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Heat Conduction Analysis of a Hollow Sphere made of Three Different Layers of Materials

S. M. Khaled Khan[1,*], Md. Mashrur Islam[2]

[1]Department of Mechanical Engineering, CUET, Chittagong, Bangladesh

[2]Department of Electrical and Electronic Engineering, RUET, Rajshahi, Bangladesh

*E-mail address: khaledcuet1@gmail.com

ABSTRACT

A systematic arrangement has been gotten for the transient issue of three-dimensional multilayer heat conduction in a circle with layers in the spiral course. The arrangement strategy can be connected to an empty circle or a strong circle made out of a few layers of different materials. By and large, the partition of factors connected to 3D round directions has interesting attributes because of the nearness of related Legendre works as the eigenfunctions. Additionally, an eigenvalue issues the azimuthal way likewise requires arrangement; once more, its properties are one of a kind inferable from periodicity the azimuthal way. In this way, broadening existing arrangements in 2D round directions to 3D circular directions isn't clear. In a round facilitate framework, one can explain a 3D transient multilayer heat conduction issue without the nearness of fanciful eigenvalues. A 2D tube shaped polar facilitate framework is the main other case in which such multidimensional issues can be tackled without the utilization of nonexistent eigenvalues. The nonappearance of nonexistent eigenvalues renders the arrangement philosophy fundamentally increasingly valuable for pragmatic applications. The strategy portrayed can be utilized for all the three kinds of limit conditions in the external and inward surfaces of the circle. The arrangement method is shown on an illustrative issue for which results are gotten.

***Keywords*:** Naiver-Strokes, Heat transfer, Spherical Contour, Material Science

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1. INTRODUCTION

As of late, utilizations of cutting-edge inhomogeneous materials, practically evaluated materials (FGMs), have happened to incredible significance in high-innovation businesses, for example, aviation, heat motor parts and warmth exchanger tubes. These materials as far as mechanical properties change consistently in thickness [1]. In the previously mentioned applications, FGMs are exposed to warm loadings, so the examination of the temperature dissemination in FGMs is imperative in building configuration process. To anticipate the hardware dependability, the temperatures at which the gadget disappointments will happen must be known. Taking into account that the warm examination is imperative in various businesses, various explanatory answers for the examinations of warmth conduction have been led. Some scientific arrangements of warmth conduction issues are identified with empty barrels and have incredible significance for various enterprises [2].

Warmth conduction for practically reviewed materials (FGMs) has been examined by various systematic arrangements. Hosseini et al. [3] contemplated the transient warmth conduction in a (FGMs) barrel utilizing explanatory arrangement, division of variable technique, and during the time spent taking care of this issue, utilized Bessel works in a 1-D round and hollow shell. The transient warmth conduction in a practically evaluated round and hollow board without considering the warmth source, as indicated by the double stage slack (DPL) hypothesis, is explored by Akbarzadeh and Chen [4]; they expected that inhomogeneous materials alter consistently along the outspread course dependent on the power-law detailing, and by utilizing the Laplace change strategy, changed over the outcomes acquired to time area. Sutradhar et al. [5] examined transient warmth conduction for exponentially (FGMs) in three measurements by utilizing the limit component technique; they utilized the Laplace change (LT) way to deal with wipe out the reliance on schedule. Examination of transient warmth conduction in practically reviewed (FG) empty barrels exposed to an appropriated axisymmetric heat transition in one dimensional facilitate framework is explored by Malekzadeh et al. [6].

Hassanzadeh and Pekel [7] contemplated transient warmth conduction in (FG) empty chamber for one dimensional issue by utilizing a numerical Durbin's Inverse Laplace change technique to change over the outcomes acquired in time area. Al-Nimr and Naji [8] tackled the issue of hyperbolic warmth conduction in isotropic materials. Jiang [9] utilized the Laplace change (LT) strategy for the scientific arrangement of the hyperbolic warmth conduction issue in empty circle exposed to unexpected temperature changes close behind limit surfaces. Babaei and Chen [10] researched and explained a transient hyperbolic warmth conduction issue in the inhomogeneous (FGMs) empty barrel and circle separately, prompting a one-dimensional warmth conduction issue without considering time-subordinate warmth source, and to change over the outcome acquired into time space, they utilized numerical reversal of the Laplace change; they expected that the material properties are altered just in the outspread course.

