Particle localization via measurement induced entanglement

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ABSTRACT

To explore the boundary between quantum and classical physics in the context of quantum entanglement, the particle localization via measurement induced entanglement on photons incident onto a distinguishable, massive non-interacting two-particle system was studied. The specific case of how particles acquire well defined spatial localization when light is scattered off and detected was considered. The localization process studied both qualitatively and quantitatively was observed to be valid for particles which were initially localized as well as initially completely delocalized. The localization scheme was also observed to be extremely sensitive to its initial conditions. Furthermore, a difference in the localization scheme in terms of the number of scattering events was observed between monochromatic photons and photons with variable frequencies. From all these results it was apparent that we can interpret the uniquely quantum features of entanglement in terms of classicality.

Keywords: Quantum entanglement, spatial localization, Fluffy-Bunny entanglement, position probability density, relative position, photons, variance, scattering angle

1. INTRODUCTION

With the advent of the Quantum mechanical era, the early twentieth century physicists came to a profound realization that a different set of rules governed the mechanics of the microscopic regime. To perceive why different sets of rules should be applicable to different
size scales many studies had been carried out to understand the boundary between the quantum and classical physics. The observation that the canonical commutation relations become Poisson brackets are presented as providing a definite answer to the problem of understanding how the classical laws of Newtonian mechanics emerge from the more basic laws of quantum mechanics [1].

There is also an enormous amount of mathematical work, called semiclassical analysis or, in more modern terms, microlocal analysis in which the limit $\hbar \to 0$ of Schrödinger evolutions is rigorously studied [2]. According to the Heisenberg’s uncertainty principle in the quantum world, the position of a particle cannot be precisely measured, and the object could be in a superposition of states spread across space. But classical physics is based on a local realism so that these kinds of states are not present, and objects always have well-defined positions in the classical world. One key question that is central to understanding the boundary between these regimes is the localization of objects in the position space [3].

Macroscopic systems are never isolated from their environments. Therefore, they should not be expected to follow Schrödinger’s equation, which is applicable only to a closed system [4]. As a result, systems usually regarded as classical suffer or benefit from the natural loss of quantum coherence which leaks out to the environment [5, 6]. Since no system is truly isolated but interacts with an environment which constantly measures the position of the particle and produces in this way a narrow wave packet [7], an effect called decoherence [8]. Therefore, one of the successful descriptions in explaining the quantum and classical transition is the theory of decoherence, the loss of quantum coherence of macroscopic objects due to their coupling with the environments [9, 10].

On the other hand, this theory is quite difficult to apply since the nature of the interaction between the system and the environment is complex. But if the measurements are done only on the quantum subsystem and if all the measurements in position are done in the relative space it eliminates the need for the quantum information about the environmental states correlated with the quantum subsystem. Such a measurement process can induce entanglement between two particles which leads to robust semi classical states with well-defined relative localization.

Because of this measurement process we can see that the quantum subsystem appears to exhibit classical behavior as it begins to interact more and more with its environment. Entanglement is defined as a quantum mechanical phenomenon in which the quantum states of two or more objects are described with reference to each other, even though the individual objects may be spatially separated. Because of this the objects are linked and changing one invariably affects the other. This leads to correlations between observable physical properties of a quantum system [11].

Rau et al. have shown that the unique quantum features of entanglement are associated with classical behavior. But this interpretation lacks a clear experimental signature that allows the theory to be tested. This was further studied and resolved by Knott et al. [12] providing a simple accessible scheme that enables experimentalists to unambiguously determine whether scattering events can induce, relative position localization for quantum particles. The localization process is driven due to the measurement induced entanglement between the particles.

This is a very specific type of entanglement known as the ‘Fluffy-Bunny’ entanglement [13]. Because of this a whole different class of states called the fluffy-bunnies arises which are robust entanglements due to the measurements and interactions between particles. These fluffy-bunny entanglements reveal how classical like behavior can emerge from quantum systems and
they are the kind of entanglement that is responsible for the way the classical world is seen. Odd as it may seem, this suggests that the process of entanglement which is thought to be a pure quantum feature is associated with the emergence of classicality from the quantum regime.

