been calculated on a first-order valuation basis with conservative model elements, i.e., with elements chosen on the safe side by the standards of the second-order basis. Retrospective reserves at the end of the first year [26]:

$$_1V_x = \frac{(l_x P)(1+i) - 1d_x}{l_{x+1}}. \tag{2.11}$$

Parts at the end of the year to *t* are:

$$_tV_x = \frac{(l_{x+t-1}(_{t-1}V) + l_{x+t-1}P)(1+i) - 1d_{x+t-1}}{l_{x+1}}. \tag{2.12}$$

3. OBJECTS AND METHODS OF RESEARCH

3. 1. Research Object

The object of this research is to determine the reserves of combined term life insurance premiums using the retrospective method based on the assumption of De Moivre. To determine the parameters in making a Mortalita Table assuming De Moivre refers to the 2011 Indonesian Mortality Table to determine ω.

3. 2. Research Methods

The method used to determine the amount of combined futures insurance premiums, which is using the retrospective method based on the De Moivre Mortalita Table assumption referring to the 2011 Indonesian Mortality Table.

3. 2. 1. Mortality Law

The law of mortality here is the relationship that exists between l_x, q_x and also μ_x Scientists researched to obtain several theories regarding mortality law, namely De Moivre's law, Gompertz's law, and Makeham's law [30]. This study uses the law of De Moivre.

3. 2. 2. De Moivre's Law

In 1725 De Moivre stated that the number living at the age of x can be expressed in the following formula:

$$l_x = \frac{l_0(\omega - x)}{\omega} \tag{3.1}$$

by ω expressing a person's maximum age, the acceleration of mortality is:

$$\mu_x = \frac{1}{\omega - x} \tag{3.2}$$

to calculate the life chances of an x-year old person will live in another year, namely:

$$_t p_x = \frac{\omega - x - t}{\omega - x} \tag{3.3}$$

so for the chance of dying someone x years old will die in another year, namely:

$$_t q_x = \frac{t}{\omega - x} \tag{3.4}$$

3. 3. Retropective methods

Based on Equation (2.12), the end of first year reserves with compensation of Rp1 are

$$_1V_{xy} = \frac{(l_{xy}P)(1+i)-1d_{xy}}{l_{x+1,y+1}} \quad (3.5)$$

So in general, reserves at the end of the year t are

$$_tV_{xy} = \frac{((l_{x+t-1,y+t-1}\ _{t-1}V)+(Pl_{x+t-1,y+t-1})(1+i)-1d_{x+t-1,y+t-1}}{l_{x+1,y+1}} \quad (3.6)$$

Based on Equation (3.1), reserves at the end of the year t using the De Moivre assumption are:

For example $(\omega - x - t + 1) = K$ and $(\omega - y - t + 1) = L$, then

$$_tV_{xy} = \frac{((\frac{l_0}{\omega})^2 K.L_{t-1}V + (\frac{l_0}{\omega})^2 KLP)(1+i) - 1(\frac{l_0}{\omega})^2(KL-(K-1)(L-1))}{(\frac{l_0}{\omega})^2(K-1)(L-1)} \quad (3.7)$$

$$_tV_{xy} = \frac{(KL_{t-1}V + (KLP)(1+i)) - 1(KL-(K-1)(L-1))}{(K-1)(L-1)} \quad (3.8)$$

4. RESULTS AND DISCUSSION

4. 1. Research Data

The data used in this study is simulation data, namely four types of data of policyholders. The amount of compensation for each policyholder is equalized, which is Rp100000000. The life insurance coverage period is equal to 20 years with a 6.5% interest rate. The following is the policy holder data presented in Table 1,

Table 1. Data of Policy Holders.