The multilayer parts are utilized in wide scope of enterprises, e.g., common, atomic, synthetic, aviation, and so on account of the additional favorable position of joining thermophysical properties. Numerical just as investigative strategies can be utilized to acquire temperature dispersion in such frameworks. The logical answers for such issues are very helpful because of the way that it can prompt better knowledge through its numerical structure. Besides, the scientific arrangement can likewise be utilized to check the numerical codes. The expository answers for one-dimensional time-subordinate warmth conduction in multilayer frameworks have been produced for a very long while. These arrangements are substantial for Cartesian, tube shaped, and round facilitate frameworks. The arrangements were created by the utilization of division of factors [11-17], symmetrical development [18, 19], Green's capacity approach [20], Laplace change [21, 22], and the Galerkin strategy [23]. These arrangements were additionally reached out to multidimensional geometries in Cartesian facilitate frameworks [24-27]. Yener and Ozisik [28] utilized limited indispensable change to show a general answer for time-subordinate warmth conduction in limited media made out of various areas.

As of late, partition of factors and limited necessary change were utilized to take care of time-subordinate warmth conduction issues in 2D round and hollow polar directions with numerous layers in the spiral course [29-32]. Moreover, an arrangement utilizing partition of factors has been produced for comparative issues having 2D circular polar directions [33, 34]. It was demonstrated that no fanciful eigenvalues were available in the arrangement of these issues in tube shaped and round polar directions. Stretching out 2D circular directions to 3D isn't clear, on the grounds that the eigenvalue issue for the polar course is changed from the Legendre condition to a related Legendre condition. Also, an extra eigenvalue issue must be tackled for the azimuthal bearing. Contrasted with ordinary materials, materials with temperature-subordinate warm conductivities as a rule serve in the high-temperature condition, for instance, hard-headed materials for an impact heater, and subsequently the warm examination is essential for such materials. In any case, the temperature subordinate element of material properties prompts the nonlinearity of the administering condition, and in this way the troubles of the induction of logical arrangements increment.

2. MATHEMATICAL MODELLING

Figure 1 demonstrates the organize arrangement of the issue just as the n concentric isotropic round layers in impeccable warm contact. The temperature-autonomous warm conductivity and warm diffusivity of the ith layer are given as ki and ai, individually. The underlying temperature, at t = 0, of the ith layer is viewed as fi (r, theta, phi). Nonuniform, time-free warmth sources gi (r, theta, phi) are thought to be available in each layer for t>0. Any of the three sorts of limit conditions (Dirichlet, Neumann, or blended) are pertinent in the inward (I = 1 and r = r0) and the external (I = n and r = rn) surfaces in the outspread bearing. The strategy introduced in the present work is relevant just for time-free warmth source and limit conditions.

The heat conduction equation for the temperature in the i-th layer can be written as;

$$\frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_i}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T_i}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T_i}{\partial\phi^2} + \frac{g_i(r,\theta,\phi)}{k_i} \quad r_{i-1} \le r \le r_i;\ 1 \le i \le n \tag{1}$$

Eqn-1 after successive differentiations can be written as;

$$\frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_i}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\mu}\left((1-\mu^2)\frac{\partial T_i}{\partial\theta}\right) + \frac{1}{r^2(1-\mu^2)}\frac{\partial^2 T_i}{\partial\phi^2} + \frac{g_i(r,\theta,\phi)}{k_i} \quad r_{i-1} \le r \le r_i;\ 1 \le i \le n \tag{2}$$

A general form of inhomogeneous boundary conditions at the innermost and outermost radial surfaces of the multilayer structure is given as follows:

- Inner surface of first layer ($i = 1$)

$$A_{\text{in}} \frac{\partial T_1}{\partial r}(r_0, \mu, \phi, t) + B_{\text{in}} T_1(r_0, \mu, \phi, t) = C_{\text{in}}(\mu, \phi) \tag{3}$$