In this paper we consider photons incident onto a distinguishable, massive non-interacting two-particle system and the resulting variation of the position probability densities of the relative position of the two-particle system were studied. Initially a qualitative study was done for a pair of delocalized particles and then the validity of the localization process and the sensitivity of the localization process to its initial conditions was examined by extending the localization scheme for a pair of particles having an initial localization.

Furthermore, a quantitative study was done for the localization scheme in terms of changing variance of the position probability density of the relative position of the two particles. Also the localization process was tested by using photons of variable frequencies and it was compared with the results obtained using monochromatic photons.

The localization process was valid for particles with initial localization and even for particles which were completely delocalized and the localization scheme was extremely sensitive to its initial conditions and thereby proved both qualitatively and quantitatively. Furthermore, a difference in the localization scheme, in terms of the number of scattering events was observed between monochromatic photons and photons with variable frequencies.

2. LOCALIZATION

Photons were incident onto a massive, distinguishable, non-interacting two-particle system. The schematic diagram of the experimental setup [12] used for this study is shown in Fig. 1. These particles were initially considered to be delocalized in a finite region $d$ along the $x$-axis while they were tightly confined to the $y$ and $z$ axes. Plane waves with wave number $k$ and wavelength $\lambda$ were incident on to the two particles and the scattered photons were detected at some angle $\theta$ on a screen located at a distance $L$ ($L \gg d$) away from the two particles. The initial wave function of the two particles was considered as $C(x)$ where $x$ is the relative position between the two particles and $\sum |C(x)|^2 = 1$. During each scattering event a photon of definite momentum was scattered off from the two particles imparting a momentum change of $\Delta k = \pm h \sin \theta / \lambda$ in the relative momentum space. But the detection of this scattered photon did not provide any information about the way the particle was scattered. The photon may have scattered off either from the first particle or the second one.

Therefore, it was in a superposition of these two possible states. It is this lack of information that entangles the two particles and drives the localization process. With the scattering of photons, the two particles get entangled with a definite centre of mass momentum. Throughout the entire localization process, the centre of mass of the two particles remains in a shifted momentum eigenstate, neither particle having a well-defined absolute position.

Therefore, all calculations were done in the relative space rather than in the absolute space. As the centre of mass remains un-entangled during the whole localization process, it can be ignored.

We assumed that all the photons are incident on the two particles in a very short time so that, the consideration of the dynamics of the particles between detection events is not necessary [3].
Figure 1. Plane waves with wave number $k$ are incident on the pair of particles and the photons scattered off from particles 1 and 2 are detected at angle $\theta$ in the far field.

With the scattering of photons, the two particles get more and more entangled with each other. This is a very specific type of entanglement known as the Fluffy-Bunny entanglement [13] made up of robust states of well-defined relative positions that arise naturally due to the measurements and interactions between particles. We consider a system of two-particles in a momentum eigenstate with undefined relative position. After the detection of a single scattering event, the resultant system is a linear combination of the photon interacting with each particle and can be represented as

$$\begin{align*}
|p_1\rangle|p_2\rangle & \rightarrow c_1|p_1 + \Delta k\rangle|p_2\rangle + c_2|p_1\rangle|p_2 + \Delta k\rangle
\end{align*}$$

(1)

where $p_1$ and $p_2$ are the initial momenta of the two particles given by $p_i = \hbar k_i$ and $c_i$ are the probability amplitudes and $\Delta k$ is the momentum imparted by each photon on the particles.