No	Name	Age (year)	Period of engagement (year)	Compensation
1	A & B	50 & 40	20	Rp100000000
2	C & D	35 & 40	20	Rp100000000
3	E & F	50 & 55	20	Rp100000000
4	G &H	20 & 60	20	Rp100000000

4. 2. The Forming of Mortality Tables Based on De Moivre

Based on the Indonesian Mortality Table 2011. Determined the upper limit of the age of the cohort (ω) is 111 and the number of initial cohorts (l_0) is 100000, then a Mortality Table can be formed based on the assumption of De Moivre. Calculation using Ms. Excel 2010, By Using Equation (3.1), the number of people living at the age of x is obtained, for x = 1

$$l_1 = 99099.0991 \approx 99099$$

So based on De Moivre's assumption the number of people living at the age of 1 year is 99099, and calculations are made up to x = 111. Using equation (3.3), we get ${}_tp_x$ as follows,

$${}_1p_1 = 0.990909$$

So the opportunity for 1-year-olds to live for another year is 0.990909.

4. 3. Calculating Joint Term Life Annuities

Calculation of a combined initial n-year life annuity denoted by $\ddot{a}_{xy:\overline{n}|}$ and finally annuity denoted by $a_{xy:\overline{n}|}$ for data Table 2, can be calculated using the formula in Equation (2.7) and (2.8) as follows: Data of the 1st policy holder is based on the De Moivre:

$$\ddot{a}_{50,40:\overline{20}|} = 9.300917894$$

$$a_{50,40:\overline{20}|} = 8.43793465$$

Table 2. Life Annuity.

No	Policy holder	$\ddot{a}_{xy:\overline{n}\vert}$		$a_{xy:\overline{n}\vert}$	
		De Moivre	TMI 2011	De Moivre	TMI 2011
1	A & B	9.300917894	10.65	8.43793465	9.83690575
2	C & D	9.537936115	11.29	8.68814448	10.52023610
3	E & F	9.033388187	9.83	8.1560124	8.96166706
4	G &H	9.274871055	9.92	8.40946215	9.05294631

4. 4. Single Net Combined Premium Calculation

Based on Table 1 the calculation of net premium is denoted by $A^{\ 1}_{\widehat{xy}:\overline{n}|}$ for data Table 3, can be calculated using the formula in Equation (2.9) as follows, Data of the 1st policy holder is based on the assumption of De Moivre,

$$A^{\ 1}_{\widehat{50,40}:\overline{20|}} = 0.29532158$$

Table 3. Single Net Premium

No	Policy holder	$A^{\ 1}_{\widehat{xy}:\overline{n|}}$	
		De Moivre	TMI 2011
1	A & B	0.295321585	0.164146074
2	C & D	0.267664083	0.076553172
3	E & F	0.326042205	0.268475281
4	G &H	0.299336965	0.261056092

4. 5. Annual Premium Calculation of Joint Term Life Insurance

Using Equation (2.10), the annual premium value is obtained as follows:

For data on the 1st policy holder based on the De Moivre Assumption:

$$P^{1}_{50,40:\overline{20|}} = 3175188$$

For data on the 1st policy holder based on the TMI 2011:

$$P^{1}_{50,40:\overline{20|}} = 1541116$$

by using the same method, an annual premium is obtained for four policy holders,

Table 4. Annual Premium

No	Policy holder	Premium (*De Moivre)*	Premium (TMI 2011)
1	A & B	Rp3175188	Rp1541116
2	C & D	Rp2806310	Rp678327
3	E & F	Rp3609301	Rp2731155
4	G & H	Rp3227398	Rp2631770

Annual premiums based on the De Moivre assumption are greater than the annual premiums based on the 2011 TMI, this is because the living opportunities are based on De Moivre's assumptions for each policyholder smaller than the life chances based on TMI 2011.