- Outer surface of nth layer ($i = n$)

$$A_{\text{out}} \frac{\partial T_1}{\partial r}(r_n, \mu, \phi, t) + B_{\text{out}} T_n(r_n, \mu, \phi, t) = C_{\text{out}}(\mu, \phi) \tag{4}$$

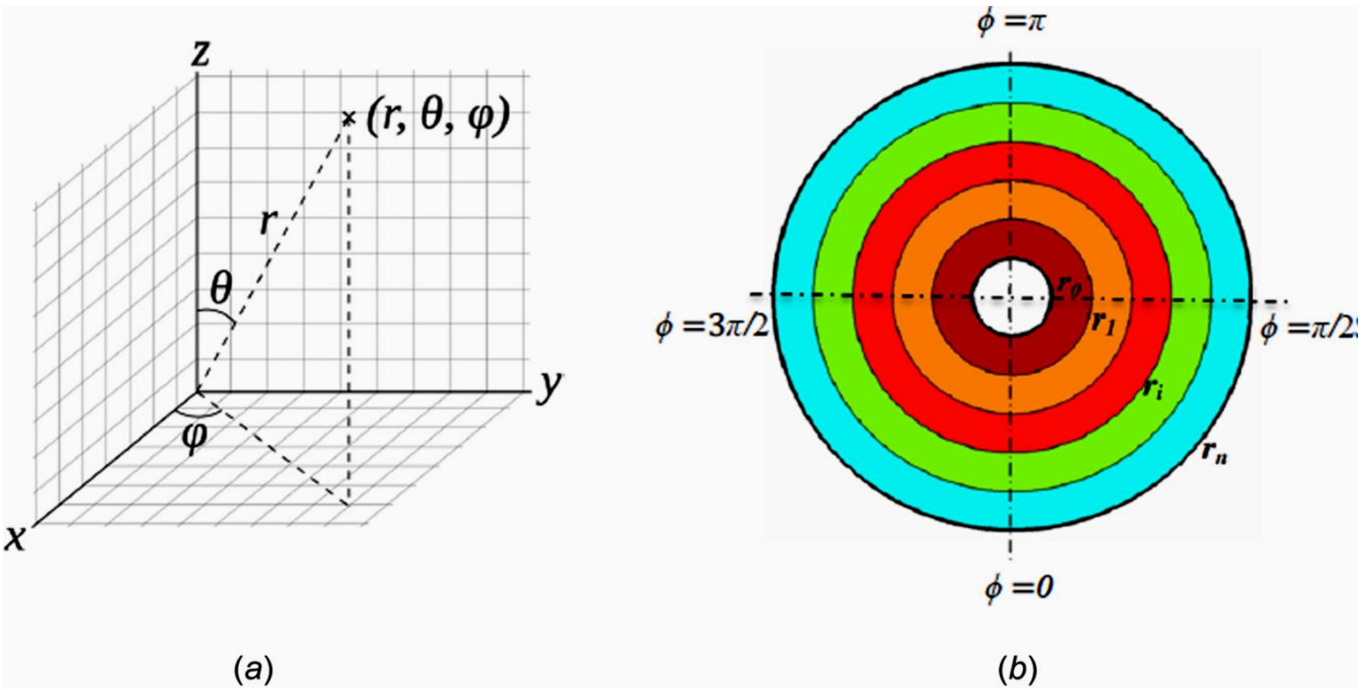


Fig. 1. (a) Coordinate system for the problem and (b) 2D cross-sectional view of n concentric spherical layers at 90-degree plane [25].

3. SOLUTION METHODOLOGY

An overall objective of our work is to improve building and systems performances in terms of durability, comfort and economics. In order to predict, improve and meet a certain set of performance requirements related to the indoor climate of buildings, the associated energy demand, the heating, venting and air conditioning systems and the durability of the building and its interior, simulation tools are indispensable. In the field of heat, air and moisture transport in building and systems, much progress on the modeling and simulation tools has been established. However, the use of these tools in an integrated building simulation environment is still limited. Also, a lot of modeling work has been done for energy related building systems, such as solar systems, heat pump systems and heat storage systems. Often, these models focus on the systems and not on the coupled problem of building and systems.