The two particles are considered to act as a perfect point scatterer. Thus the common phase could be removed. Both particles are assumed to scatter light with equal probability in
all directions, i.e., $|c_1| = |c_2|$. By imposing these conditions, eq. 1 could be reduced to the following relation in the relative coordinate space as

$$|p\rangle = \frac{1}{\sqrt{2}} \left( |p + \Delta k/2 \rangle + e^{i\phi} |p - \Delta k/2 \rangle \right)$$

(2)

where $p = (p_1 - p_2)/2$ and $\phi$ is the phase depending on the angle at which the photon is detected. This equation shows that the measurement has broadened the relative momentum wave function of the two particles. According to the Heisenberg’s uncertainty principle, this broadening of the momentum should be accompanied by a reduction of the uncertainty of the relative position space due to the conjugate relation between momentum and spatial space.

Therefore, with each scattering event, the two particles get more and more entangled and the momentum wave function broadens more and more thereby reducing the uncertainty of the relative position between the two particles. This localization scheme is continued through a feedback process and with each subsequent measurement, the initially delocalized particles become more and more localized in the relative space. Each of these relative position states belongs to the class of Fluffy-Bunny. Therefore, these states are well defined and robust since subsequent measurements do not change the relative position. It only reinforces localization at the existing separation. These fluffy bunny states are responsible for the emergence of the classicality from the quantum regime through the unique process of quantum entanglement [13]. When photons are scattered onto the two particles, each scattered photon will impart a momentum of $\Delta k = \pm \hbar \sin \theta / \lambda$ on each particle in the relative momentum space. After a single scattering event the overall wave function of the system just before detection can be written as [3]:

$$\Psi(x, \theta) = \sum_x c_x |x\rangle \left[ A(x) |0\rangle + \int_0^{2\pi} \frac{1}{\sqrt{2\pi}} (e^{i2\pi x \sin \theta / \lambda} + e^{-i2\pi x \sin \theta / \lambda}) |\theta\rangle d\theta \right]$$

(3)

$|\theta\rangle$ represents the state of the photon when it is scattered at an angle $\theta$ and $|0\rangle$ represents the state of a photon when it is not scattered.

The term $A$ is defined as:

$$A(x) = \left( \frac{1}{2\pi} \int_0^{2\pi} \sin^2 (2\pi x \sin \theta / \lambda) d\theta \right)^{1/2} = \left( 1 - J_0(4\pi x / \lambda) \right)^{1/2}$$

(4)

where $J_0$ is the Bessel function of the first kind of order zero and $\lambda$ is the wavelength of the photons. In this study, the state of the whole system including the two particles and photons was represented by $\Psi$, while the state of the two particles was represented by $\psi$. Equation 3 can be further simplified as:

$$\Psi(x, \theta) = \begin{cases} 
\frac{c(x)}{2\sqrt{2\pi}} (e^{i2\pi x \sin \theta / \lambda} + e^{-i2\pi x \sin \theta / \lambda}) & \theta \neq 0 \\
(c(x)A(x)) & \theta = 0 
\end{cases}$$

(5)
If the localization process is executed considering only the scattered photons then, the two particles will always localize to a zero separation. This suggests that a non-scattered photon also gives information about the relative position between the two particles and therefore it is crucial for the successful localization of the two particles. Fig. 2 represent the variation of non-scattering term $A(x)$ with respect $x$ for unit wavelength.

The probability for a photon to be detected at an angle $\theta$ can be written from eq. 5 as

$$P(\theta \neq 0) = \langle \Psi | \hat{a}_\theta \hat{a}_\theta^\dagger | \Psi \rangle = \frac{1}{2\pi} \int_0^d |c(x)|^2 \cos^2 \left(\frac{2\pi x \sin \theta}{\lambda}\right) dx$$

and the probability for detecting a non-scattered photon as:

$$P(\theta = 0) = \int_0^d |c(x)|^2 A(x)^2 dx = 1 - \int_0^{2\pi} P(\theta \neq 0) d\theta$$

Figure 2. Variation of the non-scattering term $A(x)$ with $x$ for unit wavelength

With each scattering event, the overall wave function of the system was updated. Therefore, with each scattering event, the probability density distribution of the scattering angle of a photon changed. This process was continued through a feedback process until at some angles higher probabilities were detected.
3. TWO-PARTICLE SYSTEM