The following is a comparison of the probability life of each policy holder,

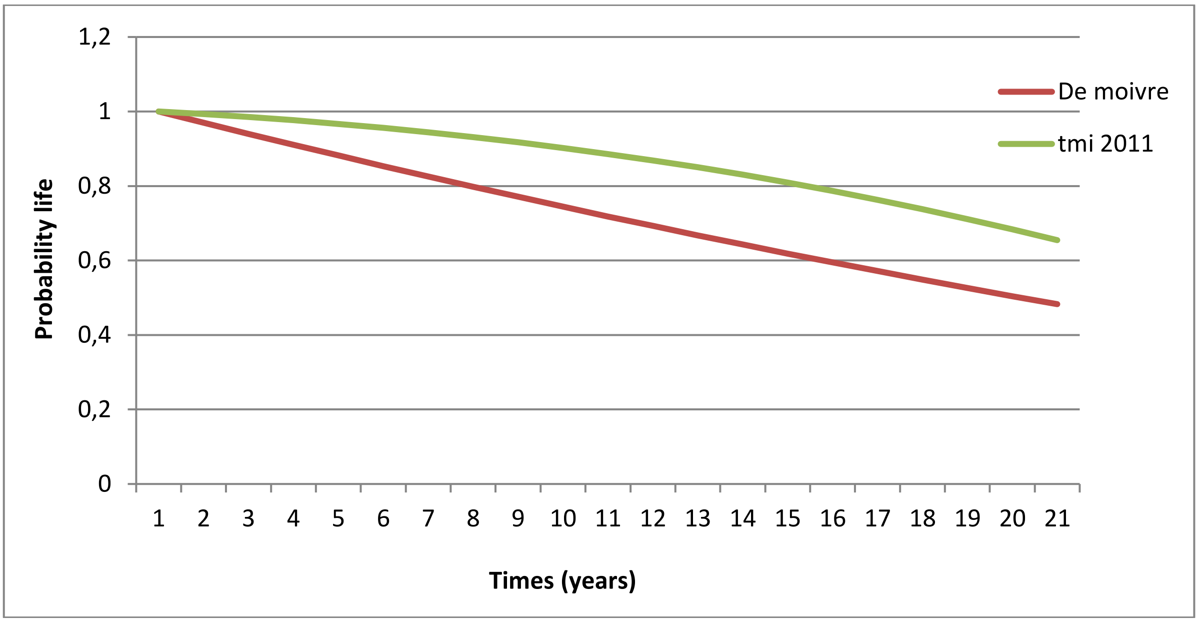

Figure 1. Life Opportunities Chart for Policy Holders aged 40 years and 50 years

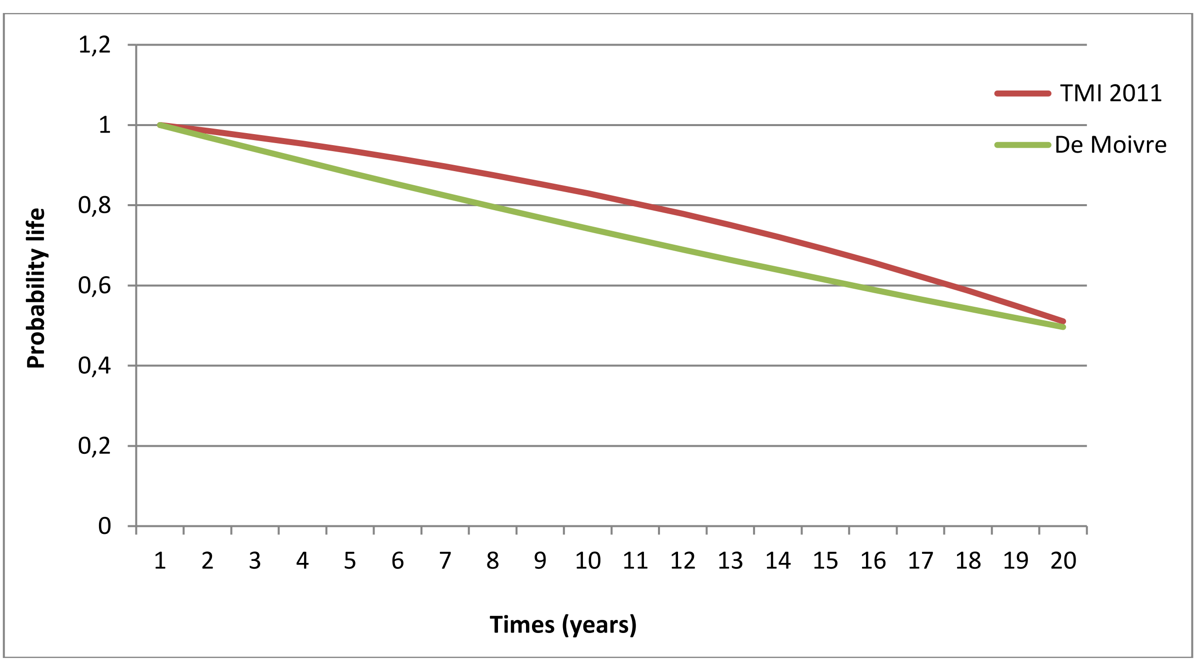

Figure 2. Life Opportunities Chart for Policy Holders aged 20 years and 60 years

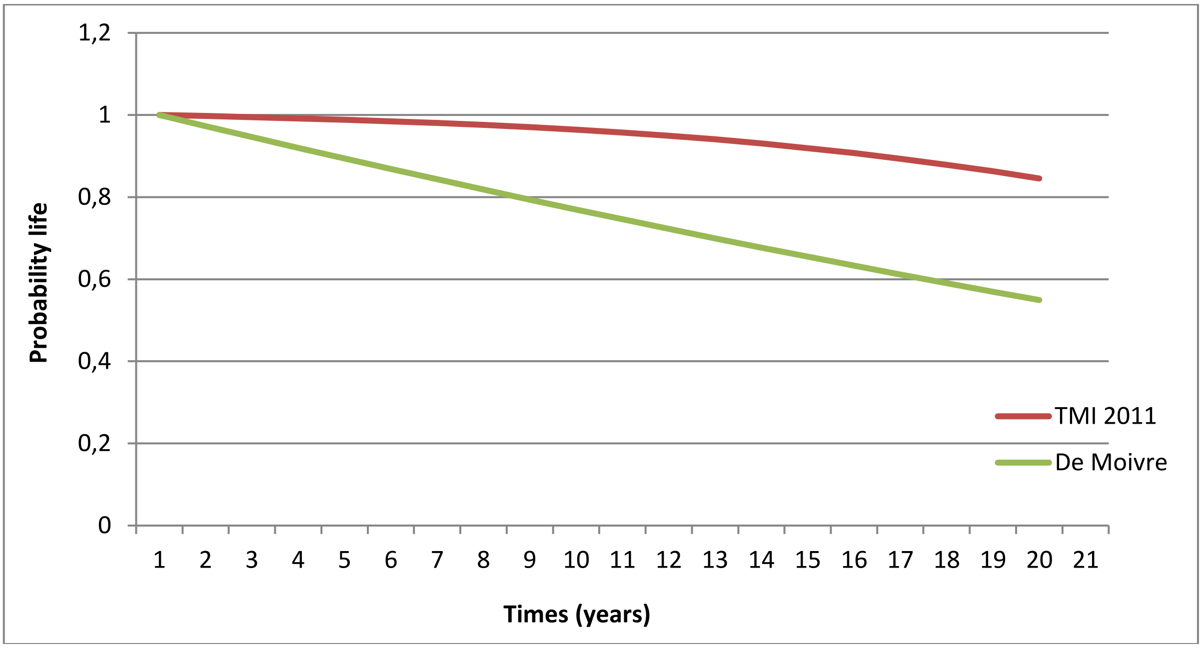

Figure 3. Life Opportunities Chart for Policy Holders aged 40 years and 35 years

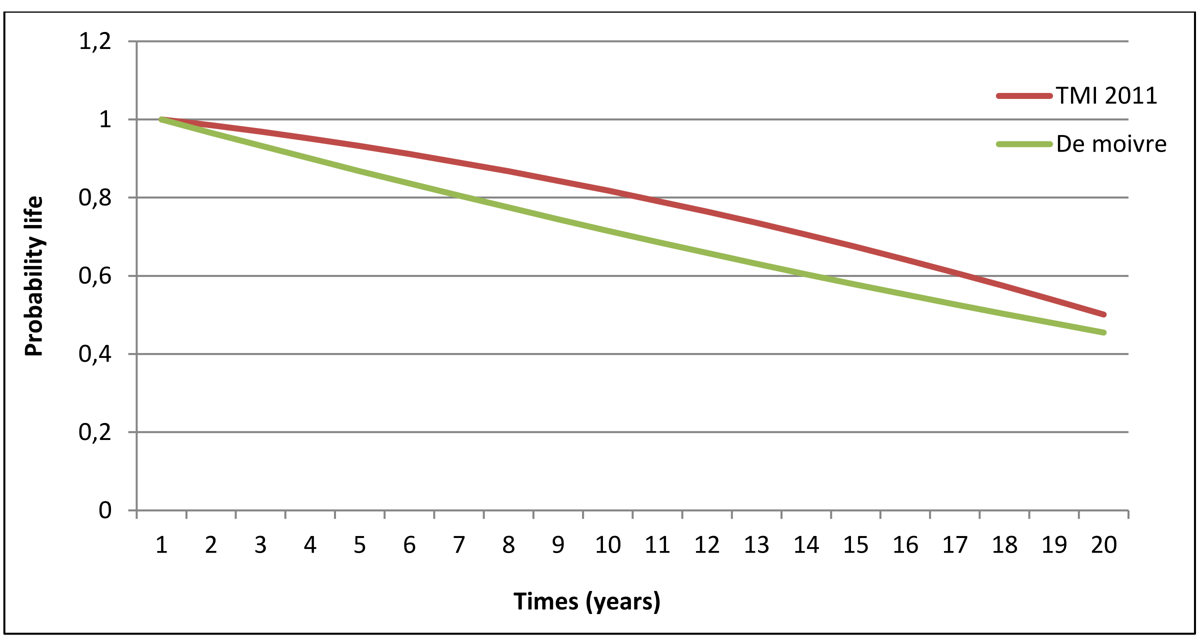

Figure 4. Life Opportunities Chart for Policy Holders aged 50 years and 55 years

4. 6. Calculation of Retrospective Reserves Based on De Moivre Assumptions

Based on the explanation in chapter 3, the premium reserves of combined term life insurance with the sum assured for the four policies are Rp. 100,000,000, and the period for 20 years, using the retrospective method based on Equation (3.6),

Table 5. First Year Premium Reserves.

No	Policholder	First Premium Reserve	
		De Moivre	TMI 2011
1	A & B	Rp368000	Rp957721
2	C & D	Rp290883	Rp479728
3	E & F	Rp463874	Rp1436305
4	G & H	Rp411543	Rp1357419

5. CONCLUSIONS

Based on the results of the research and discussion, it can be concluded that, based on De Moivre, the annual premium of combined term life insurance products is obtained, the annual premium based on the De Moivre assumption is greater than the annual premium based on TMI 2011, due to the De Moivre assumption smaller than TMI 2011. The results of the calculation of premium reserves are based on the De Moivre assumption smaller than premium reserves based on the Indonesian Mortality Table 2011, this is due to the large amount of reserves depending on the development of premiums, meaning that the greater the number of policyholders. The amount of premium reserves based on assumptions and TMI 2011 on average from the first to mid reserves has increased, then down to zero value because at the beginning of the insurance contract, the risk of claims experienced by the company is relatively small. Can be seen from the life chances of policyholders at the beginning of the contract is relatively large. But in the mid to the end of the insurance contract, the risk of claims experienced by the company is very large so that the reserve value decreases to zero.

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