The physical properties and geometry of this problem are described in Singh, Jain, and Rizwan Uddin [24, 25], the inner face of the sphere has a temperature of zero at all times. The outer hemisphere with positive y value has a nonuniform heat flux defined by;

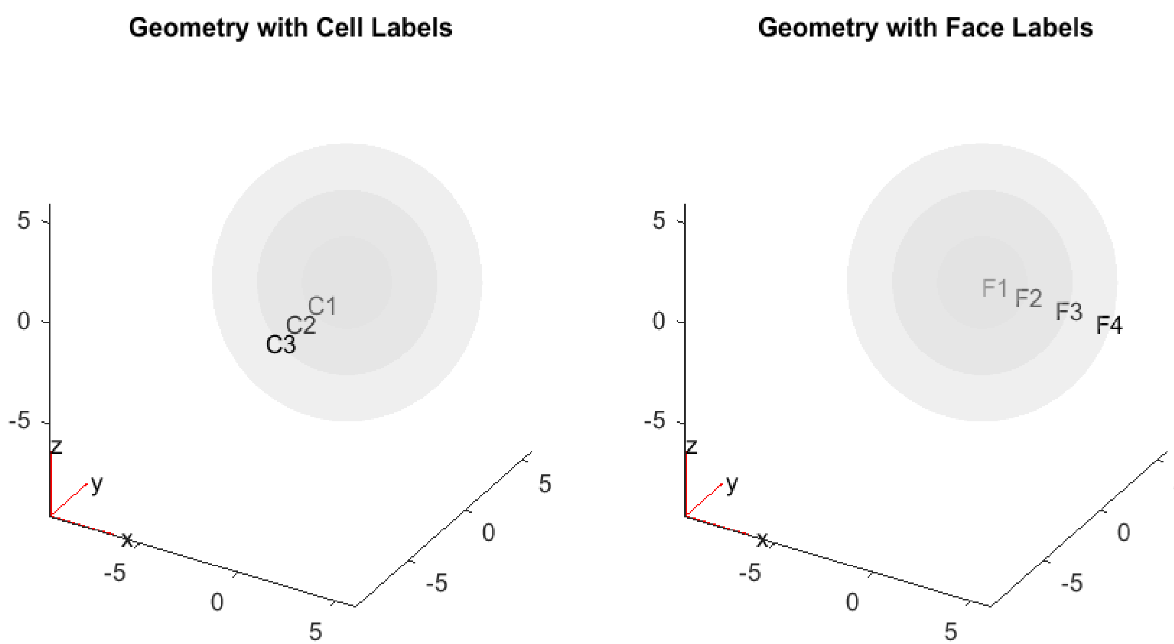
$$q_{\text{outer}} = \theta^2(\pi - \theta)^2\phi^2(\pi - \phi)^2 \tag{5}$$

$$0 \le \theta \le \pi, 0 \le \phi \le \pi. \tag{6}$$

θ and ϕ are azimuthal and elevation angles of points in the sphere. Initially, the temperature at all points in the sphere is zero.

4. RESULTS

The simulation was performed using MATLAB environment. The very basic Naiver-Strokes along with basic heat equations are used and the contour plots of the sphere consisting with 3 different materials have been plotted. Figure 2 depicts the modeling outputs from the simulation where the design and construction of the sphere is shown.



(a) Cell Level Geometry (b) Face level Geometry

Fig. 2. Geometry of the sphere with 3 different materials.

After modeling, the heat flux conditions using eqn- (3) to (5) the heat flux characteristics had been obtained (Figure 3). To plot shapes at a few times, with the form levels being the equivalent for all plots, decide the scope of temperatures in the arrangement. The base temperature is zero since it is the limit condition on the internal substance of the circle. Figure-4 demonstrates distinctive shape plots for various occasions as for temperature increase.