The initial wave function of the two-particle system was considered as $\psi_0(x) = c(x)$ where $x$ is the relative position between the two particles. If a photon scattered at an angle $\theta = \theta_1$ was detected then, the new un-normalized state of the two-particle system is:

$$\psi_1 = \hat{a}_\theta |\Psi_0\rangle = c(x) \cos \left( \frac{2\pi x}{\lambda} \sin \theta_1 \right)$$

Then, the position probability density for the two particles to have a relative separation of $x$ will be given by:

$$|\psi_1(x, \theta)|^2 = p(x) = |c(x)|^2 \cos^2 \left( \frac{2\pi x}{\lambda} \sin \theta_1 \right)$$

If the photon is not scattered, the un-normalized new state of the two particles is given by

$$|\psi_1(x)\rangle = c(x)A(x)$$

and the position probability density for the two particles for a non-scattering event is:

$$P(x) = |c(x)|^2 A(x)^2$$

If a flat distribution is taken for the initial wave function $c(x)$ of the two-particle system, then the position probability density has a maximum at:

$$x = \frac{\pi n}{2\pi \sin \theta_1 / \lambda}$$

Equation 8 shows that by detecting a photon scattered from two initially delocalized pair of particles at an angle $\theta_1$, a relative localization is induced. By renormalizing and iterating equations 6 and 7, the localization of the two particles can be simulated.

4. DELOCALIZED PARTICLES

For the initial wave function of the two particles, a flat distribution was considered and 150 monochromatic photons were scattered onto them. By detecting these scattered photons, the variation of the probability density of the scattering angle was studied. For a photon that got scattered, the scattered angle $\theta \neq 0$ was randomly chosen by the probability density curve.

The simulated probability density curves for $n = 6, 7$ and 149 scattering events with scattering angles of 0.7, 2.3 and 2.5 rad are presented in Fig. 3. The probability density curve for the scattering angle was updated with each scattering event and after the sixth scattering event a scattering angle of 0.7 rad was randomly selected from the distribution.
Due to this choice, the probability density curve for the next scattering event, the seventh scattering event gets adjusted and the probable scattering angle is increased to 2.3 rad. As the scattering events increased up to 149, the probability density curve became more and more refined and a clear probability distribution for the most probable scattering angles emerged with the increase in the number of scattering events.

\[3(a): n = 6, \theta = 0.7 \text{rad}\]

\[3(b): n = 7, \theta = 2.3 \text{rad}\]
Figure 3. Probability density vs. the scattering angle for the scattering events 
\((a) n = 6, \theta = 0.7 \text{ rad} \hspace{0.5cm} (b) n = 7, \theta = 2.3 \text{ rad} \hspace{0.5cm} \text{and} \hspace{0.5cm} (c) n = 149, \theta = 2.5 \text{ rad}

Figure 4(a). Probability density vs. the scattering angle for the first five scattering events
The probability density curve for the first five scattering events for another simulation is shown by Fig. 4(a). After the first scattering event, a scattering angle of 0.6 rad was randomly selected from the distribution. For the subsequent scattering events, the probability density peaks corresponding to the randomly selected scattering angles of 3.3, 0.8, 0.8 and 4.2 rad were observed respectively. With each scattering event, the emerging probability density distribution increased the likelihood of a detection of a photon at the same scattering angle as in the previous scattering event. With subsequent detection of a scattered photon, this feedback process is strengthened, further refining the probability density curve of the scattering angle. In Fig. 4(b) the three-dimensional representation of the variation of the probability density with the scattering angle for $n = 9, 12, 14, 17$ scattering events is presented. The emergence of the most probable scattering angles with the number of scattering events is clearly depicted clarifying the feedback process of the localization scheme.