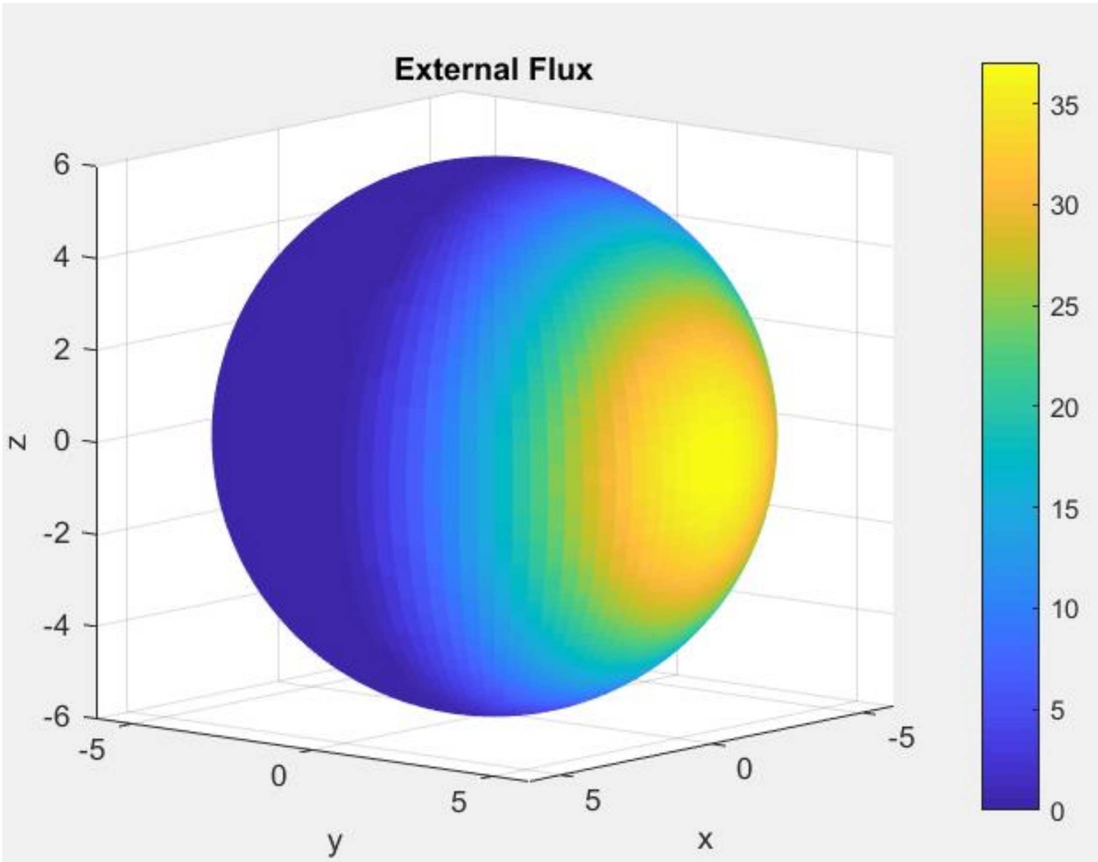
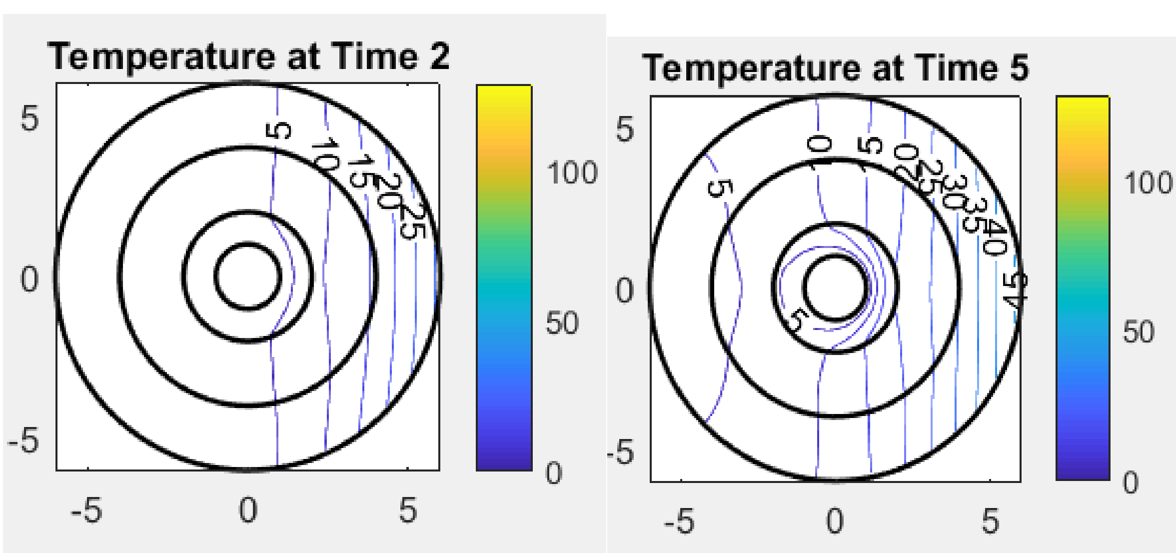
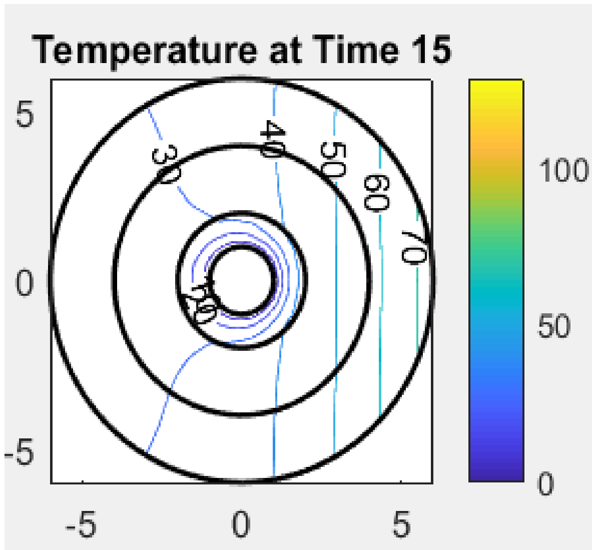


Fig. 3. External heat flux density plot of the sphere (3D).

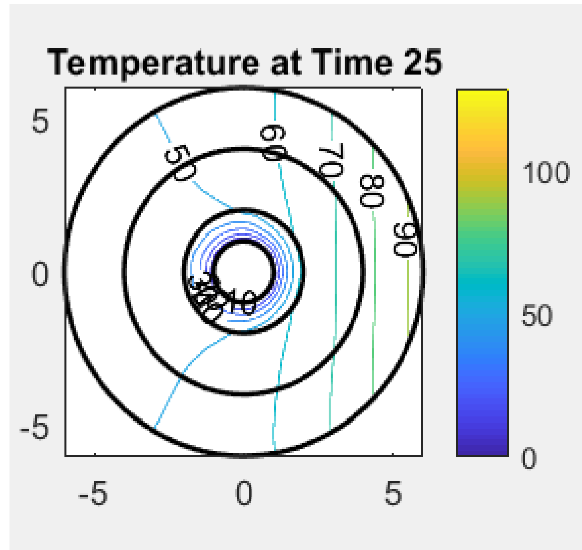


(a) (b)

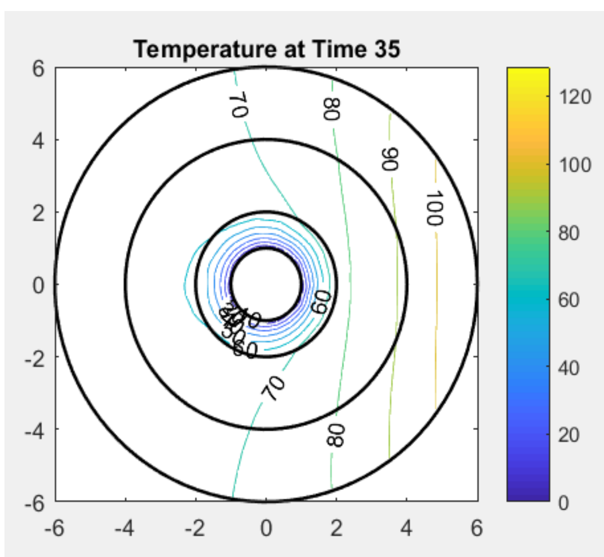
Fig. 4(a,b). Contour plots for the sphere with time and temperature.