The photon scattering simulation was carried out for 150 scattering events. With each scattering and non-scattering event, the state of the two particles $\psi_n(x)$ was updated and each state was normalized for the continuation of the process for the next photon. Fig. 5(a), 5(b) and 5(c) show the position probability density (PPD) with respect to the relative position in terms of wavelength of the two particles after $n = 1, 25, 150$ scattering photon events respectively.
Prior to the photon scattering, the position probability density against relative position was a flat curve and with the detection of the scattered photon, a sinusoidal interference pattern emerged.

\[ n = 1 \]

\[ n = 25 \]
A relative localization was induced by detecting a photon scattered from two delocalized particles. With this detection of the scattered photon, the two particles get entangled and with further scattering of photons, a relative localization between the two particles was induced. Fig. 5(d) represents the position probability density after 500 scattering events. This shows that the localization process was robust to further measurements. Fig. 5(e) presents, a three-dimensional representation of the variation of the probability density of the relative position with the number of scattering events. The initial photon scattering events $n = 10$ and 25 represents the two-particle system in a superposition of states of relative position within the region 0 to $10\lambda$ exhibiting the features of a quantum system. As the scattering events increases the two particles begin to localize around $9.2\lambda$, although there is still a substantial uncertainty in relative position space. Furthermore, it could be seen that with each scattering event, the emerging pattern of the relative localization increased the likelihood of obtaining the relative position of the two particles at $x = 9.2\lambda$. The feedback process of each scattering event increased the probability of the relative position to be at $x = 9.2\lambda$ thus narrowing the peak of the wave function in position space.

The system of particles emerges into the classical regime from the quantum regime. After the scattering of 150 photons a sharp peak could be seen which corresponds to a well define relative localization between the two particles. Once the probability distribution of the scattered photons is stabilized, the feedback process ensures that further scattering events only reinforce localization at the existing separation.
**Figure 5(d).** Probability density vs. relative position of the two particles after scattering 500 photons

**Figure 5(e).** The position probability density vs. the relative position of the two particles after scattering 10, 25, 50, 75, 100, 150 photons
The entanglement driven localization process was further verified by considering the change in variance of the position probability density of the relative position of the two particles and this is presented in Fig. 6. The variance of each of the position probability density curves corresponding to each scattering event has reduced significantly with the number of scattering events and this reduction is almost identical for the three separate simulations. This confirms the successful relative localization of the two particles through the feedback process.

5. INITIALLY LOCALIZED PAIR

A Gaussian distribution $e^{-(x-\mu)^2/2\sigma^2}/\sqrt{2\pi}\sigma$ with mean $\mu = 4\lambda$ and standard deviation $\sigma = 1$ was considered as the initial position probability density (PPD) of the two particles and 150 photons were scattered onto them. By observing the scattered photons, the variation of the position probability density distributions for the relative position between the two particles was studied. Fig. 7 represents the position probability density with relative position distributions of the two-particle system after 10, 25, 50, 75, 100 and 150 scattering events. The possible states for the relative positions were within an envelope of the initial position probability density distribution. Even after few photon scattering events, the probability of finding a state outside the initial Gaussian distribution was almost zero.
Figure 7. Probability density with the relative position of the two-particle system after 10, 25, 50, 75, 100 and 150 photon scattering events.

Figure 8. Probability density vs. the relative position for the two-particle system after 1, 3, 5, 7 and 10 scattering events
This confirmed the validity of the localization process. Since there was some initial relative localization between the two particles prior to the photon scattering, the scattering process should give a well-defined localized state with a lesser number of scattering events compared to a system of particles having an initial delocalization. As expected the simulation revealed that after 75 scattering events the two particles acquired a well-defined localization. This is nearly half the scattering events when compared with the initially delocalized case.

To validate the localization scheme, a Dirac-Delta distribution generated by Gaussian distribution $\mu = 5\lambda$ and standard deviation $\sigma = 0.1$ was considered for the initial PPD of the two-particle system. In this case the two-particle system has a well-defined relative position prior to photon scattering. If the localization scheme is valid, then scattering of photons on this system should not have any effect on the initial relative position of the two-particle system. Fig. 8 represents the probability density with the relative position of the two-particle system after $n = 1, 3, 5, 7$ and $10$ scattering events. Just after one scattering event the relative positions of the two particles are the same as the initial one. The subsequent measurements do not change the value of the relative position. This in fact proves the validity of the localization scheme used in the experiments.