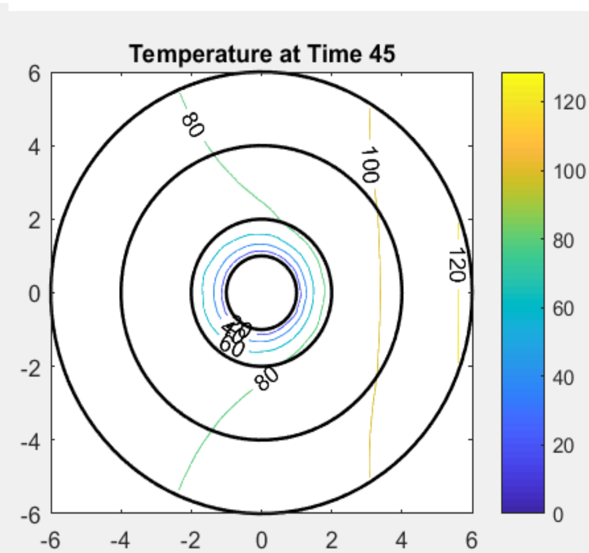
(c)



(d)

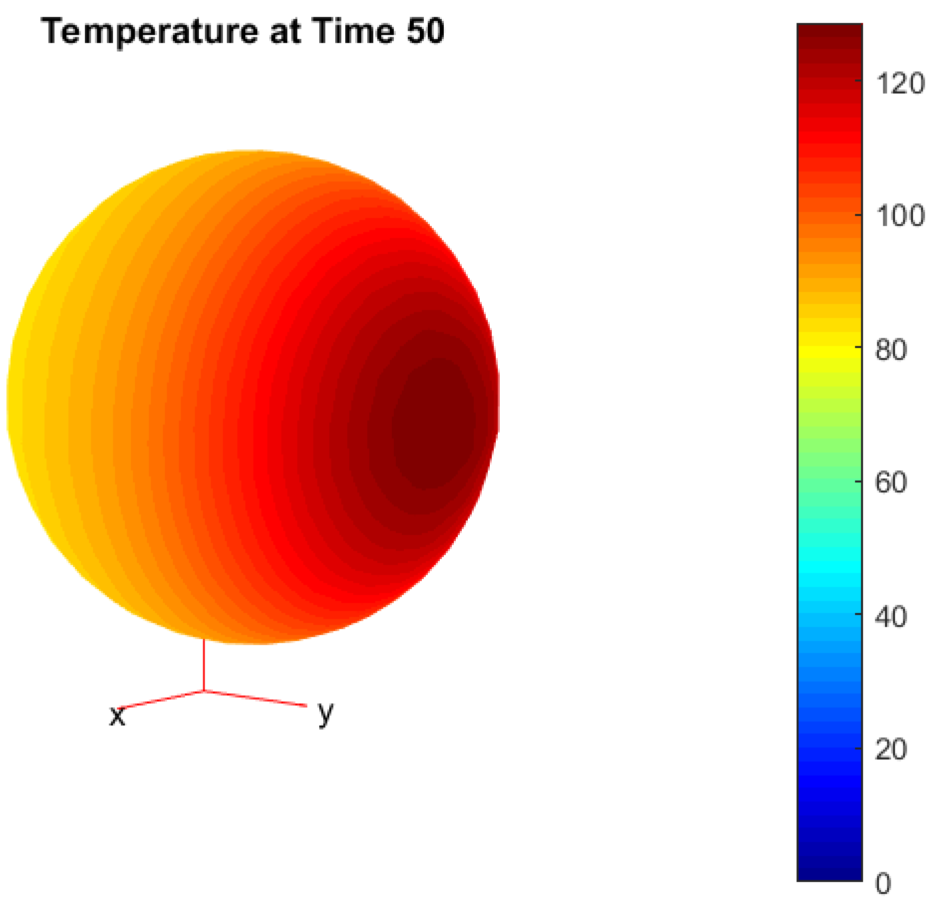


(e)



(f)

Fig. 4(c-f). Contour plots for the sphere with time and temperature.



(g)

Fig. 4(g). Contour plots for the sphere with time and temperature.

Numerical results show that the error progressively decreases with the increasing values of M and P. Moreover, for M = 9 and P = 10, the truncation error becomes reasonably small (of order 0.001), so the series is truncated for these values of M and P. Temperature variations along radial directions are shown in Figs. 4-5 for different polar and azimuthal planes.

In the detailing of the present half and half limited component, the central arrangement of the issue assumes a critical job. In view of the scientific meaning of them, they are utilized to develop the component inside the field, which precisely fulfills the administering condition, and all the while convert the space indispensable into the limit necessary in the half and half utilitarian. The present investigation proposes the principal arrangement-based mixture limited component definition to take care of the nonlinear warmth conduction issues with temperature-subordinate warm conductivity. It is an immediate augmentation of the HFS-FEM for straight warmth conduction issues by joining the Kirchhoff change and it is anything but difficult to be numerically actualized.

The numerical outcomes demonstrate that the present mixture limited component definition is exceptionally compelling and helpful to take care of the nonlinear warmth conduction issue. It is appeared for the examination of nonlinear warmth conduction issues, particularly those identified with the roundabout opening, the enhancements of both the effectiveness of work discretization and the numerical precision of the present mixture limited component show are critical, for the utilization of general or unique central arrangements.

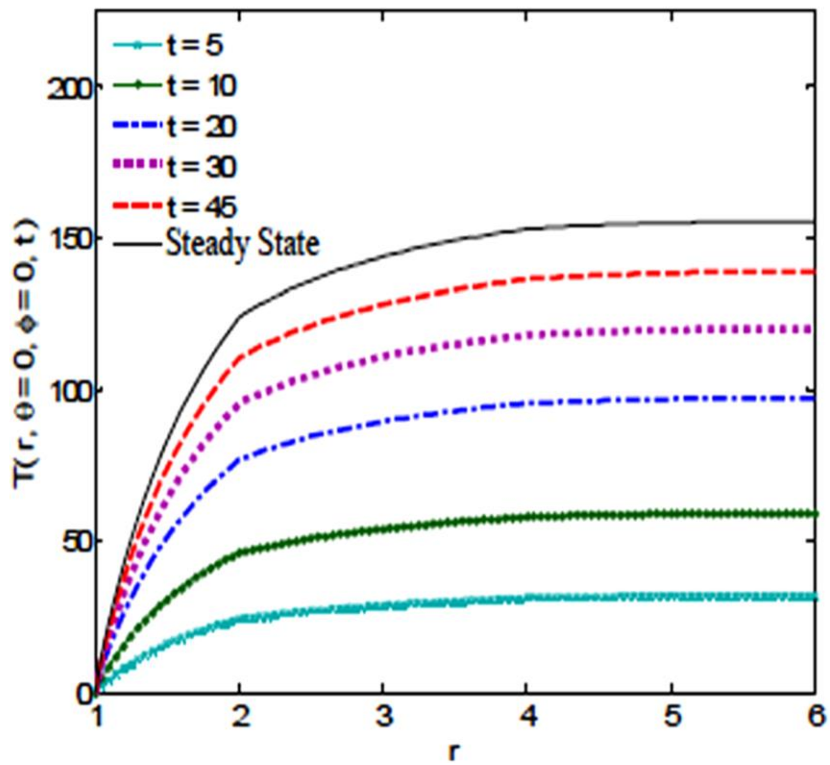


Fig. 5. Transient temperature distribution in the radial direction along theta = 0.

5. CONCLUSION

A precise explanatory arrangement of the 3D transient warmth conduction issue in round directions with various layers the outspread way is displayed here. Since taking care of the multilayer heat conduction issue in 3D round directions delivers no fanciful eigenvalues, this arrangement is manageable for application to different down to earth circumstances. The nonexistent eigenvalues are missing in this issue in light of the fact that in circular organizes, the spiral eigenvalues are not unequivocally reliant on those in the polar and azimuthal bearings. At the point when the proposed arrangement was connected to an illustrative model, the arrangement was sensibly exact with a genuinely modest number of terms in the triple arrangement.

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