Figure 9. Variance of the position probability density with the number of scattering events for the three separate cases of $\sigma = 0.5$, $1$, $3$ and $\mu = 5$

One of the main observations was that the sensitivity of the localization scheme to its initial conditions. This was examined by considering three Gaussian distributions for the initial position probability density of the two-particle system with the same mean $\mu = 5\lambda$ and standard deviations of $\sigma = 0.5$, $1$, $3$. Photons were scattered for each case in equal amounts and the resulting variation of the variance of the position probability density distributions with number of scattering events are presented in Fig. 9. The variance drops rapidly with the number of
scattering events and the decrease of the variance with the number of scattering events is higher for the initial position probability density distribution having the lowest standard deviation of 0.5, while it’s comparatively lower for the position probability density distribution with highest standard deviation of $\sigma = 3$. An exponential curve of the form $ae^{-bx}$ was fitted to each plot and the values of the constants $a$ and $b$ were determined for each case. The values of the coefficients $a$ and $b$ with their error bounds and coefficient of determination for each run of the standard deviation is tabulated in Table 1.

<table>
<thead>
<tr>
<th>Standard deviation ($\sigma$)</th>
<th>Coefficient of determination % ($R^2$)</th>
<th>$a$ (error bounds)</th>
<th>$b$ (error bounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>97.9% 99.9% 100%</td>
<td>0.033 (0.030,0.036) 0.041 (0.040,0.042) 0.044 (0.044,0.045)</td>
<td>1.05 (1.14,0.97) 1.27 (1.29,1.24) 1.34 (1.34,1.33)</td>
</tr>
<tr>
<td>1</td>
<td>99.3% 99.6% 99.6%</td>
<td>0.025 (0.024,0.027) 0.014 (0.013,0.015) 0.014 (0.013,0.015)</td>
<td>1.14 (1.20,1.09) 1.14 (1.18,1.10) 1.13 (1.17,1.09)</td>
</tr>
<tr>
<td>3</td>
<td>98.3% 93.5% 95.0%</td>
<td>0.0027 (0.0025,0.0028) 0.0023 (0.0020,0.0025) 0.0028 (0.0024,0.0031)</td>
<td>0.66 (0.70,0.62) 0.62 (0.70,0.54) 0.78 (0.87,0.69)</td>
</tr>
</tbody>
</table>

The value of $b$ reduces as the standard deviation increases. Therefore, the rate of change of variance rapidly increases as the standard deviation of the initial position probability density distribution reduces. This implies that having more information about the initial relative position of the two particles prior to photon scattering will strengthen the feedback process, thus reducing the number of scattering events needed to obtain well defined relative localization.

6. PHOTONS OF VARIABLE FREQUENCIES

The localization process was studied for a pair of delocalized particles by scattering photons of variable frequencies. For each scattering event the wavelength of the photon was selected randomly from a blackbody distribution.

Then a simulation was executed to examine the way monochromatic photons and photons of variable frequencies affect the localization process. The position probability density distribution of the relative position for photons with variable frequencies is presented in Fig. 10 (a) and with monochromatic photons in Fig. 10(b).
Figure 10. Position probability density with the relative position using (a) photons with variable frequencies and (b) monochromatic photons
The measurement induced localization can be also observed for the photons of variable frequencies. In this case the two particles obtained well defined relative localization with a lesser number of scattering events than when the monochromatic photons were used. Therefore, it is evident that the localization process is dependent on the type of photons used in the scattering events. This could be further verified by considering the change in variance of the position probability density distributions for each of the two cases and then by fitting exponential curves for both cases. The values of the coefficients $a$ and $b$ with their error bounds and coefficient of determination for monochromatic photons and photons with variable frequency is tabulated in Table 2. The value of $b$ was larger for photons with variable frequencies than the values obtained for the monochromatic photons. This confirms that the localization process is dependent on the type of photons used in the scattering events.

**Table 2.** The values of the coefficients $a$ and $b$ with their error bounds and coefficient of determination for monochromatic and variable frequency photons

<table>
<thead>
<tr>
<th>Type of photons</th>
<th>Coefficient of determination $%$ ($R^2$)</th>
<th>$a$ (error bounds)</th>
<th>$b$ (error bounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monochromatic photons</td>
<td>91.9%</td>
<td>$0.0021 (0.19,0.23) \times 10^{-2}$</td>
<td>$0.66 (0.73,0.60)$</td>
</tr>
<tr>
<td></td>
<td>97.2%</td>
<td>$0.0024 (0.23,0.26) \times 10^{-2}$</td>
<td>$0.63 (0.67,0.60)$</td>
</tr>
<tr>
<td></td>
<td>90.1%</td>
<td>$0.0020 (0.17,0.22) \times 10^{-2}$</td>
<td>$0.63 (0.70,0.56)$</td>
</tr>
<tr>
<td>Photons of variable frequency</td>
<td>97.3%</td>
<td>$1.09 \times e^{-08} (1.02,1.16) \times e^{-08}$</td>
<td>$0.81 (0.86,0.77)$</td>
</tr>
<tr>
<td></td>
<td>97.5%</td>
<td>$1.16 \times e^{-08} (1.086,1.24) \times e^{-08}$</td>
<td>$0.88 (0.92,0.82)$</td>
</tr>
<tr>
<td></td>
<td>96.8%</td>
<td>$6.68 \times e^{-09} (6.20,7.17) \times e^{-09}$</td>
<td>$0.83 (0.88,0.77)$</td>
</tr>
</tbody>
</table>

7. DISCUSSION

The study of the relative localization of a pair of delocalized particles showed that by scattering photons of fixed wavelength relative localization could be induced between two particles. The variation of the probability density of the scattering angle with the number of scattering events showed how the feedback process of the localization scheme increased, the likelihood of the detection of a photon at the same scattering angle. As the number of scattering events increased, the likelihood of detecting a separation of the two particles began to increase. Another important observation was one of the inherent properties of the classical world which is responsible for the way that we see the world around, the robustness of the relative position measurement. Position probability density distribution curves showed that the initial separation between the two particles took a range of values. This may be interpreted as the relative position of the two particles being in a superposition of states while having a well-defined momentum, a clear depiction of the initial quantum state of the two-particle system. As the localization process continued, the relative positions became increasingly well-defined showing the emergence of the classical states from the quantum regime. After obtaining the well-defined
relative position state, the subsequent measurements were robust to further measurements proving the emergence of the classical nature of the two-particle system. The study of the relative localization of a pair of particles with an initial localization enabled to investigate the validity of the localization scheme as well as the sensitivity of the localization scheme to its initial conditions. The resulting relative position between the two particles was in accordance with the initial relative position of the two particles. As the initial conditions changed, the localization scheme got adjusted accordingly. The study of the change in variance of the position probability density distributions showed the dependence and the sensitivity of the localization scheme to its initial conditions. The localization scheme was valid for even photons having different wavelengths. The entanglement driven localization process may be used as a successful tool to understand the boundary between the quantum and the classical world.

8. CONCLUSIONS

From this study, it is evident that the unique quantum features of entanglement could be used to understand the emergence of classicality in quantum systems. A simple photon scattering experiment can be used to study the way in which the particles acquire well-defined localization in the relative position space. The validity of the localization scheme was verified by changing the initial conditions of the localization process. The transition of the two-particle system from its quantum states to the robust semi classical states was clearly observed through the graphical representation of the position probability density distributions with respect to the relative position of the two particles. A quantitative analysis of the localization process carried out in terms of changing variance of the position probability density distributions of the two-particle system enabled, the study of the variations in the localization scheme in a quantitative manner thus clarifying the link between localization and entanglement.

References


